ACDA 2025 Mini-tutorial on Learning-Augmented Algorithms Part 2

Algorithmic Ideas from Learning-Augmented Algorithms

Vaidehi Srinivas

Northwestern University

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Yes and yes!

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- Exciting time to get involved!

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Formalism: Learning-Augmented "warm starts" help us design and reason about algorithms for sequences of instances

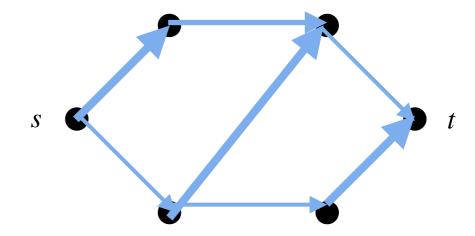
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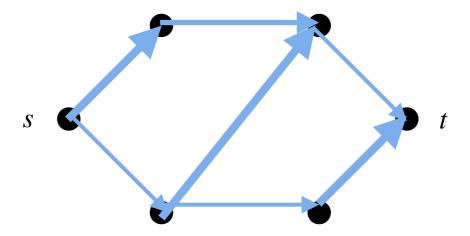
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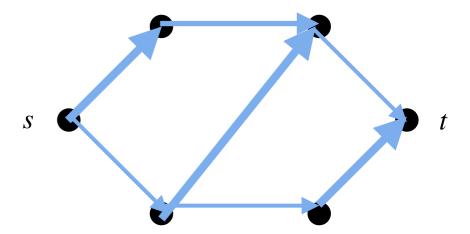
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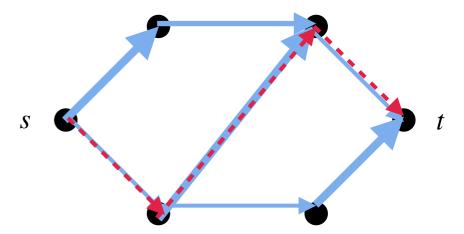
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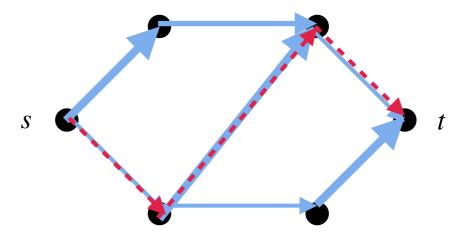
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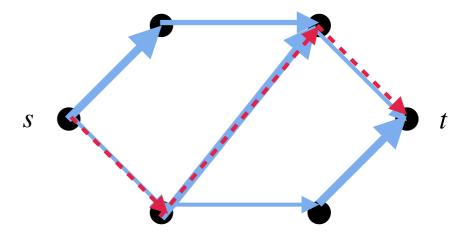
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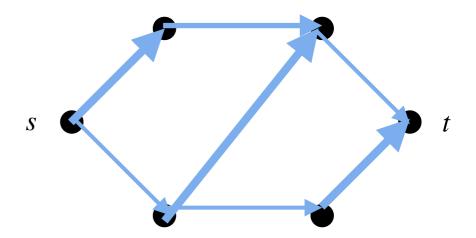
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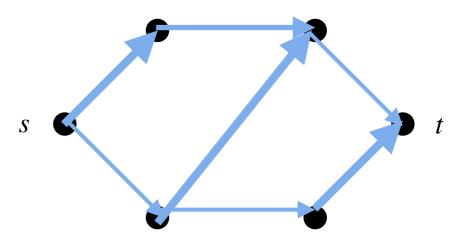


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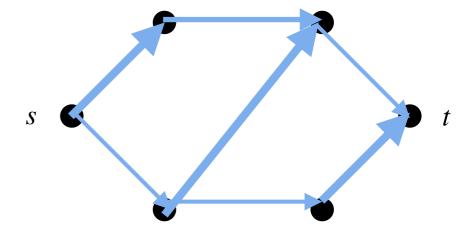
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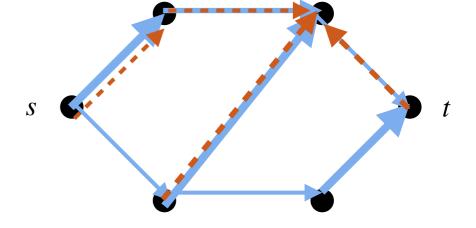
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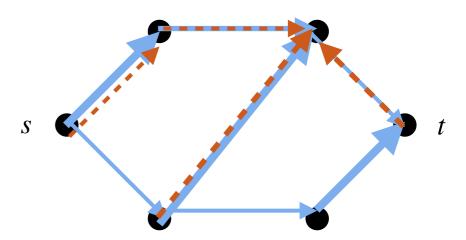
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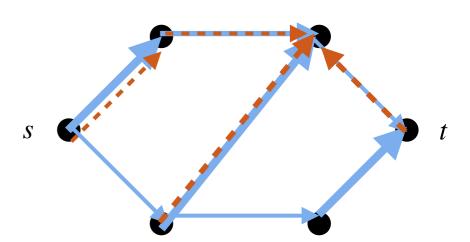
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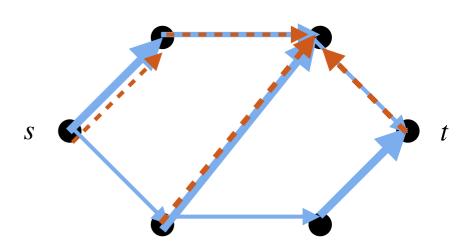
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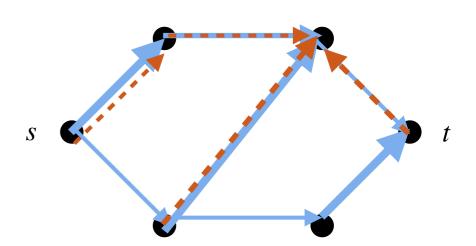
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Always outputs correct answer! Can we optimize runtime?

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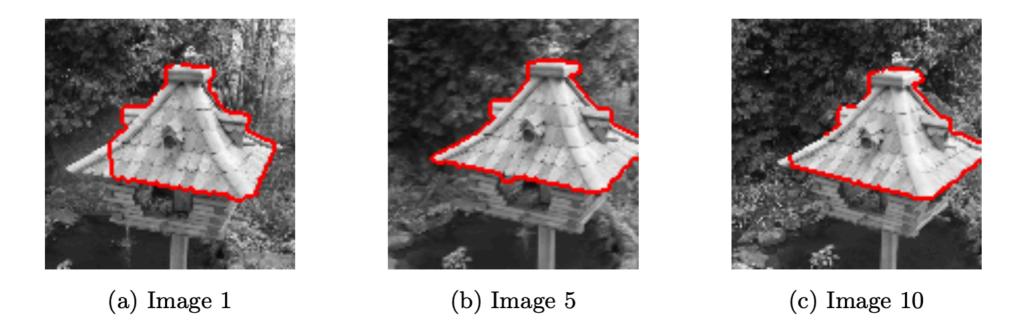


Figure 4: Cuts (red) on the first, fifth, and last images from the 120 × 120 pixels BIRDHOUSE sequence.

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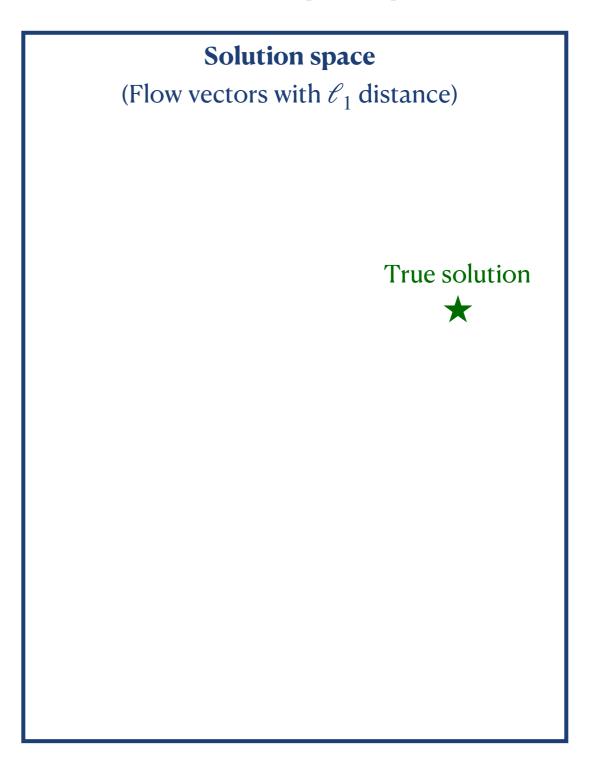
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Solution space
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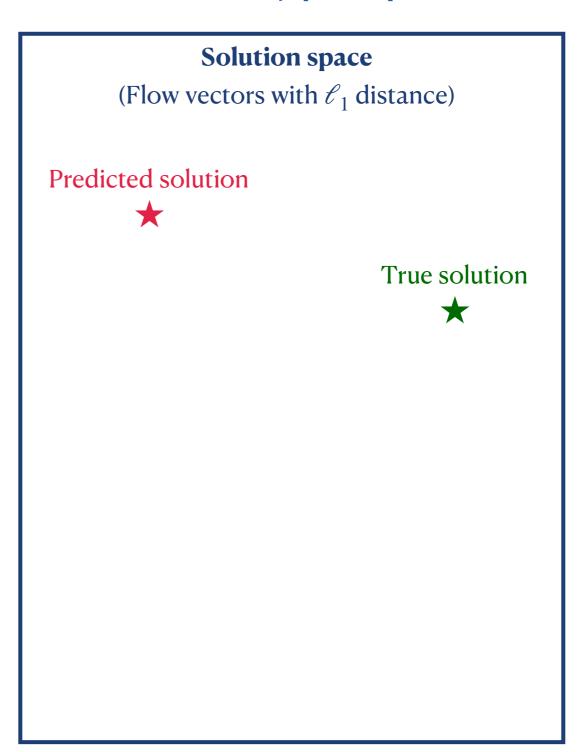
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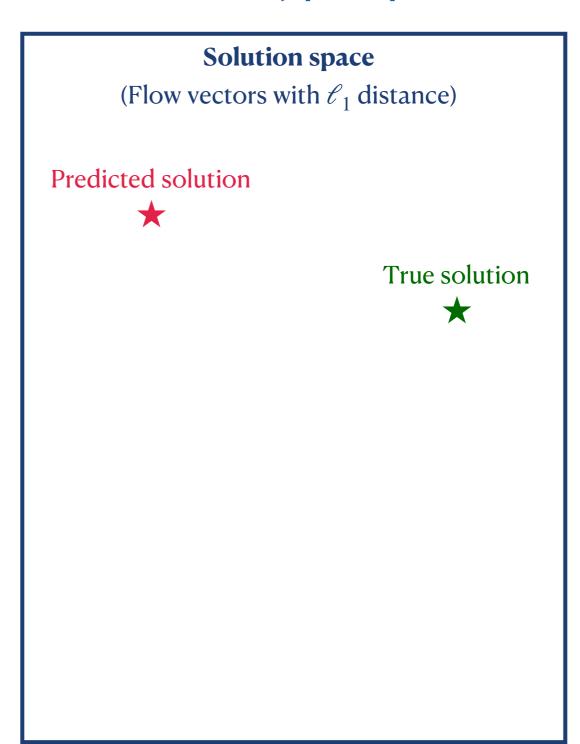
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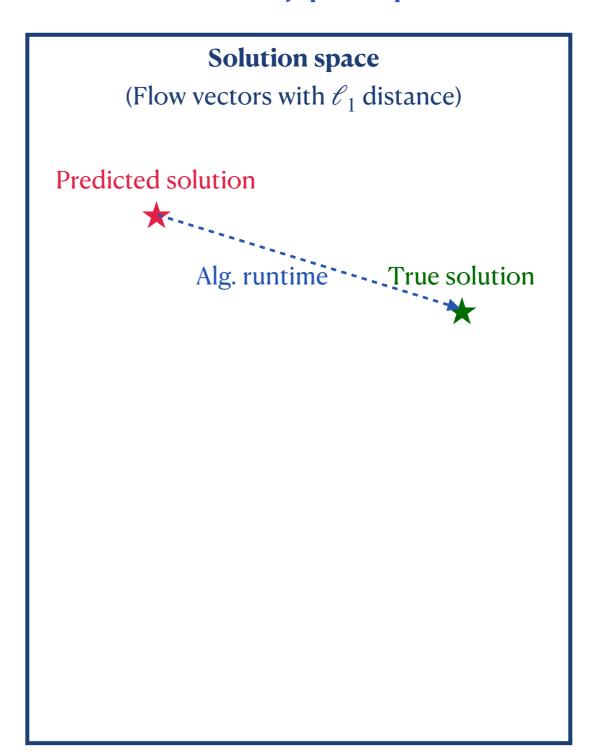
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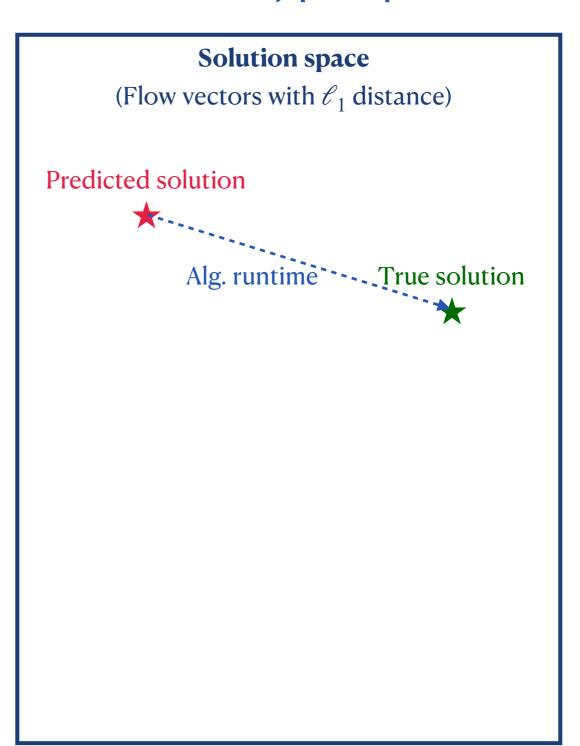
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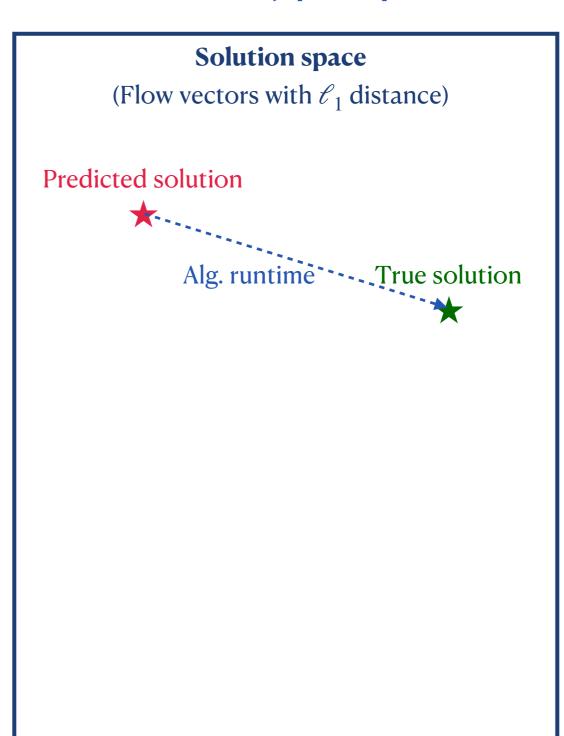
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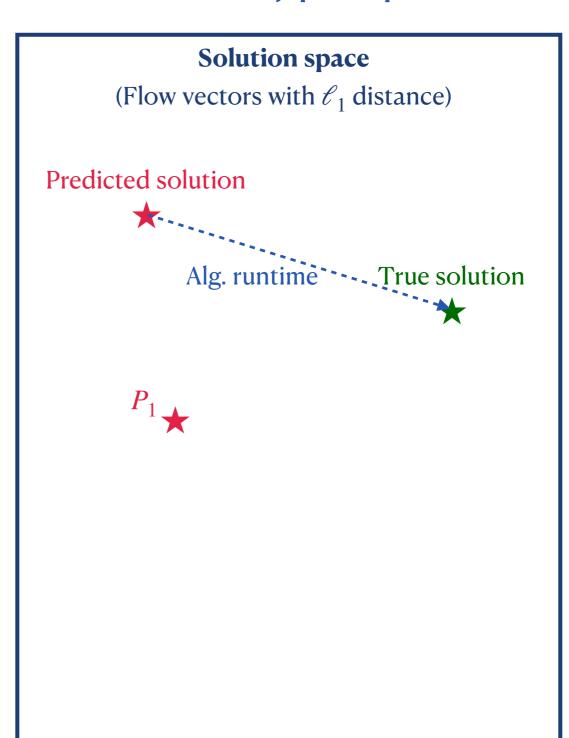
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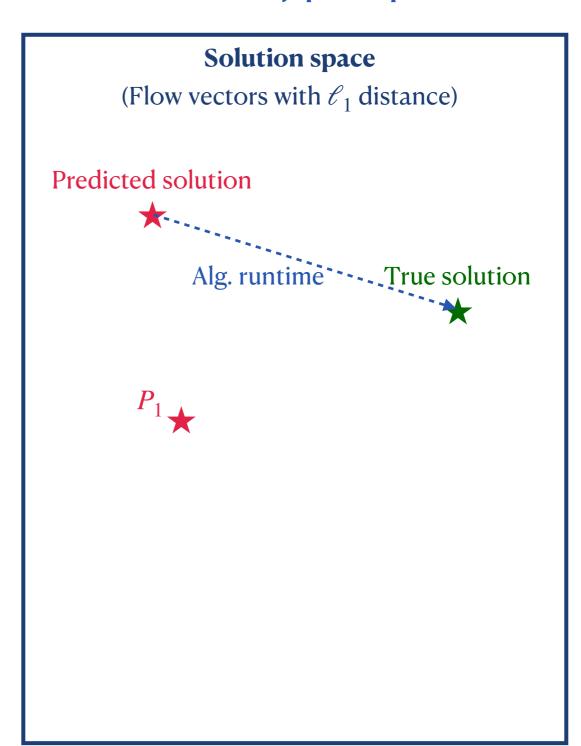


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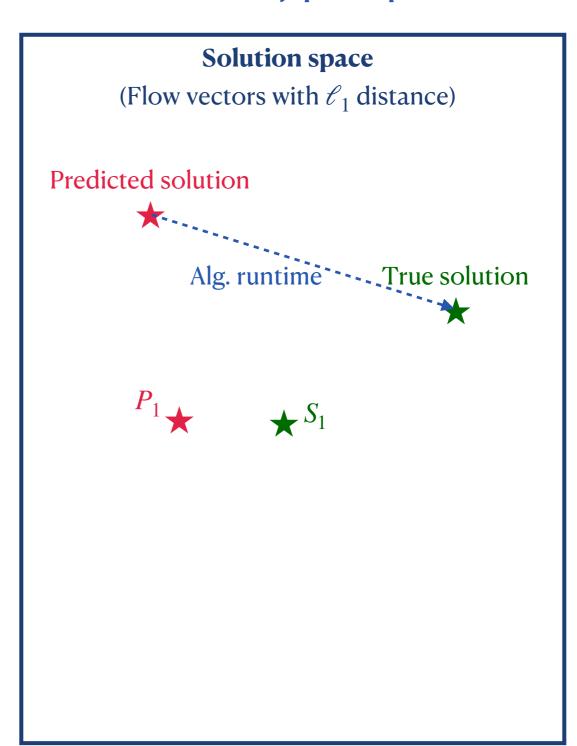


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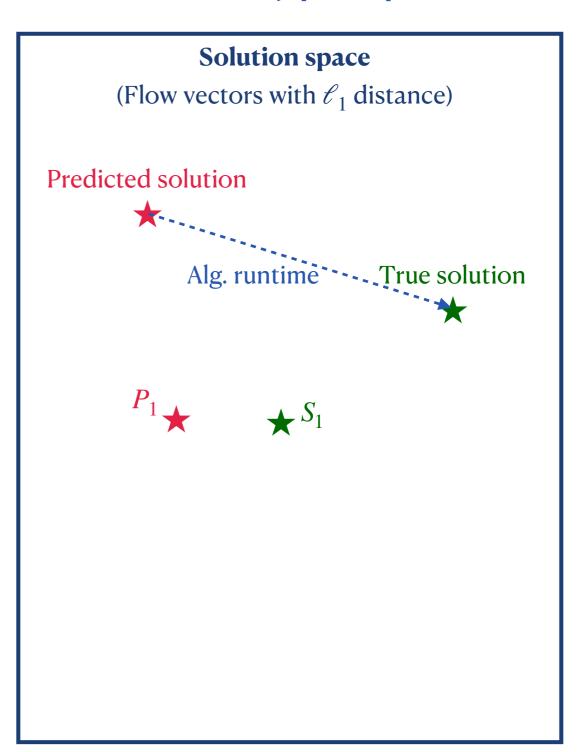


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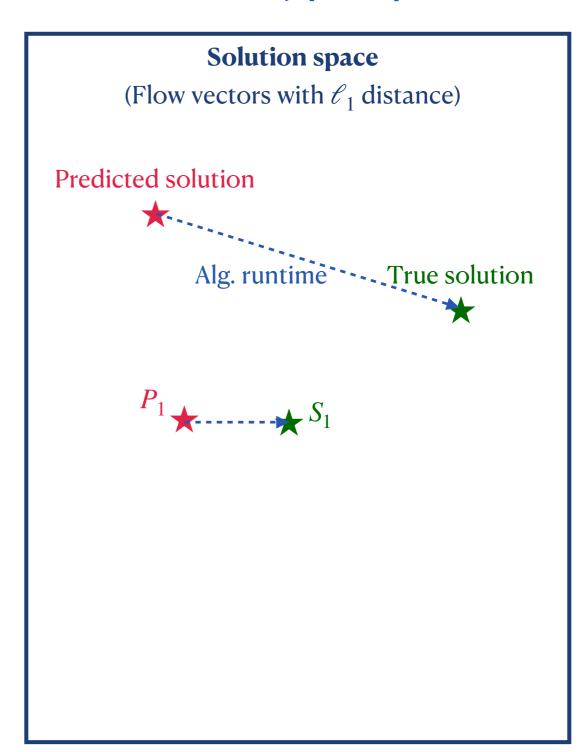


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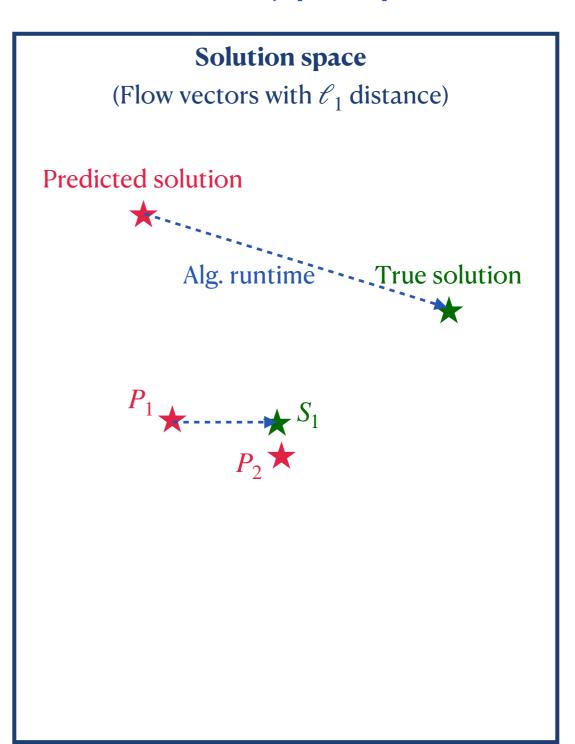


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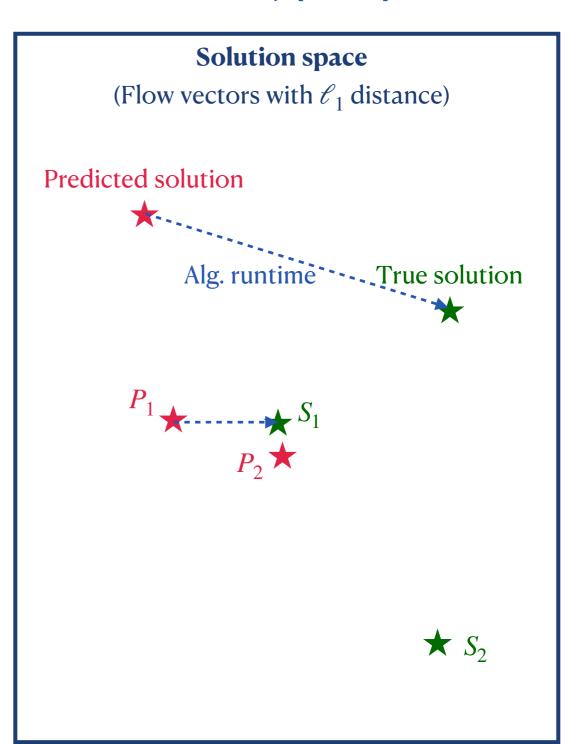


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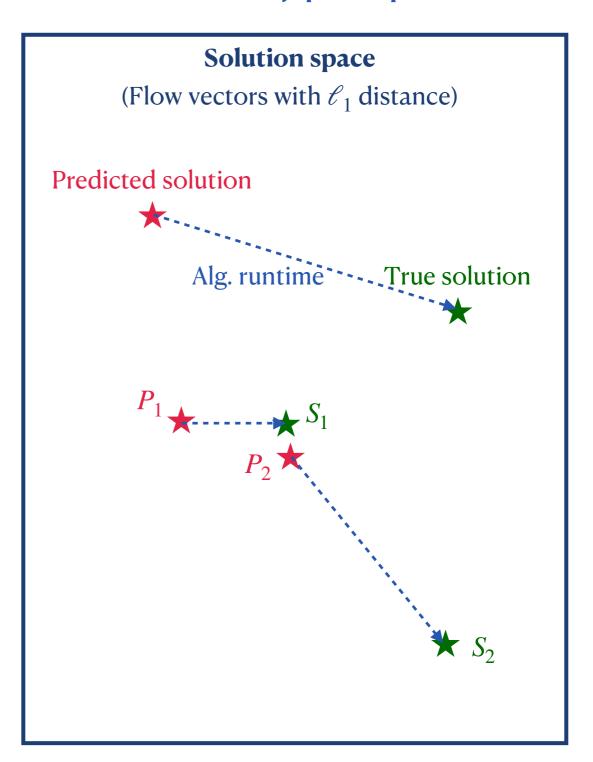


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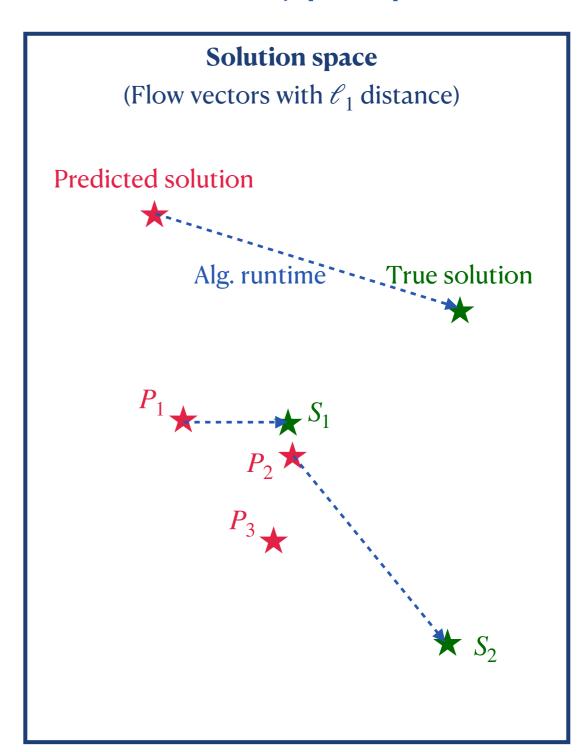


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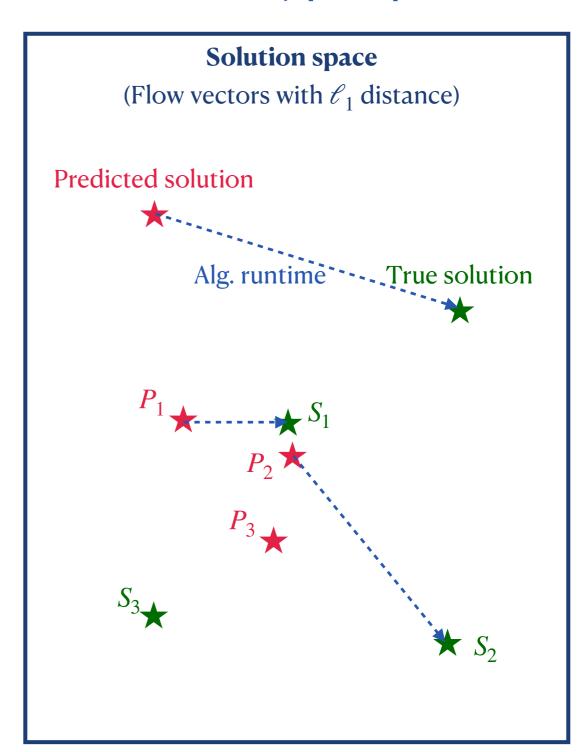


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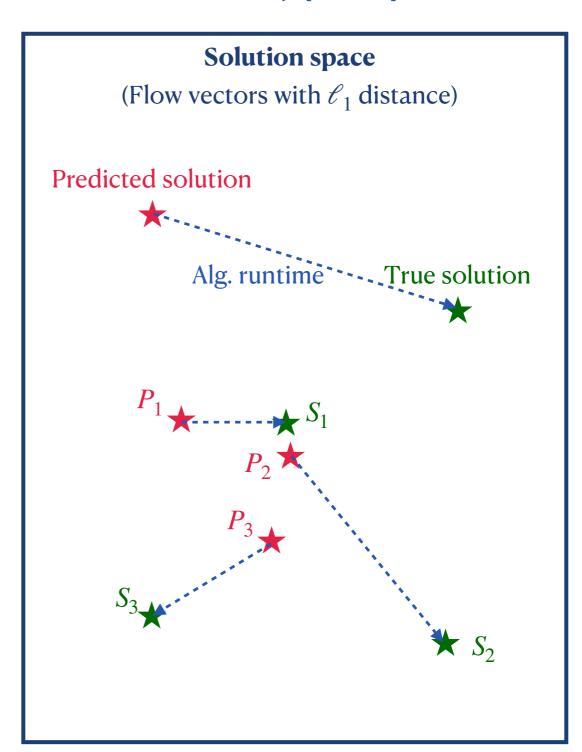


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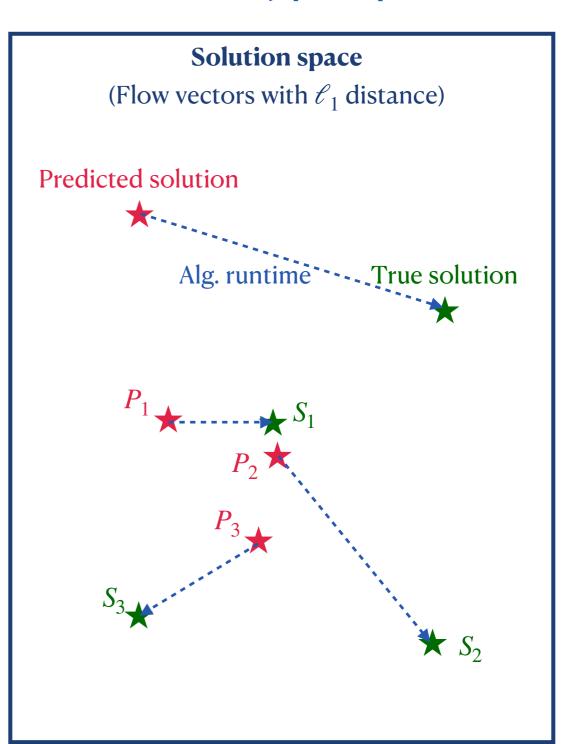
Ford-Fulkerson Warm Start algorithm [DMVW '23]:

- Flow solution is a vector indexed by edges
- Runtime of algorithm proportional to \mathcal{C}_1 distance between the predicted flow and the true solution

Meta problem: On each day *t*:

- Algorithm predicts a point P_t in the solution space
- True solution S_t is revealed
- Algorithm pays $d(P_t, S_t)$

Unlocks online algorithms toolkit!



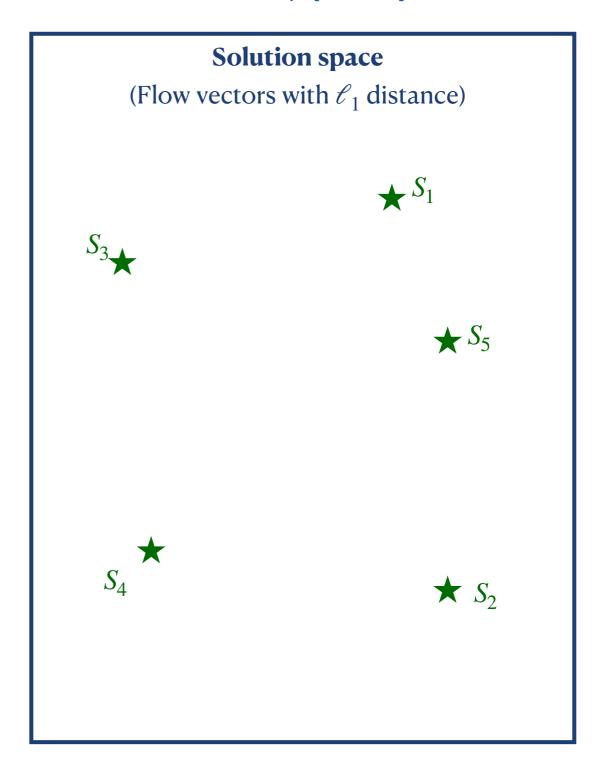
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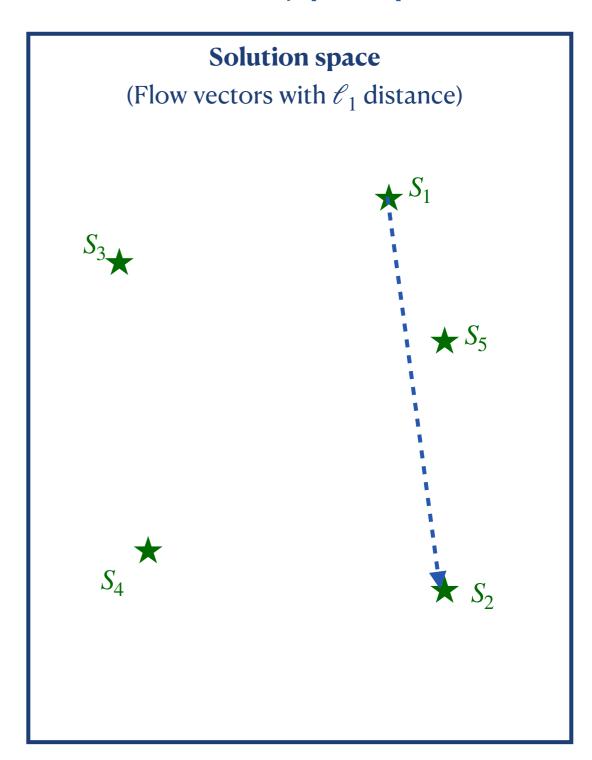
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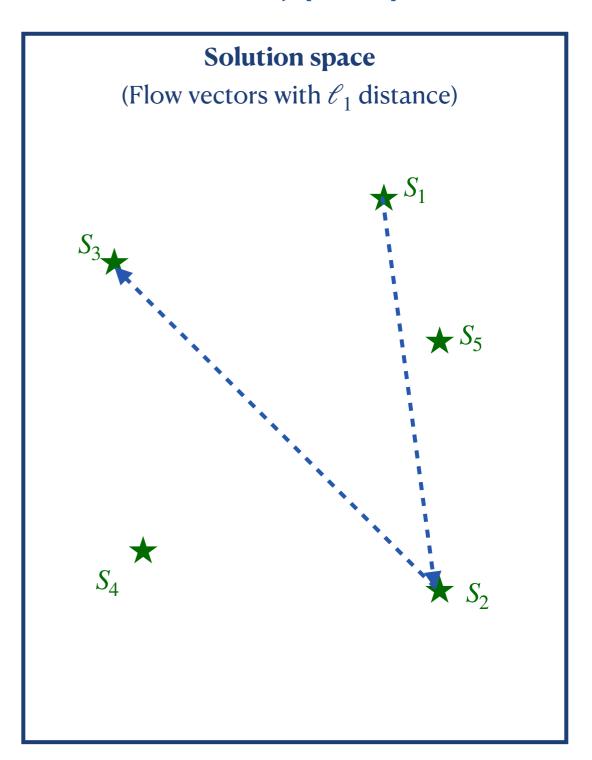
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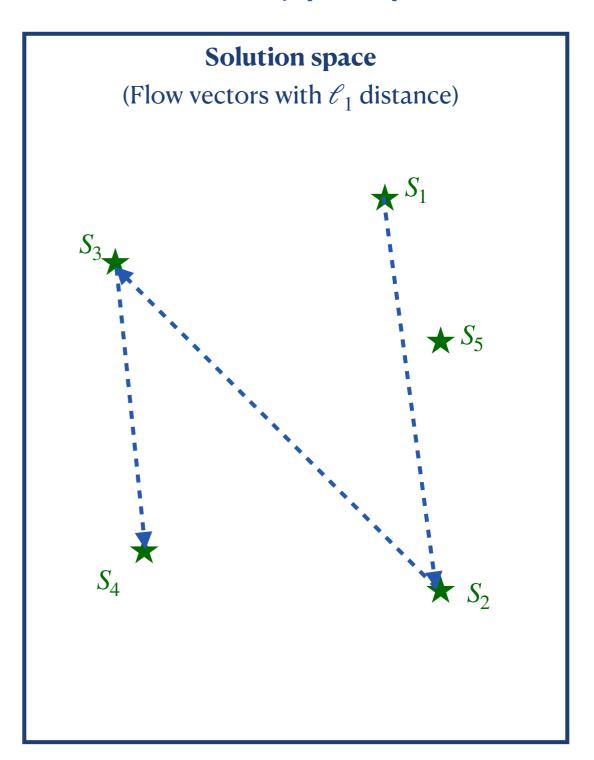
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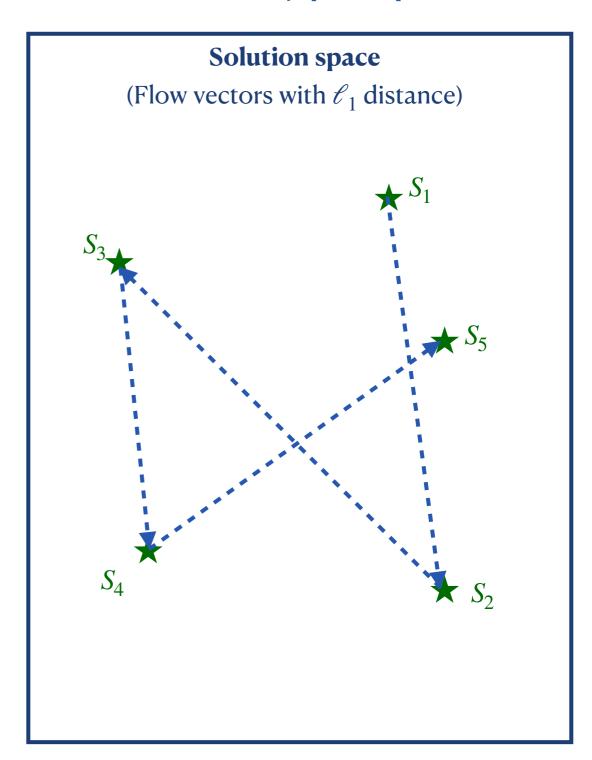
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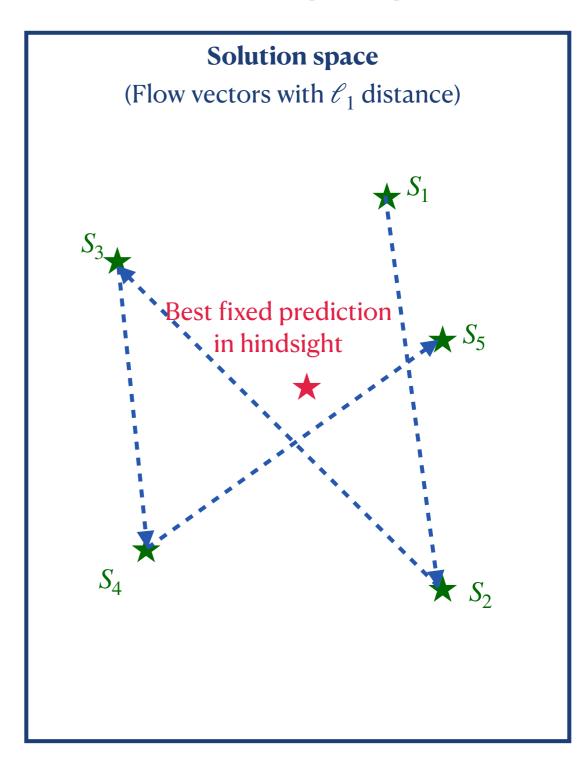
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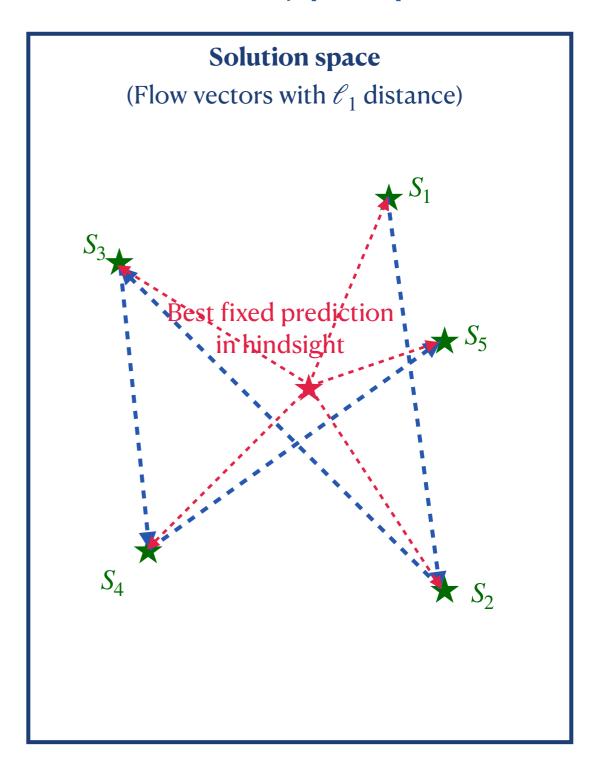
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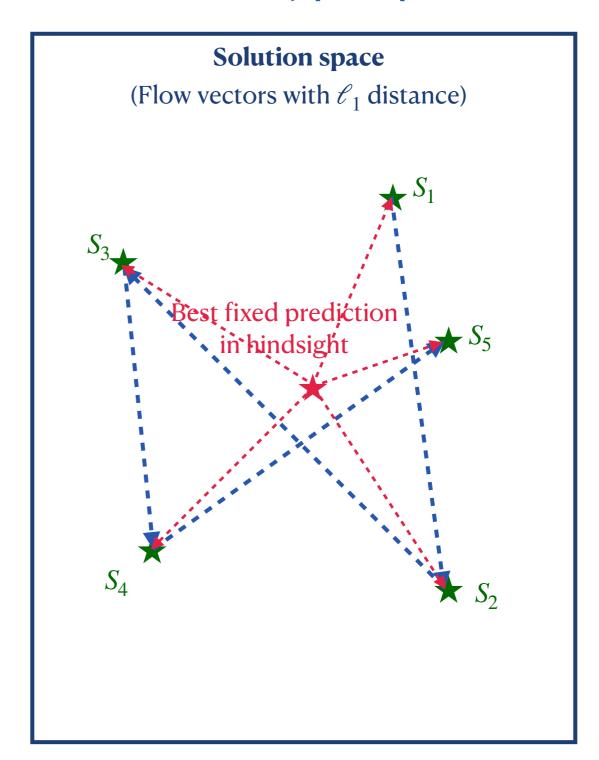


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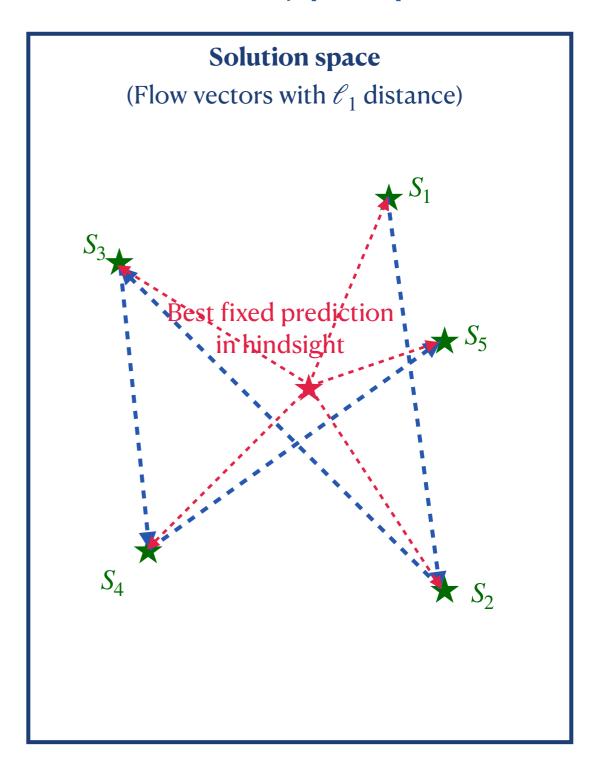
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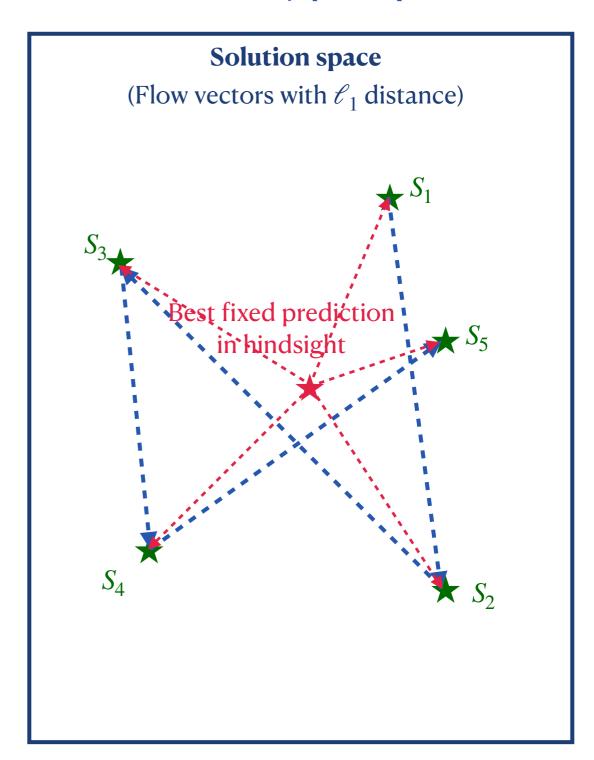
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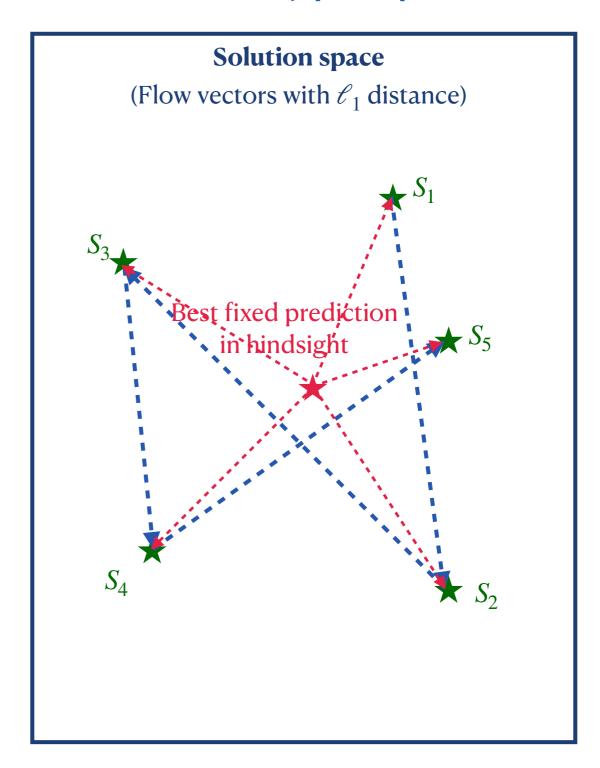


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Can we do even better?

 Yes! Can design algorithms to take advantage of other forms of structure (multiple clusters) and compete against adaptive baselines [Blum Srinivas '25]



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- Analyze and compare strategies for solving sequences of related instances
- See new algorithmic opportunities
- Modularize (warm start vs. meta problem)

References

(1) Repeated Computations: Sequences of related instances of a problem can be solved faster than one at a time

Learning-augmented warm start algorithms:

- [Davies Moseley Vassilvitskii Wang '23][Polak Zub '22] Max-flow via Ford-Fulkerson
- [Dinitz Im Lavastida Moseley Vassilvitskii '21][Chen Silwal Vakilian Zhang '22] Primal dual bipartite matching
- [Davies Vassilvitskii Wang '24] Max-flow via Push Relabel

Prediction strategies for warm start algorithms:

- [Khodak Balcan Talwalkar Vassilvitskii '22] Compete with best fixed prediction in hindsight
- [Blum Srinivas '25] Take advantage of weaker structure in sequences

Three ideas

(1) Repeated Computations: Sequences of related instances of a problem can be solved faster than one at a time

(2) Dynamic Algorithms/Data Structures: Dynamic problems are easier with information about future updates

(3) Randomized Algorithms: Randomized algorithms and data structures can be hedged to take advantage of extra information by incorporating a prior

Dynamic Algorithms

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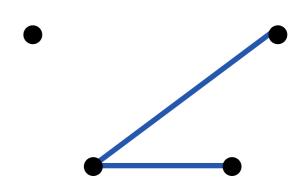
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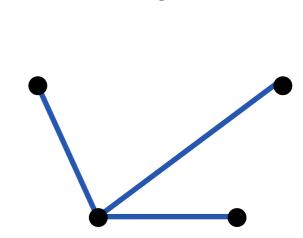
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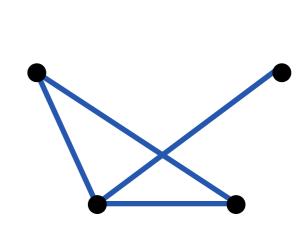
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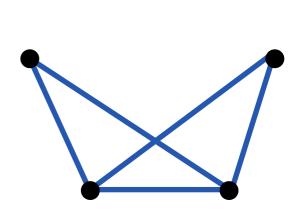
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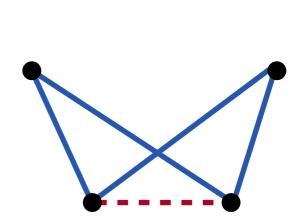
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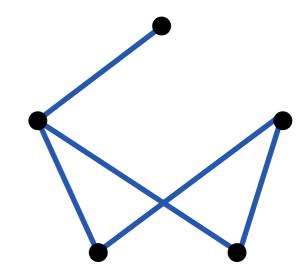
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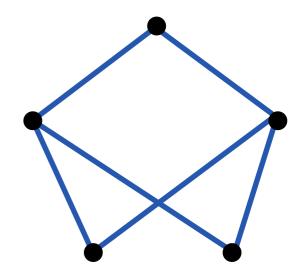
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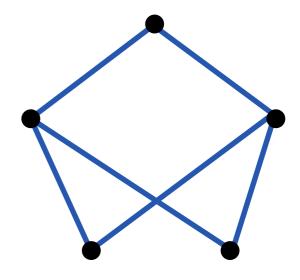
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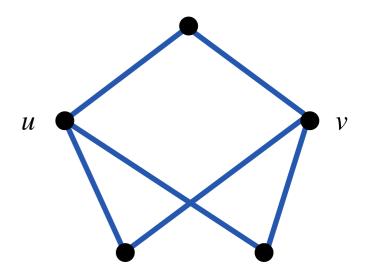
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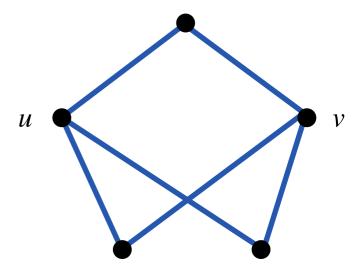
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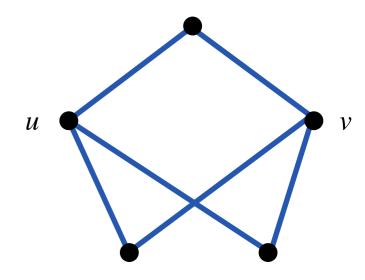


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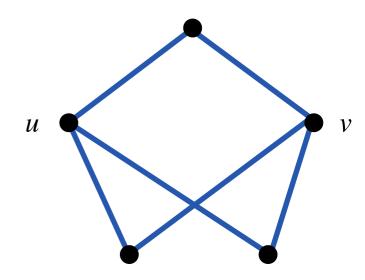
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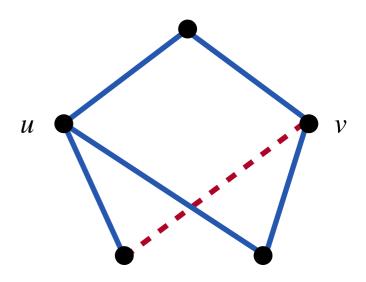
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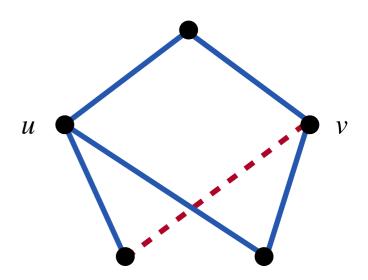
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- Design data structures to "reuse" work to solve subproblems



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 $O(n^{2/3})$ (worst-case) update time [Galil Italiano Sarnak '99]

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Gap between fully-dynamic and offline dynamic exists for many problems

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Predicted-Updates

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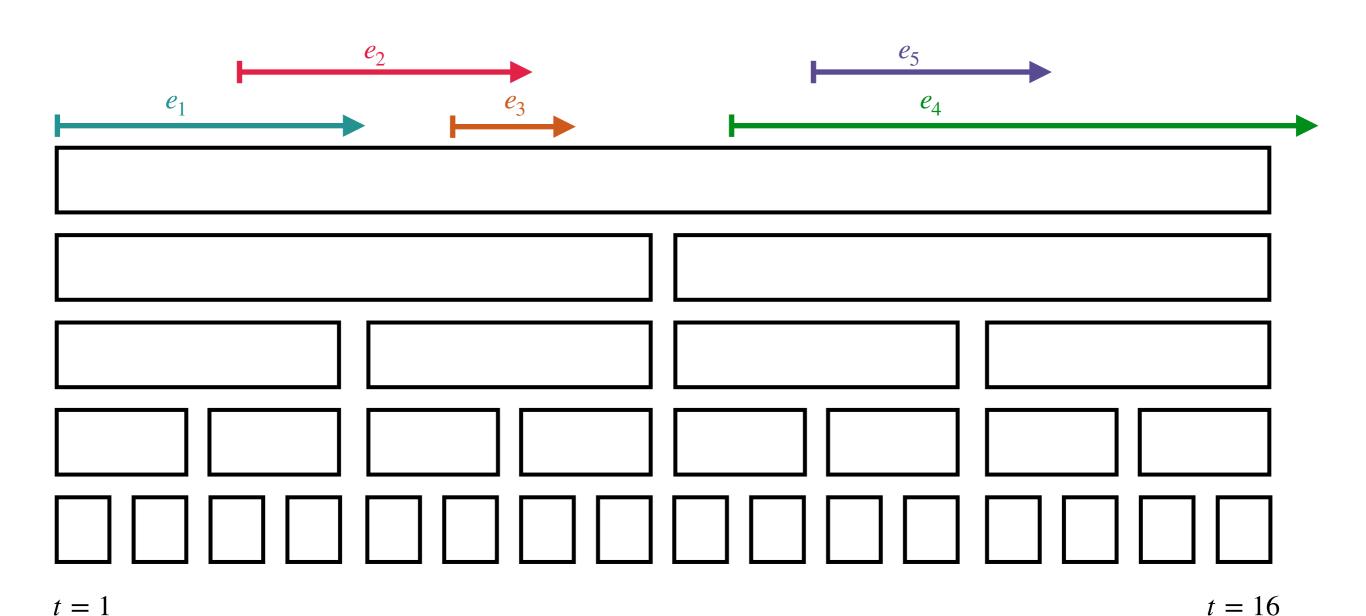
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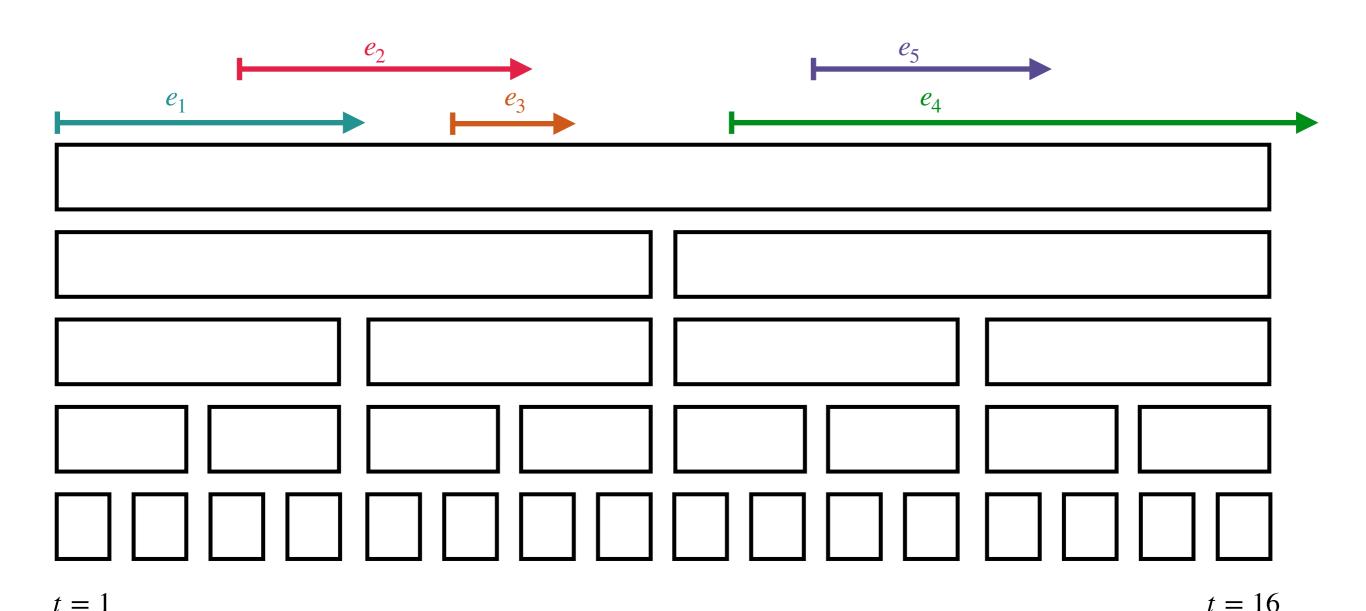
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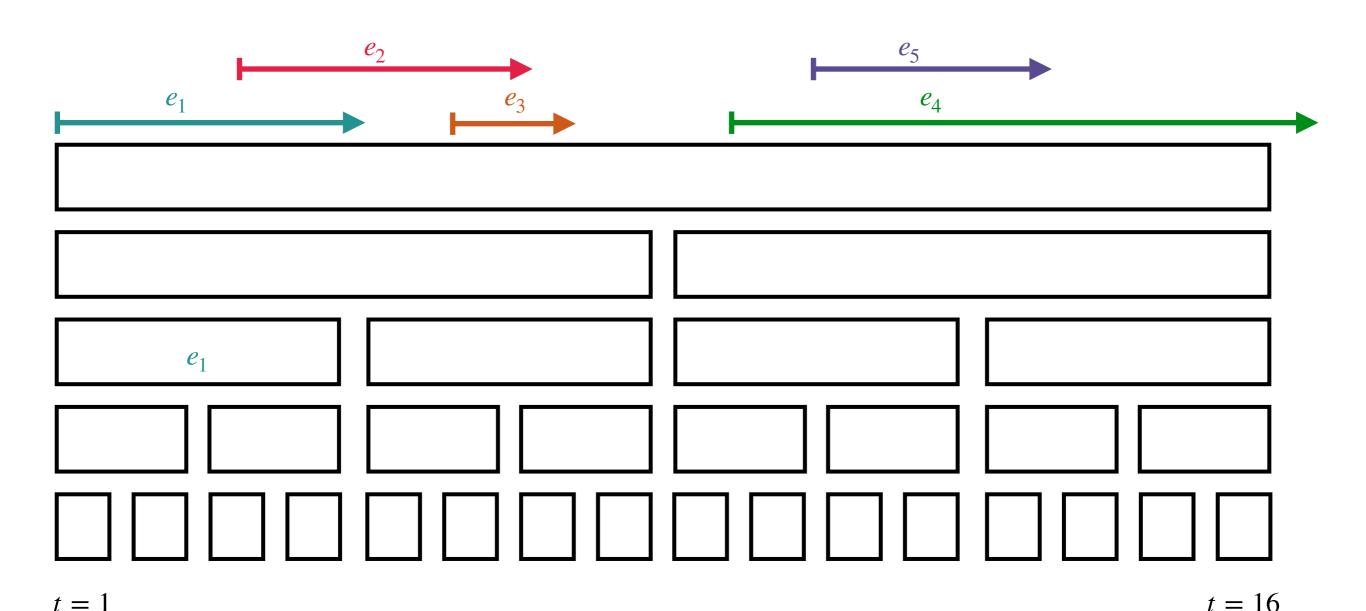




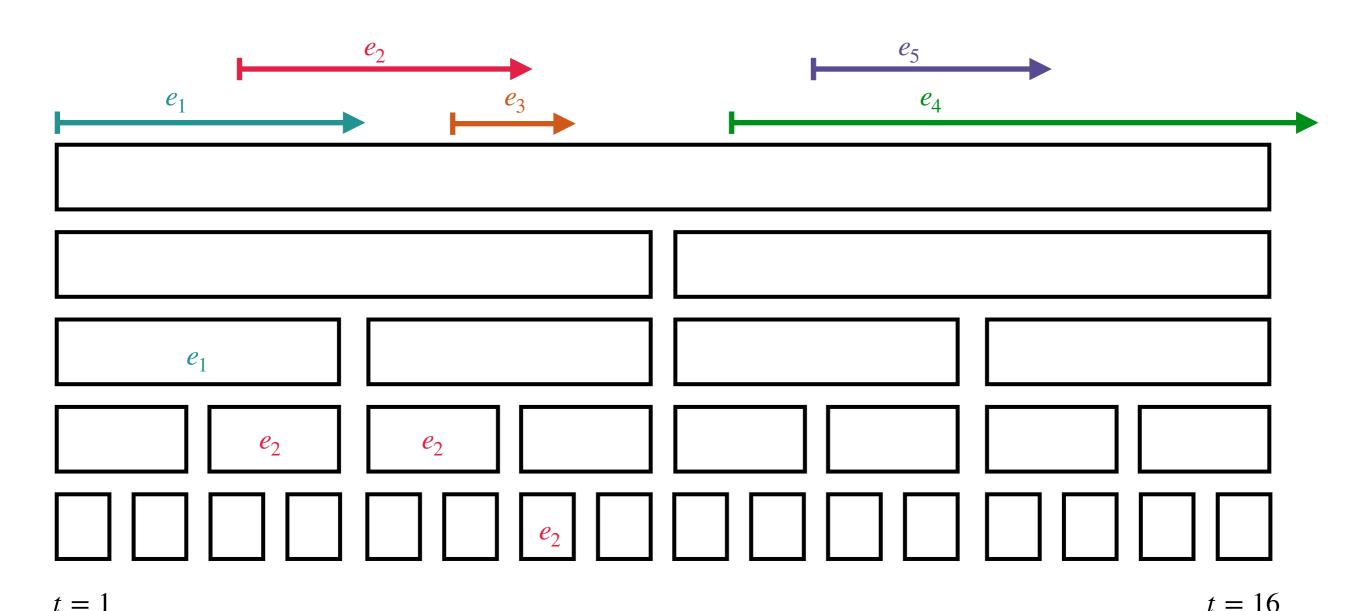
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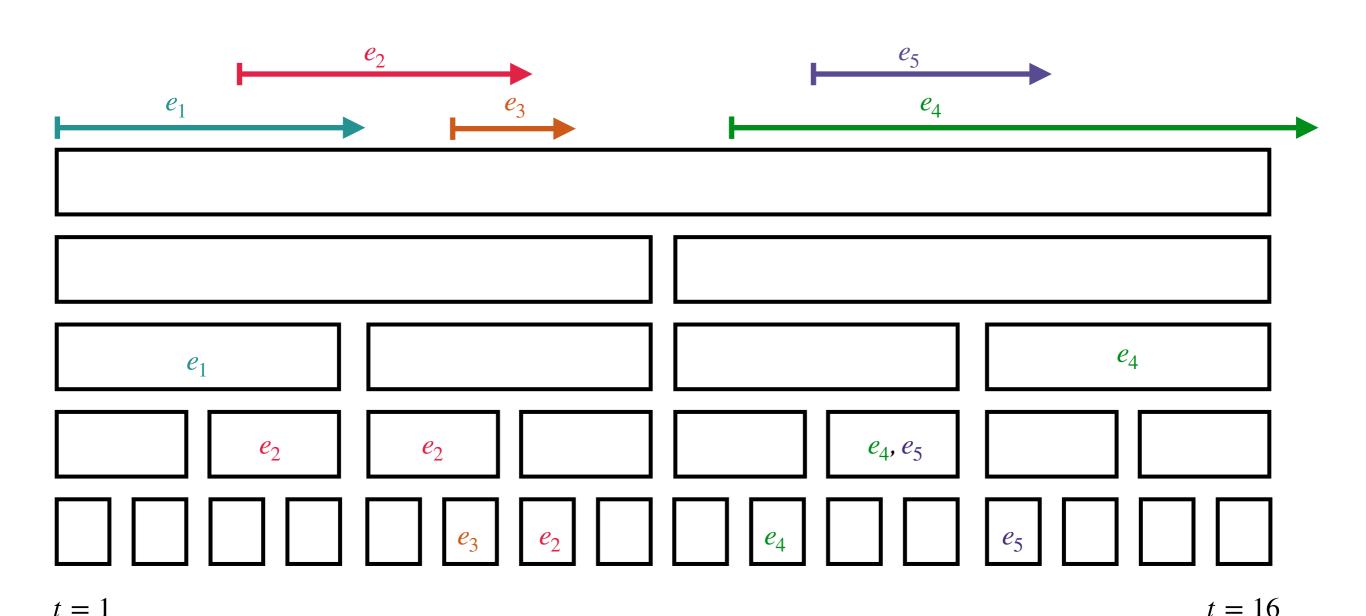
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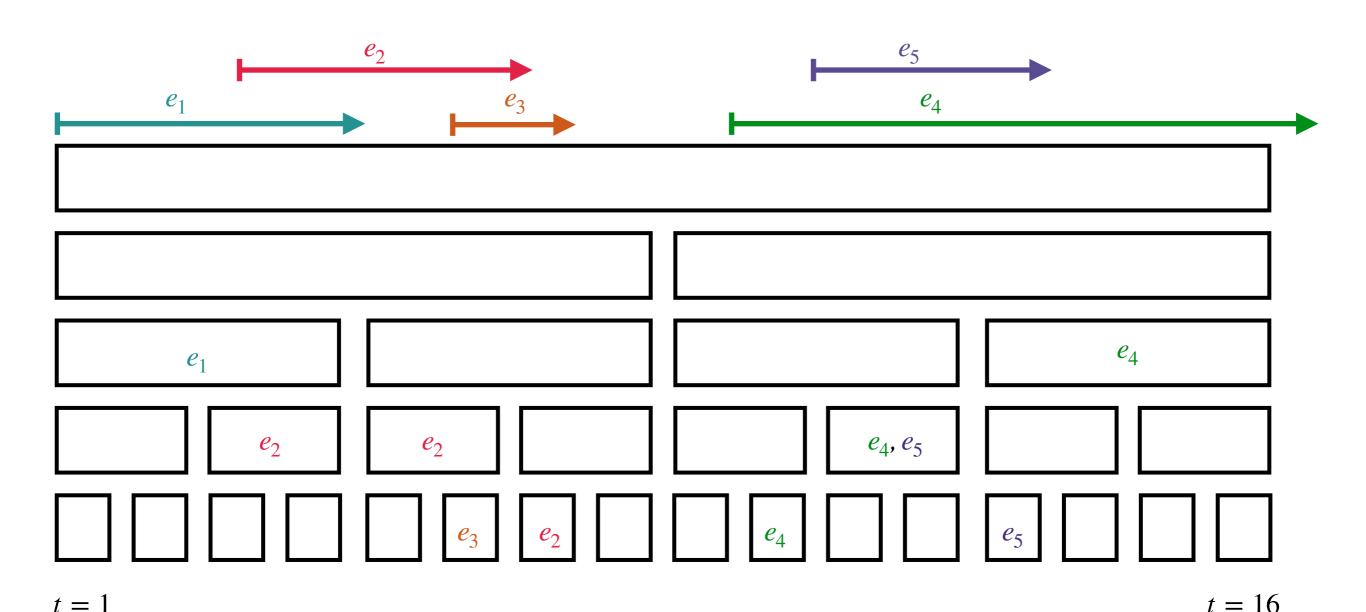
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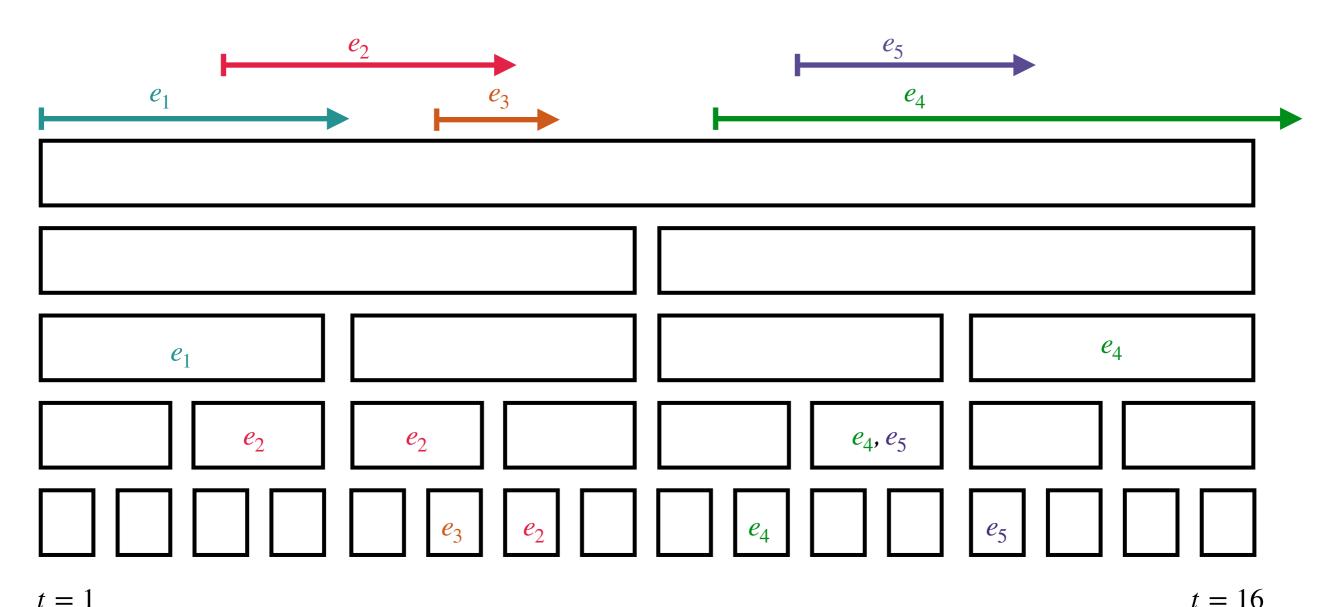
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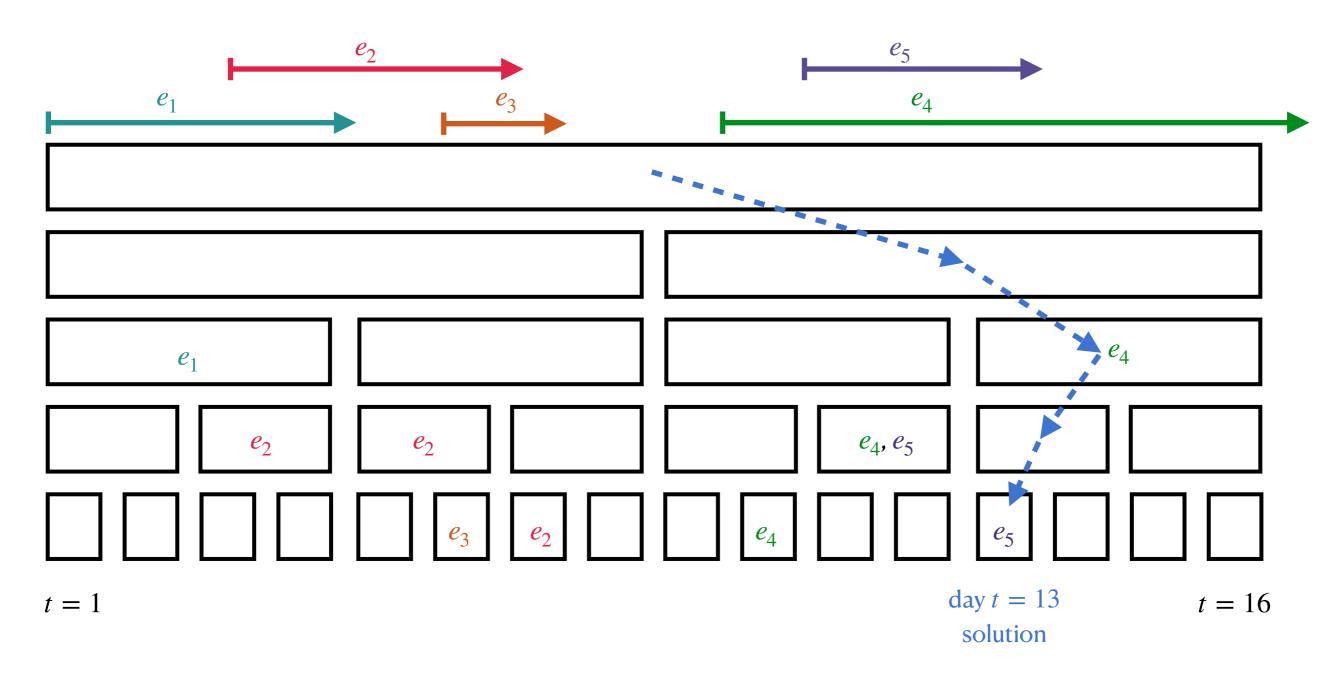
- Associate each edge with "windows" for which it is **permanent**
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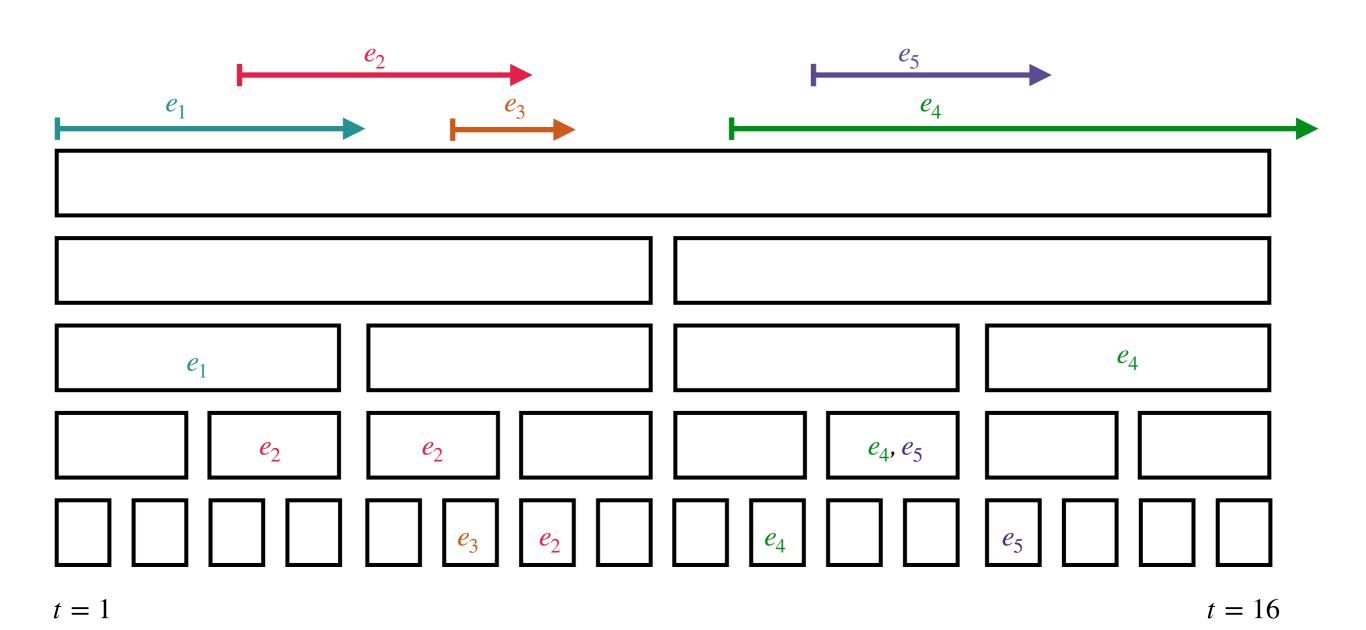


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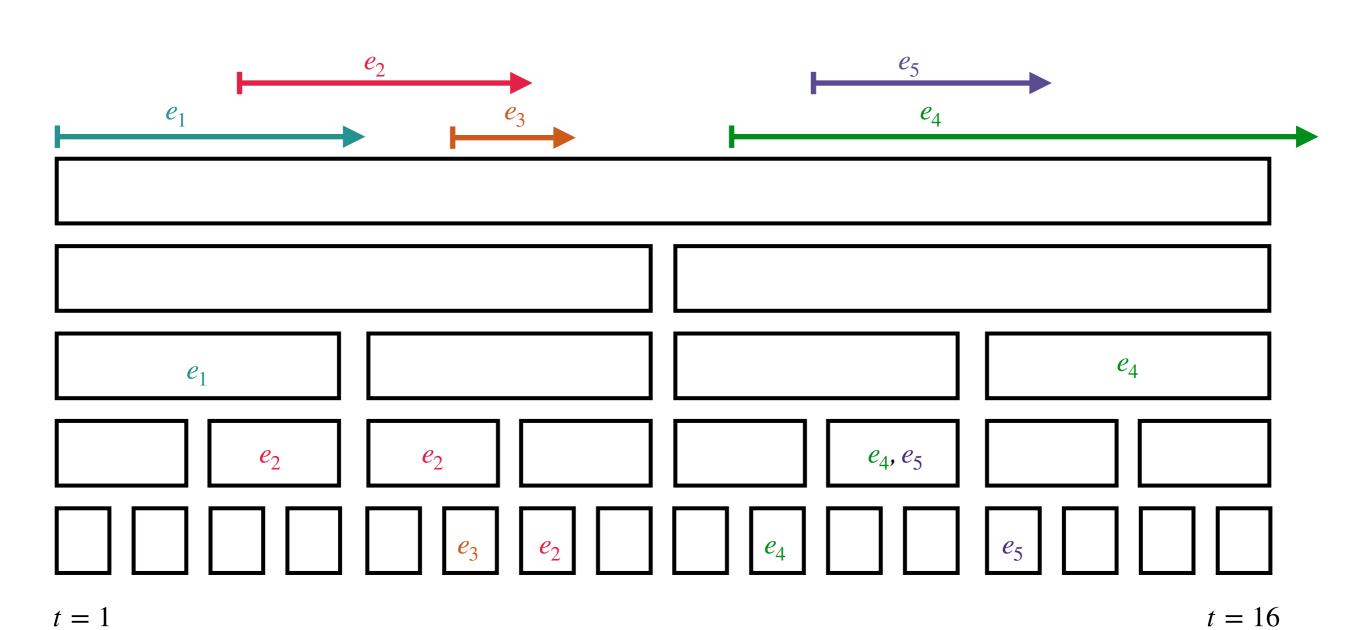


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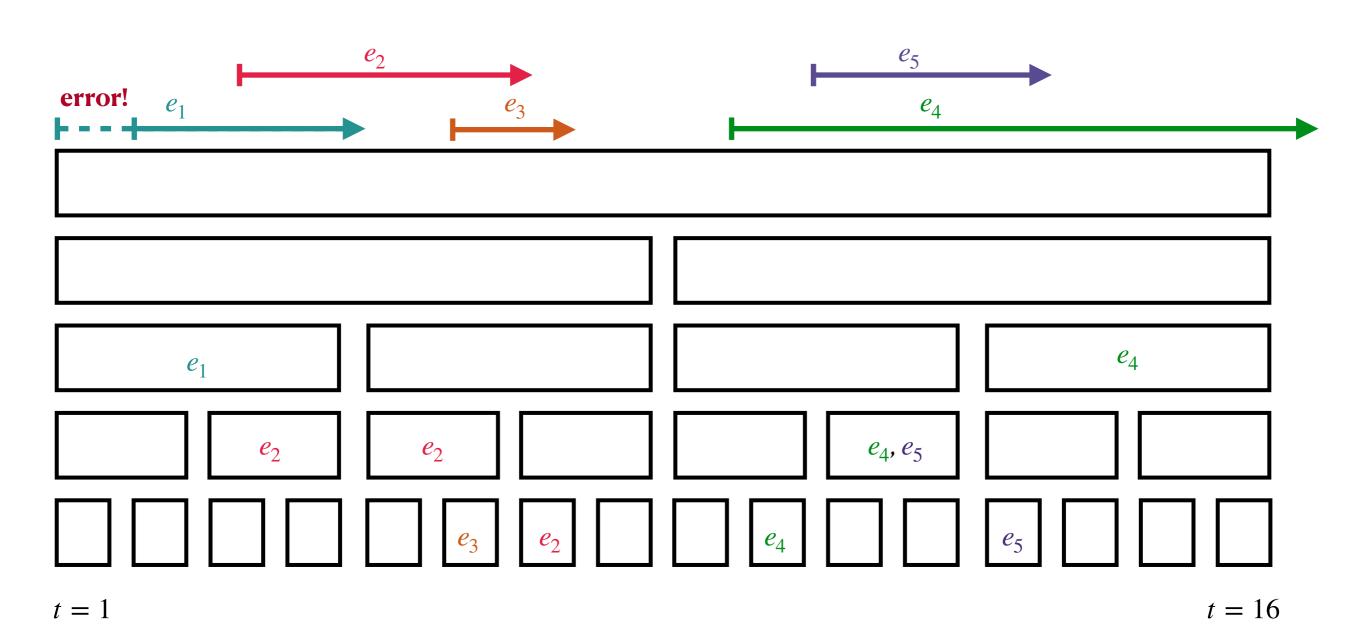




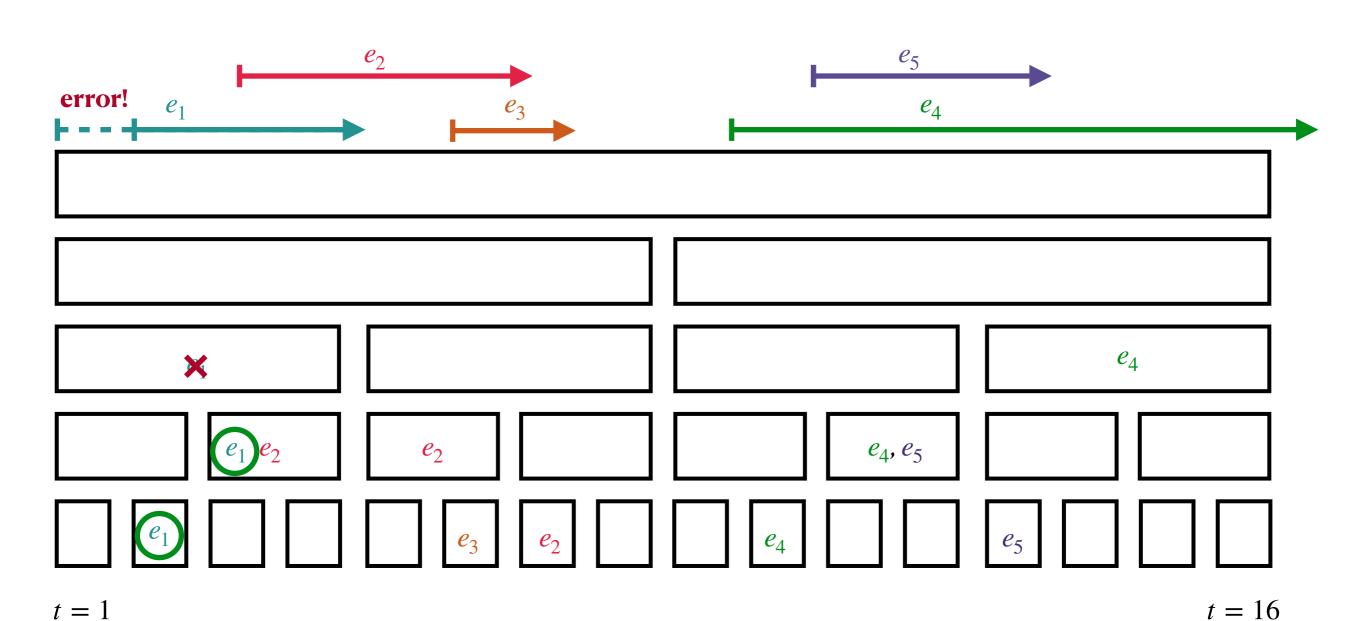
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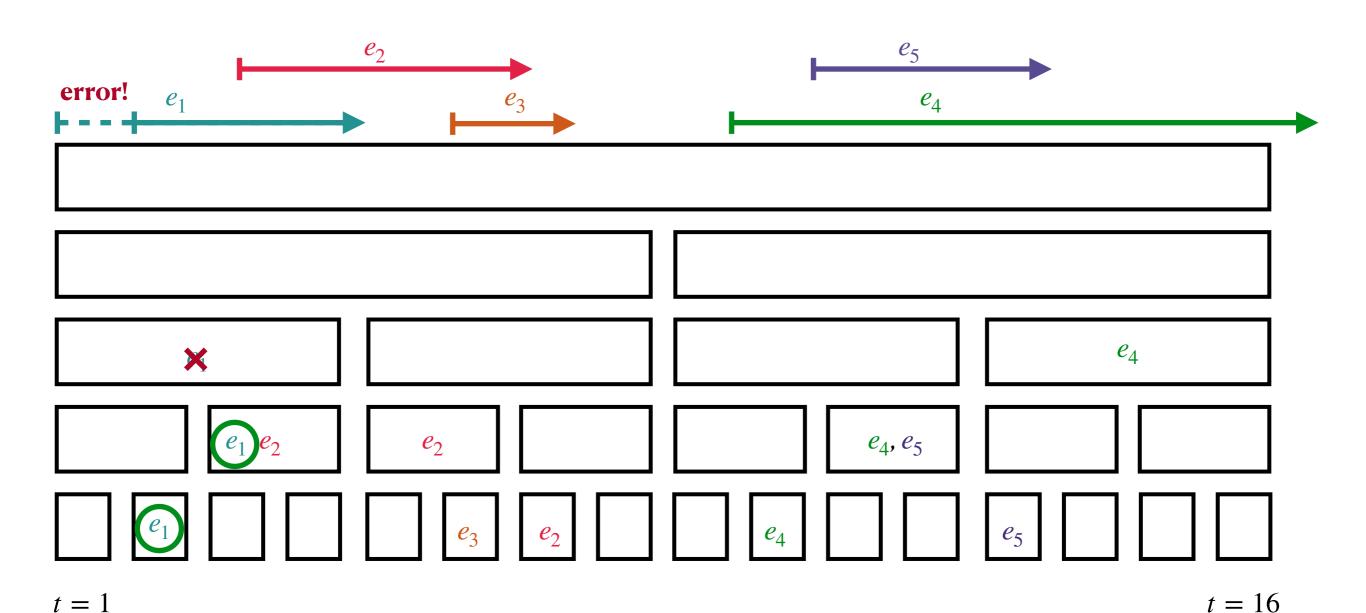
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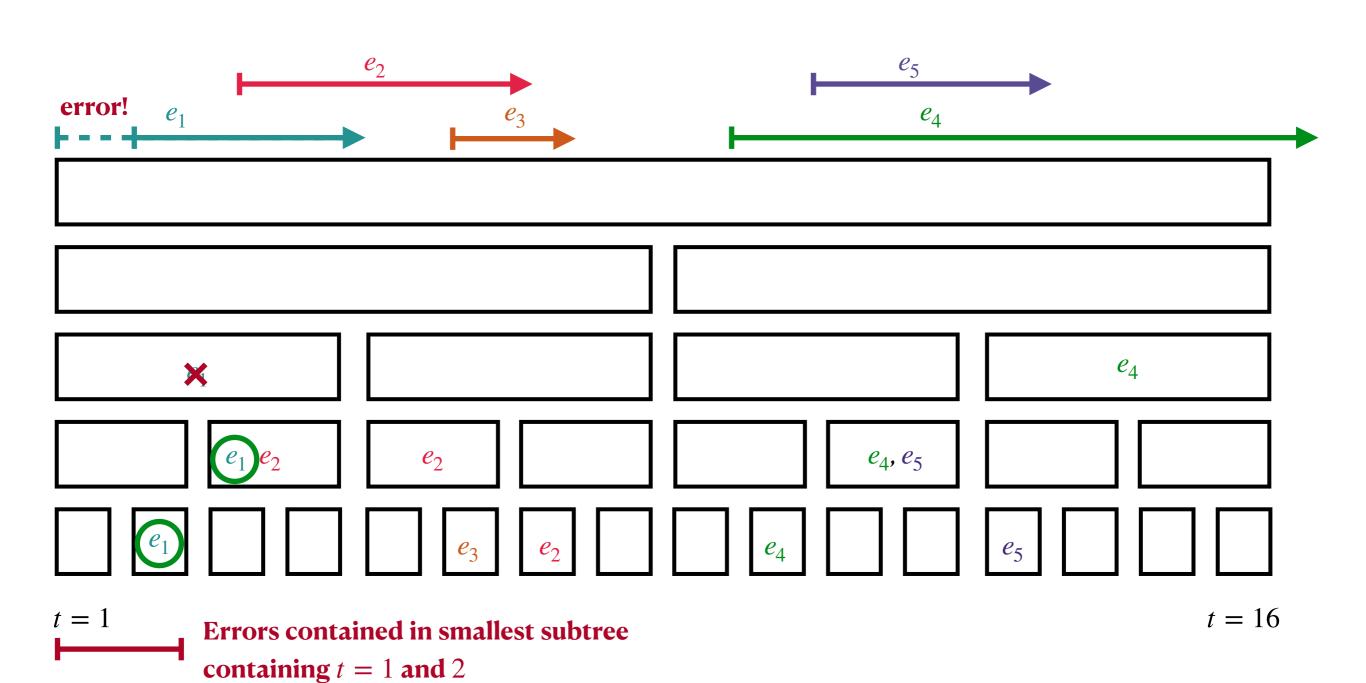
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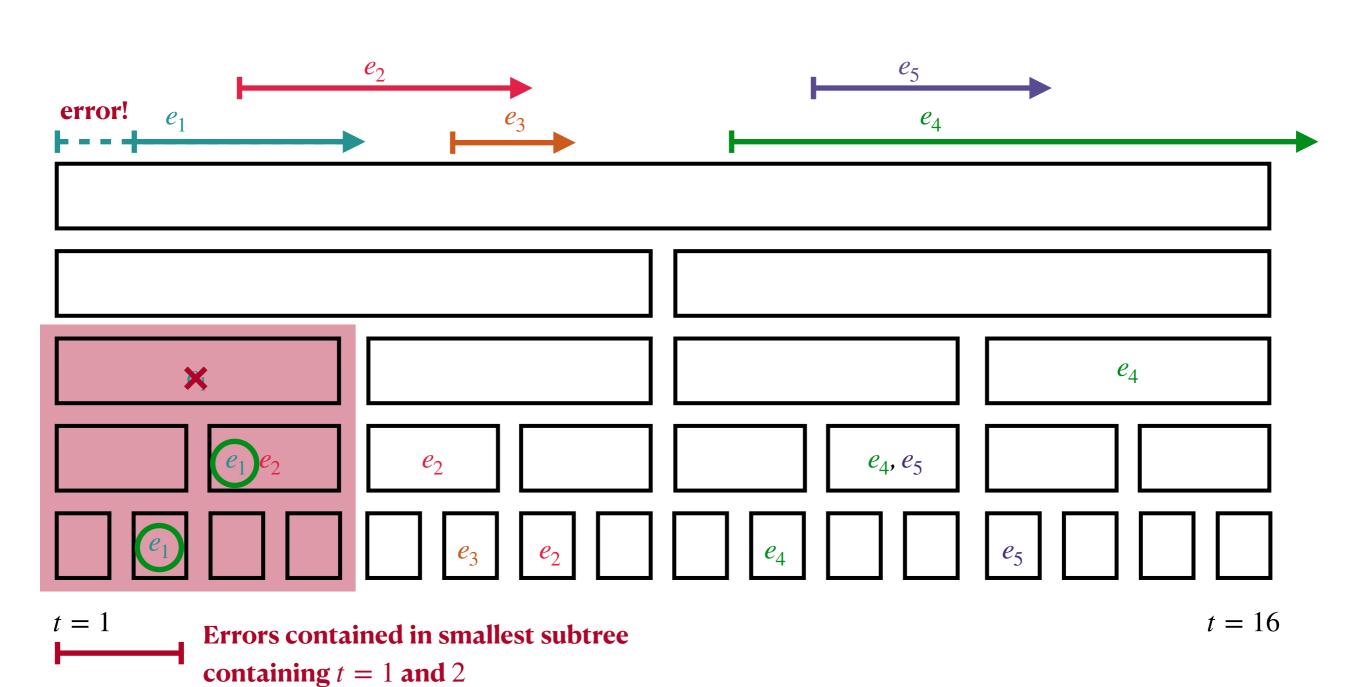
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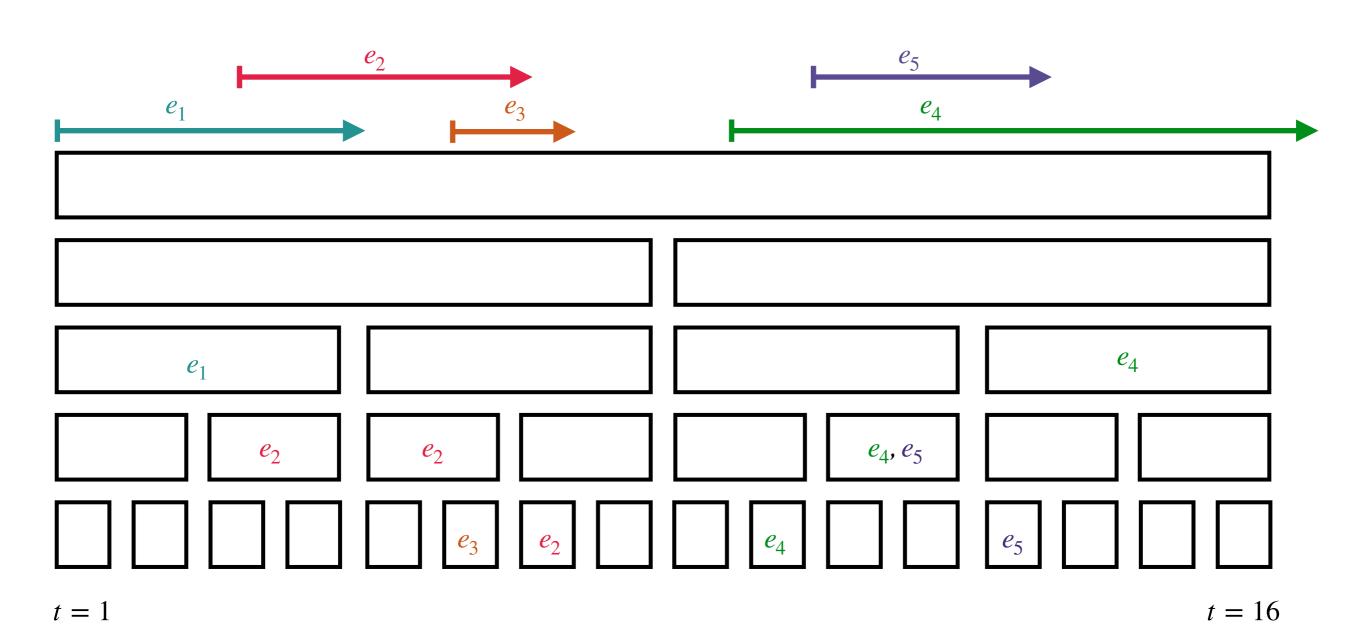


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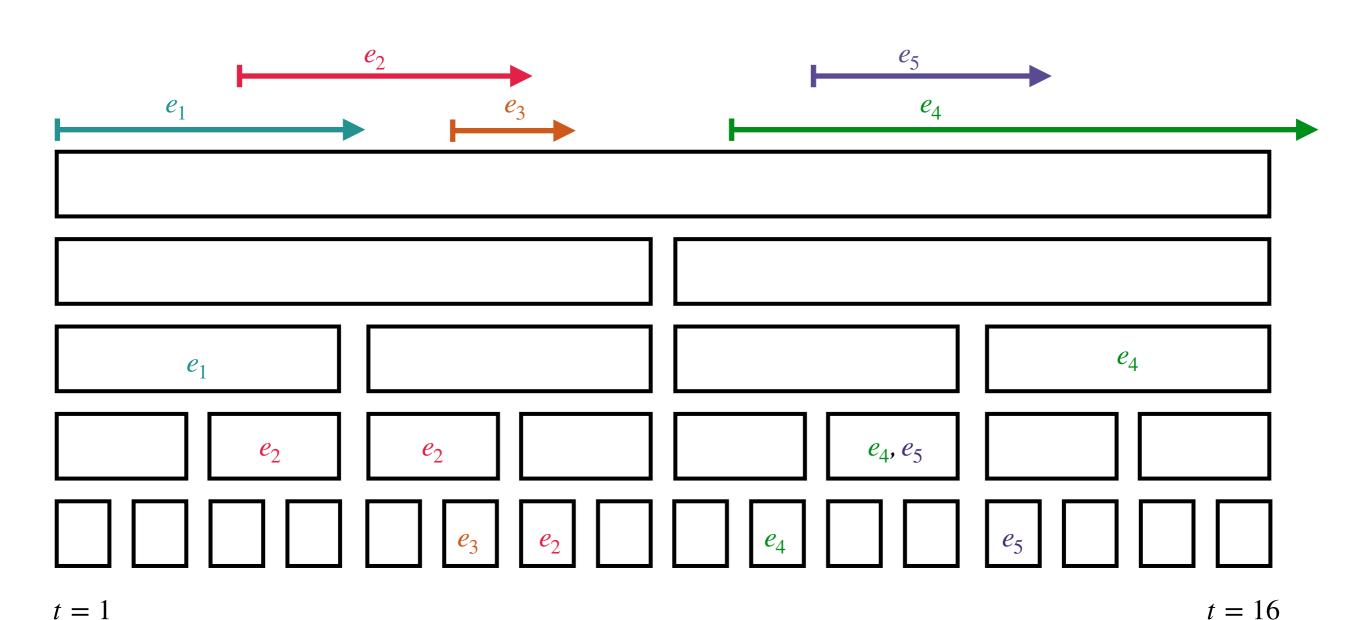


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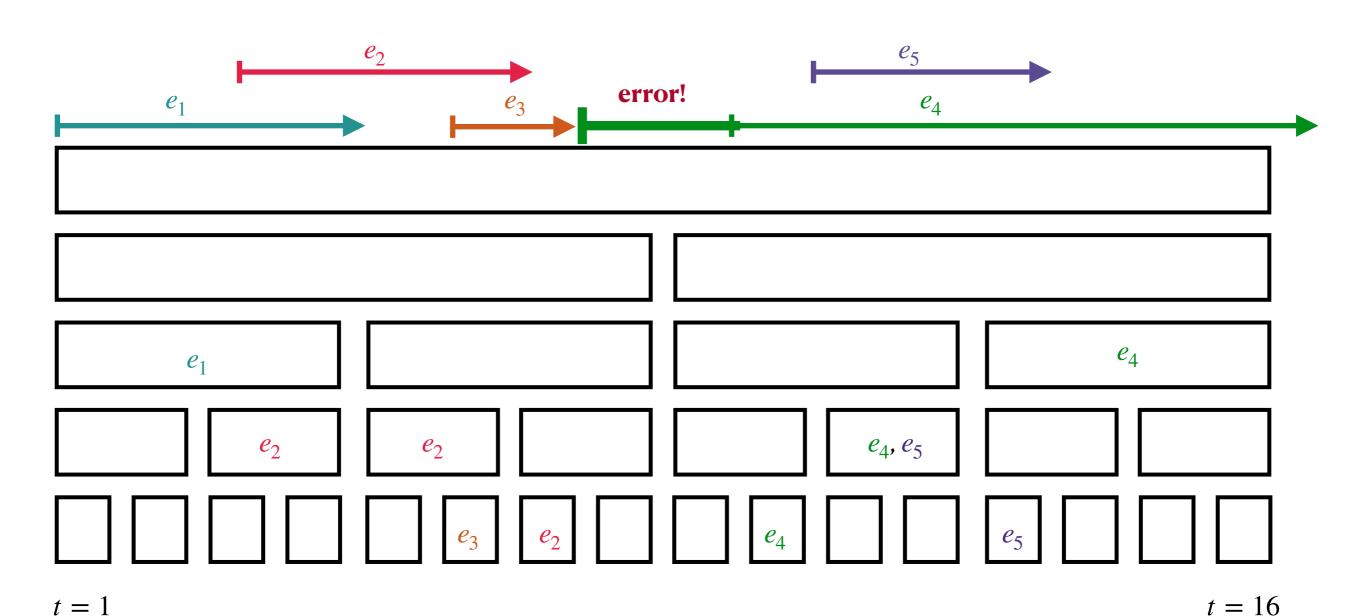




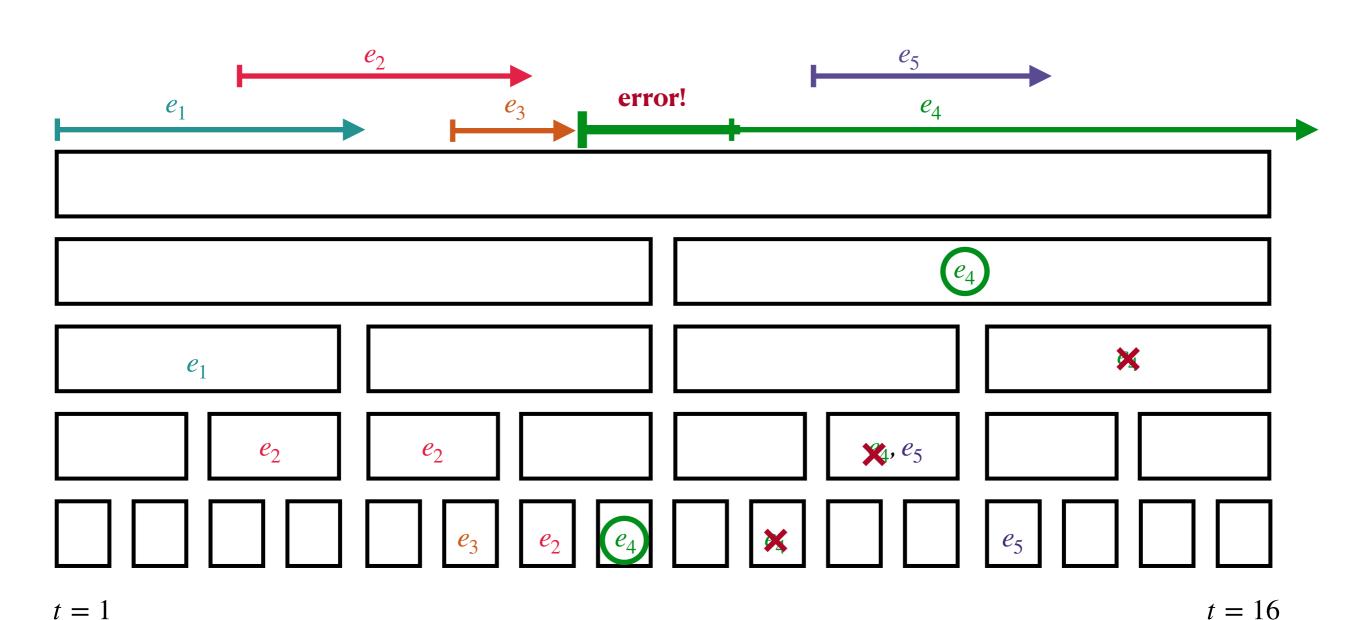
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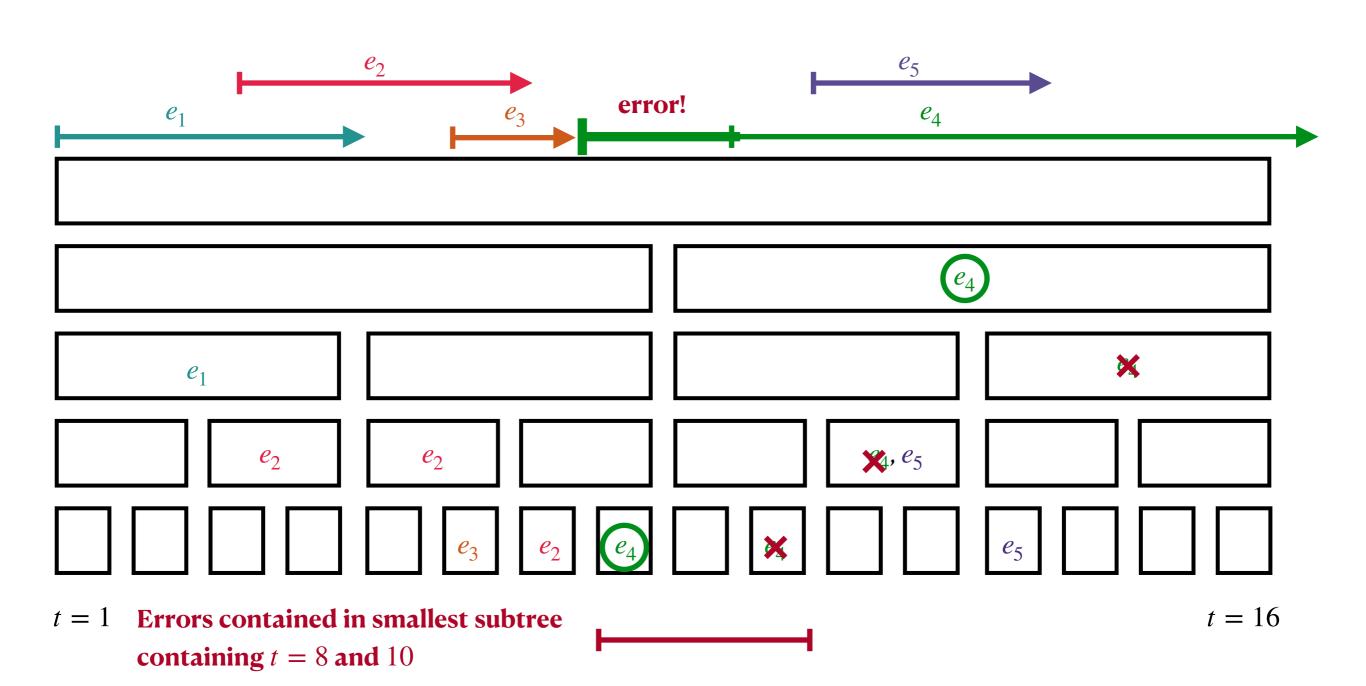
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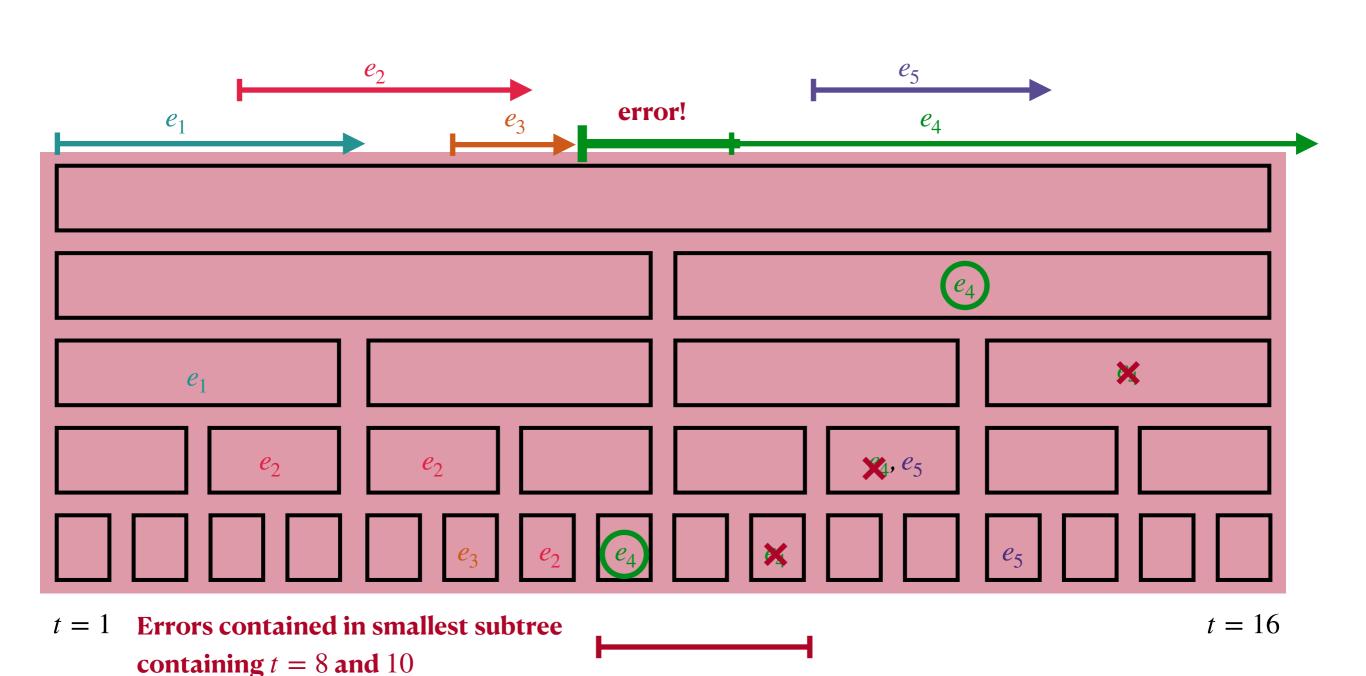
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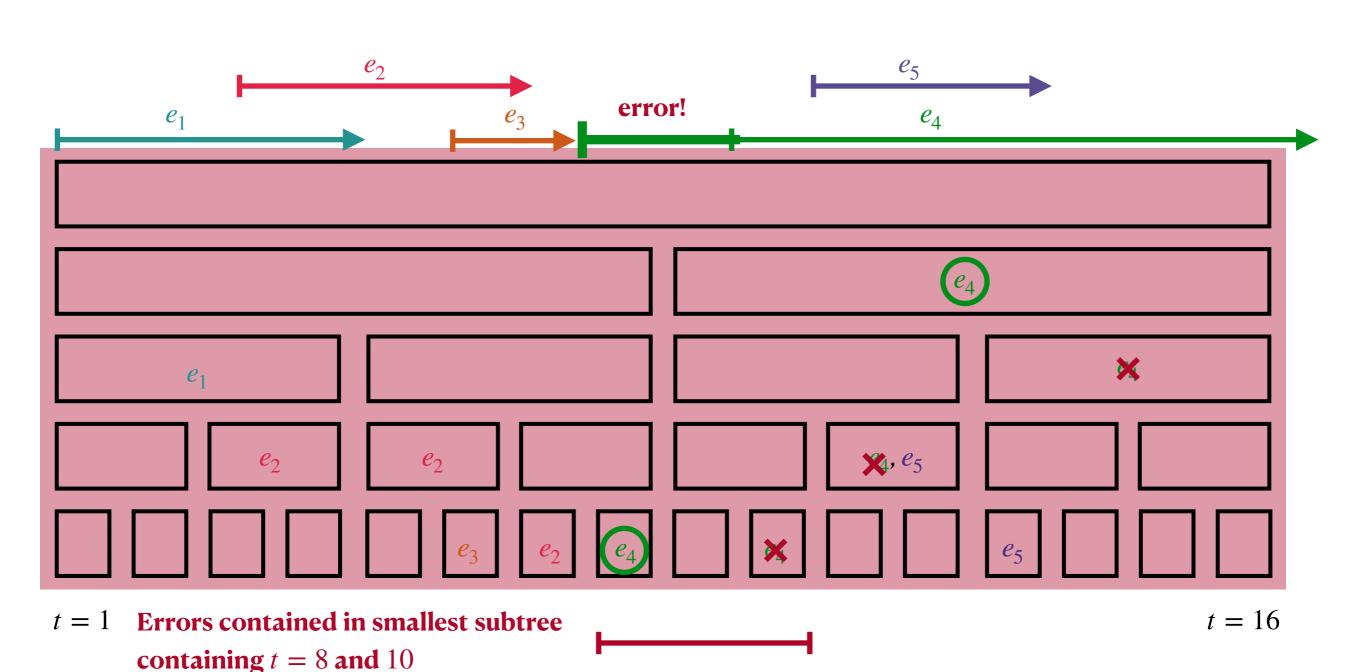
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Observation: errors are localized!

...but not very localized, small errors can trigger large recomputations

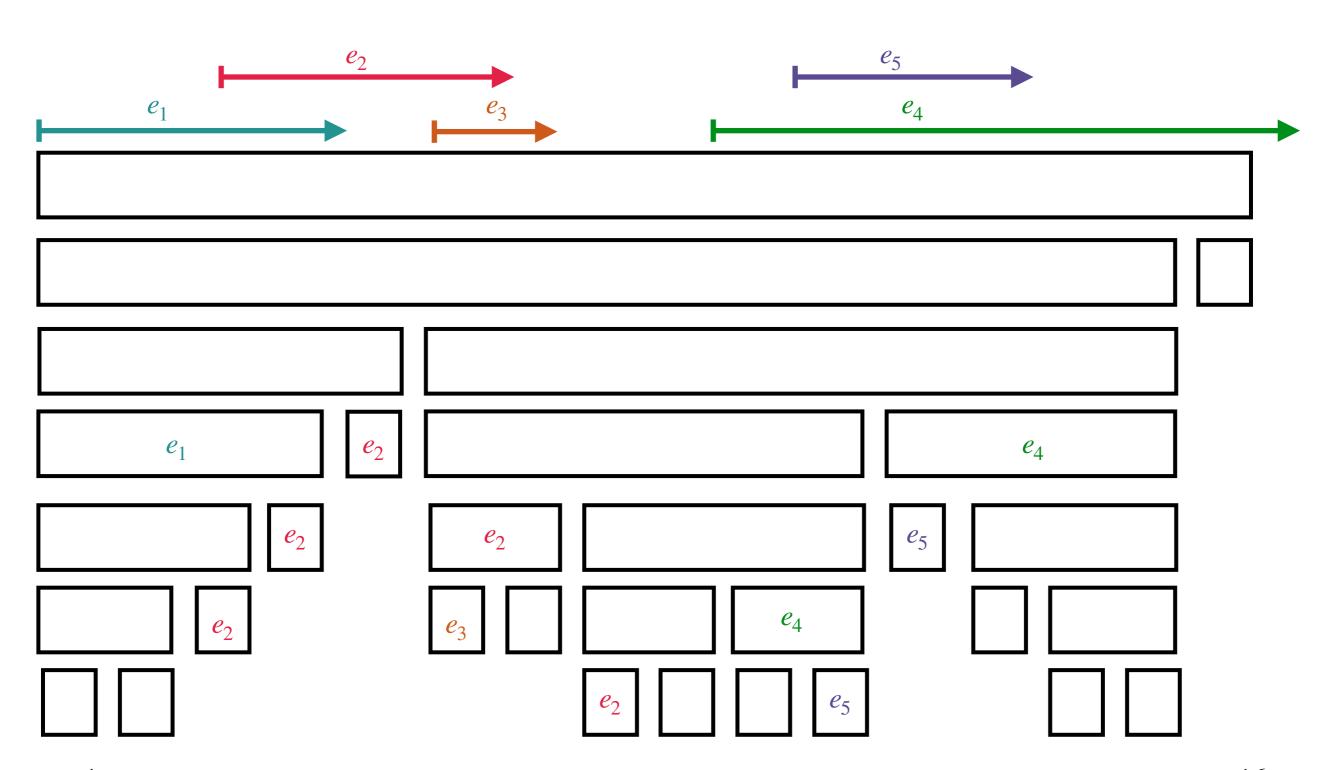


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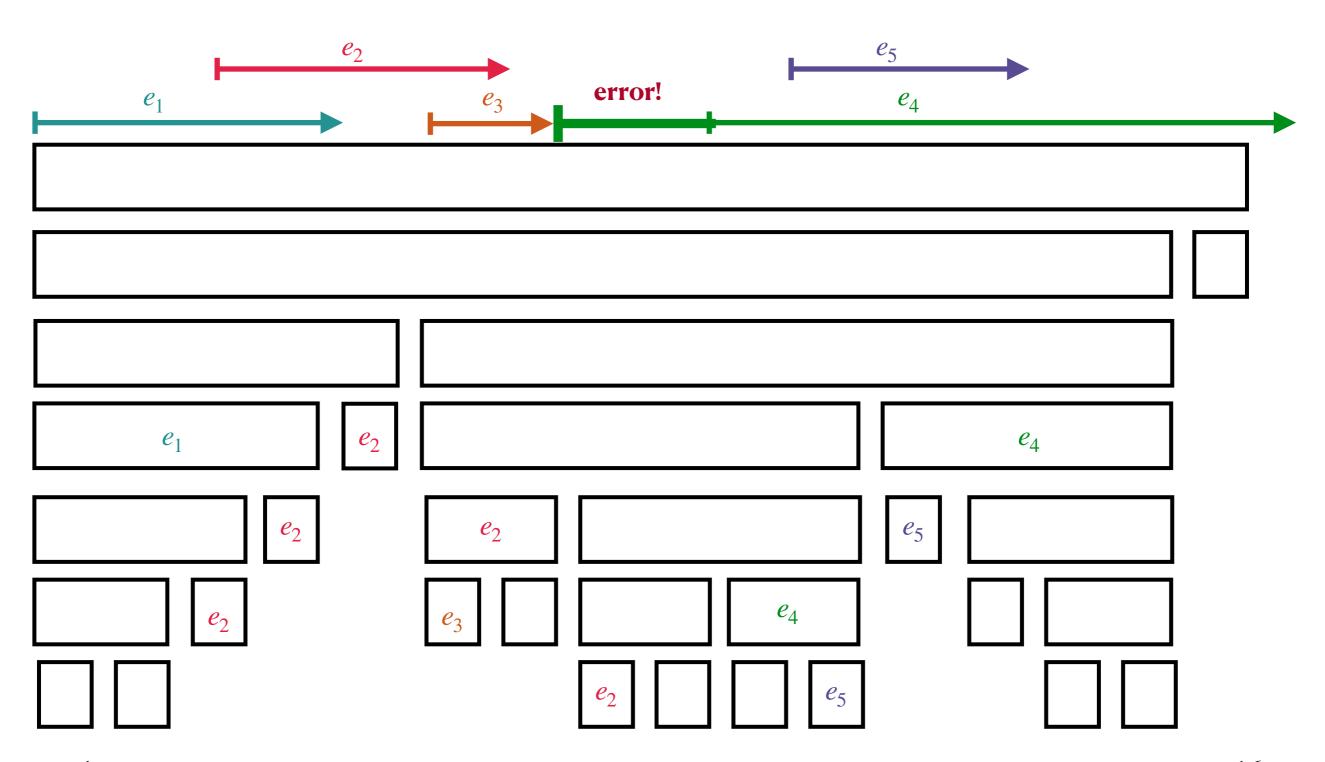


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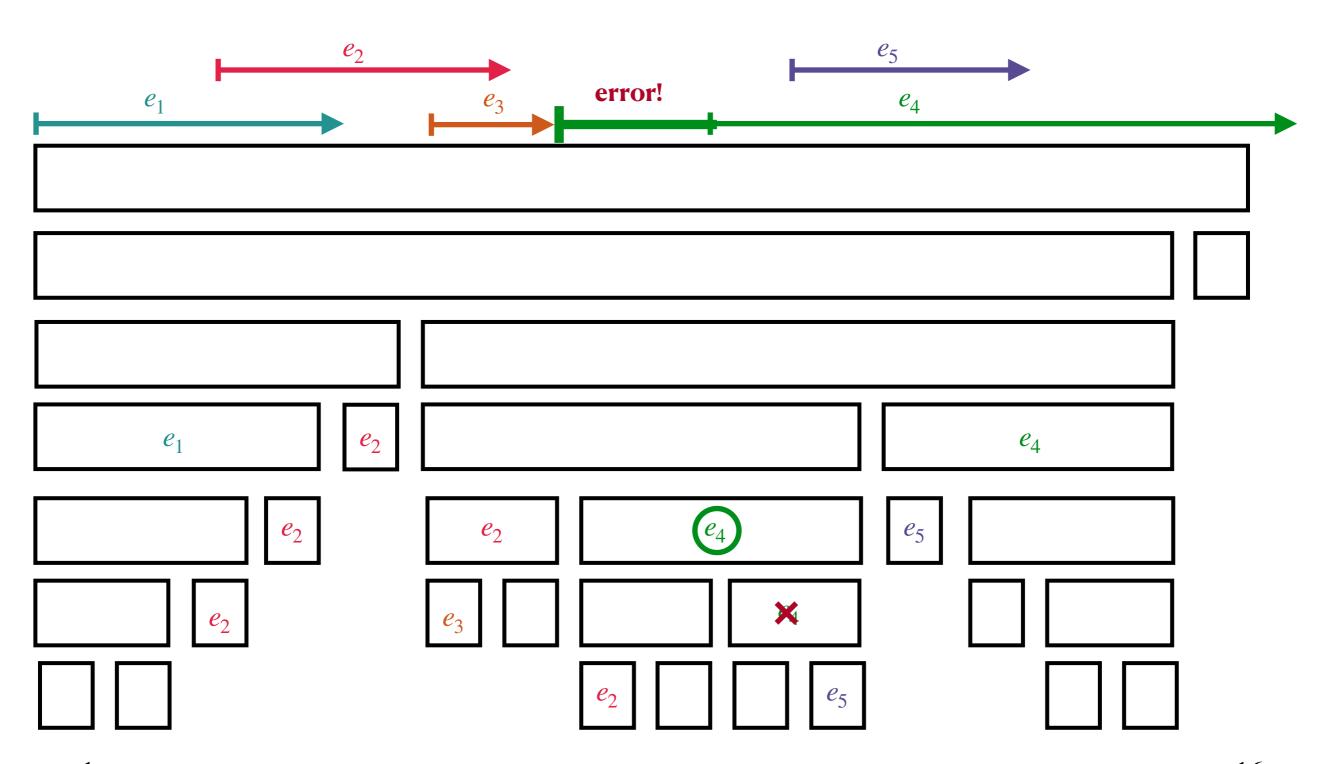
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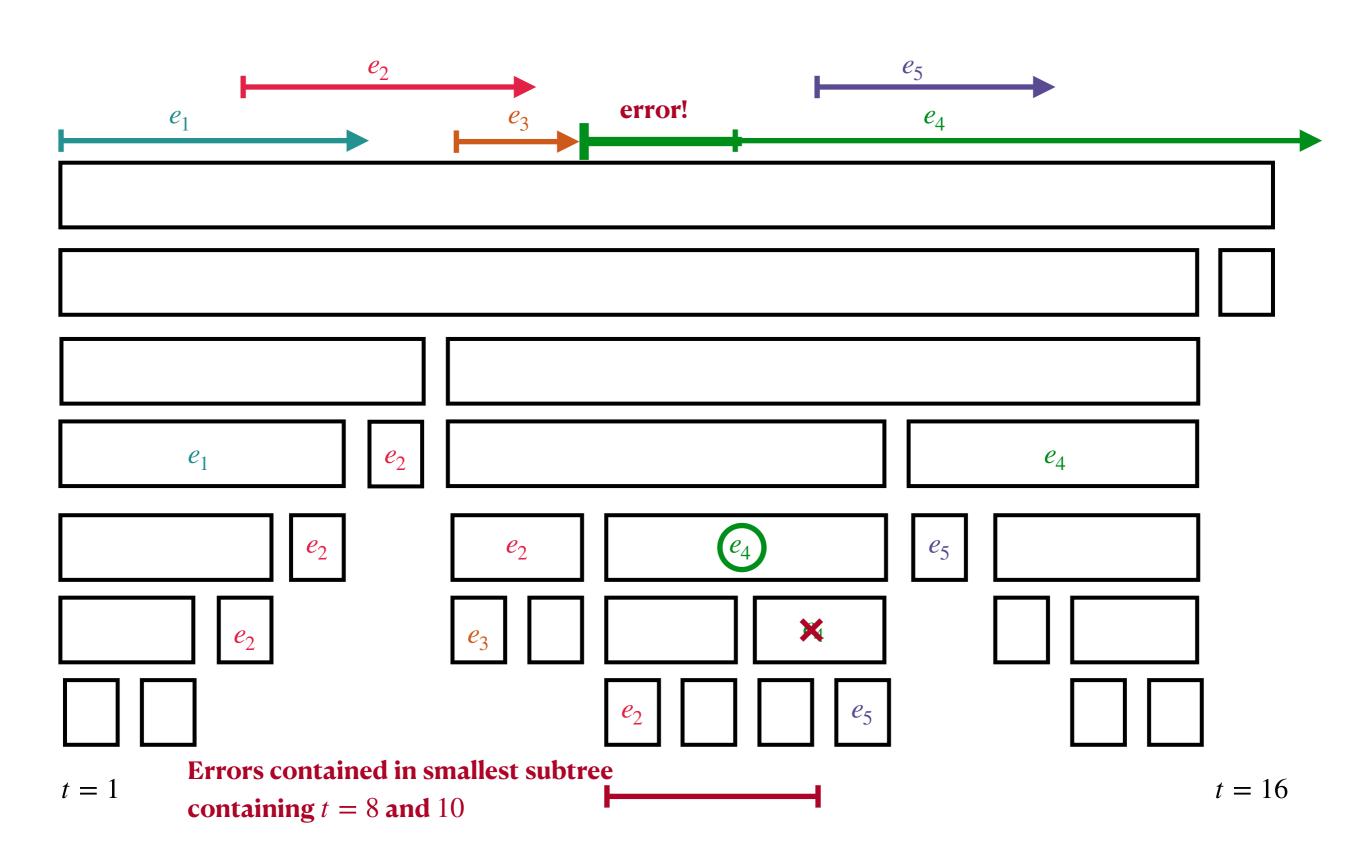
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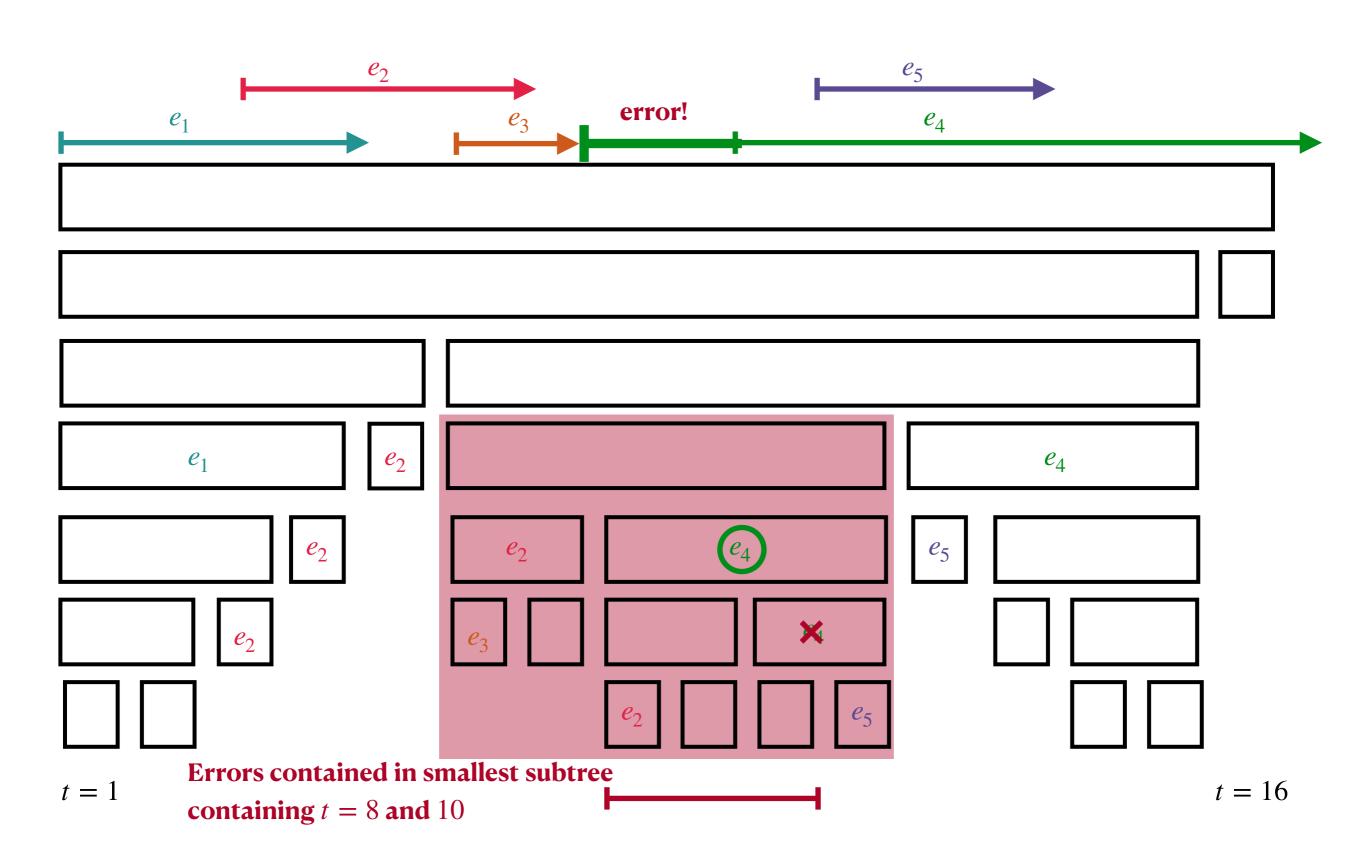
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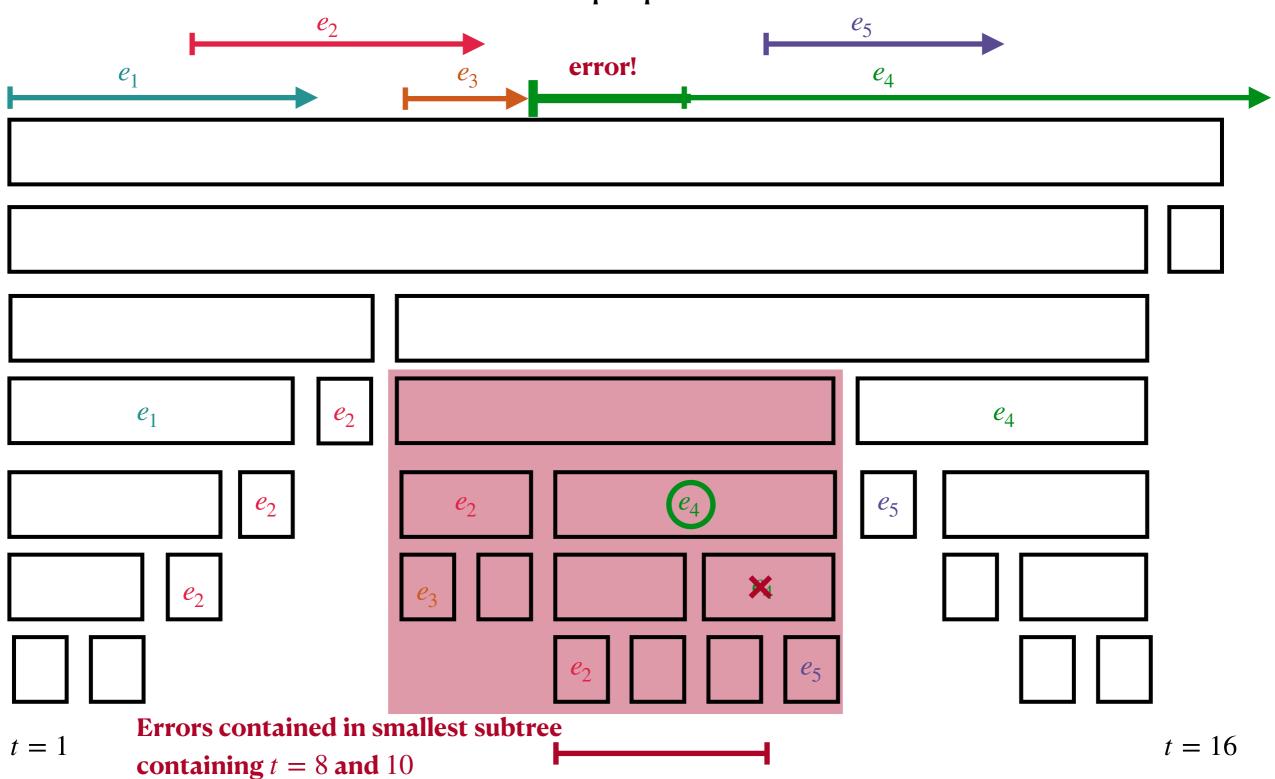
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⇒ size of error subtree proportional to size of error



Use this (and other) ideas to get an fully-dynamic to offline reduction with predictions

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Informal Theorem [Liu Srinivas '24]

Given a predicted sequence of events in advance, we can solve dynamic

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$$\widetilde{O}(\min\{T+\eta, T\cdot n^{2/3}\}),$$

Where η is the ℓ_1 error of the prediction (sum over events of the absolute prediction error in time).

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Takeaway: asymptotically lose nothing by taking advantage of predictions!

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Many recent works use similar and different techniques to use predictions to get improved dynamic algorithms:

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Warm Starts

- Unstructured input sequence
- Potentially large changes in input, take advantage of similarities in solutions
- At bare minimum, must read input on each day ⇒ no sublinear update time

References

(2) Dynamic Algorithms/Data Structures: Dynamic problems are easier with information about future updates

Dynamic algorithms with predictions:

- [Liu Srinivas '24] Offline to online transformations
- [van den Brand Forster Nazari Polak '23] Graph and matrix problems
- [Agarwal Balkanski '24] Submodular maximization
- [McCauley Moseley Niaparast Singh '24] Incremental Topological Ordering
- [McCauley Moseley Niaparast Niaparast Singh '25] Incremental Approximate Single Source Shortest Paths

Related ideas:

- [Peng Sandlund Sleator '17] Designing offline-dynamic algorithms
- [Peng Rubinstein '22] Fully-dynamic to incremental reduction with known deletion error

Three ideas

(1) Repeated Computations: Sequences of related instances of a problem can be solved faster than one at a time

(2) **Dynamic Algorithms/Data Structures:** Dynamic problems are easier with information about future updates

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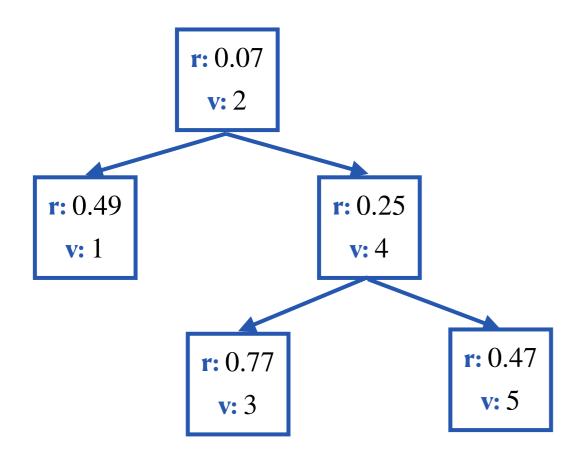
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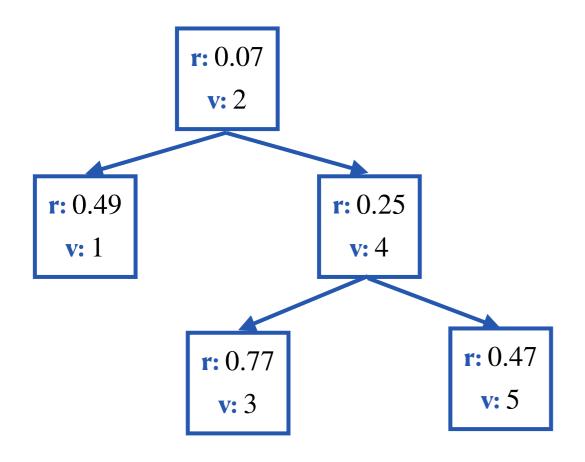
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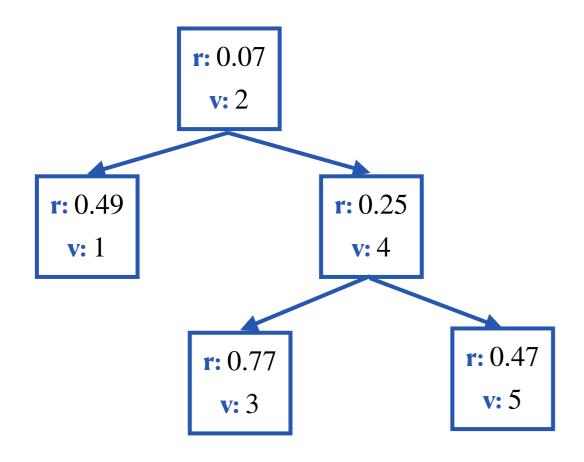
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- log(*n*) depth with high probability (quicksort type analysis)
- Coupling between ranks and random tree allows dynamic updates

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 - Dynamic optimality: long standing open problem
- What if you had **predictions** about the access sequence in advance? Can you get a **provable** optimality guarantee, without sacrificing worst-case update time?

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- (Meets provable benchmarks for splay trees, when frequencies are estimated on-the-fly)

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Algorithmic challenge: How do you design the right distribution?

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 - Use predictions about the min-cut to inform what vertices to contract in Karger-Stein

References

(3) Randomized Algorithms: Randomized algorithms and data structures can be hedged to take advantage of extra information by incorporating a prior

Learning-Augmented Search Trees:

- [Lin Luo Woodruff '23] Learning-Augmented Treaps with guarantees for stochastic accesses
- [Chen Cao Stepin Chen '25] Guarantees for general access sequences

An incomplete list of other places to see this idea:

- [Lykouris Vassilvitskii '18] Caching
- [Kumar Purohit Svitkina '18], and follow up work, Ski-rental (proof of concept for online algorithms in general)
- [Moseley Niaparast Singh '25] Min-cut via Karger-Stein

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- Join us!

- (1) Repeated Computations: Sequences of related instances of a problem can be solved faster than one at a time
- (2) Dynamic Algorithms/Data Structures: Dynamic problems are easier with information about future updates
- (3) Randomized Data Structures: Randomized data structures can be hedged to take advantage of extra information by incorporating a prior

...and so much more!

Takeaway: Learning-Augmented algorithms gives us tools and frameworks to reason about interesting and practical new problems

Exciting time to join the field!

- Lots of great work over the last 5-10 years laying the groundwork
- Seeing the payoff in new results that take advantage of a new ways of algorithmic thinking
- Join us!

THANKS!

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