ACDA 2025 Mini-tutorial on Learning-Augmented Algorithms
Part 2

Algorithmic Ideas from Learning-Augmented Algorithms

Vaidehi Srinivas
Northwestern University

Recap

Learning-Augmented Algorithms (a.k.a. Algorithms with Predictions):

Take advantage of **unreliable predictions**, without sacrificing worst-case guarantees

Consistency	Graceful Degradation	Robustness
If prediction is good, take advantage!	in magnitude of prediction error	If prediction is bad, revert to worst-case guarantee

Ok great, but...

- Can we actually achieve this for interesting problems?
- Does it help develop **new** algorithmic ideas?

Yes and yes!

Part 2

How to utilize extra **unreliable** information in algorithm design?

Worst-case analysis
Pessimistically ignore anything unreliable

Heuristic
Optimistically assume information is good, because it often is

Learning-Augmented Algorithms
Do both simultaneously!

Goal of talk:

- Highlight a few interesting examples where **new theoretical frameworks** and abstractions lead to **new algorithmic ideas**
- Focus on data structures and optimizing runtime (though there's lots of exciting work in other relevant areas, like streaming, online algorithms, and more!)
- Exciting time to get involved!

Three ideas

(1) **Repeated Computations:** Sequences of related instances of a problem can be solved faster than one at a time

(2) **Dynamic Algorithms/Data Structures:** Dynamic problems are easier with information about future updates

(3) **Randomized Algorithms:** Randomized algorithms and data structures can be hedged to take advantage of extra information by incorporating a prior

Three ideas

(1) **Repeated Computations:** Sequences of related instances of a problem can be solved faster than one at a time

(2) **Dynamic Algorithms/Data Structures:** Dynamic problems are easier with information about future updates

(3) **Randomized Algorithms:** Randomized algorithms and data structures can be hedged to take advantage of extra information by incorporating a prior

Sequences of Instances

Setting: solve a **sequence** of instances $I_1, \ldots, I_T$ of some algorithmic problem

Example: max-flow

Goal: minimize the **total runtime** to solve all of them

Challenge: If could solve sequences asymptotically faster than one at a time, would get an asymptotically faster worst-case algorithm

Hope: Adapt to **structure** in the sequence, when it exists!

(like dynamic algorithms, but want to impose less structure)

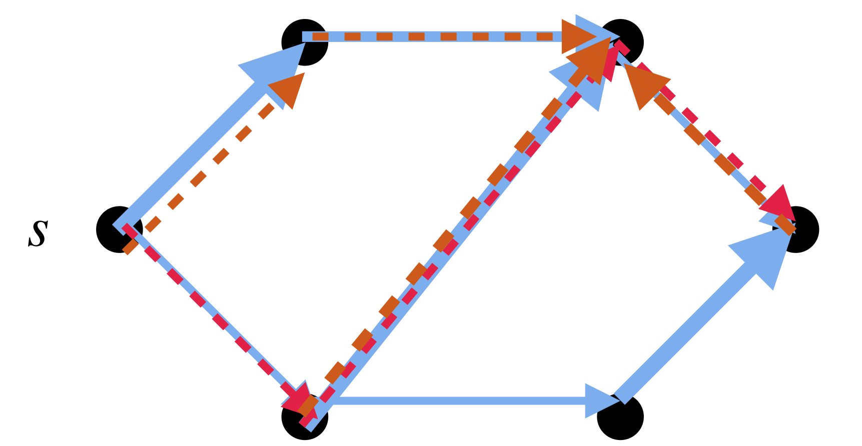
Example: suppose flow instances came from a predictable traffic network

Formalism: Learning-Augmented **"warm starts"** help us design and reason about algorithms for sequences of instances

Warm Start Example: Max-Flow via Ford-Fulkerson

Recall Ford-Fulkerson:

- Start with the all-0 flow
- Each iteration: find **augmenting path** to increase flow in residual graph by 1, in time $O(|E|)$
- # of iterations: bounded by max flow in graph f



Total cost: $O(|E|f)$

Warm start algorithm [Davies Moseley Vassilvitskii Wang '23]

- Start with a potentially **infeasible prediction** p for the flow on every edge
- Each iteration: augmenting-path like procedure in time $O(|E|)$
- # of iterations: bounded by $\eta = O(\sum_{e \in E} \left| p(e) - f(e) \right|)$, where $f(e)$ is the true flow on every edge (ℓ_1 error)

Total cost: $O(|E| \cdot \min\{\eta, f\})$

Always outputs correct answer!

Can we optimize runtime?

What to do with a warm start algorithm?

Challenge: a warm-start algorithm is only as good as the prediction!

For new instance, must predict the optimal flow

$\longrightarrow$ very high-dimensional learning problem

Simple idea: For a **sequence** of related instances, use

"yesterday's solution as today's prediction"

[DMVW '23] Experiments on image segmentation tasks from frames of a video

(a) Image 1 (b) Image 5 (c) Image 10

Figure 4: Cuts (red) on the first, fifth, and last images from the 120×120 pixels BIRDHOUSE sequence.

Theoretical Interpretation

Is “yesterday’s solution as today’s prediction” theoretically principled?

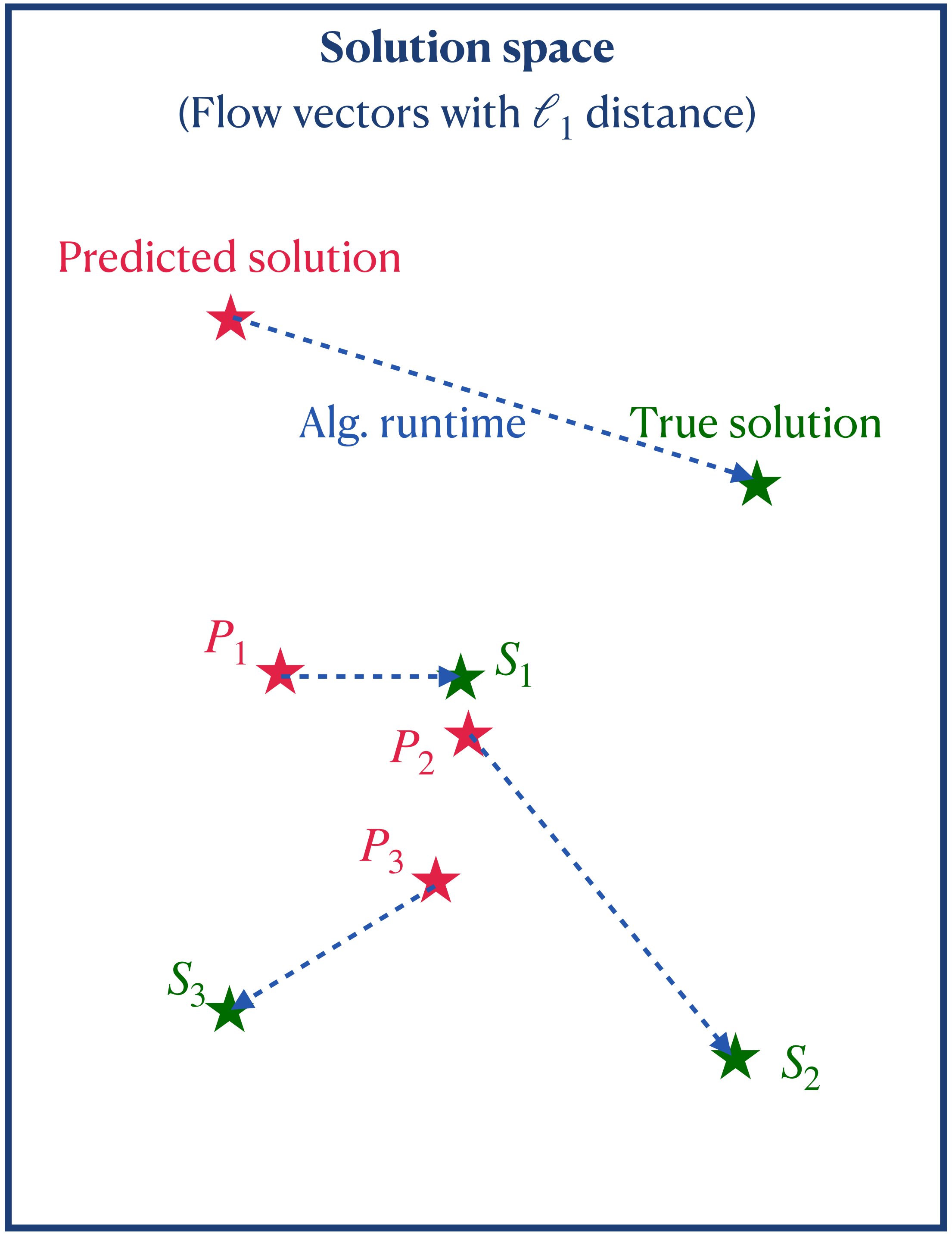
Ford-Fulkerson Warm Start algorithm [DMVW ’23]:

- Flow solution is a vector indexed by edges
- Runtime of algorithm proportional to ℓ_1 **distance** between the predicted flow and the true solution

Meta problem: On each day t:

- Algorithm predicts a point P_t in the solution space
- True solution S_t is revealed
- Algorithm pays $d(P_t, S_t)$

Unlocks online algorithms toolkit!



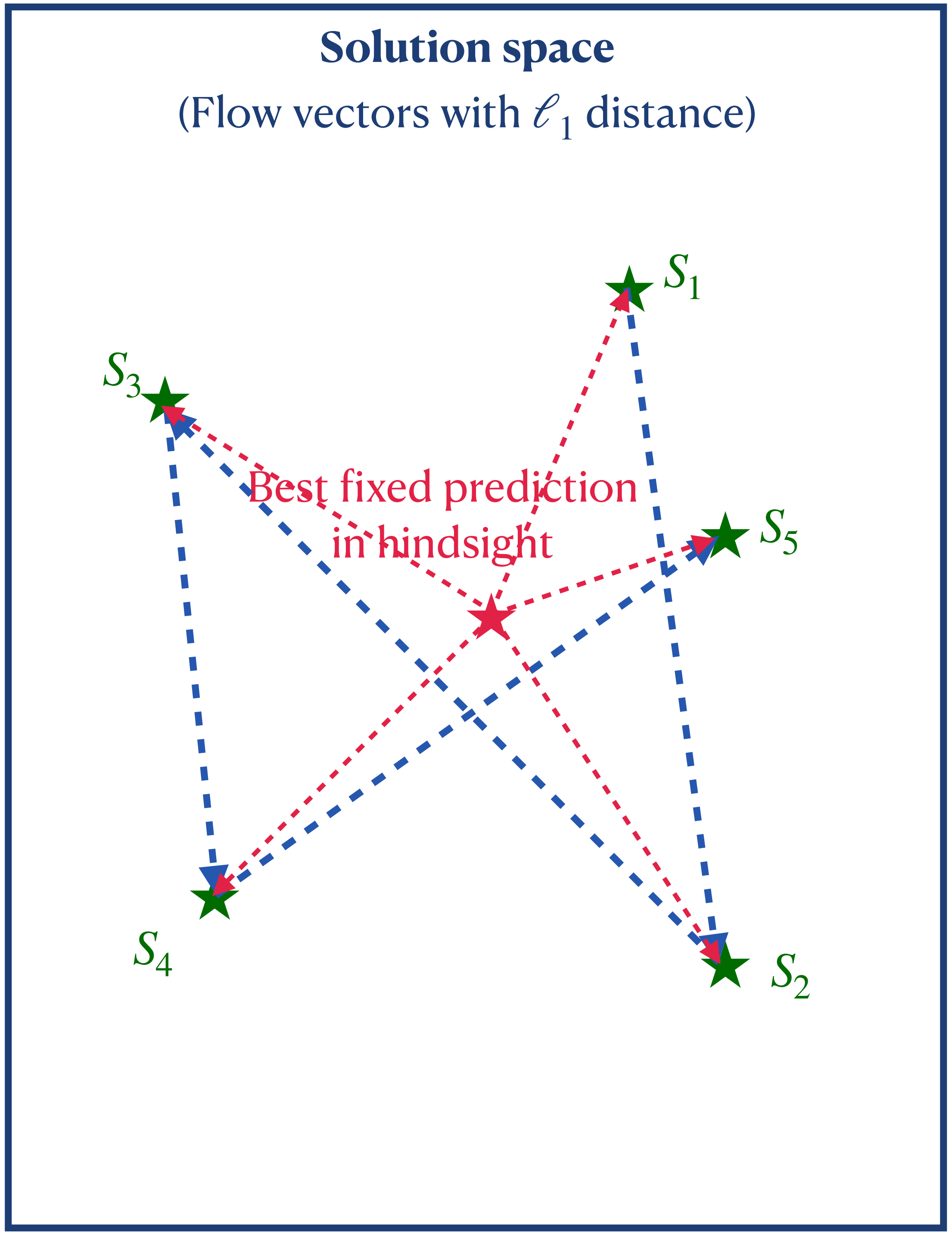
Theoretical Interpretation

Is "yesterday's solution as today's prediction" theoretically principled?

- "Yesterday's solution as today's prediction" is competitive with the best fixed prediction in hindsight! [Khodak Balcan Talwalkar Vassilvitskii '22]
  - Natural baseline from online algorithms literature
  - When solutions in sequence form a tight **cluster**, faster than solving instances one at a time

Can we do even better?

- Yes! Can design algorithms to take advantage of other forms of structure (multiple clusters) and compete against adaptive baselines [Blum Srinivas '25]



It's not just max-flow!

Warm start algorithms (with Learning-Augmented formalism) studied for:

- Ford-Fulkerson [Polak Zub '22][Davies Moseley Vassilvitskii Wang '23]
- Bipartite matching [Dinitz Im Lavastida Moseley Vassilvitskii '21][Chen Silwal Vakilian Zhang '22]
- Max-Flow via Push-Relabel [Davies Vassilvitskii Wang '24]

Can hope to prove for other **local search** algorithms, main new contribution is dealing with infeasibility

In practice, warm starts are used for many optimization problems!

Takeaways: Theoretical formalism of Learning-Augmented algorithms allows us to

- Analyze and compare strategies for solving sequences of related instances
- See new algorithmic opportunities
- **Modularize** (warm start vs. meta problem)

References

(1) Repeated Computations: Sequences of related instances of a problem can be solved faster than one at a time

Learning-augmented warm start algorithms:

- [Davies Moseley Vassilvitskii Wang '23][Polak Zub '22] Max-flow via Ford-Fulkerson
- [Dinitz Im Lavastida Moseley Vassilvitskii '21][Chen Silwal Vakilian Zhang '22] Primal dual bipartite matching
- [Davies Vassilvitskii Wang '24] Max-flow via Push Relabel

Prediction strategies for warm start algorithms:

- [Khodak Balcan Talwalkar Vassilvitskii '22] Compete with best fixed prediction in hindsight
- [Blum Srinivas '25] Take advantage of weaker structure in sequences

Three ideas

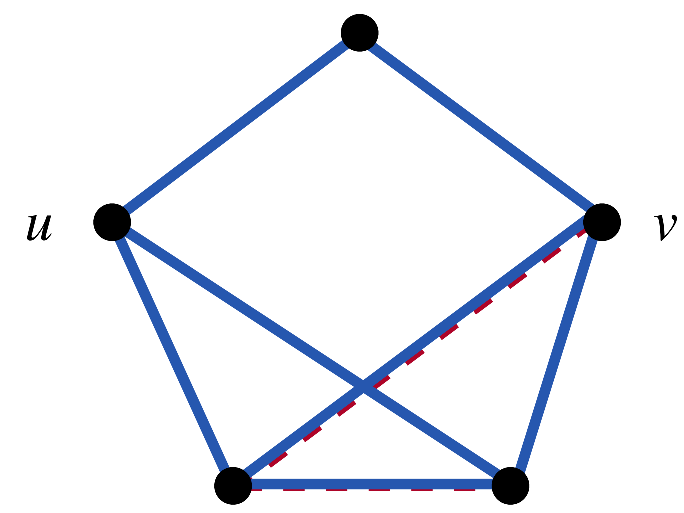
(1) **Repeated Computations:** Sequences of related instances of a problem can be solved faster than one at a time

(2) **Dynamic Algorithms/Data Structures:** Dynamic problems are easier with information about future updates

(3) **Randomized Algorithms:** Randomized algorithms and data structures can be hedged to take advantage of extra information by incorporating a prior

Dynamic Algorithms

Example: 3-vertex connectivity (triconnectivity) in dynamic graphs



Dynamic model:

- Graph on n vertices undergoes edge insertions and deletions (fully-dynamic)
- Maintain data structure to efficiently query whether pairs of vertices are 3-vertex connected
- Minimize **amortized** time per update/query

Challenge:

- Instance only changes a little, but solution can change a lot!
- Design data structures to "reuse" work to solve subproblems

Example: Triconnectivity

Best known fully dynamic algorithm for triconnectivity:

$O(n^{2/3})$ (worst-case) update time [Galil Italiano Sarnak '99]

Baseline: better than recomputing solution from scratch every day

Good: update time sublinear in graph size

Gold standard: polylog(n) update time (exponential improvement over baseline)

Compare to "offline dynamic" setting:

- Sequence of updates and queries are given in one batch
- Algorithm returns answers to all queries at once

Best known **offline dynamic** algorithm for triconnectivity:

polylog(n) (amortized) update time [Peng Sandlund Sleator '17]

with slick divide-and-conquer algorithm!

Knowledge of future updates lets us reuse computation more efficiently!

Gap between fully-dynamic and offline dynamic exists for many problems

Imperfect Information

Fully Dynamic Model	Predicted-Updates Model	Offline Dynamic Model
No information about future updates	Imperfect information about future updates	Perfect information about future updates

Fully dynamic is **pessimistic**, but offline dynamic is too **optimistic**…

Can we instead use an **imperfect prediction** of future updates?

Learning-Augmented approach [Liu Srinivas '24]**:**

- Ask for a (potentially infeasible) prediction of the input sequence
- Perform fully dynamic updates on the true sequence that may or may not be close to the predicted sequence

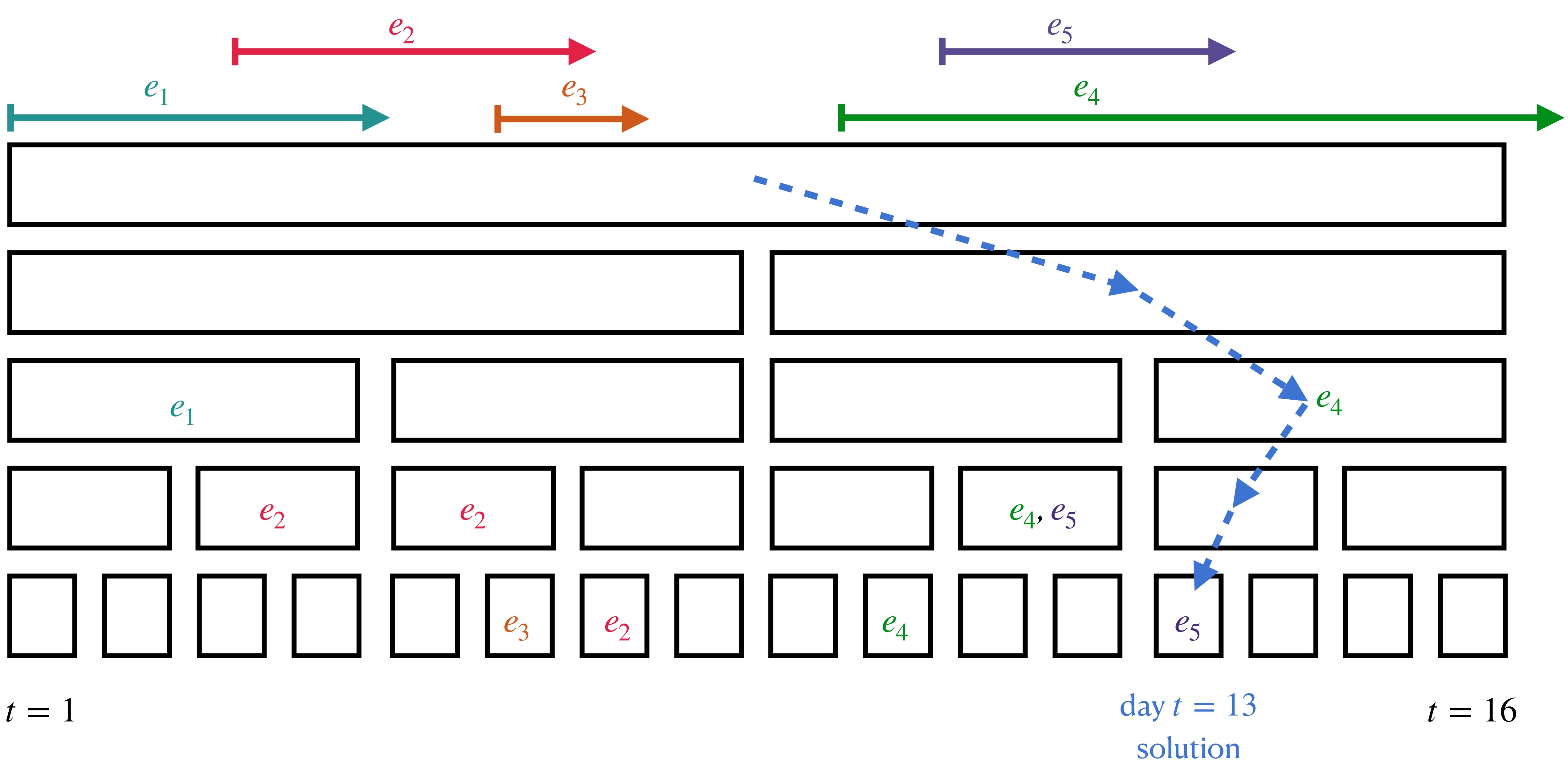
Goal:

- **Consistency:** when predictions are good, recover offline dynamic performance
- **Robustness:** always do at least as well as fully dynamic performance
- **Graceful degradation:** smooth tradeoff in error of prediction

Under the Hood of Offline Dynamic Algorithms

Divide-and-conquer approach:

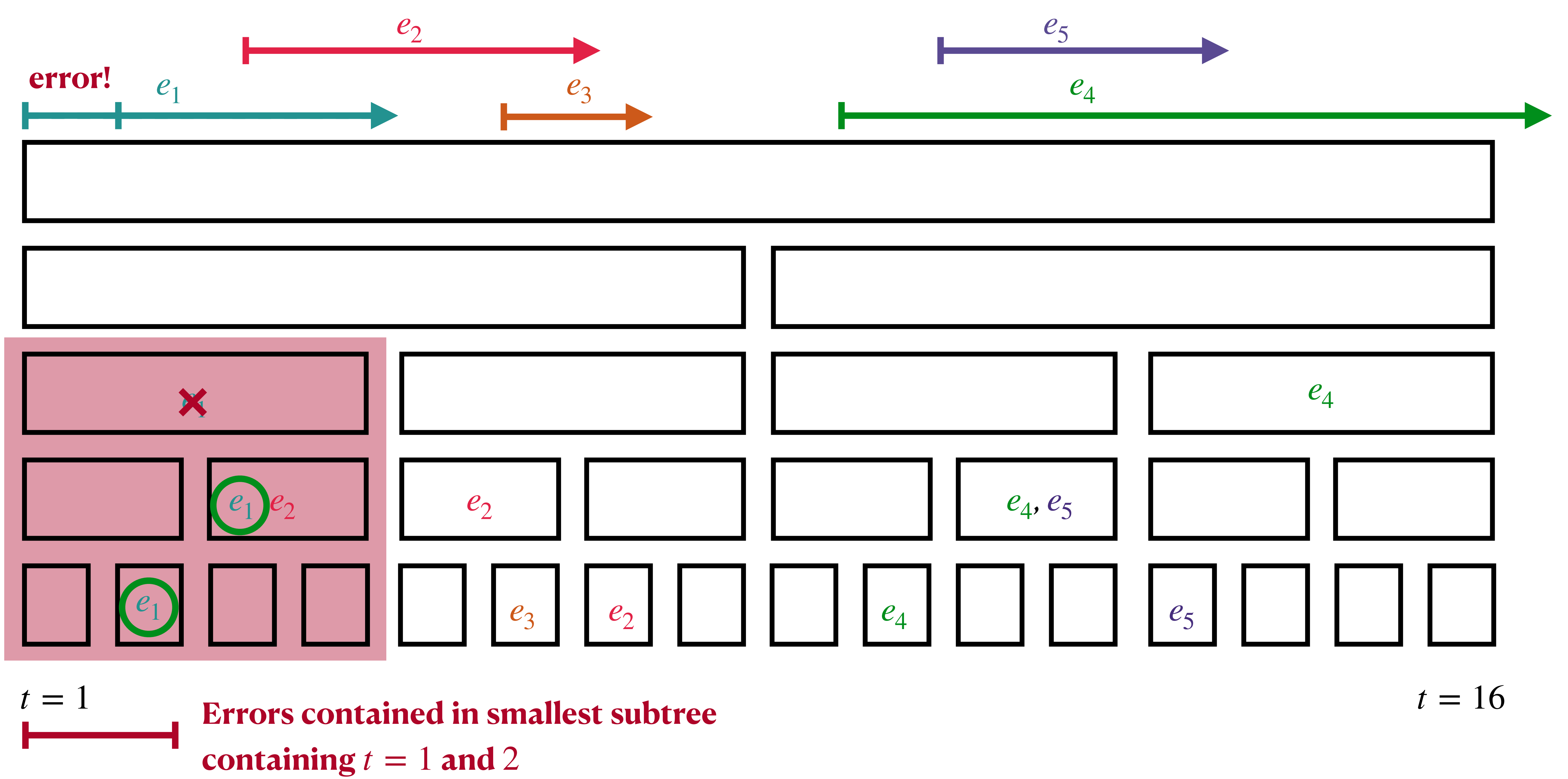
- Associate each edge with "windows" for which it is **permanent**
- Each window is associated with at most twice as many edges as its size
- Process edges in batches from root to leaf, using **sparsifier** for problem



What if the prediction is wrong?

Predicted e_1 insertion on day 1, but happens on day 2 instead.

Observation: errors are localized!

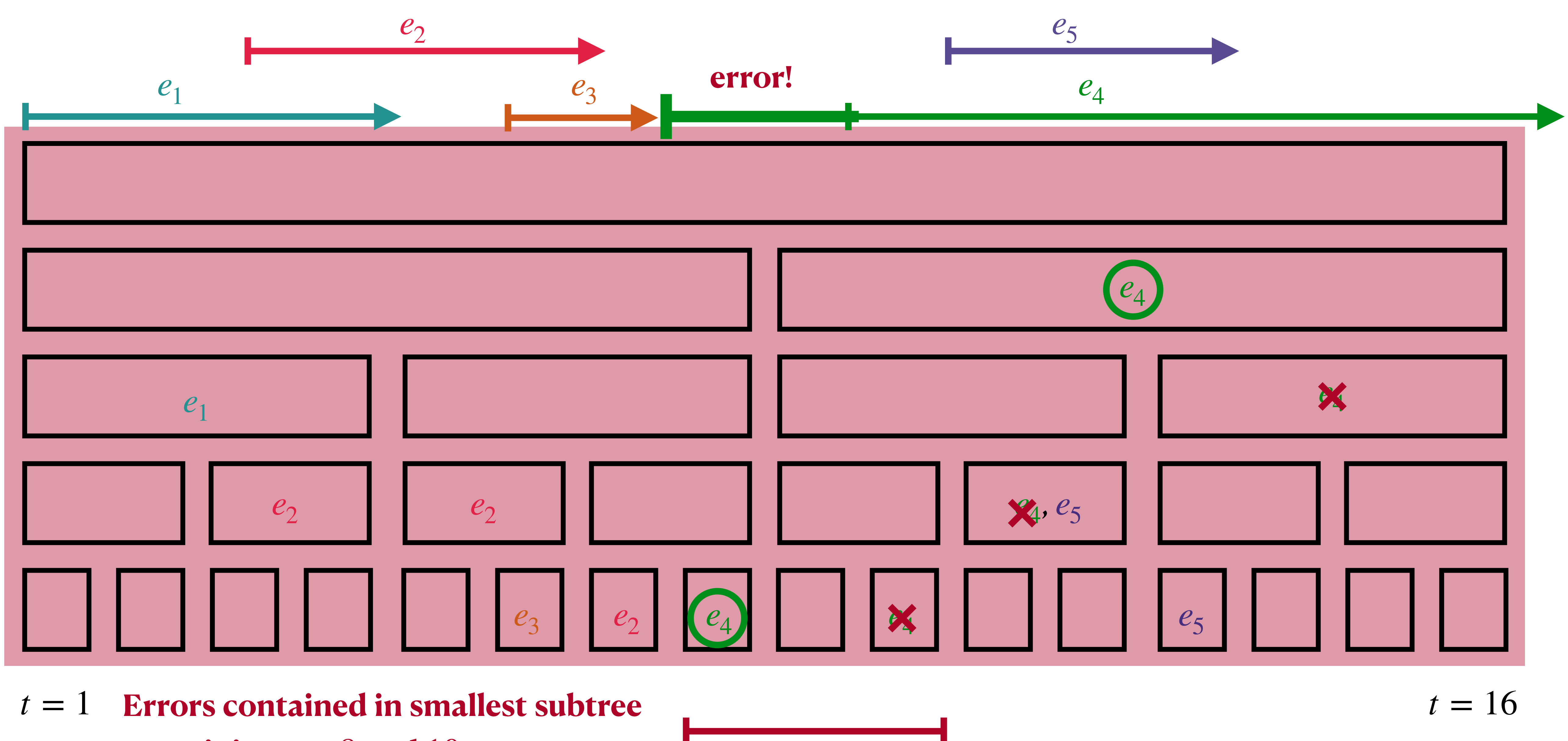


What if the prediction is wrong? (Part 2)

Predicted e_4 insertion on day 10, but happens on day 8 instead.

Observation: errors are localized!

...but not very localized, small errors can trigger large recomputations

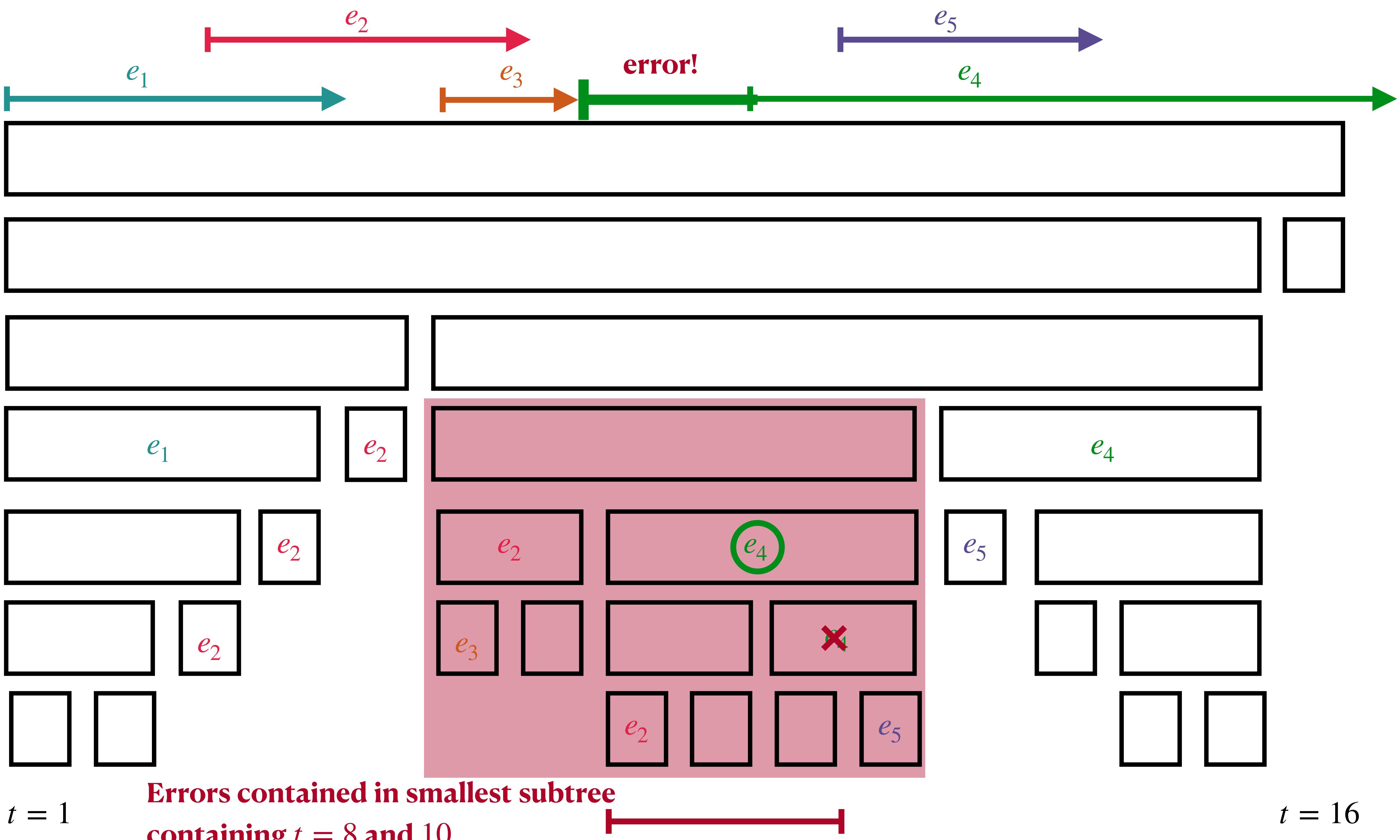


Errors contained in smallest subtree containing $t = 8$ and 10

Workaround

Use **random** divide-and-conquer tree

$\Longrightarrow$ size of error subtree proportional to size of error



Guarantees

Use this (and other) ideas to get an

fully-dynamic to offline reduction with predictions

Informal Theorem [Liu Srinivas '24]

Given a predicted sequence of events in advance, we can solve dynamic triconnectivity* in total time over T updates

$$\widetilde{O}(\min\{T+\eta,\ T \cdot n^{2/3}\}),$$

Where η is the ℓ_1 error of the prediction (sum over events of the absolute prediction error in time).

*Can do reduction for many problems with similar offline dynamic algorithms!

Takeaway: asymptotically lose **nothing** by taking advantage of predictions!

Big Picture

Many recent works use similar and different techniques to use predictions to get improved dynamic algorithms:

- [van den Brand Forster Nazari Polak '23] Other graph and matrix problems
- [Agarwal Balkanski '24] Dynamic submodular maximization
- [McCauley Moseley Niaparast Singh '24] Incremental Topological Ordering
- [McCauley Moseley Niaparast Niaparast Singh '25] Approximate SSSP

Spiritually similar but **incomparable** to warm starts

Dynamic Algorithms

- Structured input sequence (edge insertion/deletions)
- Small changes in input, potentially large changes in solution
- Can achieve sublinear update time

Warm Starts

- Unstructured input sequence
- Potentially large changes in input, take advantage of similarities in solutions
- At bare minimum, must read input on each day $\Rightarrow$ no sublinear update time

References

(2) Dynamic Algorithms/Data Structures: Dynamic problems are easier with information about future updates

Dynamic algorithms with predictions:

- [Liu Srinivas '24] Offline to online transformations
- [van den Brand Forster Nazari Polak '23] Graph and matrix problems
- [Agarwal Balkanski '24] Submodular maximization
- [McCauley Moseley Niaparast Singh '24] Incremental Topological Ordering
- [McCauley Moseley Niaparast Niaparast Singh '25] Incremental Approximate Single Source Shortest Paths

Related ideas:

- [Peng Sandlund Sleator '17] Designing offline-dynamic algorithms
- [Peng Rubinstein '22] Fully-dynamic to incremental reduction with known deletion error

Three ideas

(1) **Repeated Computations:** Sequences of related instances of a problem can be solved faster than one at a time

(2) **Dynamic Algorithms/Data Structures:** Dynamic problems are easier with information about future updates

(3) **Randomized Algorithms:** Randomized algorithms and data structures can be hedged to take advantage of extra information by incorporating a prior

Example: Treaps

Self-balancing tree data structure

Goal: Support element insertions, lookups, and deletions in $O(\log(n))$ time, n is # of elements in the tree at any given time

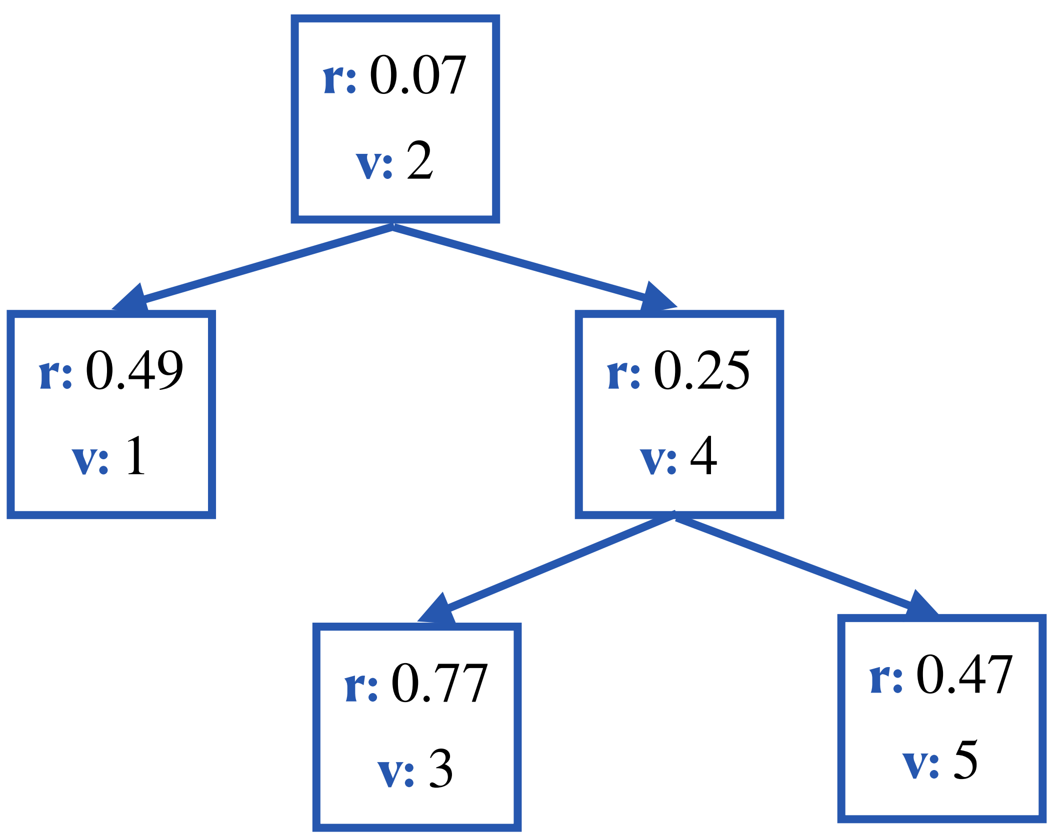
Rank $\in [0,1]$
unif. random!

Value $\in \mathbb{R}$

Treap = **Tree** + **Heap** [Aragon Seidel '89]

Min-heap w.r.t. rank: parent always has lower rank than children

Search tree w.r.t. value: left subtree contains only elements of smaller value, right subtree contains only elements of larger value



- $\log(n)$ depth with high probability (quicksort type analysis)
- Coupling between ranks and random tree allows dynamic updates

Room for improvement?

- $\log(n)$ update/query time optimal for a **worst-case** sequence
- For a given update sequence, could minimize lookup time by storing frequently accessed elements closer to the root
    - E.g., splay trees rebalance with accesses to do this dynamically
    - Splay trees have conjectured dynamic optimality (within a constant factor of best self-balancing tree in hindsight for every access sequence)
    - Dynamic optimality: long standing open problem
- What if you had **predictions** about the access sequence in advance? Can you get a **provable** optimality guarantee, without sacrificing worst-case update time?

Incorporate the prior

Idea: incorporate frequency information into the random rank! [Lin Luo Woodruff '23]

For an element x, let $p(x)$ be the **predicted** frequency with which x will be accessed

Rank $\in [0,1]$

Value $\in \mathbb{R}$

Standard Treap: Choose rank $\sim U[0,1]$

Learning-Augmented Treap: $\text{rank}(x) \sim -\lfloor \log \log(p(x)) \rfloor + U[0,1]$

Hedge frequently accessed items to the top!

Provable guarantees [Chen Cao Stepin Chen '25]**:**

- Static optimality, when predictions are perfect
- Worst-case guarantees from Treap, with tradeoff
- (Meets provable benchmarks for splay trees, when frequencies are estimated on-the-fly)

Other Applications

Takeaway: Hedging is a natural idea. Learning-Augmented algorithms gives us the language to reason about it!

Algorithmic challenge: How do you design the **right** distribution?

- Caching [Lykouris Vassilvitskii '18]
  - Use frequency information to inform randomized cache eviction decisions
- Ski-Rental [Kumar Purohit Svitkina '18] …
  - Use information about the future to make decisions in the present for online algorithms
- Min-cut [Moseley Niaparast Singh '25]
  - Use predictions about the min-cut to inform what vertices to contract in Karger-Stein

References

(3) Randomized Algorithms: Randomized algorithms and data structures can be hedged to take advantage of extra information by incorporating a prior

Learning-Augmented Search Trees:

- [Lin Luo Woodruff '23] Learning-Augmented Treaps with guarantees for stochastic accesses
- [Chen Cao Stepin Chen '25] Guarantees for general access sequences

An incomplete list of other places to see this idea:

- [Lykouris Vassilvitskii '18] Caching
- [Kumar Purohit Svitkina '18], and follow up work, Ski-rental (proof of concept for online algorithms in general)
- [Moseley Niaparast Singh '25] Min-cut via Karger-Stein

Conclusion

(1) **Repeated Computations:** Sequences of related instances of a problem can be solved faster than one at a time

(2) **Dynamic Algorithms/Data Structures:** Dynamic problems are easier with information about future updates

(3) **Randomized Data Structures:** Randomized data structures can be hedged to take advantage of extra information by incorporating a prior

...and so much more!

Takeaway: Learning-Augmented algorithms gives us tools and frameworks to reason about interesting and practical new problems

Exciting time to join the field!

- Lots of great work over the last 5-10 years laying the groundwork
- Seeing the payoff in new results that take advantage of a new ways of algorithmic thinking
- Join us!

THANKS!

vaidehi@u.northwestern.edu