EUR/SGD Option Pricing and Volatility Analysis

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1. Introduction

This report presents a comprehensive analysis of the EUR/SGD exchange rate, focusing on volatility estimation and European option pricing. The study utilizes both historical and advanced econometric models to estimate volatility, and applies three distinct methods—Black-Scholes, Cox-Ross-Rubinstein (CRR) binomial tree, and Monte Carlo simulation—for pricing European-style options. The primary objectives are to:

- Analyze the statistical properties of EUR/SGD log-returns.
- Evaluate the suitability of model assumptions, particularly normality and independence.
- Compare option prices derived from different quantitative models.
- Assess the practical implications of volatility modeling for derivative pricing.

2. Data Collection and Preparation

Historical daily exchange rate data for EUR/SGD was sourced from Investing.com using the `yfinance` API. The dataset was cleaned, sorted by date, and validated. It includes: date, opening price, closing price, lowest price, percentage change, and log-returns.

Data Processing Steps

- Downloaded daily EUR/SGD rates.
- Sorted data chronologically and removed missing values.

- Calculated log-returns as:
$$\ln\left(\frac{P_t}{P_{t-1}}\right)$$
where P_t is the closing price at time t.

3. Price and Log-Return Analysis

EUR/SGD Price Trend

- Fluctuations are typical of developed currency pairs.
- Sharp movements coincide with macroeconomic events.

Log-Returns: Calculation and Interpretation

- Preferred due to time additivity and continuous compounding.
- Exhibit volatility clustering.
- Presence of outliers suggests non-normal behavior.

4. Normality Testing of Log-Returns

Visual Analysis

- **Histogram:** Indicates leptokurtosis.
- **QQ Plot:** Deviations from normality, especially in tails.

Statistical Tests

- **Jarque-Bera Test:** Rejects normality.
- **Kolmogorov-Smirnov Test:** Significant departure from normality.
- **Anderson-Darling Test:** Confirms fat tails.

Summary: All tests reject normality, indicating limitations of Gaussian-based models.

5. Volatility Estimation

Volatility represents the uncertainty of asset price movements.

Historical Volatility

- Standard deviation of daily log-returns, annualized as:

$$\sigma_{\text{annual}} = \sigma_{\text{daily}} \times \sqrt{252}$$

- Result: ≈ 0.046

GARCH(1,1) Volatility Forecast

The GARCH (Generalized Autoregressive Conditional Heteroskedasticity) model is a time-series technique that accounts for volatility clustering in financial returns—where high-volatility periods are followed by high volatility, and low by low. The GARCH(1,1) model is defined as:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Where:

- σ_t^2: Conditional variance at time t
- ϵ_{t-1}^2: Lagged squared residuals (shock)
- σ_{t-1}^2: Lagged variance
- ω, α, β: Model parameters

This model captures time-varying volatility by modeling the current variance as a function of past squared shocks and past variances.

- The GARCH(1,1) model was fitted to the log-return series of EUR/SGD.
- Daily volatility was forecasted up to May 30, 2025.
- The forecast adapts to recent volatility patterns better than historical rolling standard deviation.
- The model's responsiveness to market shocks makes it suitable for option pricing.
- Visualization of GARCH vs. historical volatility shows more accurate and timely reflection of recent risk conditions.

The volatility derived by this method is 0.050 which is approximately equal to our estimated volatility (0.046).

6. Risk-Free Rate Calculation

Currency option pricing uses the difference in bond yields:

$$r = r_{\text{EUR}} - r_{\text{SGD}} = 0.036 - 0.035 = 0.001 = 0.1\%$$

This is used as the effective risk-free rate in all pricing models.

7. Independence of Log-Returns

- **ACF:** Minor lags show some dependence.
- **Ljung-Box Test:** Rejects full independence.

Implication: Weak autocorrelation challenges assumptions of market efficiency.

8. Option Pricing: Methodology and Results

ATM European call and put options priced using consistent parameters across models.

Black-Scholes Model

The Black-Scholes model assumes that the price of the underlying asset follows a geometric Brownian motion with constant drift and volatility. It provides a closed-form solution for the pricing of European call and put options under the assumption of no arbitrage, no dividends, and constant interest rate.

Call Option Price:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

Put Option Price:

$$P = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$

Where:

$$d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$$

This model assumes log-normal prices, constant volatility, frictionless markets, and continuous trading.

CRR Binomial Tree Model

The Cox-Ross-Rubinstein (CRR) binomial model discretizes the life of the option into n intervals and models the underlying asset's price evolution as an up or down move at each step. The model uses risk-neutral valuation and backward induction to compute the option price.

Up/Down Factors:

$$u = e^{\sigma \Delta t}, \quad d = \frac{1}{u}$$

Risk-Neutral Probability:

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

Option Value via Backward Induction:

$$V_{i,j} = e^{-r\Delta t} \left[pV_{i+1,j+1} + (1-p)V_{i+1,j} \right]$$

For n time steps: $\Delta t = \frac{T}{n}$

Monte Carlo Simulation

Monte Carlo methods simulate numerous possible future asset price paths based on stochastic differential equations, then estimate the option's value by averaging the discounted payoffs. It is particularly useful for pricing options where closed-form solutions are not available.

Simulated Stock Price Paths:

$$S_T^{(i)} = S_0 \exp\left[\left(r - \frac{\sigma^2}{2}\right)T + \sigma\sqrt{T}Z^{(i)}\right]$$

Call Option Price:

$$C = e^{-rT} \cdot \frac{1}{M}\sum_{i=1}^{M} \max(S_T^{(i)} - K, 0)$$

Where $Z^{(i)} \sim \mathcal{N}(0,1)$, and M is the number of simulations.

Results: All models yield consistent prices for ATM European options.

Table 1: European Option Prices for EUR/SGD (ATM)

Method	Call Price	Put Price
Black-Scholes	0.011783	0.011635
CRR Binomial	0.011753	0.011606
Monte Carlo	0.011753	0.011566

9. Discussion and Insights

- Non-normal returns suggest traditional models are limited.
- GARCH improves volatility estimation accuracy.
- Model comparison favors flexibility of simulation-based approaches.
- Mild autocorrelation implies potential for predictability.

10. Conclusion

- Log-returns are non-normal and autocorrelated.
- GARCH(1,1) enhances pricing and risk assessment.
- All pricing models are valid, but differ in assumptions and complexity.
- Advanced modeling ensures better pricing and hedging decisions.

References

1. Hull, J. C. (2018). *Options, Futures, and Other Derivatives* (10th ed.). Pearson Education.

2. Yahoo Finance – Source of historical EUR/SGD exchange rate data.

3. Python libraries: `numpy`, `pandas`, `matplotlib`, `arch`, `scipy`, `statsmodels`, `yfinance`.