TOP ONE PERCENT GMAT 99TH PERCENTILE CLUB

Quant Concepts Session 1 – Inequalities

Key Concepts:

1. If a > b, then am > bm for m > 0 and am < bm for m < 0, that is, when we multiply both sides of inequality by a negative quantity, the sign of inequality is reversed. Do not cancel anything from both sides of an inequality unless you are sure that the cancelled quantity is positive, so ax > ay does not necessarily mean x > y, etc. Also, don't cross multiply unless you are sure of the sign of the multipliers.
2. The concept of number line is very useful in checking inequalities. The common values to check are x = 0, 1, –1, >1 (preferred value = 2), between 0 and 1 (preferred value = 1/2), between – 1 and 0 (preferred value = –1/2), and less than –1 (preferred value = –2). So in short, there are 7 points: –2, –1, –1/2, 0, 1/2, 1, 2.
3. If (x – a) (x – b) < 0, then x lies between a and b. OR a < x < b. If (x – a) (x – b) > 0, then x lies outside a and b. OR x < a, x > b.
4. If $x^2 > x$, then either x > 1 or x is negative (x < 0).
5. If $x^2 < x$, then x lies between 0 and 1. (0 < x < 1).
6. If $x^2 = x$, then x = 0 or x = 1.
7. If $x^3 > x$, then either x > 1 or x is between –1 and 0 (either x > 1 or –1 < x < 0).
8. If $x^3 < x$, then either x lies between 0 and 1 or x is less than –1. (Either 0 < x < 1 or x < –1)
9. If $x^3 = x$, then x = 0 or x = 1 or x = –1.
10. If x > y, it is not necessary that $x^2 > y^2$ or etc. So, even powers can't be predicted.
11. If x > y, it is necessarily true that $x^3 > y^3$ or etc. So, odd powers and roots don't change sign.
12. ab > 0 means a/b > 0 and vice versa. The two are of the same sign.
13. ab < 0 means a/b < 0 and vice versa. The two are of the opposite sign.
14. If X is positive, then
    (1) (a + X) / (b + X) > a/b if a < b (2) (a + X) / (b + X) < a/b if a > b

Problems:

1. If x is positive which of the following could be correct ordering of 1/x, 2x, x^2?
   I. $x^2 < 2x < 1/x$ II. $x^2 < 1/x < 2x$ III. $2x < x^2 < 1/x$
   A. None B. I Only C. III Only D. I and II only E. I, II & III

2. If $x > y^2 > z^4$, which of the following statements could be true?
   I. x > y > z II. z > y > x III. x > z > y
   A. I only B. I and II only C. I and III only D. II and III only E. I, II, and III

3. Is M + Z > 0 (1) M – 3Z > 0 (2) 4Z – M > 0

4. If k is not equal to 0, 1, or –1, is 1/k > 0? (1) 1 / (k – 1) > 0 (2) 1 / (k + 1) > 0

5. The numbers x and y are not integers. The value of x is closest to which integer?
   (1) 4 is the integer that is closest to x + y (2) 1 is the integer that is closest to x – y

6. Are x and y both positive (1) 2x – 2y = 1 (2) x/y >1

7. Is $1 / p > r/(r^2 + 2)$ (1) $p = r$ (2) $r > 0$

8. Is $X + Y < 1$? (1) $X < 8 / 9$ (2) $Y < 1 / 8$

9. Is $x – y + 1$ greater than $x + y – 1$? (1) $x > 0$ (2) $y < 0$

10. Is z the median of any 3 positive integers x, y and z? (1) $x < y + z$ (2) $y = z$

11. On the number line, the distance between x and y is greater than the distance between x and z. Does z lie between x and y on the number line? (1) $xyz < 0$ (2) $xy < 0$

12. If $mv < pv < 0$, is $v > 0$? (1) $m < p$ (2) $m < 0$

13. If n is a nonzero integer, is $x^n < 1$? (1) $x > 1$ (2) $n > 0$

14. If x is an integer, is 3^x less than 500? (1) $4^{x-1} < 4^x – 120$ (2) $x^2 = 36$

15. Is $x > 10^{10}$? (1) $x > 2^{34}$ (2) $x = 2^{35}$

16. Is $XY>0$? 1). $X–Y>–2$ 2). $X–2Y<–6$

17. If x and y are integers and xy does not equal 0, is $xy < 0$? (1) $y = x^4 – x^3$ (2) $–12y^2 – y^2x + x^2y^2 > 0$

18. If $r + s > 2t$, is $r > t$? (1) $t > s$ (2) $r > s$

19. If $p < q$ and $p < r$, is $(p)(q)(r) < p$? (1) $pq < 0$ (2) $pr < 0$

20. Is $5^n < 0.04$? (1) $(1/5)^n > 25$ (2) $n^3 < n^2$

21. Is $p^2q > pq^2$? (1) $pq < 0$ $p < 0$

22. Is $m > n$? (1) $n – m + 2 > 0$ (2) $n – m – 2 > 0$

23. Is $3^p > 2^q$? (1) $q = 2p$ (2) $q > 0$

24. Is mp greater than m? (1) $m > p > 0$ (2) p is less than 1

25. Is $2X–3Y< X^2$? 1). $2X–3Y=–2$ 2). $X>2$ and $Y>0$