Quant Concepts: Session 3: Numbers + General Algebra

Solutions:

1. If x, y, and z are integers and xy + z is an odd integer, is x an even integer?
(1) xy + xz is an even integer (2) y + xz is an odd integer

Main condition: xy + z is odd can be satisfied in the following ways

x	y	z
E	E	O
E	O	O
O	E	O
O	O	E

(1) x (y + z) is even

x	y	z	x (y + z) = even
E	E	O	Yes
E	O	O	Yes
O	E	O	No
O	O	E	No

Whenever x (y + z) is even, x is even … sufficient.

(2) y + xz is odd

x	y	z	y + xz = odd
E	E	O	No
E	O	O	Yes
O	E	O	Yes
O	O	E	Yes

Whenever y + xz is odd, x is even or odd … NS

Ans. A

2. If n is an integer between 10 and 99 is n < 80?
(1) The sum of the two digits of n is a prime number.
(2) Each of the two digits of n is a prime number.

(1) Let's take, say, a sum of 13, which is prime, and somewhat big.
This could be 9+4 —> 94 (more than 80), or 4+9 —> 49 (less than 80). Not sufficient.

(2) The maximum can be 77 … sufficient.

Ans. B

3. For all positive integers m, [m] = 3m when m is odd and [m] = ½ m when m is even. Which of the following is equivalent to [9] × [6]?
[81] [54] [36] [27] [18]

The answer is D) [27]

since 9 is odd, [9] =(3)(9) = 27
and 6 is even, [6] = (1/2)(6) = 3
27 x 3 = 81 So, the correct answer is (D) [27] = 3 x 27 = 81

Problems:

1. The integers m and p are such that 2 < m < p, and m is not a factor of p. If r is the remainder when p is divided by m, is r > 1?
(1) the greatest common factor of m and p is 2
(2) the least common multiple of m and p is 30

statement (1)

let's just PICK A WHOLE BUNCH OF NUMBERS WHOSE GCF IS 2 and watch what happens. let's try to make the numbers diverse.
say,
4 and 6
6 and 8
8 and 10
10 and 12
...
4 and 10
6 and 14
6 and 16
8 and 18
8 and 22
...
in all nine of these examples, the remainders are greater than 1. in fact, there is an obvious pattern, which is that **they're all even**, since the numbers in question must be even. **in statement 1, both m and p are even. therefore, the remainder is even, so it's greater than 1.**

done.

sufficient.

--

statement (2)
just pick various numbers whose lcm is 30.
notice the numbers selected above:
5 and 6 --> remainder = 1
10 and 15 --> remainder = 5 > 1
insufficient.

ans (a)

2. If the integer n is greater than 1, is n equal to 2?
(1) n has exactly two positive factors
(2) The difference between any two distinct positive factors is odd.

Answer: B

(1) this is a disguised way of saying 'n is prime' … n could be 2 or 3 or 5 or 7 … therefore, insufficient. We can't say for sure that n = 2.

(2) **The most important word here is ANY.**

Imagine n = 3 … factors are 1, 3. The difference will be even
Imagine n = 5 … factors are 1, 5. The difference will be even
Imagine n = 9 … factors are 1, 3, 9. The difference of 1 and 3, 1 and 9, and 3 and 9, will all be even **Notice the word ANY)**.

So for all odd numbers, (2) can't be satisfied … as each number will have 1 and itself as a factor … so the difference between the number (odd) and 1 will always be even.

Imagine n = 4 … factors = 1, 2, 4 … difference of 2 and 4 = even … OUT
Imagine n = 12 … factors = 1, 2, 3, 4, 6, 12 … differences (2, 4) (2, 6), (2, 12), (4, 6), (4, 12), (6, 12) etc… are all even … OUT.
So all even numbers > 2 cannot satisfy (2) … each number will have 2 and itself (even) as a factor and the difference of 2 and an even number will always be even.

If n = 2, the factors are 1 and 2 and the difference is odd.

By the above analysis, 2 is the only such number. Ans. B

3. The function f is defined for all positive integers n by the following rule: f(n) is the number of positive integers each of which is less than n and also has no positive factor in common with n other than 1. If p is a prime number then f(p) = ?
p – 1 p – 2 (p + 1) / 2 (p – 1) / 2 2

Pick a prime number for p. Let's say p=5.

The positive integers less than 5 are 4, 3, 2, and 1.

5 and 4 share only 1 as a factor
5 and 3 share only 1 as a factor
5 and 2 share only 1 as a factor
5 and 1 share only 1 as a factor

There are four positive integers, therefore, that are both less than 5 and share only 1 as a factor.

Ans. p – 1.

4. For every positive even integer n, the function h(n) is defined to be the product of all the even integers from 2 to n, inclusive. If p is the smallest prime factor of h(100) + 1, then p is
   A. between 2 and 10 B. between 10 and 20 C. between 20 and 30
   D. between 30 and 40 E. greater than 40

   The basic logic behind this question: If k and n are both integers greater than 1 and if k is a factor of n, k cannot be a factor of (n + 1).

   Let's first consider the prime factors of h (100). According to the given function,
   h (100) = 2*4*6*8*...*100
   By factoring a 2 from each term of our function, h(100) can be rewritten as
   h (100) = 2^{50} (1*2*3*...*50).

   2, 3, 4 … 50 are factors of h (100) … so 2, 3, 4 … 50 can't be factors of h (100) + 1. So the smallest factor will be more than 50 (which is always more than 40). Ans. E

5. Is the integer n odd?
   (1) n is divisible by 3 (2) 2n is divisible by twice as many positive integers as n

   (1) n could be 6 or 9. NS
   (2) 2n has twice as many factors as n

   n = 1 … one factor; 2n = 2 … two factors

   n = 2 … two factors; 2n = 4 … three factors

   n = 3 … two factors; 2n = 6 … four factors

   n = 4 … three factors; 2n = 8 … four factors

   n = 9 … three factors; 2n = 18 … six factors

   … we can see that for each of the odd numbers (n), the number of factors of 2n is exactly double but for an even number, it is never the case. So n must be odd … Sufficient.
   Ans. B

   Let's say a number has "n" different factors.
   When you multiply this number by 2, you POTENTIALLY create "n" MORE factors - by doubling each factor.
   HOWEVER,
   the only way that ALL of these factors can be NEW (i.e., not already listed in the original *n* factors) is if they are ALL ODD.
   If there are ANY even factors to start with, then those factors will be repeated in the original list. Therefore, **if the number is even, then the number of factors will be less than doubled** because of the repeat factors.

   Thus if statement (2) is true, then the number must be odd.
   Ans. B

6. How many different prime numbers are factors of the positive integer n?
   (1) four different prime numbers are factors of 2n
   (2) four different prime numbers are factors of n^2.

The minimum value of a product of four different prime integers = 2 * 3 * 5 * 7 = 210
(1) if 2n = 210 (4 prime factors), n = 105 (3 prime factors)
if 2n = 420 (4 prime factors), n = 210 (4 prime factors) … so we are not sure … n may have 3 or 4 prime factors.
NS
(2)If n is a positive integer, n and n^2 will have an exact number of factors.
Imagine … n^2 has four factors 2, 3, 5, 7 … then the minimum value of n^2 will be $2^2 * 3^2 * 5^2 * 7^2$ … when we take the square root, we must have even powers of each of the factors.
So n will have the same number of prime factors as n^2.
Ans. B

7. Does the integer k have a factor p such that 1 < p < k?
(1) k > 4! (2) 13! + 2 ≤ k ≤ 13! + 13.

The question is whether k is prime or not?

HOW?

the question is asking whether *k* has a factor that is *greater than 1, but less than itself*.
if you're good at these number property rephrasing, then you can realize that this question is equivalent to "is k non–prime?"

(1) k > 24
if k = 25, it is not prime
if k = 29, it is prime
NS

(2) The possible values of k are each of the positive integers from 13! + 2 to 13! + 13.

key realization:
every one of the numbers 2, 3, 4, 5, ..., 12, 13 is a factor of 13!.

13! + 2 can be written as 2k + 2 = multiple of 2 … not prime
13! + 3 can be written as 3k + 3 = multiple of 3 … not prime
.
.
.
13! + 13 can be written as 13k + 13 = multiple of 13 … not prime

So each of the values of k is a non prime number. Sufficient.
Ans. B

8. The positive integer k has exactly two positive prime factors, 3 and 7. If K has a total of 6 positive factors, including 1 and k, what is the value of K?
(1) 3^2 is a factor of k (2) 7^2 is NOT a factor of k

We know that the number of divisors (factors) of a given number N (including one and the number itself) where $N = a^m \times b^n \times c^p$ ..... where a, b, c are prime numbers is given by (1 + m) (1 + n) (1 + p) ......

Here $k = 3^m * 7^n$ so the number of factors = (1 + m) (1 + n) = 6
6 can be written as 1 × 6 or 2 × 3 or 3 × 2 or 6 × 1.
If 1 + m = 1, m = 0 … means we will not have any power of 3 … so this is unacceptable.
If 1 + n = 1, n = 0 … means we will not have any power of 7 … so this is unacceptable.

If $1 + m = 2$ and $1 + n = 3$, $m = 1$, $n = 2$ and the number will become $3^1 \times 7^2$
If $1 + m = 3$ and $1 + n = 2$, $m = 2$, $n = 1$ and the number will become $3^2 \times 7^1$
So the only possible values of k can be $3^1 \times 7^2$ or $3^2 \times 7^1$

(1) says 3^2 is a factor of k … so $k = 3^2 \times 7^1$
(2) says 7^2 is a not factor of k … so $k = 3^2 \times 7^1$

Each of the statements gives the value of k … Ans. D

Problems:

1. If t is a positive integer and r is the remainder when $t^2 + 5t + 6$ is divided by 7, what is the value of r?
   (1) When t is divided by 7, the remainder is 6 (2) when t^2 is divided by 7, the remainder is 1

   (1) $t = 7k + 6$

   $(t^2 + 5t + 6) / 7 = [(7k + 6)^2 + 5 (7k + 6) + 6] / 7 = (49k^2 + 84k + 35k + 72) / 7$ … the first three terms will give 0 remainders as all of them are multiples of 7 … so the remainder = the remainder when 72 is divided by 7 = 2. Sufficient

   (2) t = 1 and t = 6 satisfy the conditions.
   If t = 1, then 1 + 5 + 6 = 12, which yields a remainder of 5 upon division by 7.
   If t = 6, then 36 + 30 + 6 = 72, which yields a remainder of 2 upon division by 7.
   Insufficient …
   Ans. A

2. If p is a positive odd integer, what is the remainder when p is divided by 4?
   (1) When p is divided by 8, the remainder is 5.
   (2) p is the sum of the squares of two positive integers.

   (1) $p = 8k + 5$ … $8k + 5$ divided by 4 will give a remainder = 1 … 8k/4 will give 0 remainder and 5/4 will give a remainder of 1. Sufficient.
   (2) $p = x^2 + y^2$ … p is odd … this means that one of the numbers is even and the other number is odd.
   Let x be even = 2a and y be odd = 2b + 1
   $x^2 + y^2 = 4a^2 + 4b^2 + 4b + 1$ … which when divided by 4 will give a remainder of 1. Sufficient.
   Ans. D

3. If n is a positive integer and r is the remainder when $(n – 1)(n + 1)$ is divided by 24, what is the value of r?
   (1) n is not divisible by 2 (2) n is not divisible by 3

   (1) if n = 3, then $(n – 1)(n + 1) = 8$, so the remainder is 8
   if n = 5, then $(n – 1)(n + 1) = 24$, so the remainder is 0
   insufficient

   (2) if n = 2, then $(n – 1)(n + 1) = 3$, so the remainder is 3
   if n = 5, then $(n – 1)(n + 1) = 24$, so the remainder is 0
   insufficient

   (together)
   the best approach, unless you're really good at number properties, is to try the first few numbers that satisfy

both statements, and watch what happens.
if n = 1, then (n – 1)(n + 1) = 0, so the remainder is 0
if n = 5, then (n – 1)(n + 1) = 24, so the remainder is 0
if n = 7, then (n – 1)(n + 1) = 48, so the remainder is 0
if n = 11, then (n – 1)(n + 1) = 120, so the remainder is 0
...you can see where this is headed.

here's the theory:
– if n is not divisible by 2, then n is odd, so both (n – 1) and (n + 1) are even. Moreover, since every other even number is a multiple of 4, one of those two factors is a multiple of 4. So the product (n – 1)(n + 1) contains one multiple of 2 and one multiple of 4, so it contains at least 2 x 2 x 2 = three 2's in its prime factorization.
– if n is not divisible by 3, then exactly one of (n – 1) and (n + 1) is divisible by 3, because every third integer is divisible by 3. Therefore, the product (n – 1)(n + 1) contains a 3 in its prime factorization.
– thus, the overall prime factorization of (n – 1)(n + 1) contains three 2's and a 3.
– therefore, it is a multiple of 24.
– sufficient

answer = c

4. If N is a positive integer, is (N^3 – N) divisible by 4?
(1) n = 2k + 1, where K is an integer. (2) n^2 + n is divisible by 6

N^3 – N = (N – 1) * N * (N + 1).
(1) N = Odd … so (N – 1) and (N + 1) both will be even and their product will be divisible by 4.
Sufficient
(2) n (n + 1) is divisible by 6 … N = 2 and N = 3 can be taken as 2 trial values.

If N = 2, N^3 – N = 6 … not divisible by 4.
If N = 3, N^3 – N = 24 … divisible by 4.

Not sufficient.
Ans. A

Problems:

1. If d is a positive integer, f is the product of the first 30 positive integers, what is the value of d?
(1) 10^d is a factor of f (2) d > 6

f = 30! …
(1) **<u>Number of Zeroes at the end of a Factorial = Maximum Power of 10 in the factorial:</u>** It is given by the maximum power of 5 in the number.

Maximum power of 5 in 30! = (30/5) + (30/5^2) + (30/5^3) + …
= 6 + 1 + 0 … = 7
But 10^1 to 10^7 all the powers can be factors of 30!
So d can be any number from 1 to 7.
NS

(2) NS

Combining: d = 7

Ans. C

Detailed background theory behind the above logic.
So how many 10's are in f?
write down the numbers that contain 2s and 5s (only those)
30*28*26*25*24*22*20*18*16*15*14*12*10*8*6*5*4*2
Now ask yourself Is my limiting factor going to be 5 or is it going to be 2?
It's going to be 5 because there are many more 2's up there. So circle the numbers that contain 5's:
30, 25, 20, 15, 10, 5
How many 5's do you have? Seven 5's (don't forget – 25 has two 5's!), so you can make seven 10's.
"limiting factor" means "which is least common or likely." Think of it this way: there are many more multiples of 2 than there are multiples of 5. In probability terms, a number is more likely to be even than to be a multiple of 5. In divisibility terms, take some large number that is divisible by both 2 and 5, and it is likely to have more factors of 2 than 5.
For example: 400 = 4*10*10 = (2*2)(2*5)(2*5) = (2^4)(5^2).
I know, numbers with more factors of 5 than factors of 2 exist...this is just a bet we make to ease the computation.
In general, the larger the factor, the less likely it is to divide evenly into a number. The larger the factor, the more of a "limiting factor" it is.
here's all you have to do:
forget entirely about 10, 20, and 30, and **ONLY THINK ABOUT PRIME FACTORIZATIONS**.
(TAKEAWAY: this is the way to go in general – when you break something down into primes, you should not think in hybrid terms like this. instead, just translate *everything* into the language of primes.)
each PAIR OF A '5' AND A '2' in the prime factorization translates into a '10'.
there are **seven 5's**: one each from 5, 10, 15, 20, and 30, and two from 25.
there are waaaaaaayyyyy more than seven 2's.
therefore, **30! can accommodate as many as seven 10's** before you run out of fives.

2. If n and m are positive integers, what is the remainder when $3^{4n+2} + m$ is divided by 10?
(1) n = 2 (2) m = 1

Unit's digit of a number is the same as the remainder when the number is divided by 10.

REMAINDERS UPON DIVISION BY 10 are simply UNIT'S DIGITS.

For instance, when 352 is divided by 10, the remainder is 2.
So we need to know the unit's digit of $3^{4n+2} + m$.

Unit's digit of 3^{4n+2} can be divided by the rule of cyclicity.

In all such questions, divide the power by 4 and check the remainder.
If the remainder is 1, 2 or 3, then convert the question to LAST DIGIT RAISED TO REMAINDER.
If the remainder is 0, convert the question to LAST DIGIT RAISED TO FOUR.

So (4n + 2) / 4 gives a remainder of 2.
So the power will be reduced to 2.
So now the question becomes "What's the unit's digit of 3^2 + m?" or "What's the unit's digit of 9 + m?" … the answer will depend only on the value of m … so Ans. B

Problems:

1. If each term in the sum *a1 + a2+… an* is either 7 or 77 and the sum equals 350, which of the following could be the value of n?
   38 39 40 41 42

   Well, first, think about the qualitative aspects of the sequence: if the sequence consisted entirely of 7's, then there would be fifty terms in the sequence. these answer choices are reasonably close to fifty, so it stands to reason that by far the majority of the terms will be 7's. therefore, try as few 77's as possible.

   Try only one 77:
   Remaining terms = 350 - 77 = 273
   This would be 273 / 7 = 39 sevens
   So ... you'd have one '77' and thirty-nine '7's … this works! Answer = c … 40 terms.

2. When a certain tree was first planted, it was 4 feet tall, and the height of the tree increased by a constant amount each year for the next 6 years. At the end of the 6th year, the tree was 1/5 taller than it was at the end of the 4th year. By how many feet did the height of the tree increase each year?
   3/10 2/5 ½ 2/3 6/5

   The correct answer is 2/3

   This is essentially a sequence problem in disguise. Let x = amount of yearly growth, in feet.

   Yr0 = 4
   Yr1 = 4+x
   Yr2 = 4+x+x=4+2x
   Yr3 = 4+x+x+x=4+3x
   Yr4 = 4+x+x+x+x=4+4x
   Yr5 = 4+x+x+x+x+x=4+5x
   Yr6 = 4+x+x+x+x+x+x=4+6x

   We are told the amount at the end of Year 6 is 6/5 of the amount at the end of year 4. Thus we can write:

   4+6x = 6/5 (4+4x)
   5(4+6x) = 6(4+4x)
   20+30x = 24+24x
   6x=4
   x=2/3

3. For a finite sequence of nonzero numbers, the number of variations in sign is designed as the number of pairs of consecutive terms of the sequence for which the product of the two consecutive terms is negative. What is the number of variations in sign for the sequence 1, –3, 2, 5, –4, –6?
   One Two Three Four Five

   1*–3 = negative –3*2 = negative 5*–4 = negative Ans. 3

4. $2 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8$?
   2^9 2^{10} 2^{16} 2^{35} 2^{37}

   Write the series as

   $1 + 1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8$ … terms after the first '1' are in GP with a = 1, r = 2, and n = 9.

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_9 = \frac{1(2^9 - 1)}{2 - 1} = 2^9 - 1$$

So the final answer = $1 + 2^9 - 1 = 2^9$.

There can be many other methods to solve it:

(1) **PATTERN RECOGNITION**

it should be clear that there's nothing special about 2^8 as an ending point; in other words, they just cut the sequence off at a random point. therefore, if we **investigate smaller "versions" of the sequence, we should be able to detect a pattern**.
let's look:
first term = 2
sum of first 2 terms = 4
sum of first 3 terms = 8
sum of first 4 terms = 16
ok, it's clear what's going on: each new term doubles the sum. **if you see a pattern this clear, it doesn't matter whether you understand WHY the pattern exists; just continue it.**
so, i want the sum of nine terms, so i'll just double the sum five more times:
32, 64, 128, 256, 512.
this is choice (a).
this is a general rule, by the way: IF SOMETHING CONTAINS MORE THAN 4–5 IDENTICAL STEPS, YOU SHOULD BE ABLE TO EXTRACT A PATTERN FROM LOOKING AT SIMILAR EXAMPLES WITH FEWER STEPS.

(2) **ALGEBRA WITH EXPONENTS** ("textbook method")

the first two terms are 2 + 2. this is 2(2), or 2^2.
now, using this combined term as the "first term", the first two terms are $2^2 + 2^2$. this is $2(2^2)$, or 2^3.

now, using *this* combined term as the "first term", the first two terms are $2^3 + 2^3$. this is $2(2^3)$, or 2^4.

you can see that this will keep happening, so it will continue all the way up to $2^8 + 2^8$, which is $2(2^8) = 2^9$.

(3) ESTIMATE

these **answer choices are ridiculously far apart**, so you should be able to estimate the answer.
memorize some select powers of 2. notably, 2^10 = 1024, which is "about 1000". 2^9 = 512, which is "about 500". and of course you should know all the smaller ones (2^6 and below) by heart.
thus we have 2^8 is about 250, and the other terms are 128, 64, 32, 16, 8, 4, 2, 2.
looking at these numbers, i'd make a ROUGH ESTIMATE WITHIN A FEW SECONDS:
250 is 250.
128 is ~130.
64 and 32 together are ~100.
the others look like thirty or so together.
so, 250 + 130 + 100 + 30 = 510.
the only answer choice within shouting range is (a); the others are absurdly huge.

5. A certain list contains several different integers. Is the product of the integers in the list positive?
(1) The product of the greatest and the smallest of the integers in the list is positive
(2) There is an even number of integers in the list

Statement (1) means that the smallest and largest elements of the list have the same sign, i.e., are both positive or both negative.
But, since those are *the smallest and largest elements of the list*, that means that all the elements *between* have to have that same sign, too.
or:
you can't have 0 between two positive numbers, or between two negative numbers.
either *everything* in the list is positive, or *everything* in the list is negative.

If all the terms are positive: the product of integers will be positive.

If all the terms are negative:

Case 1:
If there is an even number of negative terms … product will be positive.
Case 2:
If there is an odd number of negative terms … product will be negative.

NS

(2) If the terms are –ve, +ve, +ve, +ve … the product will be negative.
If the terms are +ve, +ve, +ve, +ve … the product will be positive.

NS

Combining:

If there is an even number of terms (all positive or all negative), the product will surely be positive.

Ans. C

6. If there are more than two numbers in a certain list, is each of the numbers in the list equal to 0?
   (1) The product of any two numbers in the list is equal to 0.
   (2) The sum of any two numbers in the list is equal to 0.

   (1) The numbers can be 0, 0, 0, 0 OR 0, 0, 0, 2

   NS

   (2) **The most important words: any two.**

   In order for the sum of "any 2 numbers" to = 0, all the numbers must equal 0

   Imagine a, b, c are the numbers: then we have

   a + b = 0
   b + c = 0
   c + a = 0

   Adding 2 (a + b + c) = 0 or a + b + c = 0 … so all three are zero.

   The logic can be extended to any number of terms.

SUFFICIENT

Questions based on Number Line

1. If n denotes a number to the left of 0 on the number line such that the square of n is less than 1/100, then the reciprocal of n must be
A. less than –10 B. between –1 and –1/10 C. between –1/10 and 0
D. between 0 and –1/10 E. greater than 10

Given that n < 0 … A
Also, $n^2 < 1/100$ … i.e. |n| < 1/10 … i.e. –1/10 < n < 1/10 … using A from above,
–1/10 < n < 0 … taking reciprocal, (applies since both are on same side of 0) …
–10 > 1/n or 1/n < –10. Ans. A

2. If s and t are two different numbers on the number line, is s + t = 0?
(1) The distance between s and 0 is the same as the distance between t and 0
(2) 0 is between s and t

(1) |s| = |t| ⇒ s = t or s = -t.
But s ant t are different so s = t is not possible.
So s = -t or s + t = 0 … Sufficient.
(2) 0 is between s and t. But that means s can equal –5 and t can equal +3. In such a case, 0 is still between s and t but that does not make them equidistant from 0. Or, s and t can be –4 and +4 respectively in which case they are equidistant from 0. Therefore, this statement doesn't necessarily answer the question because it can have different results.
Ans. A

Miscellaneous questions

1. The symbol * represents one of the four arithmetic operations: addition, subtraction, multiplication, and division. Is (5 * 6) * 2 = 5 * (6 * 2)?
(1) 5 * 6 = 6 * 5 (2) 2 * 0 = 2

(1) tells us that * = + or ×
But in both the cases,
(5 + 6) + 2 = 5 + (6 + 2)
AND
(5 × 6) × 2 = 5 × (6 × 2)
So as per statement (1)
(5 * 6) * 2 = 5 * (6 * 2)
Sufficient

(2) tells us that * can be either + or – .

(5 + 6) + 2 = 5 + (6 + 2)

(5 – 6) – 2 ≠ 5 – (6 – 2)

Not sufficient
Ans. A

2. If k is a positive integer and the ten's digit of k + 5 is 4, what is the ten's digit of k?
   (1) k > 35 (2) The units digit of k is greater than 5.

   The correct answer is B
   Your first task is to decode the given fact that the tens digit of (k+5) is 4. This means $40 \leq k+5 \leq 49$, which means that $35 \leq k \leq 44$.

   (1) All this tells you is that k isn't 35. That still leaves everything from 36 to 44, so the tens digit could still be either 3 or 4: insufficient.

   (2) Of the aforementioned possibilities, only 35, 36, 37, 38, 39 fit this bill. therefore the tens digit must be 3.

   Sufficient.

3. If the operation ^ is one of the four arithmetic operations addition, subtraction, multiplication, and division, is (6 ^ 2) ^ 4 = 6 ^ (2 ^ 4)?
   (1) 3 ^ 2 > 3 (2) 3 ^ 1 = 3

   With statement 1:
   this function can only be addition or multiplication
   with either of these two operations the left side does indeed equal the right...sufficient
   with statement 2
   this function can be either multiplication or division
   with multiplication the left and right side equal one another
   with division it doesn't...
   hence 2 is insufficient.
   Ans. A

4. Is the hundredth digit of decimal d greater than 5?
   (1) The tenth digit of 10d is 7 (2) The thousandth digit of d/10 is 7

   Let us say the number is 0.ABCD (the decimal d)

   (1) d = 0.ABCD, hence 10d = A.**B**CD 10th digit of 10d = B = 7. which is 100th digit of d (=0.A7CD and thus bigger than 5) Sufficient

   (2) d = 0.ABCD, hence d/10 = 0.0A**B**CD 1000th digit of 10d = B = 7. which is 100th digit of d (=0.A7CD and thus bigger than 5) Sufficient
   Ans. D

5. What is the tens digit of the positive integer r?
   (1) The tens digit of r/10 is 3. (2) The hundreds digit of 10r is 6.

   (1) Suppose r/10 = abc3d.e
   So r = abc3de and ten's digit = d, which is not known to us.

   (2) Let the number be
   10r = abcd6ef
   So r = abcd6e.f

so ten's digit has to be 6 in original number.

Ans. B

6. In the table above, z = 20q?

q	q	q	q
q	r	s	t
q	u	v	w
q	x	y	z

(1) q = 3
(2) Each value in the table other than q is equal to the sum of the value immediately above it in the table and the value immediately to its left in the table.

(1) alone:
obviously INSUFFICIENT, because we have no rule dictating how the table works.

(2) alone:
given all the Q's in the table, use this rule to fill in the other squares of the table **in terms of Q**. you can fill in the table from left to right and/or from top to bottom, following the rule repeatedly.

Q ..... Q ..... Q ..... Q

Q ... 2Q ... 3Q ... 4Q

Q ... 3Q ... 6Q ... 10Q

Q ... 4Q ... 10Q ... 20Q

as you can see, the bottom right square must contain 20Q, so it follows that Z must equal 20Q.
we don't have a *numerical* value for Z or Q unless we take both statements together, but note that this is immaterial: all that matters is the relationship between Z and Q, which is completely determined by (2) alone.

Ans. B

7. For which of the following functions is f(a+b)=f(a)+f(b) for al positive numbers a and b?
$f(x)=x^2$ f(x)=x+1 f(x)= √x f(x)=2/3 f(x)=–3x

OA (E)
just pick numbers.
a=1 b=3
A. f(1+3) = f(1)+f(3)
4^2 = 1^2 + 3^2
16=2+9 => NO
B. (4+1) = (1+1) + (3+1)
5=2+4 => NO
C. √4 = √1 + √3
2=1+√3 => NO
D. 2/4 = 2/1 + 2/3
1/2 = 8/3 => NO
E. –3(4) = –3(1) + –3(3)
–12 = –12 => YES

8. What is the result when x is rounded to the nearest hundredth?
   (1) When x is rounded to the nearest thousandth the result is 0.455
   (2) The thousandth digit is 5

   To solve this problem most efficiently, you should rephrase the first statement.
   When x is rounded to the nearest thousandth the result is 0.455 —> this just means that **$0.4545 \leq x < 0.4555$**. if you don't understand why the left–hand one is "$\leq$" and the right–hand one is just "<", think about the rounding rules when the last digit is 5.
   that's insufficient, because all the values from 0.4545 to 0.454999..... will round to 0.45, but all the values from 0.4550 to 0.4554999999... will round to 0.46. (you don't have to figure out these entire ranges; it's good enough to try the largest and smallest values and note that they round to different numbers.)

   ONCE AGAIN:

   x could be 0.4546, which would round up to 0.455 when rounding to the thousandths place (per the statement), but would round down to 0.45 when rounded to hundredths (one possible answer to the question). On the other hand, x could be 0.4551, which would round up to 0.455 when rounding to the thousandths place (per the statement), and would round up to 0.46 when rounded to hundredths (another possible answer to the question). The value of x rounded to the nearest hundredth could be 0.45 or 0.46.

   (2) is obviously insufficient, because you have no idea how big the number is at all. The 5 in the thousandths place does tell us to round up to the next hundredth, but we have no idea what that is.

   combined, though: the only numbers in the range $0.4545 \leq x < 0.4555$ such that the thousandths digit is 5 (i'm assuming that's what it's supposed to say) are the numbers from 0.4550 to 0.4554999999..., which are precisely the ones that round to 0.46.

   0.455 <= x < 0.456 (the inequality symbols are awkward to format here, so read them carefully)

   The thousandths digit is 5 or greater for all of these values, therefore we will round up to the next hundreth: 0.46.

   Sufficient