TOP ONE PERCENT GMAT 99TH PERCENTILE CLUB

Quant Concepts Session 1 – Inequalities

1. If x is positive which of the following could be correct ordering of $1/x$, $2x$, x^2?
   I. $x^2 < 2x < 1/x$ II. $x^2 < 1/x < 2x$ III. $2x < x^2 < 1/x$
   A. None B. I Only C. III Only D. I and II only E. I, II & III

   I. $x^2 < 2x$ … cancel x from both sides (only because it is positive), we have $x < 2$
   $2x < 1/x$ or $2x^2 < 1$ or $x^2 < 1/2$ or $x < 1/\sqrt{2}$ or $x < 0.707$
   Combining, we have $x < 2$ and $x < 0.7$ … the common solution is $x < 0.7$ so statement I is possible.

   II. $x^2 < 1/x$ or $x^3 < 1$ or $x < 1$
   $1/x < 2x$ or $2x^2 > 1$ or $x^2 > 1/2$ or $x > 1/\sqrt{2}$ or $x > 0.707$
   Combining, we have $0.707 < x < 1$ … so statement II is possible.

   III. $2x < x^2$ … cancel x from both sides (only because x is positive) so we have $x > 2$.
   $x^2 < 1/x$ or $x^3 < 1$ or $x < 1$.
   Combining: $x > 2$ and $x < 1$ … these 2 can't be true together … so statement III is impossible.
   Ans. D

2. If $x > y^2 > z^4$, which of the following statements could be true?
   I. $x > y > z$ II. $z > y > x$ III. $x > z > y$
   A. I only B. I and II only C. I and III only D. II and III only E. I, II, and III

   If $x > y^2$ we may have $x > y$ or $x < y$ both as valid.

   $5 > 4$ … so $5 > 2^2$ and $5 > 2$ … this shows that if $x > y^2$, $x > y$ is possible.

   $1/3 > 1/4$ … so $1/3 > (1/2)^2$ but $1/3 < 1/2$ … this shows that if $x > y^2$, $x < y$ is also possible.

   For statement I
   Imagine: $x = 100$, $y = 5$, $z = 1$ so $x > y^2 > z^4$ and $x > y > z$ (possible)

   For statement II
   Imagine: $x = 1/4$, $y = 1/3$, $z = 1/2$ so $x > y^2 > z^4$ and $z > y > x$ (possible)

   For statement III
   Imagine: $x = 1/2$, $y = 1/4$, $z = 1/3$ so $x > y^2 > z^4$ and $x > z > y$ (possible)

   Ans. E

3. Is $M + Z > 0$ (1) $M – 3Z > 0$ (2) $4Z – M > 0$

   Each statement alone is not sufficient.
   Combining… add the two statements…

(1) + (2), we can know that $z > 0$, then, $m > 3z$ so $m > 0$.
Together, $m + z > 0$ Answer is C

4. If k is not equal to 0, 1, or –1, is $1/k > 0$?
(1) $1 / (k - 1) > 0$ (2) $1 / (k + 1) > 0$

(1) tells that $(k - 1)$ must be positive so $(k - 1) > 0$ so $k > 1$ … so $1/k > 0$ always … sufficient.
(2) tells that $(k + 1)$ must be positive so $(k + 1) > 0$ so $k > -1$ … k can be -1/2 or 2 … so 1/k can be positive or negative. Ans. A

5. The numbers x and y are not integers. The value of x is closest to which integer?
(1) 4 is the integer that is closest to $x + y$ (2) 1 is the integer that is closest to $x - y$

This question deals with rounding-off.
$3.5 \le x + y < 4.5$
$0.5 \le x - y < 1.5$
____

$4 \le 2x < 6$

therefore
$2 \le x < 3$
But x could be 2.1 and the nearest integer will be 2
x could also be 2.9 and the nearest integer will be 3. NS
Ans. E

6. Are x and y both positive (1) $2x - 2y = 1$ (2) $x/y > 1$
Statement (1) can be rephrased: $x - y = ½$. We only know that $x > y$, since the difference is positive. (1) alone is insufficient.
Statement (2) has two options.
If x and y are both positive, then x must be larger than y, so $x > y > 0$.
If x and y are both negative, then x is more negative than y, so $x < y < 0$.
Because we don't know whether they are both positive or both negative, (2) alone is insufficient.
From (1), we know that $x > y$. The only option in (2) for this to be true is if they are both positive. (1) and (2) together are sufficient. (C) is the answer.

7. Is $1 / p > r/(r^2 + 2)$ (1) $p = r$ (2) $r > 0$

(1) imagine $p = r = 2$

$1 / p > r/(r^2 + 2)$ becomes $1/2 > 2 / (4 + 2)$ or $1/2 > 1/3$ … YES

Imagine $p = r = -2$

$1 / p > r/(r^2 + 2)$ becomes $-1/2 > -2 / (4 + 2)$ or $-1/2 > -1/3$ … NO

NS.

(2) doesn't talk about p … NS

Combining: p = r and both p and r are positive.

The question becomes: "Is $1/p > p / (p^2 + 2)$ … we can cross multiply here (all values positive)
$\Rightarrow p^2 + 2 > p^2$ or $2 > 0$, which is always true.
Ans. C

8. Is $X + Y < 1$? (1) $X < 8 / 9$ (2) $Y < 1 / 8$

Combining (1) and (2) $x + y < 73/72$ … $x + y$ can be $72.5 / 72$ … > 1 or can be $71 / 72$ … < 1. NS. Ans. E

9. Is $x – y + 1$ greater than $x + y – 1$? (1) $x > 0$ (2) $y < 0$

Q. Is $x – y + 1 > x + y – 1$ or is $y < 1$?
(1) NS
(2) Sufficient … if $y < 0$, y will surely be less than 1.
Ans. B

10. Is z the median of any 3 positive integers x, y and z? (1) $x < y + z$ (2) $y = z$

this is the same as asking: **is *z* equal to the middle number of the three numbers?**

statement (1)
this statement tells nothing about the order of the three numbers. it could be true regardless of the order of the 3 numbers, and, more to the point, regardless of the position of z in the ordered list.
examples:
x = 1, y = 2, z = 3: z is not the median
x = 1, y = 3, z = 2: z is the median
insufficient

statement (2)
if y and z are equal, there are three possibilities:
–– they are the two largest #s in the list. in this case, both of them equal the median of the list.
–– they are the two smallest #s in the list. in this case, both of them equal the median of the list.
–– all three numbers in the list are the same. in this case, all of them equal the median.
in any of these cases, z is the median.
sufficient

answer = b

11. On the number line, the distance between x and y is greater than the distance between x and z. Does z lie between x and y on the number line?
(1) $xyz < 0$ (2) $xy < 0$

___-1________0_________1________2______________
x z y

z lies between x and y.

OR

___-1________0_________1________2______________
y x z

z doesn't lie between x and y.

Ans. E

12. If $mv < pv < 0$, is $v > 0$? (1) $m < p$ (2) $m < 0$

$mv < pv < 0$ … so m and p are of the same sign and v is of an opposite sign.

Imagine if v is negative, then m and p are both positive and m has to be greater than p.
$v = -1$, $m = 3$, $p = 2$ works …

Imagine if v is positive, then m and p are both negative and m has to be less than p.
$v = 1$, $m = -3$, $p = -2$ works …

(1) gives $m < p$, so v has to be positive.
(2) gives m is negative so v has to be positive.
Ans. D

13. If *n* is a nonzero integer, is $x^n < 1$? (1) $x > 1$ (2) $n > 0$

The question asks whether x^n is less than 1. In order to answer this, we need to know not only whether *x* is less than 1, but also whether *n* is positive or negative since it is the combination of the two conditions that determines whether x^n is less than 1.

(1) INSUFFICIENT: If $x = 2$ and $n = 2$, $x^n = 2^2 = 4$. If $x = 2$ and $n = -2$, $x^n = 2^{(-2)} = 1/(2^2) = 1/4$.

(2) INSUFFICIENT: If $x = 2$ and $n = 2$, $x^n = 2^2 = 4$. If $x = 1/2$ and $n = 2$, $x^n = (1/2)^2 = 1/4$.

(1) AND (2) SUFFICIENT: Taken together, the statements tell us that x is greater than 1 and n is positive. Therefore, for any value of *x* and for any value of *n*, x^n will be greater than 1 and we can answer definitively "no" to the question.

The correct answer is C.

14. If x is an integer, is 3^x less than 500? (1) $4^{x-1} < 4^x - 120$ (2) $x^2 = 36$

Since $3^5 = 243$ and $3^6 = 729$, 3x will be less than 500 only if the integer x is less than 6. So, we can rephrase the question as follows: "Is $x < 6$?"

(1) INSUFFICIENT: We can solve the inequality for x.
$4^{x-1} < 4^x - 120$
$4^{x-1} - 4^x < -120$
$4^x(4^{-1}) - 4^x < -120$
$4^x(1/4) - 4^x < -120$
$4^x[(1/4) - 1] < -120$
$4^x(-3/4) < -120$
$4^x > 160$

Since $4^3 = 64$ and $4^4 = 256$, x must be greater than 3. However, this is not enough to determine if $x < 6$.

(2) INSUFFICIENT: If $x^2 = 36$, then x = 6 or -6. Again, this is not enough to determine if x < 6.

(1) AND (2) SUFFICIENT: Statement (1) tells us that x > 3 and statement (2) tells us that x = 6 or -6. Therefore, we can conclude that x = 6. This is sufficient to answer the question "Is x < 6?" (Recall that the answer "no" is sufficient.)

The correct answer is C

15. Is $x > 10^{10}$? (1) $x > 2^{34}$ (2) $x = 2^{35}$

(1) SUFFICIENT: Statement (1) tells us that $x > 2^{34}$, so we want to prove that $2^{34} > 10^{10}$. We'll prove this by manipulating the expression 2^{34}.
$2^{34} = (2^4)(2^{30})$
$2^{34} = 16(2^{10})^3$

Now $2^{10} = 1024$, and 1024 is greater than 10^3. Therefore:

$2^{34} > 16(10^3)^3$
$2^{34} > 16(10^9)$
$2^{34} > 1.6(10^{10})$.

Since $2^{34} > 1.6(10^{10})$ and $1.6(10^{10}) > 10^{10}$, then $2^{34} > 10^{10}$.

(2) SUFFICIENT: Statement (2) tells us that that $x = 2^{35}$, so we need to determine if $2^{35} > 10^{10}$. Statement (1) showed that $2^{34} > 10^{10}$, therefore $2^{35} > 10^{10}$.

The correct answer is D.

16. Is XY>0? 1). X–Y>–2 2). X–2Y<–6

X-2Y < -6 => -X + 2Y > 6
Combined X - Y > -2, we know Y > 4
X - Y > -2 => -2X + 2Y < 4
Combined X - 2Y < -6, we know -X < -2 => X > 2
Therefore, XY > 0
Answer is C

17. If x and y are integers and xy does not equal 0, is xy < 0?
(1) $y = x^4 - x^3$ (2) $-12y^2 - y^2x + x^2y^2 > 0$

Question: Do *x* and *y* have the opposite signs?

(1) INSUFFICIENT: We can factor the right side of the equation $y = x^4 - x^3$ as follows:
$y = x^4 - x^3$

$y = x^3(x - 1)$ … remember x can't be equal to 1 … which will make y = 0.

When x is negative, y will be positive. xy will be negative.

When x is positive integer (it can't be 1 … it has to be > 1), y will be positive. xy will be positive.

NS

(2) INSUFFICIENT: Let's factor the left side of the given inequality:

$-12y^2 - y^2x + x^2y^2 > 0$
$y^2(-12 - x + x^2) > 0$
$y^2(x^2 - x - 12) > 0$
$y^2(x + 3)(x - 4) > 0$

The expression y^2 will obviously be positive, but it tells us nothing about the sign of y; it could be positive or negative.

NS

(1) AND (2) INSUFFICIENT:

$y^2(x + 3)(x - 4) > 0$ or $(x + 3)(x - 4) > 0$ or $[(x - (-3)][(x - 4)] > 0$. $x > 4$ OR $x < -3$. This is obviously not enough to determine the sign of x.
The correct answer is E.

18. If r + s > 2t, is r > t ? (1) t > s (2) r > s

(1) SUFFICIENT: We can combine the given inequality $r + s > 2t$ with the first statement by adding the two inequalities:

$$\begin{array}{c} r + s > 2t \\ \underline{t > s} \\ r + s + t > 2t + s \\ r > t \end{array}$$

(2) SUFFICIENT: We can combine the given inequality $r + s > 2t$ with the second statement by adding the two inequalities:

$$\begin{array}{c} r + s > 2t \\ \underline{r > s} \\ 2r + s > 2t + s \\ 2r > 2t \\ r > t \end{array}$$

The correct answer is D.

19. If p < q and p < r, is $(p)(q)(r) < p$? (1) pq < 0 (2) pr < 0

The question tells us that $p < q$ and $p < r$ and then asks whether the product pqr is less than p.
Statement (1) INSUFFICIENT: We learn from this statement that either p or q is negative, but since we know from the question that $p < q$, p must be negative. To determine whether $pqr < p$, let's test values for p, q, and r. Our test values must meet only 2 conditions: p must be negative and q must be positive.

P	*Q*	*r*	*pqr*	Is *pqr* < *p*?
-2	5	10	-100	YES
-2	5	-10	100	NO

Statement (2) INSUFFICIENT: We learn from this statement that either *p* or *r* is negative, but since we know from the question that $p < r$, *p* must be negative. To determine whether $pqr < p$, let's test values for *p*, *q*, and *r*. Our test values must meet only 2 conditions: *p* must be negative and *r* must be positive.

p	*q*	*R*	*pqr*	Is $pqr < p$?
-2	-10	5	100	NO
-2	10	5	-100	YES

If we look at both statements together, we know that *p* is negative and that both *q* and *r* are positive. To determine whether $pqr < p$, let's test values for *p*, *q*, and *r*. Our test values must meet 3 conditions: *p* must be negative, *q* must be positive, and *r* must be positive.

p	*q*	*R*	*pqr*	Is $pqr < p$?
-2	10	5	-100	YES
-2	7	4	-56	YES

At first glance, it may appear that we will always get a "YES" answer. But don't forget to test out fractional (decimal) values as well. The problem never specifies that *p*, *q*, and *r* must be integers.

p	*q*	*r*	*pqr*	Is $pqr < p$?
-2	.3	.4	-.24	NO

Even with both statements, we cannot answer the question definitively. The correct answer is E.

20. Is $5^n < 0.04$? (1) $(1/5)^n > 25$ (2) $n^3 < n^2$

In problems involving variables in the exponent, it is helpful to rewrite an equation or inequality in exponential terms, and it is especially helpful, if possible, to rewrite them with exponential terms that have the same base.

$0.04 = 1/25 = 5^{-2}$

We can rewrite the question in the following way: "Is $5^n < 5^{-2}$?"

The only way 5^n could be less than 5^{-2} would be if *n* is less than -2. We can rephrase the question: "Is $n < -2$"?

(1) SUFFICIENT: Let's simplify (or rephrase) the inequality given in this statement.

$(1/5)^n > 25$
$(1/5)^n > 5^2$
$5^{-n} > 5^2$
$-n > 2$
$n < -2$ (recall that the inequality sign flips when dividing by a negative number)

This is sufficient to answer our rephrased question.
(2) INSUFFICIENT: n^3 will be smaller than n^2 if *n* is either a negative number or a fraction between 0 and 1. We cannot tell if *n* is smaller than -2.

The correct answer is A.

21. Is $p^2q > pq^2$? (1) $pq < 0$ $p < 0$

The question can first be rewritten as "Is $p(pq) > q(pq)$?"

If pq is positive, we can divide both sides of the inequality by pq and the question then becomes: "Is $p > q$?" If pq is negative, we can divide both sides of the inequality by pq and change the direction of the inequality sign and the question becomes: "Is $p < q$?"

Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.

(1) INSUFFICIENT: Knowing that $pq < 0$ means that the question becomes "Is $p < q$?" We know that p and q have opposite signs, but we don't know which one is positive and which one is negative so we can't answer the question "Is $p < q$?"

(2) INSUFFICIENT: We know nothing about q or its sign.

(1) AND (2) SUFFICIENT: From statement (1), we know we are dealing with the question "Is $p < q$?," and that p and q have opposite signs. Statement (2) tells us that p is negative, which means that q is positive. Therefore p is in fact less than q.
The correct answer is C.

22. Is $m > n$? (1) $n - m + 2 > 0$ (2) $n - m - 2 > 0$

We can rephrase the question: "Is $m - n > 0$?"
(1) INSUFFICIENT: If we solve this inequality for $m - n$, we get $m - n < 2$. This does not answer the question "Is $m - n > 0$?".
(2) SUFFICIENT: If we solve this inequality for $m - n$, we get $m - n < -2$. This answers the question "Is $m - n > 0$?" with an absolute NO.

The correct answer is B.

23. Is $3^p > 2^q$? (1) $q = 2p$ (2) $q > 0$

Since Statement 2 is less complex than Statement 1, begin with Statement 2 and a BD/ACE grid.

(1) INSUFFICIENT: We can substitute $2p$ for q in the inequality in the question:
$3^p > 2^{2p}$. This can be simplified to $3^p > (2^2)^p$ or $3^p > 4^p$.

If $p > 0$, $3^p < 4^p$ (for example $3^2 < 4^2$ and $3^{0.5} < 4^{0.5}$)
If $p < 0$, $3^p > 4^p$ (for example $3^{-1} > 4^{-1}$)
Since we don't know whether p is positive or negative, we cannot tell whether 3^p is greater than 4^p.
(2) INSUFFICIENT: This tells us nothing about p.
(1) AND (2) SUFFICIENT: If $q > 0$, then p is also greater than zero since $p = 2q$. If $p > 0$, then $3^p < 4^p$. The answer to the question is a definite NO.
The correct answer is C.

24. Is *mp* greater than *m*? (1) $m > p > 0$ (2) *p* is less than 1

To begin, list all of the scenarios in which *mp* would be greater than *m*. There are only 2 scenarios in which this would occur.
Scenario 1: *m* is positive and *p* is greater than 1 (since a fractional or negative *p* will shrink *m*).
Scenario 2: *m* is negative and *p* is less than 1 -- in other words, *p* can be a positive fraction, 0 or any negative number. A negative value for *p* will make the product positive, 0 will make it 0 and a positive fraction will make a negative *m* greater).

NOTE: These scenarios could have been derived algebraically by solving the inequality $mp > m$:
$mp - m > 0$
$m(p - 1) > 0$
Which means either $m > 0$ and $p > 1$ OR $m < 0$ and $p < 1$.
(1) INSUFFICIENT: This eliminates the second scenario, but doesn't guarantee the first scenario. If $m = 100$ and $p = .5$, then $mp = 50$, which is NOT greater than *m.* On the other hand, if $m = 100$ and $p = 2$, then $mp = 200$, which IS greater than *m.*

(2) INSUFFICIENT: This eliminates the first scenario since *p* is less than 1, but it doesn't guarantee the second scenario. *m* has to be negative for this to always be true. If $m = -100$ and $p = -2$, then $mp = 200$, which IS greater than *m.* But if $m = 100$ and $p = .5$, then $mp = 50$, which is NOT greater than *m.*

(1) AND (2) SUFFICIENT: Looking at statements (1) and (2) together, we know that *m* is positive and that *p* is less than 1. This contradicts the first and second scenarios, thereby ensuring that *mp* will NEVER be greater than *m*. Thus, both statements together are sufficient to answer the question. Note that the answer to the question is "No" -- which is a definite, and therefore sufficient, answer to a "Yes/No" question in Data Sufficiency.
The correct answer is C.

25. Is $2X–3Y< X^2$? 1). $2X–3Y=–2$ 2). $X>2$ and $Y>0$

SQUARE is never negative… square ≥ 0 always.
Given equation can be written as, Is $x^2 - 2x + 3y > 0$?

i. $2x-3y = -2$

The given equation becomes $x^2 - (2x - 3y) = x^2 + 2$, which is always > 0 … (SUFF)

ii. $x > 2$ and $y > 0$

Given equation can be written as $x(x - 2) + 3y$, which is also always > 0, if $x > 2$ and $y > 0$.

sufficient

Answer D.