Solution 6 – Arithmetic + Word problems:

1. For an overlapping set problem we can use a double-set matrix to organize our information and solve. Let's call *P* the number of people at the convention. The **boldface** entries in the matrix below were given in the question. For example, we are told that one sixth of the attendees are female students, so we put a value of *P*/6 in the female students cell.

	FEMALE	NOT FEMALE	TOTALS
STUDENTS	***P*/6**	*P*/6	***P*/3**
NON STUDENTS	*P*/2	**150**	2*P*/3
TOTALS	**2*P*/3**	*P*/3	***P***

The non-boldfaced entries can be derived using simple equations that involve the numbers in one of the "total" cells. Let's look at the "Female" column as an example. Since we know the number of female students (*P*/6) and we know the total number of females (2*P*/3), we can set up an equation to find the value of female non-students:

P/6 + *Female Non Students* = 2*P*/3.
Solving this equation yields: *Female Non Students* = 2*P*/3 – *P*/6 = *P*/2.

By solving the equation derived from the "NOT FEMALE" column, we can determine a value for *P*.
P/6 + 150 = P/3 so P = 900
The correct answer is E.

2.

OA: B

take it one statement at a time, and see what you can do with
the facts. (1)
taking this statement in combination with the 4% statistic cited in the prompt, we have that 16 = 4% of the total number of students studying french. this means that 400 students are studying french.
all we know is that 'at least 100' are studying japanese; that figure could be greater than, equal to, or
less than 400, so, insufficient.

(2)
let 'Q' stand for the number of students who study BOTH french and japanese. we have no idea of the size of 'Q', BUT:

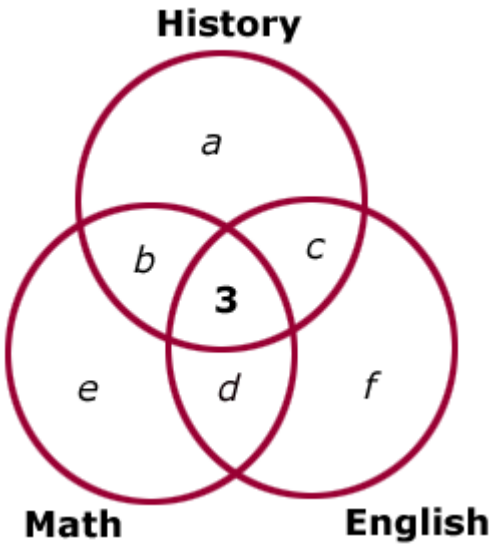
-- Q is 4% of the number of students studying french (as stated in the prompt)
-- Q is 10% of the number of students studying Japanese these two facts together imply that the number of students studying french MUST be larger, because the same quantity Q is a smaller percentage of that number. (if you're interested, the number of students taking french must be exactly 10/4 times the number of students taking japanese for these equations tohold simultaneously.)
Sufficient Answer = b

3.

if there are 60 rooms total, then the motel rented 45 rooms. this means that **15 of the rooms were not rented.**
also, 3/5 of 60, or 36, rooms have aircon; the motel rented 2/3 of these, or 24, rooms.
this means that **12 rooms with aircon weren't rented.**
12 out of 15 = 80%.

4.

For an overlapping set problem with three subsets, we can use a Venn diagram to solve.



Each circle represents the number of students enrolled in the History, English and Math classes, respectively. Notice that each circle is subdivided into different groups of students. Groups *a*, *e*, and *f* are comprised of students taking only 1 class. Groups *b*, *c,* and *d* are comprised of students taking 2 classes. In addition, the diagram shows us that 3 students are

taking all 3 classes. We can use the diagram and the information in the question to write several equations:

History students: $a + b + c + 3 = 25$
Math students: $e + b + d + 3 = 25$
English students: $f + c + d + 3 = 34$
TOTAL students: $a + e + f + b + c + d + 3 = 68$

The question asks for the total number of students taking exactly 2 classes. This can be represented as $b + c + d$.

If we sum the first 3 equations (History, Math and English) we get:
$a + e + f + 2b + 2c + 2d + 9 = 84$.

Taking this equation and subtracting the 4th equation (Total students) yields the following:

$a + e + f + 2b + 2c + 2d + 9 = 84$
$-[a + e + f + b + c + d + 3 = 68]$
$b + c + d = 10$

The correct answer is B.

5. 2%A+(100+1%A)/2=50+A/40

6. Let the least one is x. When other 10 populations have the greatest value, x will have the minimum value.
X+10*1.1X=132000
X=11000
Answer is D

7. 60%*120%/(40%+60%*120%)=64%

8. Fat in milk is x*1%, y*2% and z*3%, respectively.
So we have the equation: x*1%+y*2%+z*3%=(x+y+z)*1.5%
Simplify the equation, we can obtain that x=y+3z

9. For 1, the tip for a $15 bill will be $2, which is less than $15*15%=2.25; the tip for a $20 will be $4, which is greater than $15*15%=2.25. Insufficient.
For 2, tips is $8, means the tens digit of the bill is 4, and the largest possible value of the bill is $49. $8>49*15%=7.35. Sufficient alone.
Answer is B

10. Let's denote the formula for the money spent on computers as $pq = b$, where

p = price of computers
q = quantity of computers
b = budget

We can solve a percent question that doesn't involve actual values by using smart numbers. Let's assign a smart number of 1000 to last year's computer budget (b) and a smart number 100 to last year's computer price (p). 1000 and 100 are easy numbers to take a percent of.

This year's budget will equal 1000 × 1.6 = 1600
This year's computer price will equal 100 × 1.2 = 120

Now we can calculate the number of computers purchased each year, $q = b/p$
Number of computers purchased last year = 1000/100 = 10
Number of computers purchased this year = 1600/120 = 13 1/3 (while 1/3 of a computer doesn't make sense it won't affect the calculation)

	p	q	b
This Year	100	10	1000
Last Year	120	13 1/3	1600

The question is asking for the percent increase in quantity from last year to this year

_____ _____ = 33 1/3%

This question could also have been solved algebraically by converting the percent increases into fractions.

Last year: $pq = b$, so $q = b/p$
This year: $(6/5)(p)(x) = (8/5)b$
If we solve for x (this year's quantity), we get $x = (8/5)(5/6)b/p$ or $(4/3)b/p$
If this year's quantity is 4/3 of last year's quantity (b/p), this represents a 33 1/3% increase.

The correct answer is A.

11. 10x+2y=5Z or 5(x+y)

1. y=10 sufficient to know x
2. z=(x+y)=16 sufficient to know x

Therefore, D.

OR

FACT ABOUT WEIGHTED AVERAGES: if you have the weighted average and both endpoints, then you also have the RATIO of the weights in the problem. that ratio is the reciprocal of the ratio of thedistances between the endpoints and the weighted average.
in this problem, we have this:
(endpoint Y 2%)-------**distance=3**-----(weighted average 5%)------**distance=5**-------(endpoint X 10%)

so
since distance Y : distance X = 3 : 5, the ratio of the weights (literal "weights" in tons, in this problem) of
Y : X must be 5 : 3.

because you have this ratio, specifying even one of the quantities is sufficient to determine everything - just use the ratio to figure out the rest.
therefore, either of the choices will be sufficient.

12. It is easy to prove that statement A is alone sufficient. Lets assume that the items cost x, y and z. In statement A, we are given that x = 50 and y = 20. So, the total discount = (10 + 2 + 0.1z) = 12 + 0.1z. where the original total cost = (50 + 20 + z) = 70 + z

Hence avg. discount = (12 + 0.1z) * 100 / (70 + z) = 100 * { [(7 + 0.1z)/(70+z)] + [5 / (70+z)] }
= 100 * [0.1 + 5/(70+z)]
= 10 + 500/(70+z)

But since z is the least expensive item, z must be less than 20 and greater than 0.
Hence for z = 0 , avg discount = 17.14% and for z = 20, avg. disc = 15.55% Hence in all situations avg. disc is greater than 15%
this problem is also a classic C TRAP.

a **c trap** is a problem that's clearly written to be difficult, but on which both statements taken together are VERY CLEARLY sufficient.
and by VERY CLEARLY i don't mean "after i solve this equation, and move that over there, then ... oh yeah, my memorized rule of thumb tells me they're sufficient" - i mean it's OBVIOUS. (examples follow,
for those of you who have your og's handy.)
on these problems, you can rest assured that (c) is not the answer. also, because the two statements together are sufficient, you can also strike answer (e).
this leaves only (a), (b), (d). and in the dream situation, in which one of the answers by itself is clearly insufficient, then you can guess that the other one must be sufficient - and you'll be right a startlingly
high percentage of the time.

you should NOT use the "c trap" approach as a PRIMARY method - i'm sure that, one fine day, a difficult "c trap" problem will come along to which the answer actually is 'c' (although i've yet to see one) - but, rather, as an AID TO GUESSING.
still, if you get into a guessing situation, the c trap is one of the strongest weapons in your arsenal.

this problem:

* **identify the problem as a c trap**: if you take the two statements together, then you have the prices of ALL the items in the problem. if that's the case, then the answer to the prompt question is clearly either "yes" or "no"; hence, sufficient.
kill (c) and (e) and narrow the choices to (a), (b), and (d).

* statement (2) is insufficient. this isn't ridiculously obvious, but the presence of two remaining unknowns should convince you (remember that you're in guessing mode here).
kill (b) and (d).

answer = a.

13. Each statement alone is clearly not enough... we are trying to determine if income/population > 16,500 so we need to have values for the numerator and the denominator. So it's either C or E.

(1&2) Suppose we start with the fraction $5.5B/330K. This is greater than $16,500. However, the given values are both lower limits. If we increase the numerator while keeping the denominator constant, income per capita will certainly increase. If, instead, we increase the denominator, the income per capita will decrease. A slightly larger value for the population would bring it down below $16,500. Not knowing what the upper limit for the population is what makes it insufficient.

Answer is E.

14. first, recognize that you want to make a proportion involving 2 quantities: (a) amount of concentrate and
(b) amount of JUICE, NOT water.
to take care of (b), note that 1 can of concentrate is used to make 1 + 3 = 4 cans of juice (assuming that the volumes are additive, a necessary assumption here). therefore, **the ratio of concentrate to juice is 1:4.**

you want to make 200 x 6 = 1200 ounces of

orange juice. set up a proportion:
(ounces of concentrate) / 1200 = 1 / 4

solve:
300 ounces of concentrate
each can of concentrate contains 12 ounces, so 300/12 = 25 cans.

15. This is one way to solve this, using weighted averages:
72% male apply; therefore 28% male don't apply
80% female apply; therefore 20% female don't apply

I want to know what fraction of students is male. I could get this one of two ways:
a) the actual number of males in the class and the actual total number of students in the class

(# male /
total)
b) the proportion or percentage of male students to female (which, together, comprise the total - so I
don't necessarily need to know the actual numbers in order to know a percentage)

(1) 840 = m + f. I don't know actual numbers for male and female. I also don't know proportions for male / female. Can't do it.

(2) 75% of all students (m+f) have applied. Don't know actual numbers for male and female. I can, however, figure out proportions. If 72% of males apply, and 80% of females apply, and 75% overall apply, then I can figure out the proportion of males to females in the class - this is a weighted average problem.

Here's how: 8 percentage points separate the males and the females (80-72). The overall average, 75, is
3 points away from the males (at 72) and 5 points away from the females (at 80). This means there are more males than females and, further, it tells me the actual proportion. Of every 8 students, 5 are male and 3 are female. (If there were equal numbers of males and females, the overall average would be
76%, or 4 points away from each. Because the overall number is closer to the male percentage, the males have more of an impact on the final number. Further, the proportion of males to females is equal
to that skew - a 50/50 weighting would mean that each group "pulled" the average by 4 points. Since the males "pulled" the average one point closer to them, they have a weighting of 5, and the females lose
that point for a weighting of 3.)

So, 5/8 of the students are male and 3/8 are female.

16. **OA:** C

(1) alone:
since total cost = L + M, this means that L + M = 3M, or L = 2M.
so, rephrased, statement 1 says that **labor cost twice as much as materials**.
still, this is insufficient, as picking values will show: if M = $1 and L = $2, the answer is yes, but if M =
$150,000 and L = $300,000, the answer is no.

(2) alone:
great, but no information at all about materials. so, if L = M = $1, then yes; if L = $1 and M = $499,997
(so that profit = $2), then no. Insufficient.

together:
we have

P > L
which, using the rephrasing found above, rephrases to
P > 2M
also,
P = 500,000 - 3M (because total cost = 3M)
so
500,000 - 3M > 2M
500,000 > 5M
100,000 > M
since M is less than 100,000, it follows that profit, which is 500,000 - 3M, must be more than 500,000 -
3(100,000) =
200,000. answer =
yes
sufficient

remember: profit = revenue - cost
(this is the only thing you'll ever have to know on the gmat that pertains even remotely to business)
the whole point of statement (1) is that you have no actual dollar value for the total cost. if you had a concrete dollar value - any dollar value, regardless of the actual amount - you'd be able to calculate the profit.

17. OA is D.

(1) remember that ratios are the same as fractions in this sort of context.
this statement means that the FT employees are a smaller fraction of division Y than of the company as a whole. this means that they must be a bigger fraction of division X than of the company as a whole, because the fraction of the whole company that's employed FT must
be between the two divisions' fractions.
sufficient.

(analogy: if i mix two powders together to make a shake that's 5% fat, and the first powder is 3% fat, then the second powder must be more than 5% fat)

-- (2)
the first part means that (FT in div. X) > (FT in div. Y), and the second part means that (PT in div. X) < (PT in div. Y).
therefore, considering the ratio of FT : PT for each division, we have that FT/PT for div. X must be greater than FT/PT for div. Y. (this is the case because of either the numerator or the denominator: the numerator of X is greater, and the denominator is smaller.)
since the FT/PT fraction is bigger for div. X than for div. Y, it must be bigger for div. X than for the company as a whole (see the reasoning above under statement (1) for why this is true).
Sufficient

18. Answer is C, both statements together are sufficient.

the statement "2/3 are **either cows or pigs**" doesn't mean that there are either 40 cows or 40 pigs. it means that, if you take the cows and the pigs together, they constitute 2/3 of the animals on the farm. in other words, **cows + pigs = 40.**
(i can understand your alternate reading of the problem statement; it's reasonable enough. just remember that the gmat is their playground, not yours, and so you have to play by their rules - so
remember the way certain statements are written. as a postscript, i hope that future problems like this
one will be purged and/or rewritten for clarity before they make it into the official question pool; it would be a shame if students miss the problem just because of its ambiguity.)
thus:

(1)
this means that there are at least 27 cows (because 27 cows, 13 pigs is the least # of cows satisfying this criterion). that's all we know, though; there could be anywhere between 27 cows (and therefore 13 pigs) and 40 cows (and therefore 0 pigs).
insufficient

(2)
this means that there are at least 13 pigs, which means that there are at most 27 cows. that's all we know.
insufficient

(together)
(1) says there are at least 27 cows; (2) says there are at most 27 cows. so, there are 27 cows and 13 pigs.
sufficient

answer = c

19. We begin by figuring out Lexy's average speed. On her way from A to B, she travels 5 miles in one hour, so her speed is 5 miles per hour. On her way back from B to A, she travels the same 5 miles at 15 miles per hour. Her average speed for the round trip is NOT simply the average of these two speeds. Rather, her average speed must be computed using the formula $RT = D$, where R is rate, T is time and D is distance. Her average speed for the **whole** trip is the **total** distance of her trip divided by the **total** time of her trip.

We already know that she spends 1 hour going from A to B. When she returns from B to A, Lexy travels 5 miles at a rate of 15 miles per hour, so our formula tells us that $15T = 5$, or $T = 1/3$. In other words, it only takes Lexy 1/3 of an hour, or 20 minutes, to return from B to A. Her total distance traveled for the round trip is 5+5=10 miles and her total time is 1+1/3=4/3 of an hour, or 80 minutes.

We have to give our final answer in minutes, so it makes sense to find Lexy's average

rate in miles per minute, rather than miles per hour. 10 miles / 80 minutes = 1/8 miles per minute. This is Lexy's average rate

We are told that Ben's rate is half of Lexy's, so he must be traveling at 1/16 miles per minute. He also travels a total of 10 miles, so (1/16)*T* = 10, or *T* = 160. Ben's round trip takes 160 minutes.

Alternatively, we could use a shortcut for the last part of this problem. We know that Ben's rate is half of Lexy's average rate. This means that, for the entire trip, Ben will take twice as long as Lexy to travel the same distance. Once we determine that Lexy will take 80 minutes to complete the round trip, we can double the figure to get Ben's time. 80 × 2 = 160.

The correct answer is D.

20. There is an important key to answering this question correctly: this is not a simple average problem but a weighted average problem. A weighted average is one in which the different parts to be averaged are not equally balanced. One is "worth more" than the other and skews the "simple" average in one direction. In addition, we must note a unit change in this problem: we are given rates in miles per hour but asked to solve for rates in miles per minute.

Average rate uses the same D = RT formula we use for rate problems but we have to figure out the different lengths of time it takes Dan to run and swim along the total 4-mile route. Then we have to take the 4 miles and divide by that total time. First, Dan runs 2 miles at the rate of 10 miles per hour. 10 miles per hour is equivalent to 1 mile every 6 minutes, so Dan takes 12 minutes to run the 2 miles. Next, Dan swims 2 miles at the rate of 6 miles per hour. 6 miles per hour is equivalent to 1 mile every 10 minutes, so Dan takes 20 minutes to swim the two miles.

Dan's total time is 12 + 20 = 32 minutes. Dan's total distance is 4 miles. Distance / time = 4 miles / 32 minutes = 1/8 miles per minute.

Note that if you do not weight the averages but merely take a simple average, you will get 2/15, which corresponds to incorrect answer choice B. 6 mph and 10 mph average to 8mph. (8mph)(1h/60min) = 8/60 miles/minute or 2/15 miles per minute.

The correct answer is A.

21. Distance = Rate × Time, or *D* = *RT*.

(1) INSUFFICIENT: This statement tells us Harry's rate, 30 mph. This is not enough to calculate the distance from his home to his office, since we don't know anything about the time required for his commute.

D = *RT* = (30 mph) (*T*)

D cannot be calculated because *T* is unknown.

(2) INSUFFICIENT: If Harry had traveled twice as fast, he would have gotten to work in half the time, which according to this statement would have saved him 15 minutes. Therefore, his actual commute took 30 minutes. So we learn his commute time from this statement, but don't know anything about his actual speed.

$D = RT = (R)$ (1/2 hour)
D cannot be calculated because R is unknown.

(1) AND (2) SUFFICIENT: From statement (1) we learned that Harry's rate was 30 mph. From Statement (2) we learned that Harry's commute time was 30 minutes. Therefore, we can use the rate formula to determine the distance Harry traveled.
$D = RT$ = (30 mph) (1/2 hour) = 15 miles

The correct answer is C.

22. To determine Bill's average rate of movement, first recall that Rate × Time = Distance. We are given that the moving walkway is 300 feet long, so we need only determine the time elapsed during Bill's journey to determine his average rate.

There are two ways to find the time of Bill's journey. First, we can break down Bill's journey into two legs: walking and standing. While walking, Bill moves at 6 feet per second. Because the walkway moves at 3 feet per second, Bill's foot speed along the walkway is 6 – 3 = 3 feet per second. Therefore, he covers the 120 feet between himself and the bottleneck in (120 feet)/(3 feet per second) = 40 seconds.

Now, how far along is Bill when he stops walking? While that 40 seconds elapsed, the crowd would have moved (40 seconds)(3 feet per second) = 120 feet. Because the crowd already had a 120 foot head start, Bill catches up to them at 120 + 120 = 240 feet. The final 60 feet are covered at the rate of the moving walkway, 3 feet per second, and therefore require (60 feet)/(3 feet per second) = 20 seconds. The total journey requires 40 + 20 = 60 seconds, and Bill's rate of movement is (300 feet)/(60 seconds) = 5 feet per second.

Short-cut: This problem may also be solved with a shortcut. Consider that Bill's journey will end when the crowd reaches the end of the walkway (as long as he catches up with the crowd before the walkway ends). When he steps on the walkway, the crowd is 180 feet from the end. The walkway travels this distance in (180 feet)/(3 feet per second) = 60 seconds, and Bill's average rate of movement is (300 feet)/(60 seconds) = 5 feet per second.

The correct answer is E.

23. It is easier to break this motion up into different segments. Let's first consider the 40 minutes up until John stops to fix his flat.

40 minutes is 2/3 of an hour.

In 2/3 of an hour, John traveled 15 × 2/3 = 10 miles (*rt* = *d*)

In that same 2/3 of an hour, Jacob traveled 12 × 2/3 = 8 miles
John therefore had a two-mile lead when he stopped to fix his tire.

It took John 1 hour to fix his tire, during which time Jacob traveled 12 miles. Since John began this 1-hour period 2 miles ahead, at the end of the period he is 12 – 2 = 10 miles behind Jacob.
The question now becomes "how long does it take John to bridge the 10-mile gap between him and Jacob, plus whatever additional distance Jacob has covered, while traveling at 15 miles per hour while Jacob is traveling at 12 miles per hour?" We can set up an *rt* = *d* chart to solve this.

	John	Jacob
R	15	12
T	*t*	*t*
D	*d* + 10	*d*

John's travel during this "catch-up period" can be represented as 15*t* = *d* + 10
Jacob's travel during this "catch-up period" can be represented as 12*t* = *d*

If we solve these two simultaneous equations, we get:
15*t* = 12*t* + 10
3*t* = 10
t = 3 1/3 hours

Another way to approach this question is to note that when John begins to ride again, Jacob is 10 miles ahead. So John must make up those first 10 miles plus whatever additional distance Jacob has covered while both are riding. Since Jacob's additional distance at any given moment is 12*t* (measuring from the moment when John begins riding again) we can represent the distance that John has to make up as 12*t* + 10. We can also represent John's distance at
any given moment as 15*t*. Therefore, 15*t* = 12*t* + 10, when John catches up to Jacob. We can solve this question as outlined above. The correct answer is B.

24. 1) T= D/R

2) Question asks for T1/T2

3) Second statement says Y=20

4) Set up equation using Y=20:

(D1/R1)/(D2/R2)=

100 X [(20/R1) X (R2/20)]

5) Plug in

Rates: (20/X) X

(1.25X/20)

6) At this point, realize that the "X"'s cross out, stop calculating and realize that B is sufficient.

25.

probably the most efficient way to solve this one is with the concept of

relative rates. start at the time when the cyclist passes the hiker.
since they're both traveling in the same direction, their relative rate - i.e., the rate at which the distancebetween them is changing - is found by subtraction. so, as long as they're both moving the cyclist is gaining on the hiker at a rate of 20 - 4 = 16 mph.
after the cyclist stops, the hiker is making up that distance at a rate of 4 mph. that's only 1/4 as fast as
the relative rate during the first 5 minutes, so it will take the hiker 4 times as long to catch back up. so, 4 x 5 = 20 minutes.

if you wanted to, you could calculate the actual distance between the cyclist and the hiker when the cyclist stops: (16 mi/h)(1/12 h) = d, where "1/12 h" is five minutes. then you could find the time taken by the hiker alone to travel the same distance "d".
you don't have to do this, though, if you realize the 1:4 ratio referenced above.

26.

Let a be the number of hours it takes Machine A to produce 1 widget on its own. Let b be the number of hours it takes Machine B to produce 1 widget on its own.

The question tells us that Machines A and B together can produce 1 widget in 3 hours. Therefore, in 1 hour, the two machines can produce 1/3 of a widget. In 1 hour, Machine A can produce $1/a$ widgets and Machine B can produce $1/b$ widgets. Together in 1 hour, they produce $1/a + 1/b = 1/3$ widgets.

If Machine A's speed were doubled it would take the two machines 2 hours to produce 1 widget. When one doubles the speed, one cuts the amount of time it takes in half. Therefore, the amount of time it would take Machine A to produce 1 widget would be $a/2$. Under these new conditions, in 1 hour Machine A and B could produce $1/(a/2) + 1/b = 1/2$ widgets. We now have two unknowns and two different equations. We can solve for a.

The two equations:
$2/a + 1/b = 1/2$ (Remember, $1/(a/2) = 2/a$)
$1/a + 1/b = 1/3$

Subtract the bottom equation from the top:
$2/a - 1/a = 1/2 - 1/3$
$1/a = 3/6 - 2/6$
$1/a = 1/6$
Therefore, $a = 6$.

The correct answer is E.

27. Tom's individual rate is 1 job / 6 hours or 1/6.
During the hour that Tom works alone, he completes 1/6 of the job (using $rt = w$).
Peter's individual rate is 1 job / 3 hours.
Peter joins Tom and they work together for another hour; Peter and Tom's respective individual rates can be added together to calculate their combined rate: 1/6 + 1/3 = 1/2.
Working together then they will complete 1/2 of the job in the 1 hour they work together.

At this point, 2/3 of the job has been completed (1/6 by Peter alone + 1/2 by Peter and Tom), and 1/3 remains.

When John joins Tom and Peter, the new combined rate for all three is: 1/6 + 1/3 + 1/2 = 1.
The time that it will take them to finish the remaining 1/3 of the job can be solved:
$rt = w \longrightarrow (1)(t) = 1/3 \longrightarrow t = 1/3$.

The question asks us for the fraction of the job that Peter completed. In the hour that Peter worked with Tom he alone completed: $rt = w \longrightarrow w = (1/3)(1) = 1/3$ of the job.
In the last 1/3 of an hour that all three worked together, Peter alone completed:
(1/3)(1/3) = 1/9 of the job.
Adding these two values together, we get 1/3 + 1/9 of the job = 4/9 of the job.

The correct answer is E.

28. To find the combined rate of Machines A and B, we combine their individual rates. If

Machine A can fill an order of widgets in a hours, then in 1 hour it can fill $\frac{1}{a}$ of the order. By the same token, if Machine B can fill the order of widgets in b hours, then in 1 hour, it can fill $\frac{1}{b}$ of the order. So together in 1 hour, Machines A and B can fill $\frac{1}{a} + \frac{1}{b}$ of the order:

$$\frac{1}{a}+\frac{1}{b}=\frac{(b)1}{(b)(a)}+\frac{(a)1}{(a)(b)}=\frac{b}{ab}+\frac{a}{ab}=\frac{a+b}{ab}$$

So in 1 hour, Machines A and B can complete $\frac{a+b}{ab}$ of the order. To find the number of hours the machines need to complete the *entire* order, we can set up the following equation:

(fraction of order completed in 1 hour) x (number of hours needed to complete entire order) =
1 order.

If we substitute $\frac{a+b}{ab}$ for the fraction of the order completed in 1 hour, we get:

$\frac{a+b}{ab}(x)=1$ where x is the number of hours needed to complete the entire order. If we divide both sides by $\frac{a+b}{ab}$, we get:

$$x=\frac{ab}{a+b}$$

In other words, it will take Machines A and B $\frac{ab}{a+b}$ hours to complete the entire order working together at their respective rates.

The question stem tells us that a and b are both even integers. We are then asked whether a and b are equal. If they are equal, we can express each as $2z$, where z is a non-zero integer, because they are even. If we replace a and b with $2z$ in the combined rate, we get:

$$\frac{(2z)(2z)}{2z+2z}=\frac{4z^2}{4z}=z$$

So if a and b are equal, the combined rate of Machines A and B must be an integer (since z is an integer). We can rephrase the question as:

Is the combined rate of Machines A and B an integer?

Statement 1 tells us that it took 4 hours and 48 minutes for the two machines to fill the order (remember, they began at noon). This shows that the combined rate of Machines A and B is NOT an integer (otherwise, it would have taken the machines a whole number of hours to complete the order). So we know that a and b cannot be the same. Sufficient.

Statement 2 tells us that $(a + b)^2 = 400$. Since both *a* and *b* must be positive (because they represent a number of hours), we can take the square root of both sides of the equation without having to worry about negative roots. Therefore, it must be true that $a + b = 20$. So it is possible that $a = 10$ and that $b = 10$, which would allow us to answer "yes" to the question. But it is also possible that $a = 12$ and $b = 8$ (or any other combination of positive even integers that sum to 20), which would give us a "no". Insufficient.

The correct answer is A: Statement 1 alone is sufficient, but statement 2 alone is not.

29. Answer is B.
6 * 12 = (6 + x) * 8 so x = 3.

30.

Correct answer: (E)

remember that **rate = reciprocal of time taken to complete one job**.
also, remember that rates are additive, so rate(pumps a AND b) = rate(pump a) + rate(pump b). so:
rate(pumps a AND b) =
5/6 rate(pumps a AND
c) = 2/3
rate(pumps b AND c) = 1/2

using the above fact about additive
rates, rate(pump a) + rate(pump b) =
5/6 rate(pump a) + rate(pump c) =
2/3 rate(pump b) + rate(pump c) =
1/2

you know you want the rate for all three pumps. from the symmetry of the above equations, it becomes apparent that we can find this by **adding together all 3 equations**:
2rate(pump a) + 2rate(pump b) + 2rate(pump c) = 5/6 + 2/3 + 1/2
= 2 rate(pump a) + rate(pump b) + rate(pump c) = 1
rate(pumps a AND b AND c) = 1 (because rates are additive)
time = reciprocal of 1 = 1

31.

Compound interest is computed using the following formula:

$F = P (1 + r/n)^{nt}$, where
F = Final value
P = Principal
r = annual interest rate
n = number of compounding periods per year

t = number of years

From the question, we can deduce the following information about the growth during this period:

At the end of the x years, the final value, F, will be equal to 16 times the principal (the money is growing by a factor of 16).
Therefore, $F = 16P$.
$r = .08$ (8% annual interest rate)
$n = 4$ (compounded quarterly)
$t = x$ (the question is asking us to express the time in terms of x number of years)

We can write the equation
$16P = P(1 + .08/4)^{4x}$
$16 = (1.02)^{4x}$

Now we can take the fourth root of both sides of the equation. (i.e.the equivalent of taking the square root twice) We will only consider the positive root because a negative 2 doesn't make sense here.

$16^{1/4} = [(1.02)^{4x}]^{1/4}$
$2 = (1.02)^{x}$

The correct answer is B.

32. We need to consider the formula for compound interest for this problem: $F = P(1 + r)^x$, where F is the final value of the investment, P is the principal, r is the interest rate per compounding period as a decimal, and x is the number of compounding periods (NOTE: sometimes the formula is written in terms of the annual interest rate, the number of compounding periods per year and the number of years). Let's start by manipulating the given expression for r:

$$r = 100\left(\sqrt{\frac{v+q}{p}} - 1\right) \rightarrow \frac{r}{100} = \sqrt{\frac{v+q}{p}} - 1 \rightarrow 1 + \frac{r}{100} = \sqrt{\frac{v+q}{p}} \rightarrow$$

$$\left(1 + \frac{r}{100}\right)^2 = \left(\sqrt{\frac{v+q}{p}}\right)^2 \rightarrow \left(1 + \frac{r}{100}\right)^2 = \frac{v+q}{p} \rightarrow p\left(1 + \frac{r}{100}\right)^2 = v + q \rightarrow$$

$$v = p\left(1 + \frac{r}{100}\right)^2 - q$$

Let's compare this simplified equation to the compound interest formula. Notice that r in this simplified equation (and in the question) is not the same as the r in the compound interest formula. In the formula, the r is already expressed as a decimal equivalent of a percent, in the question the interest is r percent. The simplified equation, however, deals with this

discrepancy by dividing *r* by 100.
In our simplified equation, the cost of the share of stock (*p*), corresponds to the principal (*P*) in the formula, and the final share price (*v*) corresponds to the final value (*F*) in the formula. Notice also that the exponent 2 corresponds to the *x* in the formula, which is the number of compounding periods. By comparing the simplified equation to the compound interest formula, we see that the equation tells us that the share rose at the daily interest rate of *p* percent for TWO days. Then the share lost a value of *q* dollars on the third day, i.e. the "– *q*" portion of the expression. If the investor bought the share on Monday, she sold it three days later on Thursday.
The correct answer is B.

33. Let's say:
I = the original amount of bacteria
F = the final amount of bacteria
t = the time bacteria grows
If the bacteria increase by a factor of *x* every *y* minutes, we can represent the growth of the bacteria with the equation:
$F = I(x)^{t/y}$

To understand why, let's assign some values to *I*, x and y:

I =	10
x =	2
y =	3

If the bacteria start off 100 in number and they double every 3 minutes, after 3 minutes there will
be 100(2) bacteria. Let's construct a table to track the growth of the bacteria:

(time)	*F* (final count)
3	$100(2) = 100(2)^1$
6	$100(2)(2) = 100(2)^2$
9	$100(2)(2)(2) = 100(2)3$
12	$100(2)(2)(2)(2) = 100(2)^4$

We can generalize the *F* values in the table as
$100(2)^n$. The 100 represents the initial count, *I*.
The 2 represents the factor of growth (in this problem *x*).
The *n* represents the number of growth periods. The number of growth periods is found by dividing the time, *t*, by the amount of time it takes to complete a period, *y*.
From this example, we can extrapolate the general formula for exponential growth: $F = I$
This question asks us how long it will take for the bacteria to grow to 10,000 times their original amount.
The bacteria will have grown to 10,000 times their original amount when $F = 10{,}000I$.
If we plug this into the general formula for exponential growth, we get: $10{,}000I = I(x)^{t/y}$ or

$10,000 = (x)^{t/y}$.
The question is asking us to solve for t.
(1) SUFFICIENT: This statement tells us that $x^{1/y}=10$. If we plug this value into the equation we can solve for *t*.
$10,000 = (x)^{t/y}$
$10,000 =[(x)^{1/y}]^{t}$
$10,000 =(10)^{t}$
$t = 4$
(2) SUFFICIENT: The bacteria grow one hundredfold in 2 minutes, that is to say they grow by a factor of 10^2. Since exponential growth is characterize by a constant factor of growth (i.e. by x every y minutes), for the bacteria to grow 10,000 fold (i.e. a factor of 10^4), they will need to
grow another 2 minutes, for a total of four minutes ($10^2 \times 10^2 = 10^4$).
The correct answer is D, EACH statement ALONE is sufficient to answer the question.