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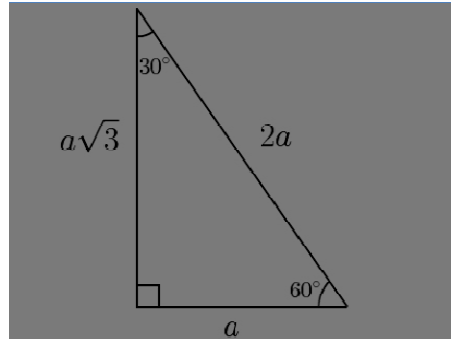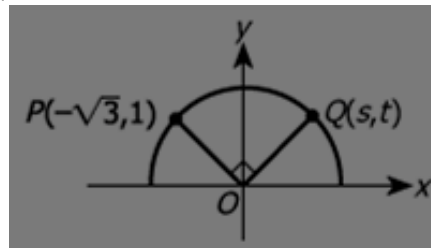Exclusive GMAT coaching by Sandeep Gupta for 99th percentile aspirants!

Quant Concepts: Session 4: Geometry
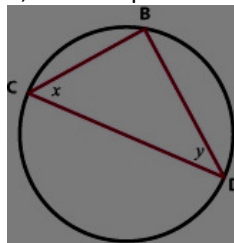
Concept # 1: 30-60-90 Right angled triangle:

Side opposite the angle 30° is the smallest (say *a*), so the side opposite the angle 60° will be *a√3* and the side the angle 90° will be *2a*.

1. In the given figure, points P and Q lie on the circle with center O. What is the value of *s*?


| ½ | 1 | √2 | √3 | 1/√2 |

2. If *CD* is the diameter of the circle, does *x* equal 30?

(1) The length of *CD* is twice the length of *BD*.   (2) *y* = 60

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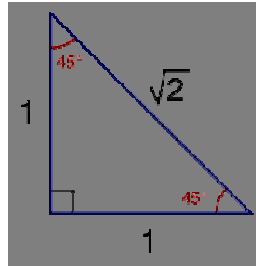
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3. For the triangle shown, where A, B and C are all points on a circle, and line segment AB has length 18, what is the area of triangle ABC?
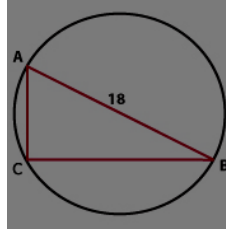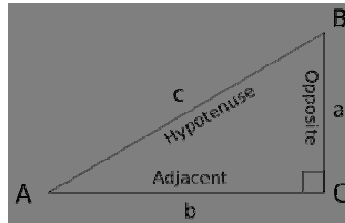

(1) Angle ABC measures 30°.   (2) The circumference of the circle is 18π.

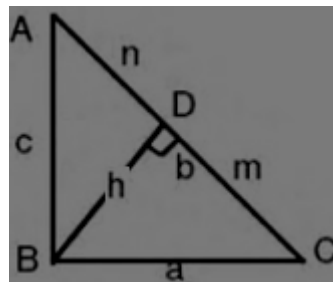**Concept # 2: 45-45-90 Right angled triangle (isosceles right triangle):**

1. Is the area of the right angled triangle ABC > 25?

(1) AC = 6   (2) AB = 10

2. The perimeter of a certain isosceles right triangle is 16 + 16√2, what is the length of the hypotenuse of the triangle?

Concept # 3: Right angled triangle

A triangle whose one angle is 90° is called a right (angled) Triangle. In the figure, b is the hypotenuse, and a & c the legs, called base and height resp.
(1) $h^2 = mn$

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(2) $AC^2 = AB^2 + BC^2$ (Pythagoras theorem)

(3) $h = ac/b$

(4) Area $= ac / 2$

Concept # 4: Similar Triangles:

1. **Proportionality Theorem:**

   Intercepts made by two transversal lines (cutting lines) on three or more parallel lines are proportional. In the figure, lines X_1Y_1 & X_2Y_2 are transversals cutting the three parallel lines AB, CD, EF. Then AC, CE, BD, DF are intercepts Also,
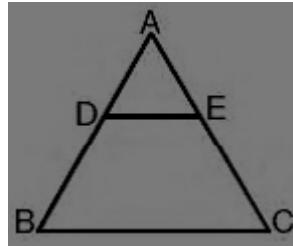

   AC/BD = CE/DF
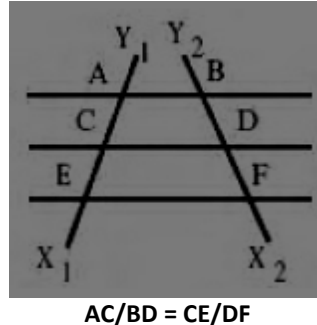
2. **Midpoint Theorem:**

   A triangle, the line joining the mid points of two sides is parallel to the third side and half of it.

3. **Basic Proportionality Theorem:**

   A line parallel to any one side of a triangle divides the other two sides proportionally. If DE is parallel to BC, then

   (a) AD/BD = AE/EC

   (b) AB/AD = AC/AE

   (c) AD/DE = AB/BC and so on.

4. **PROPERTIES OF SIMILAR TRIANGLES:**

   In SIMILAR FIGURES, the RATIO OF AREAS is (RATIO OF LENGTHS)^2

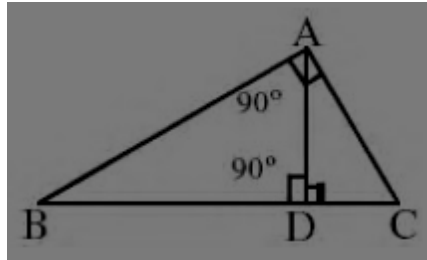   If length ratio = a : b   area ratio = $a^2 : b^2$

   In SIMILAR SOLIDS, the RATIO OF VOLUMES is (RATIO OF LENGTHS)^3

   In SIMILAR SOLIDS, the RATIO OF SURFACE AREAS is (RATIO OF LENGTHS)^2

   Length ratio = a : b   surface area ratio = $a^2 : b^2$ volume ratio = $a^3 : b^3$

5. **RIGHT TRIANGLE (similarity):**

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ABC is a Right Triangle with A as the Right angle.
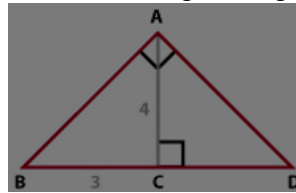

AD is perpendicular to BC then
(a) Triangle ABD ~ Triangle CBA & BA² = BC x BD
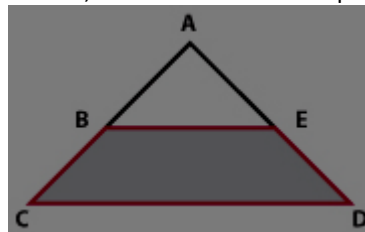(b) Triangle ACD ~ Triangle BCA & CA² = CB x CD
(c) Triangle ABD ~ Triangle CAD & DA² = DB x DC.

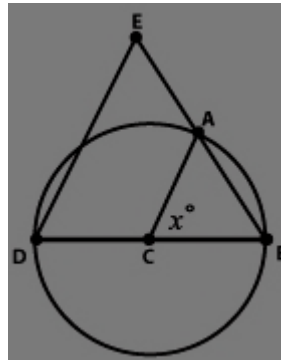1. In triangle *ABC*, if *BC* = 3 and *AC* = 4, then what is the length of segment *CD*?

    3   15/4   5   16/3   20/3

2. If *BE || CD*, and *BC = AB* = 3, *AE* = 4 and *CD* = 10, what is the area of trapezoid *BEDC*?

    12   18   24   30   48

3. In the figure, if point *C* is the center of the circle and *DB* = 7, what is the length of *DE*?

    (1) *x* = 60°   (2) *DE || CA*

4. The area of the right triangle *ABC* is 4 times greater than the area of the right triangle *KLM*. If the hypotenuse *KL* is 10 inches, what is the length of the hypotenuse *AB*?

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    (1) Angles *ABC* and *KLM* are each equal to 55 degrees.
    (2) *LM* is 6 inches.

5.   In the diagram, what is the length of *AB*?
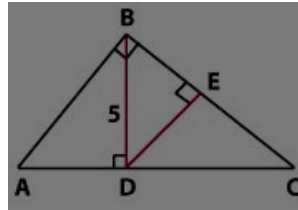
    (1) *BE* = 3   (2) *DE* = 4

6.   In the given figure, if the area of the triangle on the right is twice the area of the triangle on the left, then in terms of s, S=?

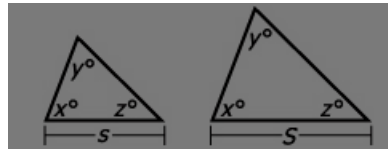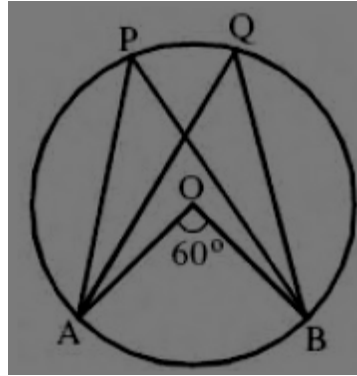$s/\sqrt{2}$ $s\sqrt{3}/2$ $s\sqrt{2}$ $s\sqrt{3}$ $2s$
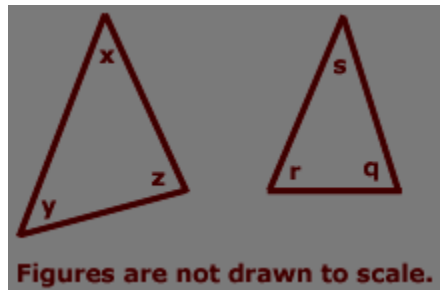
Concept # 4: Lines and Angles:

- Two angles whose sum is 90° are complementary. Each one is the complement of the other.
- Two angles whose sum is 180º are supplementary. Each one is the supplement of the other.
- Sum of the three interior angles of a triangle is 180°
- In a triangle, an exterior angle = Sum of the other two interior angles not adjacent to it
- In a triangle, sum of any two sides is greater than the third side and the difference of any two sides is less than the third side.
- In a triangle, the side opposite to the greatest angle will be the greatest and vice versa.
- If a, b, c denote the sides of a triangle then
  - if $c^2 < a^2 + b^2$, Triangle is acute angled
  - if $c^2 = a^2 + b^2$, Triangle is right angled
  - if $c^2 > a^2 + b^2$, Triangle is obtuse angled
- Sum of the four interior angles of a quadrilateral = 360°
- If a quadrilateral can be inscribed in a circle, it is called a cyclic quadrilateral. Here opposite angles are supplementary.
- In any polygon, the sum of exterior angles = 360°
- In any polygon, the sum of Interior angles = (2n – 4) 90°
- Angle in a semicircle is a right angle.
- In a circle, angle at the centre made by an arc = twice the angle made by the arc at any point on the remaining part of the circumference.

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We have $\angle APB = \frac{1}{2} \angle AOB = 30° = \angle AQB$

1. If $x - q = s - y$, what is the value of z?

Figures are not drawn to scale.

    1) $xq + sy + sx + yq = zr$ 2) $zq - ry = rx - zs$

2. What is the perimeter of isosceles triangle ABC?
    (1) The length of side AB is 9   (2) The length of side BC is 4

3. Triangle A has one side of length x. If $\sqrt{(x^8)} = 81$, what is the perimeter of Triangle A?
    (1) Triangle A has sides whose lengths are consecutive integers
    (2) Triangle A is NOT a right triangle

4. Which of the following is a possible length for side AB of triangle ABC if $AC = 6$ and $BC = 9$?
    I. 3   II. $9\sqrt{3}$ III. 13.5

    I only   II only   III only   II and III   I, II and III

5. If triangle ABD is an equilateral triangle and $AB = 6$ and $CE = 18$, what fraction of the trapezoid $BACE$ is shaded?

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6. The height of isosceles trapezoid ABDC where AC = BD and AB is parallel to CD (and AC is not parallel to BD) is 12 units. The length of diagonal AD is 15 units. What is the area of trapezoid ABDC?
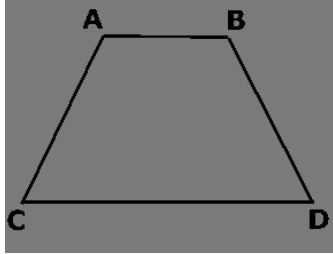
   (A) 72   (B) 90   (C) 96   (D) 108   (E) 180

7. In Quadrilateral WXYZ, is XY perpendicular to YZ?   (1) WXY is a right angle   (2) WZY is a right angle

8. In isosceles triangle RST what is the measure of angle R?
   (1) The measure of angle T is 100 degrees.   (2) The measure of angle S is 40 degrees.

Concept 5: Co-ordinates

Co-ordinates:

- If 2 points (*a*, *b*) and (*c*, *d*) lie in the same quadrant, then *a* and *c* should have the same sign; and *b* and *d* should have the same sign.
- Distance between 2 points (x_1, y_1) and (x_2, y_2) is given by
- Distance of the point (*a*, *b*) from the origin (0, 0) is given by
- Any point on the X axis can be taken as (*a*, 0)
- Any point on the Y axis can be taken as (0, *b*)
- In order to find the X–intercept of a line, put Y = 0 in the equation of the line and find X
- In order to find the Y intercept of a line, put X = 0 and find Y.
- To plot a line, first put y = 0, find the point on x axis; then put x = 0, fins the point on y axis. Join the two points to get the desired graph.


Straight line (slope):

- An equation of the form AX + BY + C = 0 is called the general equation of a straight line, where X and Y are variables and A, B, C are constants.
- Any point lying on this line will satisfy the equation of the line. i.e., the coordinates of the point when substituted by X & Y resp. in the above equation will make the LHS vanish.
- If AB is a straight line on the XY plane, then the ratio of y intercept to x intercept (with signs) is called its slope and is denoted by 'm'. The lengths OP and OQ are respectively called the intercepts on X and Y axes, made by the line.
- So slope = RISE / RUN   So,   Slope = (y_2 – y_1) / (x_2 – x_1)
- If 'm' is the slope of the line and 'c' the intercept made by the line on Y axis, the equation is Y = mX + c
- If the slope of the line is m and it passes through (X1, Y1), the equation is (Y – Y_1) = m (X – X_1)
- General form: AX + BY + C = 0 In this slope = m = –A/B, X intercept = –C/A Y intercept = –C/B
- If two lines are parallel then their slopes are equal (m_1 = m_2). If two lines do not intersect, they are parallel.
- If the slopes of two lines are equal (m_1 = m_2), then either the lines are parallel (no intersection) or they are co-incident (infinite points of intersection).

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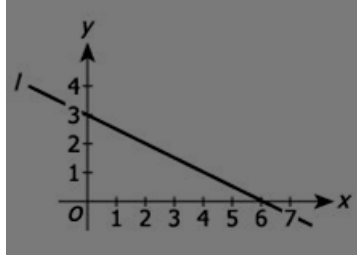
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- o If two lines are perpendicular to each other, the product of their slopes is −1. ($m_1 m_2 = −1$). **(There is one exception to this rule – when two lines are parallel to x and y axis respectively, then $m_1 m_2 ≠ −1$, even if the lines are perpendicular).**
- o $a_1X + b_1Y + c_1 = 0$ and $a_2X + b_2Y + c_2 = 0$ will represent the same straight lines if $a_1/a_2 = b_1/b_2 = c_1/c_2$. In this case, the lines are coincident and theoretically intersect at infinite points.
- o The point of intersection of two lines (X, Y) is obtained by simultaneously solving both the equations.
- o The length of perpendicular (p) from (X1, Y1) on the line AX + BY + C = 0 is P = $|ax_1 + by_1 + c /\sqrt{(a^2 + b^2)}|$ Equation of a line parallel to X axis is Y = b (b is a constant)
- o Equation of a line parallel to Y axis is X = a (a is a constant)
- o Equation of X and Y axes are Y = 0 and X = 0 respectively
- o The image of the point (a, b) in x axis is (a, -b)
- o The image of the point (a, b) in y axis is (-a, b)
- o The image of the point (a, b) in the line y = x is (b, a)
- o To plot a line, first put y = 0, find the point on x axis; then put x = 0, fins the point on y axis. Join the two points to get the desired graph.

Questions

1. If $ab ≠ 0$ and points $(−a, b)$ and $(−b, a)$ are in the same quadrant of the xy–plane, is point $(−x, y)$ in this same quadrant?　　(1) $xy > 0$　　　　(2) $ax > 0$

2. In the xy–plane, at what two points does the graph of y = (x + a) (x + b) intersect the x–axis?
(1) a + b = −1　　　　(2) The graph intersects the y–axis at (0, −6).

3. In the rectangular coordinate system, are the points (r, s) and (u, v) equidistant from the origin?
(1) r + s = 1　　　　(2) u = 1 − r and v = 1 − s

4. Point K = (A, 0), Point G = $\left(2A+4, \sqrt{2A+9}\right)$. Is the distance between point K and G prime?
(1) $A^2 − 5A − 6 = 0$　　　　(2) A > 2

5. If points *A* and *B* are on the *y*–axis in the figure, what is the area of equilateral triangle *ABC*?
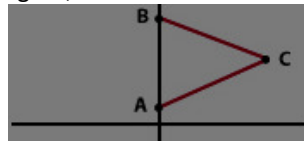
(1) Coordinates of point *B* are (0, 5√3). (2) Coordinates of point *C* are (6, 3√3).

6. The line 3*x* + 4*y* = 8 passes through all of the quadrants in the coordinate plane except:
I　　　　　　II　　　　　　III　　　　　　IV　　　　　　II and IV.

7. If *p* and *q* are nonzero numbers, and *p* is not equal to *q*, in which quadrant of the coordinate system does point (*p*, *p* − *q*) lie?　　(1) (*p*, *q*) lies in quadrant IV.　　　　(2) (*q*, −*p*) lies in quadrant III.

8. All points (x, y) that lie below the line *l*, shown above, satisfy which of the following inequalities?

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y < 2x + 3   y < −2x + 3   y < −x + 3   y < ½ x + 3   y < -½ x + 3

9.   In the XY plane, does the line with equation y = 3x + 2 contain the point (r, s)?
        (1) (3r + 2 - s) (4r + 9 - s) = 0   (2) (4r - 6 - s) (3r + 2 - s) = 0

10.   In the x–y plane, what is the y–intercept of the line l?
        (1) Slope of the line l is 3 times its y intercept.   (2) The x–intercept of line l is −1/3

11.   In the xy–plane, line k has positive slope and x–intercept 4. If the area of the triangle formed by line k and the two axes is 12, what of the y– intercept of line?

12.   Line l is defined by the equation $y - 5x = 4$ and line w is defined by the equation $10y + 2x + 20 = 0$. If line k does not intersect line l, what is the degree measure of the angle formed by line k and line w?
        0   30   60   90   It cannot be determined from the information given.

13.   Does line S intersect line segment QR?
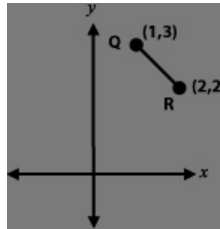
        (1) The equation of line S is $y = -x + 4$.   (2) The slope of line S is −1.

14.   In the rectangular coordinate system, does the line k (not shown) intersect quadrant II?
        (1) Slope of k is -1/6   (2) The y-intercept of k is -6

15.   In the xy-plane, the line k passes through the origin and through point (a, b), where $ab \neq 0$. Is b positive?
        (1) The slope of k is negative   (2) a < b

16.   In the XY-coordinate plane, line L and line K intersect at the point (4, 3). Is the product of their slopes negative?
        (1) The product of the x-intercepts of line L and K is positive.
        (2) The product of the y-intercepts of line L and k is negative.

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