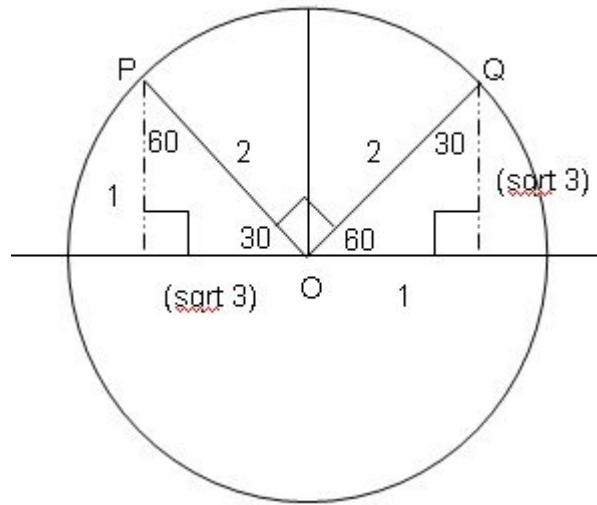




**Concept # 1: 30-60-90 Right angled triangle:**

1. First, see that after dropping perpendicular lines, we have two right triangles.



Let's begin with the triangle on the left.

We know the sides are 1 and  $(\sqrt{3})$  from point P.

If you know your special right triangles, you will quickly see that this is a 30-60-90 right triangle.

The angle opposite '1' is 30 degrees.

Let's move on to the triangle on the right.

We know that a straight line has 180 degrees.

Since we know the lower angle of the triangle on the left is 30 degrees, and we also know the angle between the two line segments is 90 degrees, the lower angle of the triangle on the right must be 60 degrees in order to sum to 180 degrees. ( $30 + 90 + x = 180$ ;  $x = 60$ )

This means the triangle on the right is also a 30-60-90 triangle. The hypotenuse of this triangle is the same as the other triangle's (which is '2' by the Pythagorean Theorem), since both are radii of the same circle.

Using the same properties of a 30-60-90 triangle, you can find the side lengths and finally the point (s,t) which gives the value for  $s = 1$ .

2. Triangle  $DBC$  is inscribed in a semicircle (that is, the hypotenuse  $CD$  is a diameter of the circle). Therefore, angle  $DBC$  must be a right angle and triangle  $DBC$  must be a right triangle.

(1) SUFFICIENT: If the length of  $CD$  is twice that of  $BD$ , then the ratio of the length of  $BD$  to the length of the hypotenuse  $CD$  is  $1 : 2$ . Knowing that the side ratios of a 30-60-90 triangle are  $1 : \sqrt{3} : 2$ , where 1 represents the short leg,  $\sqrt{3}$  represents the long leg, and 2 represents the hypotenuse, we can conclude that triangle  $DBC$  is a 30-60-90 triangle. Since side  $BD$  is the short leg, angle  $x$ , the angle opposite the short leg, must be the smallest angle (30 degrees).

(2) SUFFICIENT: If triangle  $DBC$  is inscribed in a semicircle, it must be a right triangle. So, angle  $DBC$  is 90 degrees. If  $y = 60$ ,  $x = 180 - 90 - 60 = 30$ .

The correct answer is D.

3. In order to find the area of the triangle, we need to find the lengths of a base and its associated height. Our strategy will be to prove that  $ABC$  is a right triangle, so that  $CB$  will be the base and  $AC$  will be its associated height.

(1) INSUFFICIENT: We now know one of the angles of triangle  $ABC$ , but this does not provide sufficient information to solve for the missing side lengths.

(2) INSUFFICIENT: Statement (2) says that the circumference of the circle is  $18\pi$ . Since the circumference of a circle equals  $\pi$  times the diameter, the diameter of the circle is 18. Therefore  $AB$  is a diameter. However, point  $C$  is still free to "slide" around the circumference of the circle giving different areas for the triangle, so this is still insufficient to solve for the area of the triangle.

(1) AND (2) SUFFICIENT: Note that inscribed triangles with one side on the diameter of the circle must be right triangles. Because the length of the diameter indicated by Statement (2) indicates that segment  $AB$  equals the diameter, triangle  $ABC$  must be a right triangle. Now, given Statement (1) we recognize that this is a 30-60-90 degree triangle. Such triangles always have side length ratios of

$$1 : \sqrt{3} : 2$$

Given a hypotenuse of 18, the other two segments  $AC$  and  $CB$  must equal 9 and  $9\sqrt{3}$  respectively. This gives us the base and height lengths needed to calculate the area of the triangle, so this is sufficient to solve the problem.

The correct answer is C.

### Concept # 2: 45-45-90 Right angled triangle (isosceles right triangle):

1. The question stem tells us that  $ABCD$  is a rectangle, which means that triangle  $ABE$  is a right triangle.

The formula for the area of any triangle is:  $1/2 (\text{Base} \times \text{Height})$ .

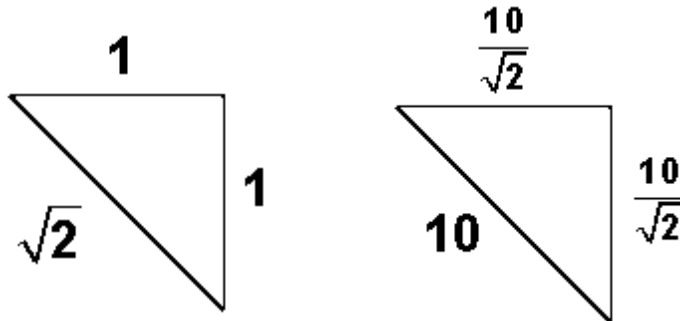
In right triangle  $ABE$ , let's call the base  $AB$  and the height  $BE$ . Thus, we can rephrase the questions as follows: **Is  $1/2 (AB \times BE)$  greater than 25?**

Let's begin by analyzing the first statement, taken by itself. Statement (1) tells us that the length of  $AB = 6$ .

While this is helpful, it provides no information about the length of BE. Therefore there is no way to determine whether the area of the triangle is greater than 25 or not.

Now let's analyze the second statement, taken by itself. Statement (2) tells us that length of diagonal AE = 10. We may be tempted to conclude that, like the first statement, this does not give us the two pieces of information we need to know (that is, the lengths of AB and BE respectively). However, knowing the length of the diagonal of the right triangle actually does provide us with some very relevant information about the lengths of the base (AB) and the height (BE).

**The right triangle with the largest area will be an isosceles right triangle (where both the base and height are of equal length).**



Now, we can calculate the area of this isosceles right triangle:

$$\frac{1}{2}(AB \times BE) = \frac{1}{2}\left(\frac{10}{\sqrt{2}} \times \frac{10}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{100}{2}\right) = \frac{1}{2}(50) = 25$$

Since an isosceles right triangle will yield the maximum possible area, we know that 25 is the maximum possible area of a right triangle with a diagonal of length 10.

Of course, we don't really know if 25 is, in fact, the area of triangle ABE, but we do know that 25 is the maximum possible area of triangle ABE. Therefore we are able to answer our original question: Is the area of triangle ABE greater than 25? *NO it is not greater than 25, because the maximum area is 25.*

Since we can answer the question using Statement (2) alone, the correct answer is B.

- Let the hypotenuse be  $x$ , then the length of the leg is  $x/\sqrt{2}$ .

$$x + 2x/\sqrt{2} = 16 + 16\sqrt{2}$$

$$x + \sqrt{2}x = 16 + 16\sqrt{2}$$

$$\text{So, } x = 16$$

### Concept # 3: Right Angled / Similar Triangles

- USE  $h^2 = mn$ , so  $4^2 = 3 * x$  so  $x = 16/3$  OR

Because angles  $BAD$  and  $ACD$  are right angles, the figure above is composed of three *similar* right triangles:  $BAD$ ,  $ACD$  and  $BCA$ . [Any time a height is dropped from the right angle vertex of a right triangle to the opposite side of that right triangle, the three triangles that result have the same 3 angle measures. This means that they are similar triangles.] To solve for the length of side  $CD$ , we can set up a proportion, based on the relationship between the similar triangles  $ACD$  and  $BCA$ :  $BC/AC = CA/CD$  or  $3/4 = 4/CD$  or  $CD = 16/3$ . The correct answer is D.

2. Since  $BE \parallel CD$ , triangle  $ABE$  is similar to triangle  $ACD$  (parallel lines imply two sets of equal angles). We can use this relationship to set up a ratio of the respective sides of the two triangles:

$$\frac{AB}{AC} = \frac{AE}{AD}$$

$$\frac{3}{6} = \frac{4}{AD}$$

So  $AD = 8$ .

We can find the area of the trapezoid by finding the area of triangle  $CAD$  and subtracting the area of triangle  $ABE$ .

Triangle  $CAD$  is a right triangle since it has side lengths of 6, 8 and 10, which means that triangle  $BAE$  is also a right triangle (they share the same right angle).

Area of trapezoid = area of triangle  $CAD$  – area of triangle  $BAE$

$$= (1/2)bh - (1/2)bh$$

$$= 0.5(6)(8) - 0.5(3)(4)$$

$$= 24 - 6$$

$$= 18$$

The correct answer is B

3. For GMAT triangle problems, one useful tool is the similar triangle strategy. Triangles are defined as similar if all their corresponding angles are equal or if the lengths of their corresponding sides have the same ratios.

(1) INSUFFICIENT: Just knowing that  $x = 60^\circ$  tells us nothing about triangle  $EDB$ . To illustrate, note that the exact location of point  $E$  is still unknown. Point  $E$  could be very close to the circle, making  $DE$  relatively short in length. However, point  $E$  could be quite far away from the circle, making  $DE$  relatively long in length. We cannot determine the length of  $DE$  with certainty.

(2) SUFFICIENT: If  $DE$  is parallel to  $CA$ , then  $(\text{angle } EDB) = (\text{angle } ACB) = x$ . Triangles  $EBD$  and  $ABC$  also share the angle  $ABC$ , which of course has the same measurement in each triangle. Thus, triangles  $EBD$  and  $ABC$  have two angles with identical measurements. Once you find that triangles have 2 equal angles, you know that the third angle in the two triangles must also be equal, since the sum of the angles in a triangle is  $180^\circ$ .

So, triangles  $EBD$  and  $ABC$  are similar. This means that their corresponding sides must be in proportion:

$$CB/DB = AC/DE$$

$$\text{radius/diameter} = \text{radius}/DE$$

$$3.5/7 = 3.5/DE$$

Therefore,  $DE = \text{diameter} = 7$ .

The correct answer is B.

4. Use  $A1 / A2 = (L1 / L2)^2$  So we have  $1/12 = (3/(3+x))^2$  or  $1/\sqrt{12} = 3/(3+x)$  or  $1/2\sqrt{3} = 3/(3+x)$  so  $x = 6\sqrt{3} - 3$ .
5. We are given a right triangle that is cut into four smaller right triangles. Each smaller triangle was formed by drawing a perpendicular from the right angle of a larger triangle to that larger triangle's hypotenuse. When a

right triangle is divided in this way, two similar triangles are created. And each one of these smaller similar triangles is also similar to the larger triangle from which it was formed.

Thus, for example, triangle  $ABD$  is similar to triangle  $BDC$ , and both of these are similar to triangle  $ABC$ . Moreover, triangle  $BDE$  is similar to triangle  $DEC$ , and each of these is similar to triangle  $BDC$ , from which they were formed. If  $BDE$  is similar to  $BDC$  and  $BDC$  is similar to  $ABD$ , then  $BDE$  must be similar to  $ABD$  as well.

Remember that similar triangles have the same interior angles and the ratio of their side lengths are the same. So the ratio of the side lengths of  $BDE$  must be the same as the ratio of the side lengths of  $ABD$ . We are given the hypotenuse of  $BDE$ , which is also a leg of triangle  $ABD$ . If we had even one more side of  $BDE$ , we would be able to find the side lengths of  $BDE$  and thus know the ratios, which we could use to determine the sides of  $ABD$ .

(1) SUFFICIENT: If  $BE = 3$ , then  $BDE$  is a 3-4-5 right triangle.  $BDE$  and  $ABD$  are similar triangles, as discussed above, so their side measurements have the same proportion. Knowing the three side measurements of  $BDE$  and one of the side measurements of  $ABD$  is enough to allow us to calculate  $AB$ .

To illustrate:

$BD = 5$  is the hypotenuse of  $BDE$ , while  $AB$  is the hypotenuse of  $ABD$ .

The longer leg of right triangle  $BDE$  is  $DE = 4$ , and the corresponding leg in  $ABD$  is  $BD = 5$ .

Since they are similar triangles, the ratio of the longer leg to the hypotenuse should be the same in both  $BDE$  and  $ABD$ .

For  $BDE$ , the ratio of the longer leg to the hypotenuse =  $4/5$ .

For  $ABD$ , the ratio of the longer leg to the hypotenuse =  $5/AB$ .

Thus,  $4/5 = 5/AB$ , or  $AB = 25/4 = 6.25$

(2) SUFFICIENT: If  $DE = 4$ , then  $BDE$  is a 3-4-5 right triangle. This statement provides identical information to that given in statement (1) and is sufficient for the reasons given above.

The correct answer is D.

#### 6. In SIMILAR FIGURES, the RATIO OF AREAS is (RATIO OF LENGTHS)<sup>2</sup>

In SIMILAR SOLIDS, the RATIO OF VOLUMES is (RATIO OF LENGTHS)<sup>3</sup>

In SIMILAR SOLIDS, the RATIO OF SURFACE AREAS is (RATIO OF LENGTHS)<sup>2</sup>

So in similar figures: if length ratio =  $a : b$ , then area ratio =  $a^2 : b^2$

In similar 3-d solids: length ratio =  $a : b$ , surface area ratio =  $a^2 : b^2$ , volume ratio =  $a^3 : b^3$

In this problem, you have  $a^2 : b^2 = 2 : 1$ . If you know the result(s) above, then it follows at once that  $a : b$  (the ratio of lengths, which is what you're looking for) is  $\sqrt{2} : 1$ . Ans. C.

#### Concept # 4: Lines and Angles:

1. We are given two triangles and asked to determine the degree measure of  $z$ , an angle in one of them.

The first step in this problem is to analyze the information provided in the question stem. We are told that  $x - q = s - y$ . We can rearrange this equation to yield  $x + y = s + q$ . Since  $x + y + z = 180$  and since  $q + s + r = 180$ , it must be true that  $z = r$ . We can now look at the statements.

Statement (1) tells us that  $xq + sy + sx + yq = zr$ . In order to analyze this equation, we need to rearrange it to facilitate factorization by grouping like terms:  $xq + yq + sx + sy = zr$ . Now we can factor:

$$\begin{aligned}xq + yq + sx + sy &= zr \rightarrow \\q(x + y) + s(x + y) &= zr \rightarrow \\(x + y)(q + s) &= zr\end{aligned}$$

Since  $x + y = q + s$  and  $z = r$ , we can substitute and simplify:

$$\begin{aligned}(x + y)(q + s) &= zr \rightarrow \\(x + y)(x + y) &= (z)(z) \rightarrow \\\sqrt{(x + y)^2} &= \sqrt{z^2} \rightarrow \\x + y &= z\end{aligned}$$

Is this sufficient to tell us the value of  $z$ ? Yes. Why? Consider what happens when we substitute  $z$  for  $x + y$ :

$$\begin{aligned}x + y + z &= 180 \rightarrow \\z + z &= 180 \rightarrow \\2z &= 180 \rightarrow \\z &= 90\end{aligned}$$

It is useful to remember that when the sum of two angles of a triangle is equal to the third angle, the triangle must be a right triangle. Statement (1) is sufficient.

Statement (2) tells us that  $zq - ry = rx - zs$ . In order to analyze this equation, we need to rearrange it:

$$\begin{aligned}zq - ry &= rx - zs \rightarrow \\zq + zs &= rx + ry \rightarrow \\z(q + s) &= r(x + y) \rightarrow \\z &= \frac{r(x + y)}{(q + s)} \rightarrow \\\frac{z}{r} &= \frac{x + y}{q + s}\end{aligned}$$

Is this sufficient to tell us the value of  $z$ ? No. Why not? Even though we know the following:

$$\begin{aligned}z &= r \\x + y &= q + s \\x + y + z &= 180 \\q + r + s &= 180\end{aligned}$$

we can find different values that will satisfy the equation we derived from statement (2):

$$\frac{90}{90} = \frac{30 + 60}{40 + 50}$$

or

$$\frac{100}{100} = \frac{40 + 40}{10 + 70}$$

These are just two examples. We could find many more. Since we cannot determine the value of  $z$ , statement (2) is insufficient.

The correct answer is A: Statement (1) alone is sufficient, but statement (2) is not.

2. The perimeter of a triangle is equal to the sum of the three sides.

(1) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

(2) INSUFFICIENT: Knowing the length of one side of the triangle is not enough to find the sum of all three sides.

Together, the two statements are SUFFICIENT. Triangle ABC is an isosceles triangle which means that there are theoretically 2 possible scenarios for the lengths of the three sides of the triangle: (1)  $AB = 9$ ,  $BC = 4$  and the third side,  $AC = 9$  OR (1)  $AB = 9$ ,  $BC = 4$  and the third side  $AC = 4$ .

These two scenarios lead to two different perimeters for triangle ABC, HOWEVER, upon careful observation we see that the second scenario is an IMPOSSIBILITY. A triangle with three sides of 4, 4, and 9 is not a triangle. Recall that any two sides of a triangle must sum up to be greater than the third side.  $4 + 4 < 9$  so these are not valid lengths for the side of a triangle.

Therefore the actual sides of the triangle must be  $AB = 9$ ,  $BC = 4$ , and  $AC = 9$ . The perimeter is 22.

The correct answer is C.

3. By simplifying the equation given in the question stem, we can solve for  $x$  as follows:

$$\begin{aligned}\sqrt{x^8} &= 81 \\ x^4 &= 81 \\ x &= 3\end{aligned}$$

Thus, we know that one side of Triangle A has a length of 3.

Statement (1) tells us that Triangle A has sides whose lengths are consecutive integers. Given that one of the sides of Triangle A has a length of 3, this gives us the following possibilities: (1, 2, 3) OR (2, 3, 4) OR (3, 4, 5). However, the first possibility is NOT a real triangle, since it does not meet the following condition, which is true for all triangles: The sum of the lengths of any two sides of a triangle must always be greater than the length of the third side. Since  $1 + 2$  is not greater than 3, it is impossible for a triangle to have side lengths of 1, 2 and 3.

Thus, Statement (1) leaves us with two possibilities. Either Triangle A has side lengths 2, 3, 4 and a perimeter of 9 OR Triangle A has side lengths 3, 4, 5 and a perimeter of 12. Since there are two possible answers, Statement (1) is not sufficient to answer the question.

Statement (2) tells us that Triangle A is NOT a right triangle. On its own, this is clearly not sufficient to answer the question, since there are many non-right triangles that can be constructed with a side of length 3.

Taking both statements together, we can determine the perimeter of Triangle A. From Statement (1) we know

that Triangle A must have side lengths of 2, 3, and 4 OR side lengths of 3, 4, and 5. Statement (2) tells us that Triangle A is not a right triangle; this eliminates the possibility that Triangle A has side lengths of 3, 4, and 5 since any triangle with these side lengths is a right triangle (this is one of the common Pythagorean triples). Thus, the only remaining possibility is that Triangle A has side lengths of 2, 3, and 4, which yields a perimeter of 9.

The correct answer is C: BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.

4. The third side of a triangle must be *less* than the *sum* of the other two sides and *greater* than their difference (i.e.  $|y - z| < x < y + z$ ).

In this question:

$$|BC - AC| < AB < BC + AC$$

$$9 - 6 < AB < 9 + 6$$

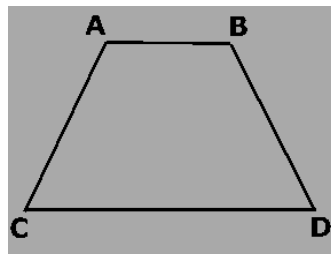
$$3 < AB < 15$$

Only 13.5 is in this range.  $9\sqrt{3}$  is approximately equal to  $9(1.7)$  or 15.3.

The correct answer is C.

5. The area of triangle ABD =  $(1/2)bh = (1/2)(6)h$   
 The area of trapezoid BACE =  $(1/2)(6 + 18)h$   
 Ratio =  $6/24 = 1/4$

6.

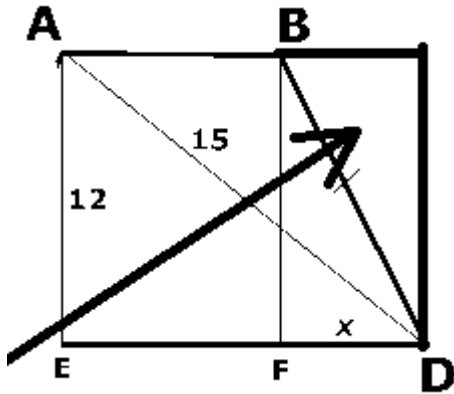


Innovative Approach

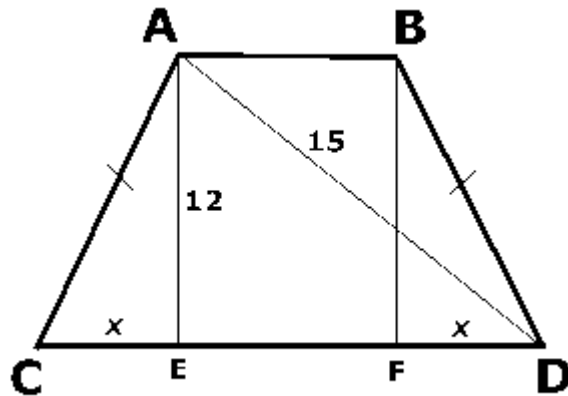
We can use the Pythagorean Theorem to see the ED = 9.

Imagine that the left triangle were to be shifted and made the way it is mentioned in the diagram below: then we just have to find the area of the rectangle with sides 12 and 9... ans.  $12 \times 9 = 108$ .





Detailed Solution:



By sketching a drawing of trapezoid ABDC with the height and diagonal drawn in, we can use the Pythagorean theorem to see the  $ED = 9$ . We also know that ABDC is an isosceles trapezoid, meaning that  $AC = BD$ ; from this we can deduce that  $CE = FD$ , a value we will call  $x$ . The area of a trapezoid is equal to the average of the two bases multiplied by the height.

The bottom base, CD, is the same as  $CE + ED$ , or  $x + 9$ . The top base, AB, is the same as  $ED - FD$ , or  $9 - x$ .

Thus the average of the two bases is  $\frac{(x+9) + (9-x)}{2} = \frac{18}{2} = 9$ .

Multiplying this average by the height yields the area of the trapezoid:  $9 \times 12 = 108$ .

The correct answer is D.

7. They're asking whether the angle at Y is a right angle. Even if you have the two statements together, you only know that the angles at X (from statement 1) and at Z (from statement 2) are right angles. This isn't good enough; the angles at W and Y can be any two angles that add to 180 degrees. Should be (E).

8.

First Statement: If T is 100 degrees, it cannot be one of the equal angles of the isosceles triangle...because  $100 + 100 = 200 > 180$ , even before taking into account the third angle. So the remaining two angles have to be the equal ones i.e.  $180 - 100 = 80 / 2 = 40 = R = S$ . SUFFICIENT

Second Statement:  $S=40$ . From this we cannot be sure if S is one of the equal angles. If S is NOT one of the equal angles, then  $S=40$  and  $R=T=70$  ( $180-40 = 140/2$ ). If S was one of the equal angles then,  $S=40 = R$  or  $T$  i.e. If  $R = 40$ , then  $T = 110$  OR if  $T = 40$ , then  $R = 110$ . INSUFFICIENT

first of all, **WE DON'T KNOW WHICH TWO ANGLES ARE EQUAL**. there are two possibilities for an isosceles triangle with a 40° angle in it:

**(case 1)** 40°, 40°, 100° (if angle S = 40° is one of the two equal angles)

**(case 2)** 40°, 70°, 70° (if angle S = 40° is NOT one of the two equal angles)

worse yet - **it would still be insufficient even if only case (1) were possible!**

this is because there are two DIFFERENT angles - 100° and the other 40° - remaining, and *you don't know which of these is angle T*. i.e., angle T could still be either 40° or 100° in this case.

Ans. A

### **Concept 5: Co-ordinates**

If 2 points  $(a, b)$  and  $(c, d)$  lie in the same quadrant, then  $a$  and  $c$  should have the same sign; and  $b$  and  $d$  should have the same sign. So here  $(-a, -b)$  and  $(a, b)$  have the same sign, so eventually **GIVEN information says that  $a$  and  $b$  are of the same sign.**

**Asked:** are  $-a, -b, -x$  of the same sign and are  $b, a, y$  of the same sign? Combined we have:

**QUESTION:** Given  $a$  and  $b$  are of the same sign, are  $a, b, x$  and  $y$  all of the same sign?

**(1)  $x$  and  $y$  are of the same sign. NOT SUFFICIENT.**

**(2)  $a$  and  $x$  are of the same sign. NOT SUFFICIENT.**

Combined  $a, b, x$  and  $y$  are of the same sign. Ans. (C).

2. From 1,  $a+b=-1$ . From 2,  $x=0$ , so  $ab=6$ .  $(x+a)*(x+b)=0$   $x^2+(a+b)x+ab=0$

So,  $x=-3, x=2$  The answer is C.

3. We need to know whether  $r^2+s^2=u^2+v^2$  or not. From statement 2,

$$u^2+v^2=(1-r)^2+(1-s)^2=r^2+s^2+2-2(r+s)$$

Combined statement 1,  $r+s=1$ , we can obtain that  $r^2+s^2=u^2+v^2$ .

Answer is C.

4.

The distance between any two points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the coordinate plane is defined by the distance formula.

$$\begin{aligned} D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2A + 4 - A)^2 + (\sqrt{2A + 9} - 0)^2} \end{aligned}$$

$$\begin{aligned}
 &= \sqrt{(A+4)^2 + (\sqrt{2A+9}-0)^2} \\
 &= \sqrt{A^2 + 8A + 16 + 2A + 9} \\
 &= \sqrt{A^2 + 10A + 25} \\
 &= \sqrt{(A+5)^2} \\
 &= A+5
 \end{aligned}$$

Thus, the distance between point K and point G is  $A + 5$ .

Statement (1) tells us that:

$$\begin{aligned}
 A - 5A - 6 &= 0 \\
 (A - 6)(A + 1) &= 0
 \end{aligned}$$

Thus  $A = 6$  or  $A = -1$ .

Using this information, the distance between point K and point G is either 11 or 4. This is not sufficient to answer the question.

Statement (2) alone tells us that  $A > 2$ , which is not sufficient to answer the question.

When we combine both statements, we see that  $A$  must be 6, which means the distance between point K and point G is 11. This is a prime number and we are able to answer the question.

The correct answer is C.

5.

To find the area of equilateral triangle  $ABC$ , we need to find the length of one side. The area of an equilateral triangle can be found with just one side since there is a known ratio between the side and the height (using the 30: 60: 90 relationship). Alternatively, we can find the area of an equilateral triangle just knowing the length of its height.  $h = \frac{a\sqrt{3}}{2}$

(1) INSUFFICIENT: This does not give us the length of a side or the height of the equilateral triangle since we don't have the coordinates of point A.

(2) SUFFICIENT: Since  $C$  has an x-coordinate of 6, the height of the equilateral triangle must be 6.

The correct answer is B.

6. Put  $y = 0$ ,  $x$  is positive and put  $x = 0$  and  $y$  is positive. So both the  $x$  and the  $y$  intercepts are positive. By plotting the line, we may say that it does not pass through Quadrant III.

The correct answer is C.

7. To determine in which quadrant the point  $(p, p - q)$  lies, we need to know the sign of  $p$  and the sign of  $p - q$ .

(1) SUFFICIENT: If  $(p, q)$  lies in quadrant IV,  $p$  is positive and  $q$  is negative.  $p - q$  must be positive because a positive number minus a negative number is always positive [e.g.  $2 - (-3) = 5$ ].

(2) SUFFICIENT: If  $(q, -p)$  lies in quadrant III,  $q$  is negative and  $p$  is positive. (This is the same information that was provided in statement 1).

The correct answer is D.

8. The line passes through  $(6, 0)$  and  $(0, 3)$ . Assume the equation as  $y = mx + c$ , substitute these 2 points in the equation of the line and we get  $m = -1/2$  and  $c = 3$ . So the equation is  $y = -\frac{1}{2}x + 3$ . As we want below this line, the answer is E.

9. If  $(r, s)$  lies on the line, then we must have  $s = 3r + 2$ .

(1) gives  $s = 3r + 2$  or  $s = 4r + 9$  NS

(2) gives  $s = 4r - 6$  or  $s = 3r + 2$  NS

Combined  $s = 3r + 2$  only. Ans. C

10.  $y = mx + c$  ... what is  $c$ ?

$$1) m = 3c \quad -c/m = -1/3 \text{ so } m = 3c$$

Can't find  $c$ . So, answer is E

11. The two intersections:  $(0, 4)$  and  $(y, 0)$  So,  $4 * y / 2 = 12 \Rightarrow y = 6$

Slope is positive  $\Rightarrow y$  is below the  $x$ -axis  $\Rightarrow y = -6$

12. First, let's rewrite both equations in the standard form of the equation of a line:

Equation of line  $l$ :  $y = 5x + 4$

Equation of line  $w$ :  $y = -(1/5)x - 2$

Note that the slope of line  $w$ ,  $-1/5$ , is the negative reciprocal of the slope of line  $l$ . Therefore, we can conclude that line  $w$  is perpendicular to line  $l$ .

Next, since line  $k$  does not intersect line  $l$ , lines  $k$  and  $l$  must be parallel. Since line  $w$  is perpendicular to line  $l$ , it must also be perpendicular to line  $k$ . Therefore, lines  $k$  and  $w$  must form a right angle, and its degree measure is equal to 90 degrees.

The correct answer is D.

13. Lines are said to intersect if they share one or more points. In the graph, line segment  $QR$  connects points  $(1, 3)$  and  $(2, 2)$ . The slope of a line is the change in  $y$  divided by the change in  $x$ , or rise/run. The slope of line segment  $QR$  is  $(3 - 2)/(1 - 2) = 1/-1 = -1$ .

(1) SUFFICIENT: The equation of line  $S$  is given in  $y = mx + b$  format, where  $m$  is the slope and  $b$  is the  $y$ -intercept. The slope of line  $S$  is therefore  $-1$ , the same as the slope of line segment  $QR$ . Line  $S$  and line segment  $QR$  are parallel, so they will not intersect unless line  $S$  passes through both  $Q$  and  $R$ , and thus the entire segment. To determine whether line  $S$  passes through  $QR$ , plug the coordinates of  $Q$  and  $R$  into the equation of line  $S$ . If they satisfy the equation, then  $QR$  lies on line  $S$ .

Point  $Q$  is (1, 3):

$$y = -x + 4 = -1 + 4 = 3$$

Point  $Q$  is on line  $S$ .

Point  $R$  is (2, 2):

$$y = -x + 4 = -2 + 4 = 2$$

Point  $R$  is on line  $S$ .

Line segment  $QR$  lies on line  $S$ , so they share many points. Therefore, the answer is "yes," Line  $S$  intersects line segment  $QR$ .

(2) INSUFFICIENT: Line  $S$  has the same slope as line segment  $QR$ , so they are parallel. They might intersect; for example, if Line  $S$  passes through points  $Q$  and  $R$ . But they might never intersect; for example, if Line  $S$  passes above or below line segment  $QR$ .

The correct answer is A.

14. The  $1/6$  is irrelevant; all that matters in (1) is that the slope is negative. Lines with negative slopes go up to the left, down to the right. this means that, if you follow ANY negatively sloped line far enough to the left, it will go up into the second quadrant. **Negatively sloped lines MUST hit quadrants 2 and 4. Positively sloped lines MUST hit quadrants 1 and 3.** Zero exceptions.

With statement (2), a horizontal or positively sloped line through (0, -6) won't hit quadrant 2, but a negatively sloped line will, so that's insufficient. ANS. (A)

15. Question : Is  $b > 0$ ?

(1) says  $(b/a) < 0$

This could mean 2 things

- a) Either  $a > 0$  &  $b < 0$  OR
- b)  $a < 0$  &  $b > 0$

Both are of opposite signs.

This is not sufficient to ans the question. This eliminated A & D

(2)  $a < b$

This could mean

- a)  $a < 0$  &  $b > 0$  (eg  $-1 < 1$ )
- b)  $a > 0$  &  $b > 0$  (eg.  $2 < 3$ )
- c)  $a < 0$  &  $b < 0$  (eg  $-2 < -1$ )

This is also not sufficient. WE can eliminate B.

If we combine (1) and (2), we get 1(b) and 2(a) give a definite solution. We can ans the question. So C is the ans.

16. Assume that the lines are  $y = m_1x + c_1$  and  $y = m_2x + c_2$ . Question: is  $m_1m_2 = -ve$ ?

(1)  $(-c_1/m_1) * (-c_2/m_2) = +ve \dots$  so  $c_1c_2/m_1m_2 = +ve \dots$  NS

(2)  $c_1c_2 = -ve \dots$

Combining ...  $m_1m_2 = -ve \dots$  Suff. C

