Quant Concepts: Session 2: Absolute Values (Modulus) + Statistics - SOLUTIONS

1. The quick way to approach will be pick a number $x < 0$. Let's pick –5. So we know $x = -5$.
   $\sqrt{(-x|x|)} = \sqrt{(-(-5)|-5|)} = \sqrt{(5*5)} = \sqrt{(25)} = 5 = -(-5) = -x$ so Answer A.

   Shortcut: Seemingly the answer can be only x or –x … but the square root can't be negative … if x is given to be negative, -x will be positive. So the answer has to be –x.

2.
   As the value of $|x|$ is never negative, we may rephrase the question as: "Is $y - z > 0$?".
   (1) gives $y - z = -x$ NS
   (2) gives $x < 0$ NS
   Combine $y - z > 0$ Ans. YES
   Ans. C

3.
   As square roots cannot be negative, the question reduces to: "Is $5 - x > 0$?"
   (1) $-x\,|x| > 0$ means x is negative so $5 - x$ is positive … Suff.
   (2) $5 - x > 0$ … Suff.
   Ans. D

4. (1) y can have any values … suppose y = 1, 2, 3 … then for each value of y, we will get various values of x. Imagine taking the equal to sign, $|x - 3| = 1$ or 2 or 3 etc. we will not get a unique value of x. There will be infinite possible values of x.
   (2) $|x - 3| \leq -y$ … we can't take y as positive as $|x - 3|$ will become negative so the only value of y can be 0. When $y = 0$, $|x - 3| \leq 0$. As $|x - 3|$ can't be less than zero (by the definition of mods), so the only value of $|x - 3|$ can be 0, so if $|x - 3| = 0$, x will have a unique value as $x = 3$. Ans. B

5. We must realize that $(a - b)$ and $(b - a)$ are of opposite signs. So
   (1) if $a < b$, $a - b < 0$ so $1 / (a - b) < 0$, so $b - a > 0$
   So the question: Is $1 / (a-b) < (b-a)$" becomes "Is –ve < +ve, Answer Yes, so (1) is sufficient.
   (2) $|a - b| > 1$ … (a, b) can be (3, 1) and (1, 3)

   If (a, b) = (3, 1), we get $(a - b)$ as +ve so $1 / (a - b)$ as +ve and $(b - a)$ as –ve and the question "is $1 / (a-b) < (b-a)$?" becomes "is +ve < -ve" Ans. NO.

   If (a, b) = (1, 3), we get $(a - b)$ as -ve so $1 / (a - b)$ as -ve and $(b - a)$ as +ve and the question "is $1 / (a-b) < (b-a)$?" becomes "is -ve < +ve" Ans. YES. So Not sufficient. Ans. A

6. We can rephrase the question: "Is $-1 < x < 1$?"
   (1) $|a| = |b|$ means $a = b$ or $a = -b$. So we have

   $x + 1 = 2(x - 1)$ or $x = 3$
   OR
   $x + 1 = 2[-(x - 1)]$ or $x = 1/3$. If $x = 1/3$, $|x| < 1$, but if $x = 3$, $|x| > 1$. Thus, we cannot answer the question.

   (2) INSUFFICIENT: $|x - 3| > 0$ means $x \neq 3$.
   This does not answer the question as to whether x is between -1 and 1. x could be ½ and $|x| < 1$ or x could be 10 and $|x| > 1$.
   (1) AND (2) SUFFICIENT: According to statement (1), x can be 3 or 1/3. According to statement (2), x cannot be 3. Thus using both statements, we know that $x = 1/3$ which IS between -1 and 1.
   Ans. C

7. $y = x + |x|$ … so y depends on x … x can be –ve, 0, or +ve.
   If x is –ve, y = 0, if x = 0, y = 0, if x = +ve, y = +ve. So y can't be negative.
   Statement 1 is sufficient: If x is –ve, y = 0.
   Statement 2 is $y < 1$, since y is an integer, and it is never negative, it can be 0 only if it is less than 1.
   So Statement 2 is sufficient too. Answer is D

8. (1)
   Let's take (x, y) as (4, 2) … we get 2 = 2 … so we get NO for the main question.
   Let's take (x, y) as (4, -2) … we get 6 > 2 … so we get YES for the main question.
   Not sufficient.

   (2) means x and y are of opposite signs.

   In this case, $|x - y|$ will result in addition of x and y (overall positive sign) but $|x| - |y|$ will result in a subtraction of 2 positive quantities … hence LHS will always be bigger than RHS.

   Take all possible cases
   (1, -2) we get 3 > -1
   (2, -1) we get 3 > 1
   (-1, 2) we get 3 > -1
   (-2, 1) we get 3 > 1.
   Sufficient … Ans. B

9. (1) INSUFFICIENT: Since x^2 is positive whether x is negative or positive, we can only determine that x is not equal to zero; x could be either positive or negative.
(2) INSUFFICIENT: By telling us that the expression $x \cdot |y|$ is not a positive number, we know that it must either be negative or zero. If the expression is negative, x must be negative ($|y|$ is never negative). However if the expression is zero, x or y could be zero.
(1) AND (2) INSUFFICIENT: We know from statement 1 that x cannot be zero, however, there are still two possibilites for x: x could be positive (y is zero), or x could be negative (y is anything).

The correct answer is E.

10. It is extremely tempting to divide both sides of this inequality by y or by the |y|, to come up with a rephrased question of "is x > y?" However, we do not know the sign of y, so this cannot be done.

(1) INSUFFICIENT: On a yes/no data sufficiency question that deals with number properties (positive/negatives), it is often easier to plug numbers. There are two good reasons why we should try both positive and negative values for y: (1) the question contains the expression |y|, (2) statement 2 hints that the sign of y might be significant. If we do that we come up with both a yes and a no to the question.

x	y	$x \cdot \|y\| > y^2$	?
-2	-4	$-2(4) > (-4)^2$	N
4	2	$4(2) > 2^2$	Y

(2) If we know that y is positive (statement 2), we can divide both sides of the original question by y to come up with "is x > y?" as a new question. Statement 1 tells us that x > y, so both statements together are sufficient to answer the question. The correct answer is C.

11. The question is: "Is $-4 < n < 4$?" (n is not equal to 0)
(1) SUFFICIENT: The solution to this inequality is $n > 4$ (if $n > 0$) or $n < -4$ (if $n < 0$). This provides us with enough information to guarantee that n is definitely NOT between -4 and 4. Remember that an absolute no is sufficient!
(2) INSUFFICIENT: n can be any negative value. This is already enough to show that the statement is insufficient because n might not be between -4 and 4.
The correct answer is A.

12.
(1) Sufficient
$x + 3 = 4x – 3$ or $x = 2$ … valid solution
$-(x + 3) = 4x – 3$ or $x = 0$ … invalid solution.

We know that 2 is the only solution possible and we can say that x is definitely positive.

(2) INSUFFICIENT:
$x - 3 = 2x - 3$ so $x = 0$ (valid solution)
$x - 3 = -(2x - 3)$ … $x = 2$ (valid solution)
Therefore, both 2 and 0 are valid solutions and we cannot determine whether x is positive, since one value of x is zero, which is not positive, and one is positive. The correct answer is A.

13. Note that the question is asking for the absolute value of x rather than just the value of x. Keep this in mind when you analyze each statement.

(1) SUFFICIENT: Since the value of x^2 must be non-negative, the value of $(x^2 + 16)$ is always positive, therefore $|x^2 + 16|$ can be written $x^2 + 16$. Using this information, we can solve for x:
$|x^2 + 16| - 5 = 27$
$x^2 + 16 - 5 = 27$
$x^2 + 11 = 27$
$x^2 = 16$
$x = 4$ or $x = -4$
Since $|-4| = |4| = 4$, we know that $|x| = 4$; this statement is sufficient.

(2) SUFFICIENT:
$x^2 = 8x - 16$
$x^2 - 8x + 16 = 0$
$(x - 4)^2 = 0$
$(x - 4)(x - 4) = 0$
$x = 4$
Therefore, $|x| = 4$; this statement is sufficient. The correct answer is D.

14. Since $|r|$ is always positive, we can multiply both sides of the inequality by $|r|$ and rephrase the question as:
Is $r^2 < |r|$? The only way for this to be the case is if r is a nonzero fraction between -1 and 1.
(1) INSUFFICIENT: This does not tell us whether r is between -1 and 1. If $r = -1/2$, $|r| = 1/2$ and $r^2 = 1/4$, and the answer to the rephrased question is YES. However, if $r = 4$, $|r| = 4$ and $r^2 = 16$, and the answer to the question is NO.
(2) INSUFFICIENT: This does not tell us whether r is between -1 and 1. If $r = 1/2$, $|r| = 1/2$ ans $r^2 = 1/4$, and the answer to the rephrased question is YES. However, if $r = -4$, $|r| = 4$ and $r^2=16$, and the answer to the question is NO.
(1) AND (2) SUFFICIENT: Together, the statements tell us that r is between -1 and 1. The square of a proper fraction (positive or negative) will always be smaller than the absolute value of that proper fraction.
The correct answer is C.

Part 2: Statistics

Mean (Average)

For the below Q 1 to 3, follow the result:
For the situation given in the diagram below (for 2 groups having Means as M1 and M2 and Number of items as N1 and N2, so that the combination of the 2 groups results in Mean = M and Number of items = N1 + N2):

$$\begin{array}{ccccc} M1 & & M & & M2 \\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ & & | & & \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ N1 & & N1+N2 & & N2 \end{array}$$

$$\frac{N_1}{N_2} = \frac{M_2 - M}{M - M_1} = \frac{D_2}{D_1}$$

So to get the ratio of the number of items of the two groups, we need to know only M1, M2, and M or we need to know D2 (Difference M2 – M) and D1 (Difference M – M1)

1.

$$\begin{array}{ccccc} Managers & given\ D_1 = 5000 & M & given\ D_2 = 15000 & Directors \\ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ & & | & & \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ N_M & & N_M + N_D & & N_D \end{array}$$

So

$$\frac{N_M}{N_D} = \frac{D_2}{D_1} = \frac{15000}{5000} = \frac{3}{1}$$

Directors to Managers = 1: 3 or 25% and 75%. So Directors will be 25% always. Answer C.

2.

$$\frac{N_X}{N_Y} = \frac{M_2 - M}{M - M_1} = \frac{29.3 - 26.6}{26.6 - 25.7} = \frac{3}{1}$$

So X will have more members. Ans. C

3.

(1)

$$\frac{N_{sh}}{N_{sw}} = \frac{M_2 - M}{M - M_1} = \frac{25 - 21}{21 - 15} = \frac{2}{3}$$

So sweaters > shirts.

(2) $15x + 25y = 420$ or $3x + 5y = 84$. $(x, y) = (23, 3)$ and $(3, 15)$ both satisfy so $x > y$ or $x < y$. Not certain. Not sufficient.

Ans. A

Median

1. Let T, J, and S be the purchase prices for Tom's, Jane's, and Sue's new houses. Given that the average purchase price is 120,000, or T + J + S = (3)(120,000), determine the median purchase price.
   (1) Given T = 110,000, the median could be 120,000 (if J = 120,000 and S = 130,000) or 125,000 (if J = 125,000 and S = 125,000); NOT sufficient.
   (2) Given J = 120,000, the following two cases include every possibility consistent with T + J + S = (120,000), or T + S = (2)(120,000). (i) T = S = 120,000 (ii) One of T or S is less than 120,000 and the other is greater than 120,000. In each case, the median is clearly 120,000; SUFFICIENT. The correct answer is B; statement 2 alone is sufficient.

2. In order to solve the question easier, we simplify the numbers such as 150, 000 to 15, 130,000 to 13, and so on.
   I. Median is 13, so, the greatest possible value of sum of eight prices that no more than median is 13*8=104. Therefore, the least value of sum of other seven homes that greater than median is (15*15-104)/7=17.3>16.5. It's true.
   II. According the analysis above, the price could be, 13, 13, 13, 13, 13,13,13,13, 17.3, 17.3, 17.3... So, II is false.
   III. Also false.
   Answer: only I must be true.

3. To find the mean of the set {6, 7, 1, 5, *x*, *y*}, use the average formula: A = S/n where *A* = the average, *S* = the sum of the terms, and *n* = the number of terms in the set. Using the information given in statement (1) that *x* + *y* = 7, we can find the mean as 4.33: Regardless of the values of *x*

   and *y*, the mean of the set is 4.33 because the sum of *x* and *y* does not change. To find the median, list the possible values for *x* and *y* such that *x* + *y* = 7. For each case, we can calculate the median.

x	*y*	DATA SET	MEDIAN
1	6	1, 1, 5, 6, 6, 7	5.5
2	5	1, 2, 5, 5, 6, 7	5
3	4	1, 3, 4, 5, 6, 7	4.5
4	3	1, 3, 4, 5, 6, 7	4.5
5	2	1, 2, 5, 5, 6, 7	5
6	1	1, 1, 5, 6, 6, 7	5.5

   Regardless of the values of *x* and *y*, the median (4.5, 5, or 5.5) is always greater than the mean (4.33). Therefore, statement (1) alone is sufficient to answer the question. Now consider statement (2). Because the sum of *x* and *y* is not fixed, the mean of the set will vary. Additionally, since there are many possible values for *x* and *y*, there are numerous possible medians. The following table illustrates that we can construct a data set for which *x* – *y* = 3 and the *mean* is greater than the median. The table ALSO shows that we can construct a data set for which *x* – *y* = 3 and the *median* is greater than the mean.

x	Y	DATA SET	MEDIAN	MEAN
22	19	1, 5, 6, 7, 19, 22	6.5	10
4	1	1, 1, 4, 5, 6, 7	4.5	4

Thus, statement (2) alone is not sufficient to determine whether the mean is greater than the median. The correct answer is (A): Statement (1) alone is sufficient, but statement (2) alone is not sufficient.

4. This is another problem about a topic that is one of the darlings of the test authors: namely, sets of consecutive integers, especially as pertaining to the averages of such sets.

   Here is the fact that you absolutely must know about these sets:
   the **mean** and the **median** of a set of consecutive integers are **the same**; both of them are the middle number (for a set with an odd # of numbers in it) or halfway between the two middle numbers (for a set with an even # of numbers in it).
   --
   remember that average = sum / number of data points. you should be ultra-aware of this relationship; the vast majority of problems about the sum of a set are really concerned with the average - and vice versa.

   Let 'X' stand for the sum of each of the sets.

   (1) is clearly insufficient, as we know nothing whatsoever about set t.
   still, take the time to interpret it: it says that the middle number of set s is 0, which also means that the sumof the elements in set s is 0 (by the fact above).

   (2)
   INCORRECT LOGIC: "the sum of these two sets(sets of consecutive integers) will be equal only when the sum is zero.

   Ex: Set S could be 5,6,7,8 & 9 while Set T could be 2,3,4,5.6.7 & 8 and these sets have equal sums. There are umpteen other examples.

   Using the fact above, we have that the average (whether mean or median - they're the same) of the numbers in set s is X/5, and the average (again, mean or median) of the numbers in set t is X/7.
   It's tempting to say 'sufficient' here, because at first glance X/5 and X/7 appear to be necessarily different, but they aren't: in the singular case X = 0, the two will be identical.
   Therefore, insufficient.

   (together)
   this tells us that X = 0, which means that the median of both sets is 0/7 = 0/5 = 0.
   sufficient.

5.

   Statement (1) tells us that 25 percent of the projects had 4 or more employees assigned. There is no information given about the middle values of the number of employees per project. Hence statement (1) is insufficient.
   Statement (2) tells us that 35 percent of the projects have 2 or fewer employees but there is no information about the middle values of the number of employees per project. Hence statement (2) is insufficient.
   Combining statement (1) and (2), we can gather than 100 – (25+35) = 40 percent of the projects have exactly 3 employees. Therefore , when listing the number of employees per project in ascending order,

35 percent of the numbers are less than 2 and 36th to the 75th projects (overall 40 projects in the middle) will have 3 employees each …

So (35 + 40) = 75 percent are 3 or less. Since the median lies in that middle 40 percent, the median is 3. Hence the correct answer is (C).

Range + SD

1. Before analyzing the statements, let's consider different scenarios for the range and the median of set A. Since we have an even number of integers in the set, the median of the set will be equal to the average of the two middle numbers. Further, note that integer 2 is the only even prime and it cannot be one of the two middle numbers, since it is the smallest of all primes. Therefore, both of the middle primes will be odd, their sum will be even, and their average (i.e. the median of the set) will be an integer. However, while we know that the median will be an integer, it is unknown whether this integer will be even or odd. For example, the average of 7 and 17 is 12 (even), while the average of 5 and 17 is 11 (odd). Next, let's consider the possible scenarios with the range. Remember that the range is the difference between the greatest and the smallest number in the set. Since we are dealing with prime numbers, the greatest prime in the set will always be odd, while the smallest one can be either odd or even (i.e. 2). If the smallest prime in the set is 2, then the range will be odd, otherwise, the range will be even. Now, let's consider these scenarios in light of each of the statements.

   (1) SUFFICIENT: If the smallest prime in the set is 5, the range of the set, i.e. the difference between two odd primes in this case, will be even. Since the median of the set will always be an integer, the product of the median and the range will always be even.

   (2) INSUFFICIENT: If the largest integer in the set is 101, the range of the set can be odd or even (for example, 101 – 3 = 98 or 101 – 2 = 99). The median of the set can also be odd or even, as we discussed. Therefore, the product of the median and the range can be either odd or even. The correct answer is A.
2. **You must read the word 'different'**
   Prior to median 25, there are 7 numbers.
   To make the greatest number as greater as possible, these 7 numbers should cost the range as little as possible. They will be, 24, 23, 22, 21, 20, 19, 18.
   So, the greatest value that can fulfill the range is: 18+25=43

3. Let SD of A be σ.
   SD of X will remain the same σ.
   SD of Y will be multiplied by 1.5 so 1.5σ
   SD of Z will be first divided by –ve sign … no change … then divided by 4 … so $\sigma/4$. As SD is always +ve,
   $1.5\sigma > \sigma > \sigma/4$ … Ans. D

4. SD can't be negative. SD is zero only when all numbers are same (which is impossible in this case). Ans. E

5. The denominator in SD formula (N) will become 102, so we have to find numbers that will change the SD by the least amount: such numbers should be as close to the mean as possible. So the answer is E.

6. * if you **ADD OR SUBTRACT A CONSTANT** to/from all the values in a set, then the **standard deviation will remain exactly the same**.

   * if you **INCREASE OR DECREASE ALL THE VALUES BY A FIXED FACTOR / PERCENTAGE**, then the **standard deviation will increase or decrease by the same percentage**.

   Make sure you know that, when ALL numbers in a set are multiplied or divided by some number, the mean and standard deviation are multiplied /divided by the same number.
   This includes increasing or decreasing all the numbers in the set by some percentage (which can be accomplished by multiplication: e.g., 30% increase = multiplication by 1.3).

   Using this principle, statement (1) tells us that both the mean and the standard deviation of the set will decrease by 30%. Therefore, the new standard deviation will decrease to 7 gallons. SUFFICIENT.

   Statement (2) tells us nothing about standard deviation, which measures SPREAD of numbers. If we achieved the 63 gallons by taking most of the water out of the tanks that were already lowest, then the standard deviation will be huge (because you'll have some tanks almost full and some almost empty). If we got there by taking most of the water out of the fullest tanks, then the standard deviation will be a lot smaller. INSUFFICIENT.