

### Quant Session 5 – Permutations and Combinations, Probability

**Permutations and Combinations concepts are used for Complex Counting**

**Arrangements** - keywords – seating, sitting, sequence, order, alphabets, schedule, ranking, itinerary, codes

**Order important** – gives unique arrangements

For e.g. A and B sitting on chair can be AB or BA so these are two distinct arrangements

It is basically selection followed by arrangement. So  ${}^n P_r = {}^n C_r \times r!$

$${}^n P_r = \frac{n!}{(n-r)!}$$

**Selection** - keywords – team, committee, balls, handshakes, matches, picking

**Order not important** – For example choosing A and B from a group of 3 or four alphabets. The order does not matter. India playing a match against Australia is the same as Australia playing against India.

$${}^n C_r = \frac{n!}{(n-r)! \times r!}$$

**Different formulae**

1.  ${}^n P_r = \frac{n!}{(n-r)!}$

When to use? When n distinct items present and r have to be selected and then arranged.

E.g – how many ways can you arrange 4 people in 5 chairs =  ${}^5 P_4$

2.  $n^r$   
All n distinct selection of r but repetition is allowed.

In how many ways can you wear three different rings on four fingers?  
 $4^3$

3.  $\frac{n!}{p! q! r!}$

Arranging n things in which p are of one type, q of a second type and r of third type:

Ex: In how many ways can you arrange the letters of word Banana?

$$\text{Ans. } \frac{6!}{3! 2!}$$

#### 4. Special Cases

5 people A, B, C, D, E to be arranged in which A and B are together.

$$4! \times 2!$$

5 people A, B, C, D, E to be arranged in which A and B are not together.

$$5! - 4! \times 2!$$

#### 5. Block diagrams - Some problems can not be done with any formula but with a block diagram

#### Combinations

1. Select 5 people out of 10      Ans.  $^{10}C_5$

Particular Cases – Select 5 out of 10 people such that A and B are always selected. This means only 3 of the remaining 8 are to be selected  $^8C_3$

Select 5 out of 10 such that A and B are never selected. This means that out of remaining 8, 5 have to be selected so it is  $^8C_5$

2. Select 5 out of 10 so that A and B are never together.

$$= \text{Total} - \text{Together} = ^{10}C_5 - ^8C_3$$

AND denotes Multiplication

OR denotes Addition

Circular Permutations:  $(n - 1)!$

**Multiple trials of a single event:** If multiple independent trials of a single event are performed, then the probability of r successes out of a total of n trials can be determined by  $^nC_r \times p^r \times q^{n-r}$

Where

n = number of times the event is performed

r = number of successes

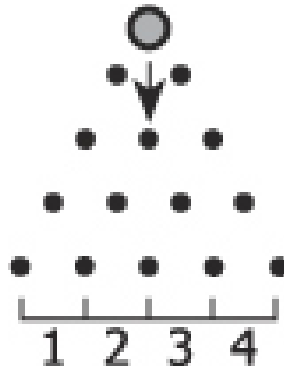
p = probability of success in one trial

q = probability of failure in one trial =  $1 - p$ .

## Questions

1. A password contains at least 8 distinct digits. It takes 12 seconds to try one combination, what is the minimum amount of time required to guarantee access to the database?
2. An engagement team consists of a project manager, team leader, and four consultants. There are 2 candidates for the position of project manager, 3 candidates for the position of team leader, and 7 candidates for the 4 consultant slots. If 2 out of 7 consultants refuse to be on the same team, how many different teams are possible?
3. A university cafeteria offers 4 flavors of pizza – pepperoni, chicken, Hawaiian and vegetarian. If a customer has an option to add, extra cheese, mushrooms, or both to any kind of pizza, how many different pizza varieties are available?
4. If 6 fair coins are tossed, how many different coin sequences will have exactly 3 tails, if all tails have to occur in a row?
5. A telephone company needs to create a set of 3-digit area codes. The company is entitled to use only digits 2, 4 and 5, which can be repeated. If the product of the digits in the area code must be even, how many different codes can be created?
6. There are 10 women and 3 men in room A. One person is picked at random from room A and moved to room B, where there are already 3 women and 5 men. If a single person is then to be picked from room B, what is the probability that a woman will be picked?  
(A) 13/21 (B) 49/117 (C) 15/52 (D) 5/18 (E) 40/117
7. A telephone number contains 10 digits, including a 3-digit area code. Bob remembers the area code and the next 5 digits of the number. He also remembers that the remaining digits are not 0, 1, 2, 5, or 7. If Bob tries to find the number by guessing the remaining digits at random, the find probability that he will be able to find the correct number in at most 2 attempts.
8. Tanya prepared 4 different letters to be sent to 4 addresses. For each letter she prepared an envelope with its correct address. If the 4 letters to be put in to 4 envelopes at random, what is the probability that only one letter will be put in to the envelope with the correct address?
9. In a certain group of 10 members, 4 members teach only French and the rest teach only Spanish or German. If the group is to choose a 3-person committee, which must have at least one member who teaches French, how many different committees can be chosen?  
40 50 64 80 100
10. To furnish a room in model home, an interior decorator is to select 2 chairs and 2 tables from a collection of chairs and tables in a warehouse that are all different from each other. If there are 5 chairs in the warehouse and if 150 different combinations are possible, how many tables are in the warehouse?  
6 8 10 15 30
11. From a bag containing 12 identical blue balls, y identical yellow balls, and no other balls, one ball will be removed at random. If the probability is less than  $\frac{2}{5}$  that the removed ball will be blue, what is the least number of yellow balls that must be in the bag?  
A. 17 B. 18 C. 19 D. 20 E. 21

12. Each of the 25 balls in a certain box is red, blue or white and has a number from 1 to 10 painted on it. If one ball is to be selected at random from the box, what is the probability that the ball selected will either be white or have an even number painted on it?
- (1) The probability that the ball will both be white and have an even number painted on it is 0.  
(2) The probability that the ball will be white minus the probability that the ball will have an even number painted on it is 0.2.
13. A certain stock exchange designates each stock with a one, two or three letter code, where each letter is selected from the 26 letters of the alphabet. If the letters may be repeated and if the same letters used in a different order constitute a different code, how many different stocks is it possible to uniquely designate with these codes?
- a) 2,951      b) 8,125      c) 15,600      d) 16,302      e) 18,278
14. The figure shown represents a board with four rows of pegs, and at the bottom of the board are four cells numbered 1 to 4. Whenever the ball shown passes through the opening between two adjacent pegs in the same row, it will hit the peg directly beneath the opening. The ball then has probability  $\frac{1}{2}$  of passing through the opening immediately to the left of that peg and probability  $\frac{1}{2}$  of passing through the opening immediately to the right. What is the probability that when the ball passes through the first two pegs at the top it will end up in cell 2?



- a)  $\frac{1}{16}$       b)  $\frac{1}{8}$       c)  $\frac{1}{4}$       d)  $\frac{3}{8}$       e)  $\frac{1}{2}$
15. A certain office supply store stocks 2 sizes of self-stick notepads, each in 4 colors: Blue, Green, Yellow or Pink. The store packs the notepads in packages that contain either 3 notepads of the same size and the same color or 3 notepads of the same size and of 3 different colors. If the order in which the colors are packed is not considered, how many different packages of the types described above are possible?
- A) 6      B) 8      C) 16      D) 24      E) 32
16. A certain junior class has 1000 students and a certain senior class has 800 students. Among these students there are 60 sibling pairs, each consisting of 1 junior and 1 senior. If 1 student is to be selected at random from each class, what is the probability that 2 students selected will be sibling pair
- 1)  $\frac{3}{40,000}$       2)  $\frac{1}{3,600}$       3)  $\frac{9}{2,000}$       4)  $\frac{1}{60}$       5)  $\frac{1}{15}$
17. How many integers between 324,700 and 458,600 have tens digit 1 and units digit 3?
- (A) 10,300      (B) 10,030      (C) 1,353      (D) 1,352      (E) 1,339
18. On his drive to work, Leo listens to one of 3 radio stations, A, B, or C. He first turns to A. If A is playing a song he likes, he listens to it; if not, he turns to B. If B is playing a song that he likes, he listens to it; if not, he turns to C. If C is playing a song he likes, he listens; if not, he turns off the radio. For each station, the probability is 0.3 that at any given moment the station is playing a song Leo likes. On his drive to work, what is the probability that Leo ill

- hear a song he likes?  
a. 0.027                      b. 0.09                      c. 0.417                      d. 0.657                      e. 0.9
19. A company that ships boxes to a total of 12 distribution centers uses color coding to identify each center. If either a single color or a pair of two different colors is chosen to represent each center and if each center is uniquely represented by that choice of one or two colors, what is the minimum number of colors needed for the coding? (Assume that the order of the colors in a pair does not matter.)  
(A) 4                      (B) 5                      (C) 6                      (D) 12                      (E) 24
20. A contest consists of  $n$  questions, each answered either True or False. Anyone who answers all  $n$  correctly will be a winner. What is the least value of  $n$  for which the probability is Less than  $1/1000$  that a person who randomly guesses the answer to each will be a winner.
21. There are 8 magazines lying on a table; 4 are fashion magazines and the other 4 are sports magazines. If 3 magazines are to be selected at random from 8 magazines, what is the probability that at least one of the fashion magazines will be selected?  
a)  $1/2$                       b)  $2/3$                       c)  $32/35$                       d)  $11/12$                       e)  $13/14$
22. If a 3-digit integer is selected at random from the integers 100 thru 199, inclusive, what is the probability that the first digit and the last digit of the integer are each equal to one more than the middle digit?  
A)  $2/225$                       B)  $1/111$                       C)  $1/110$                       D)  $1/100$                       E)  $1/50$
23. All of the stocks on the over-the-counter market are designated by either a 4-letter or a 5-letter code that is created by using the 26 letters of the alphabet. Which of the following gives the maximum number of different stocks that can be designated with these codes?  
A.  $2(26^5)$                       B.  $26(26^4)$                       C.  $27(26^4)$                       D.  $26(26^5)$                       E.  $27(26^5)$
24. A certain restaurant offers 6 kinds of cheese and 2 kinds of fruit for its dessert platter. If each dessert platter contains an equal number of kinds of cheese and kinds of fruit, how many different dessert platters could the restaurant offer?  
a. 8                      b. 12                      c. 15                      d. 21                      e. 27
25. A box contains 10 light bulbs, fewer than half of which are defective. Two bulbs are to be drawn simultaneously from the box. If  $n$  of the bulbs in box are defective, what is the value of  $n$ ?  
(1) The probability that the two bulbs to be drawn will be defective is  $1/15$ .  
(2) The probability that one of the bulbs to be drawn will be defective and the other will not be defective is  $7/15$ .
26.  $M = \{-6, -5, -4, -3, -2\}$                        $T = \{-2, -1, 0, 1, 2, 3\}$   
If an integer is to be randomly selected from set  $M$  above and an integer is to be randomly selected from set  $T$  above, what is the probability that the product of the two integers will be negative?  
A. 0                      B.  $1/3$                       C.  $2/5$                       D.  $1/2$                       E.  $3/5$
27. If a code word is defined to be a sequence of different letters chosen from the 10 letters A, B, C, D, E, F, G, H, I, and J, what is the ratio of the number of 5-letter code words to the number of 4-letter code words?  
A. 5 to 4                      B. 3 to 2                      C. 2 to 1                      D. 5 to 1                      E. 6 to 1
28. If an integer  $n$  is to be chosen at random from the integers 1 to 96, inclusive, what is the probability that  $n(n+1)(n+2)$  will be divisible by 8?  
A.  $1/4$                       B.  $3/8$                       C.  $1/2$                       D.  $5/8$                       E.  $3/4$

29. What is the probability that a student randomly selected from a class of 60 students will be a male who has brown hair?  
(1) One-half of the students have brown hair. (2) One-third of the students are males.
30. Six cards numbered from 1 to 6 are placed in an empty bowl. First one card is drawn and then put back into the bowl; then a second card is drawn. If the cards are drawn at random and if the sum of the numbers on the cards is 8, what is the probability that one of the two cards drawn is numbered 5?  
A. 1/6 B. 1/5 C. 1/3 D. 2/5 E. 2/3
31. A gardener is going to plant 2 red rosebushes and 2 white rosebushes. If the gardener is to select each of the bushes at random, one at a time, and plant them in a row, what is the probability that the 2 rosebushes in the middle of the row will be the red rosebushes?  
A. 1/12 B. 1/6 C. 1/5 D. 1/3 E. 1/2
32. A company has assigned a distinct 3-digit code number to each of its 330 employees. Each code number was formed from the digits 2, 3, 4, 5, 6, 7, 8, 9 and no digit appears more than once in any one code number. How many unassigned code numbers are there?  
A. 6 B. 58 C. 174 D. 182 E. 399
33. On Saturday morning, Malachi will begin a camping vacation and he will return home at the end of the first day on which it rains. If on the first three days of the vacation the probability of rain on each day is 0.2, what is the probability that Malachi will return home at the end of the day on the following Monday?  
A. 0.008 B. 0.128 C. 0.488 D. 0.512 E. 0.640
34. How many 4-digit positive integers are there in which all 4 digits are even?  
A. 625 B. 600 C. 500 D. 400 E. 256
35. A string of 10 lightbulbs is wired in such a way that if any individual lightbulb fails, the entire string fails. If for each individual lightbulb the probability of failing during time period T is 0.06, what is the probability that the string of lightbulbs will fail during time period T?  
A. 0.06 B.  $(0.06)^{10}$  C.  $1-(0.06)^{10}$  D.  $(0.94)^{10}$  E.  $1-(0.94)^{10}$
36. A three-digit code for certain logs uses the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 according to the following constraints. The first digit cannot be 0 or 1, the second digit must be 0 or 1, and the second and third digits cannot both be 0 in the same code. How many different codes are possible?  
A. 144 B. 152 C. 160 D. 168 E. 176
37. In a meeting of 3 representatives from each of 6 different companies, each person shook hands with every person not from his or her own company. If the representatives did not shake hands with people from their own company, how many handshakes took place?  
A. 45 B. 135 C. 144 D. 270 E. 288
38. In a stack of cards, 9 cards are blue and the rest are red. If 2 cards are to be chosen at random from the stack without replacement, the probability that the cards chosen will both be blue is 6/11. What is the number of cards in the stack?  
A. 10 B. 11 C. 12 D. 15 E. 18