Quant Concepts: Session 2: Absolute Values (Modulus) + Statistics

Part 1: Absolute Values (Mods)

Key Concepts:

1. $|x|$ is defined as the non–negative value of x and hence is never negative.

2. |5| = 5, |−5| = 5 = − (−5) so |x| = x, if x is positive and |x| = –x, if x is negative … in this case –x is a positive number … |x| = –x still means that |x| is positive because in this case –x is a positive number.

3. $|x|$ is defined as the distance of point x from 0 on the number line. The point x can be anywhere on the line (positive or negative)

4. $|x - a|$ is defined as the distance of point x from a on the number line. The point x and a can be anywhere on the line (positive or negative).

5. $\sqrt{x^2} = |x|$, that means, the square root of any quantity is defined to be non–negative. $x^2 = |x| \times |x|$

    - So $\sqrt{36} = 6 \text{ and not} - 6$.
        - BUT if $x^2 = 36 \Rightarrow x = 6 \text{ or} - 6$ both.
        - So $\sqrt{x^2} = x \text{ or} - x$ both are possible. Square roots cannot be negative (numerically).
        - If, x is negative, then $\sqrt{x^2} = -x$ as the square root has to be positive eventually. In this case x is negative and –x is positive.

6. $|x| = x \Rightarrow x \geq 0$

7. $|x| = -x \Rightarrow x < 0$

8. $|x| > x \Rightarrow x < 0$

9. $-x|x| > x \Rightarrow x < 0$

10. $-x|x| > 0 \Rightarrow x < 0$

11. $|x - a| > 0 \Rightarrow x \neq a$

12. $\dfrac{x}{|x|} = 1$ if x is positive and $\dfrac{x}{|x|} = -1$ if x is negative.

13. $|a| = |b| \Rightarrow a = b \text{ or } a = -b$

14. If |x| = a, then x = a or x = –a. If |x| > a, then x > a or x < –a. If |x| < a, then x < a or x > –a so –a < x < a.

Problems:

1. If x < 0, then √(−x |x|) is

    A. −x   B. −1   C. 1   D. x   E. √x

2. Is |x| = y − z?   (1) x + y = z   (2) x < 0

Top One Percent – T. O. P.GMAT – GMAT 99th Percentile Club: Exclusive GMAT-Prep for 99th percentile (760-800 range) aspirants – not for everyone!
Train with Sandeep Gupta (Official 770 Score twice) in Delhi and Bangalore. Contact: Delhi: +91-85868-37769: Bangalore: +91-97395-61394.
Email ID: info@top-one-percent.com.Can't attend classes? Check out our video course on pen drive: www.top-one-percent.com
Page 1

3. Is $\sqrt{[(x-5)^2]} = 5-x$?   (1) $-x\,|x| > 0$ (2) $5 - x > 0$

4. If $y \geq 0$, what is the value of x?   (1) $|x - 3| \geq y$ (2) $|x - 3| \leq -y$

5. Is $1 / (a-b) < (b-a)$?   (1) $a < b$ (2) $1 < |a-b|$

6. Is $|x| < 1$?   (1) $|x + 1| = 2|x - 1|$ (2) $|x - 3| > 0$

7. If y is an integer and $y = |x| + x$, is $y=0$?(1) $x < 0$ (2) $y < 1$

8. Is $|x - y| > |x| - |y|$?   (1) $y < x$ (2) $xy < 0$

9. Is x a negative number?
   (1) x^2 is a positive number.   (2) $x \cdot |y|$ is not a positive number.

10. Is $x \cdot |y| > y^2$?   (1) $x > y$ (2) $y > 0$

11. If n is not equal to 0, is $|n| < 4$?   (1) $n^2 > 16$ (2) $1/|n| > n$

12. Is $x > 0$?   (1) $|x + 3| = 4x - 3$ (2) $|x - 3| = |2x - 3|$

13. What is the value of $|x|$?   (1) $|x^2 + 16| - 5 = 27$ (2) $x^2 = 8x - 16$

14. If r is not equal to 0, is $r^2 / |r| < 1$
    (1) $r > -1$ (2) $r < 1$

Part 2: Statistics

Mean (Average)

1. Average or mean or AM = Sum of n quantities (or numbers) / number of them (n) OR Arithmetic Mean (A.M.) is given by $\overline{X} = \Sigma x / N$.

2. Mean of the Combined Series If N_1 and N_2 are the sizes and M_1 and M_2 are the respective means of two series then the mean M of the combined series is given by $M = \dfrac{M_1 N_1 + M_2 N_2}{N_1 + N_2}$ or we can write:

$$\frac{N_1}{N_2} = \frac{M_2 - M}{M - M_1} = \frac{D_2}{D_1}$$ ... This is the most important result.

3. If a man (or train or boat or bus) covers some journey from A to B at X km/hr (or m/sec) and returns to A at a uniform speed for Y km/hr, then the average speed during the whole journey is [2XY / (X + Y)] km/hr. **TIP:** The average speed in such a case will be a bit less than the simple average.

4. The sum of first "n" natural numbers is given by n (n + 1)/2.

5. For consecutive integers or for equally spaced numbers (AP), Mean = (First term + Last term) / 2.

6. If the average of a few consecutive integers is 0, then there will be an odd number of integers.

7. The average of an odd number of consecutive integers is an integer and the average of an even number of consecutive integers is a non-integer.

8. If in a set of numbers, the average = the highest or the lowest number, all the numbers will have to be equal.

Top One Percent – T. O. P.GMAT – GMAT 99th Percentile Club: Exclusive GMAT-Prep for 99th percentile (760-800 range) aspirants – not for everyone!
Train with Sandeep Gupta (Official 770 Score twice) in Delhi and Bangalore. Contact: Delhi: +91-85868-37769: Bangalore: +91-97395-61394.
Email ID: info@top-one-percent.com.Can't attend classes? Check out our video course on pen drive: www.top-one-percent.com
Page 2

Problems

Use $\dfrac{N_1}{N_2} = \dfrac{M_2 - M}{M - M_1} = \dfrac{D_2}{D_1}$ to solve the following 6 questions:

1. In a work force, the employees are either managers or directors. What is the percentage of directors?
    (1) The average salary for managers is $5,000 less than the total average salary.
    (2) The average salary for directors is $15,000 more than the total average salary.

2. Committee X and Committee Y, which have no common members, will combine to form Committee Z. Does Committee X have more members than Committee Y?
    (1) The average (arithmetic mean) age of the members of Committee X is 25.7 years and the average age of the members of Committee Y is 29.3 years.
    (2) The average (arithmetic mean) age of the members of Committee Z will be 26.6 years

3. During a sale, a clothing store sold each shirt at a price of $15 and each sweater at a price of $25. Did the store sell more sweaters than shirts during the sale?
    (1) The average (arithmetic mean) of the prices of all the shirts and sweaters that the store sold during the sale was $21.
    (2) The total of the prices of all the shirts and sweaters that the store sold during the sale was $420

Median:

*   Median is the middle value or the average of two middle values when the values are arranged in an order, either ascending or descending.
*   If there are odd number of observations, median is directly the middle number.
*   If there is an even number of observations, median is the average of the two middle numbers.
*   For consecutive integers or for equally spaced numbers (AP), Median = (First term + Last term) / 2. So, Median = Mean in this case.
*   Median is the 50th percentile.

Problems

1. Tom, Jane, and Sue each purchased a new house. The average (arithmetic mean) price of the three houses was $120,000. What was the median price of the three houses?
    (1) The price of Tom's house was $110,000.   (2) The price of Jane's house was $120,000.

2. Last month 15 homes were sold in Town X. The average (arithmetic mean) sale price of the homes was $ 150,000 and the median sale price was $130,000. Which of the following statement must be true?
    I.   at least one of the homes was sold for more than $165,000
    II.   at least one of the homes was sold for more than $130,000 and less than $150,000
    III.   at least one of the homes was sold for less than $130,000
    a) I only   b) II only   c) III only   d) I and II   e) I and III

3. If x and y are unknown positive integers, is the mean of the set {6, 7, 1, 5, x, y} greater than the median of the set?

    (1) x + y = 7   (2) x – y = 3

Top One Percent – T. O. P.GMAT – GMAT 99th Percentile Club: Exclusive GMAT-Prep for 99th percentile (760-800 range) aspirants – not for everyone!
Train with Sandeep Gupta (Official 770 Score twice) in Delhi and Bangalore. Contact: Delhi: +91-85868-37769: Bangalore: +91-97395-61394.
Email ID: info@top-one-percent.com.Can't attend classes? Check out our video course on pen drive: www.top-one-percent.com
Page 3

4. Set S consists of five consecutive integers, and set T consists of seven consecutive integers. Is the median of the numbers in set S equal to the median of the numbers in set T?
(1) The median of the numbers in Set S is 0.
(2) The sum of the numbers in set S is equal to the sum of the numbers in set T.

5. What is the median number of employees assigned per project for the projects at Company Z?
(1) 25 percent of the projects at Company Z have 4 or more employees assigned to each project.
(2) 35 percent of the projects at Company Z have 2 or fewer employees assigned to each project.

Range + Standard Deviation

- Range is defined as the difference between the two extreme observations of the distribution.
- Range = $X_{max} - X_{min}$. If Range = 0, all the observations are equal. Range ≥ 0 always; it is never negative.
- Standard deviation is defined as positive square root of the A.M. of the squares of the deviations of the given observations from their A.M.
  $(x - \bar{x})^2$
- It is a measure of how much each value varies from the mean of all the values.
- Less SD implies more consistency, less variation, less spread, more compactness AND vice versa.
- If SD = 0, all the observations are equal.
- Range is always greater than SD, except when all observations are equal, when both are equal to 0.
  o To be precise $SD \leq \dfrac{Range}{2}$
- The square of SD is called Variance.

Change in respective statistical parameters:

	Addition	Subtraction	Sign Change	Multiplication	Division
Mean	Change	Change	Change	Change	Change
Median	Change	Change	Change	Change	Change
Range	NO Change	NO Change	NO Change	Change	Change
SD	NO Change	NO Change	NO Change	Change	Change

Problems

1. Set A consists of 8 distinct prime numbers. If x is equal to the range of set A and y is equal to the median of set A, is the product xy even?
(1) The smallest integer in the set is 5.   (2) The largest integer in the set is 101.

2. A set of 15 different integers has a range of 25 and a median of 25. What is greatest possible integer that could be in this set?
32   37   40   43   50

3. Set A consists of all even integers between 2 and 100, inclusive. Set X is derived by reducing each term in set A by 50, set Y is derived by multiplying each term in set A by 1.5, and set Z is derived by dividing each term in set A by −4. Which of the following represents the ranking of the three sets in descending order of standard deviation?
    (A) X, Y, Z   (B) X, Z, Y   (C) Y, Z, X   (D) Y, X, Z   (E) Z, Y, X

Top One Percent – T. O. P.GMAT – GMAT 99th Percentile Club: Exclusive GMAT-Prep for 99th percentile (760-800 range) aspirants – not for everyone!
Train with Sandeep Gupta (Official 770 Score twice) in Delhi and Bangalore. Contact: Delhi: +91-85868-37769: Bangalore: +91-97395-61394.
Email ID: info@top-one-percent.com.Can't attend classes? Check out our video course on pen drive: www.top-one-percent.com
Page 4

4. If M is a negative integer and K is a positive integer, which of the following could be the standard deviation of a set {−7, −5, −3, M, 0, 1, 3, K, 7}?

> I.   −1.5   II.   −2   III. 0
>
> (A) I only   (B) II only   (C) III only   (D) I and III only   (E) None

5. A certain list of 100 data has an average of 6 and a standard deviation of d, where d is positive. Which of the following pairs of data, when added to the list, must result in a list of 102 data with standard deviation less than d?   A. −6 and 0   B. 0 and 0   C. 0 and 6   D. 0 and 12   E. 6 and 6

6. During an experiment some water was removed from each of the 6 tanks. If the standard deviation of the volumes of the water at the beginning of the experiment was 10 gallons, what was the standard deviation of the volumes of the water after the experiment?

> (1) For each tank 30% of the volume of the water that was in the tank before the beginning of the experiment was removed during the experiment
>
> (2) The average (mean) volume of water in the tanks at the end of the experiment was 63 gallons.

Top One Percent – T. O. P.GMAT – GMAT 99th Percentile Club: Exclusive GMAT-Prep for 99th percentile (760-800 range) aspirants – not for everyone!
Train with Sandeep Gupta (Official 770 Score twice) in Delhi and Bangalore. Contact: Delhi: +91-85868-37769: Bangalore: +91-97395-61394.
Email ID: info@top-one-percent.com.Can't attend classes? Check out our video course on pen drive: www.top-one-percent.com
Page 5