Solution 5 – Permutations and Combinations, Probability

1. If a password contains at least 8 distinct digits, out of the 10 digits that are possible (0-9), the total number of potential combinations that one could try are :

   1) Choose 8 out of the 10 possible digits and arrange them – resulting in 10P8 arrangements.
   2) Choose 9 out of the possible 10 digits and arrange them – resulting in 10P9 arrangements.
   3) Choose 10 out of the possible 10 digits and arrange them – resulting in 10P10 arrangements.

   Therefore the total number of potential combinations is a sum of the above three:
   (10P8 + 10P9 + 10P10).
   Since time take for one combination is equal to 12 seconds, time taken to guarantee access to the database = total number of possible combinations * time taken for each combination
   = (10P8 + 10P9 + 10P10) * 12
   = 1212 seconds.

2. Project Manager ______ - 1 slot
   Team Leader ______ - 1 slot
   Consultants _______ - 4 slots

   Number of ways in which a project manager can be chosen = 2C1
   Number of ways in which a team leader can be chosen = 3C1

   Number of ways in which 2 of the consultants can be in the same team = number of ways in which a consultant can be chosen – Number of ways in which 2 consultants can always be together

   Number of ways in which a consultant can be chosen = 7C4
   Number of ways in which 2 consultants can always be together:

   4 consultant slots and 2 people always together.
   If two people always have to be on the team, the remaining 2 people can be picked out of the 5 available people in 5C2 ways.
   Therefore, Number of ways in which 2 of the consultants can be in the same team = number of ways in which a consultant can be chosen – Number of ways in which 2 consultants can always be together = 7C4 -5C2.

   Therefore, possible ways of picking the entire engagement team = 2C1 * 3C1 * (7C4-5C2)
   = 150 possible teams.

3. Number of flavours of pizza available = 4
   Number of options available to the customer = extra cheese, extra mushrooms, both, neither (4 options).

   Therefore, number of pizza varieties available = 4 * 4 = 16 varieties.

4. Let us list down all the possible outcomes where all three tails occur in a row :

TTTHHH
HTTTHH where T represents Tails and H represents Heads.
HHTTTH
HHHTTT
Thus there are only 4 possible ways in which this can happen.

5. Possible digits that can be used = 2,4 and 5.

The product of the digits in the area code will be even every time 2 or 4 are chosen as a digit in the area code. The only time they will not be chosen is when all three digits of the area code are 5.

The number of ways in which the product will be even = total number of possible combinations (with repetition) – 1 (the combination '555' which is the only combination with an odd product of digits)

= (3*3*3) – 1 = 26 ways.

6. In order to solve this problem, we have to consider two different scenarios. In the first scenario, a woman is picked from room A and a woman is picked from room B. In the second scenario, a man is picked from room A and a woman is picked from room B.

The probability that a woman is picked from room A is 10/13. If that woman is then added to room B, this means that there are 4 women and 5 men in room B (Originally there were 3 women and 5 men). So, the probability that a woman is picked from room B is 4/9.
Because we are calculating the probability of picking a woman from room A AND then from room B, we need to multiply these two probabilities:
10/13 x 4/9 = 40/117

The probability that a man is picked from room A is 3/13. If that man is then added to room B, this means that there are 3 women and 6 men in room B. So, the probability that a woman is picked from room B is 3/9.
Again, we multiply these two probabilities:
3/13 x 3/9 = 9/117

To find the total probability that a woman will be picked from room B, we need to take both scenarios into account. In other words, we need to consider the probability of picking a woman and a woman OR a man and a woman. In probabilities, OR means addition. If we add the two probabilities, we get:
40/117 + 9/117 = 49/117
The correct answer is B.

7. The last two digits of the telephone number can be one of the following: 3,4,6,8 and 9.
Total number of possible combinations of the digits = 25 (33,34,36,38,39,43,44.. etc)

Probability of Bob getting the last two digits right in at most two attempts = probability of Bob getting it right in the first attempt + probability of Bob getting it right in the second attempt
= 1/25 + (probability of Bob getting it wrong in the first attempt * probability of Bob getting it right the second time)
= 1/25 + (24/25 * 1/24)
= 1/25 +1/25
= 2/25.

8. Let R denote the letter in the right envelope and W denote the letter in the wrong envelope.
   We are trying to find the probability of 1R3W.

   Probability = number of ways to get 1R3W/number of ways total

   number of ways total is 4! = 24. Imagine stuffing envelopes randomly. Stacy can put any of 4 letters into the first envelope, any of the remaining 3 into the next, either of the remaining 2 into the next, and has no choice to make on the last, or 4*3*2*1.

   number of ways to get 1R3W : She could fill the first envelope with the right letter (1 way), then put either of the 2 wrong remaining letters in the next (2 ways), then put a wrong letter in the next (1 way). That's 1*2*1*1 = 2.

   But since it doesn't have to be the first envelope that has the Right letter, it could be any of the 4 envelopes (i.e. we could have RWWW, WRWW, WWRW, WWWR), the total ways to get 1R3W is 4*2 = 8.

   Probability is 8/24 = 1/3.

9. Total number of ways in which a committee of 3 can be picked from 10 people = 10C3 = 120
   Since there are 6 non-French teachers, we can calculate the probability that three non-French teachers will be selected in a row = (6/10) x (5/9) x (4/8) = 1/6.

   So, 1/6 of the 120 arrangements - or 20 - contain no French teachers. If 20 don't, then the other 100 arrangements do contain French teachers.

   Option (E) is the right answer choice.

10. No of ways of choosing 2 chairs out of 5 and 2 tables out of x tables = 150

    ie, 5C2 . xC2 = 150

    5!/2!.3! . x!/2!.(x-2)! = 150

    (x.(x-1).(x-2)!)/(x-2)! = 30

    x^2 - x -30 =0

    Solving , we get x=6 or x=-5

    Since the number of tables cannot be a –ve value, therefore, number of tables = 6.

    Option (A) is the right answer choice.

11. P(blue) = 12 / (12+Y)
    P(blue) < 2/5
    so when you solve: 12 /12+Y < 2/5
    y > 18
    [This is the solution to the equation, not the inequality. The first proper solution to the inequality is the first number greater than 18, which is 19]

12. 25 balls
    each one is red, white, or blue
    each one has a number from 1 to 10

    Requirement: white OR even (note that we DON'T want white AND even - we have to be able to strip out those that fall into both categories). So our equation will be :
    probability of white + probability of even - probability of white & even

    Statement(1) :
    Translated, this means there aren't any that are both white and even. This doesn't tell us how many are white or how many are even. Hence Insufficient.

    Statement (2):
    Pwhite - Peven = 0.2. So, Pwhite could be 0.4 which would make Peven 0.2. Or Pwhite could be 0.3 which would make Peven 0.1. And (by itself) it doesn't tell us Prob of even & white, which I'd need to subtract, so... insufficient in many ways.

    Combining statement (1) AND statement (2):
    Now we know that Peven+white = 0. BUT, we still have multiple possibilities for Pwhite and Peven (see above). 0.4+0.2-0=0.6. 0.3+0.1-0=0.4. ?? Still insufficient.
    Hence Option (E) is the right answer choice in this case.

13. The key to this problem is to remember that repetition is allowed.
    A 1 letter code can be obtained in 26 different ways.
    A 2 letter code can be obtained in 26*26 ways
    A 3 letter code can be obtained in 26*26*26 ways.

    Number of codes possible to be generated with this code = 26 + 26*26 + 26*26*26 = 18278.

    Hence option (E) is the right answer choice.

14. The ball drops between the top two pegs and hits the peg in the middle of row 2. To figure out the probability for its final location, we should look at the possible routes it could travel from row 2. There are 8 possibilities, with L meaning the ball goes left, and R meaning it goes right:

    It could go LLL--this puts it into cell 1.
    It could go LLR--this puts it into cell 2.
    It could go LRL--this puts it into cell 2.
    It could go LRR--this puts it into cell 3.
    It could go RLL--this puts it into cell 2.
    It could go RLR--this puts it into cell 3.
    It could go RRL--this puts it into cell 3.
    Or it could go RRR--this puts it into cell 4.

    There are 8 total possibilities, and 3 of them give us a result of cell 2. So the probability of cell 2 is 3/8.

15. Part I: Let's consider all the possibilities that will give us the same size (two possible ways) and the same color (4 possible ways).

Since there are two sizes and 4 colors, we can make a possible number of 8 **DIFFERENT** packages. (Remember that different packages means unique packages and with the same colour, say green, we cannot consider GGG different from another GGG).

Part II : Consider all the possibilities for a same size but three different color package. Since we have four colors to choose from, we can use the combination formula to find how many ways to choose 3 from 4 colors(4C3). This will give us 4 options, but since we have two sizes, we have a total of 8 ways to package the notes in this category.

Therefore the total number of packages available will be a sum of part I and part II = 8 + 8 = 16.

Hence option (C) is the right answer choice.

16. This is an "AND" probability question because both individuals must be a part of the sibling pair for the winning outcome to occur.

    Since this is an "AND" question, it will involve multiplication. Also keep in mind that 60 "sibling pairs" is really 120 people.

    Probability of selecting one sibling pair from class of juniors =

    60 / 1,000

    Probability of selecting a sibling pair from the senior class that is THE match to the one we selected from the junior class? There is only one person that would be the match, so winning outcomes / total possibilities =

    1/800

    Therefore: Answer = 60/1000 * 1/800 = 60/800,000 = 6/80,000 = 3/40,000.

    Hence option (A) is the right answer choice.

17. Upper Limit- 458600
    Lower Limit - 324700

    Diff 133900

    The case of the last two digits ending in ‘13’ will happen in exactly every hundredth integer. And the total pool of integers under consideration is a multiple of 100, so there won't be any pattern interrupts. Therefore, we can just divide the total number of integers by 100 and we will arrive at our answer.

    So in this case, the right answer is 1339 integers that end in ‘13’.

    Hence Option (E) is the right answer choice.

18. Probability that Leo will hear a song that he likes = 1- probability that Leo will not hear a song that he likes.

    Individual probability of leo hearing a song he likes = 0.3

Individual probability of Leo not hearing a song that he likes = 0.7

Therefore the probability that Leo will hear a song that he likes = 1 – 0.7*0.7*0.7 = 1-0.343= 0.657.

Hence Option (D) is the right answer choice.

19. The best way to answer this question is to work backward from the options in hand.

    Option (A) : With four colors we can code a total of : 4 + 4C2 DC's = 4 + 6 = 10 DC's. Not sufficient.

    Option (B) : With 5 colours we can code a total of : 5 + 5C2 = 5 + 10 = 15 DC's. Sufficient.

    There is no point proceeding to the next few options as the numbers will just get larger and we are only concerned with the minimum number of colors needed for coding. Hence Option (B) is the right answer choice.

20. Let us rephrase the questions first :
    'what is the least value of n for which there is less than a 1/1000 chance of guessing n questions in a row correctly?'
    This should be the thought process:
    * there is a 1/2 chance of guessing each question correctly
    * each question is independent of the other questions, so the chance of guessing n questions correctly is (1/2)(1/2)(1/2)...(1/2), where there are n (1/2)'s
    * this is (1/2)^n, or 1/(2^n)
    so:
    1/2^n < 1/1000
    take reciprocals:
    2^n > 1000
    n ≥ 10 (because 2^10 = 1024)

21. Probability that atleast one of the fashion magazines will be selected = 1- probability that only sports magazines are selected.

    Probability that only sports magazines are selected:

    Probability that the first magazine is a sports magazine =4/8
    Probability that the second magazine is a sports magazine = 3/7
    Probability that the third magazine is a sports magazine = 2/6
    Therefore, probability that all 3 magazines are sports magazines = 4/8*3/7*2/6 = 1/14
    Hence, probability that at least one of the fashion magazines will be selected = 1- 1/14 = 13/14.
    Option (E) is therefore the right answer choice.
22. The question indicates that the first and last digit are equal to 1 more than the middle digit indicating that the first and last digits are equal. Since the range is less than 199, the first digit cannot be greater than 1. This implies that with 0 as the middle digit and 1 as the first and last digits, 101 is the only integer possible out of the 100 integers.
    Hence probability = 1/100. Hence option (D) is the right answer choice.

23. 4 letter possibilities = 26^4
    5 letter possibilities = 26^5.
    Adding them, we get,
    26^4 + 26^5 = (26^4) * (1+26) = 27*(26^4). Option (C) is the right answer choice.

24. Equal kinds of cheese and fruits indicate that we can have platters with a maximum of 2 fruits since there are only two different kinds of fruits available.

    Remember that we can even have a 1 fruit, 1 cheese option.
    Hence:

    2C-2F = 6C2*2C2=15*1=15
    1C-1F=6C1*2C1=12

    Therefore, total number of options available = 15 + 12 = 27. Option (E) is the right answer choice.

25. Total number of ways of picking two bulbs out 10 bulbs = 10C2
    Total number of ways of picking 2 bulbs out of 'n' defective bulbs =nC2

    Statement (1) :

    It is given that nC2/10C2 = 1/15 i.e nC2 = 3. Hence n = 3.
    This statement is therefore sufficient.

    Statement (2):

    Since the two bulbs are drawn simultaneously, probability that the first will be defective and the second will not be defective = n(10-n)/10C2 = 7/15

    From the above, we get, n(10-n) = 21. Solving for n, we get n = 3. Hence this statement is sufficient.

    Option (D) is therefore the right answer choice.

26. Total product possibilities is 30 (5 set m * 6 set T).
    Any number multiplied by 0 is 0 which is neither positive nor negative.
    Every number in set M has 3 possibilities to be paired with set T numbers and the product turning out to be negative.
    There are 5 numbers so 5*3 =15 such possibilities. Probability = no of favorable outcomes/total possible outcomes = 15/30 = ½

    Option (D) is the right answer choice.

27. The five letter code : ___ ____ ____ ____ ____
    The first position can be occupied by any one of 10 letters, the second by any one of 9 , the third by any one of 8 and so on…
    Number of possible 5-letter code words = 10*9*8*7*6

    Similarly, the number of possible 4-letter code words = 10*9*8*7
    Ratio of 5-letter code words to 4-letter code words = 10*9*8*7*6/ 10*9*8*7 = 6/1.

    Hence option (E) is the right answer choice.

28. n(n + 1)(n + 2) is a product of 3 consecutive integers.
    If n is even, n(n + 1)(n + 2), will be divisible by 8.

Even integers from 1-96 inclusive = (96-2)/ 2 = 94/2 = 47, 47+1 = 48.
Also if n is odd and 1 less than multiple 8, n(n + 1)(n + 2) will be divisible by 8, because this will have at least 1 multiple of 8.
Multiples of 8 from 1-96 = (96-8)/8 = 11, 11+1 = 12.
Total number of favourable outcomes = 48 + 12 = 60.
Total number of possible outcomes = 96.
Probability = Number of favourable outcomes / Number of possible outcomes
= 60/96
= 5/8.
Hence Option (D) is the right answer choice.

29. Statement (1):
Since we have no information about the number of students who are male, we cannot answer the question prompt. Hence this statement alone is insufficient.
Statement (2):
Since we have no information about the number of students who are brown haired, we cannot answer the question prompt. Hence this statement alone is insufficient.

Combining statements (1) and (2):
We have 20 males and 40 females.We have 30 students with brown hair.
So we could have 20 males and all of them brown haired (probability would be 1/3), or 20 males non brown haired (30 females brown haired) and then probability would be 0.

Hence the combination of the statements is insufficient as well. Option (E) is therefore the right answer choice.

30. Let's look at all the possible combinations that can give a sum of 8 :

First Card	Second Card
6	2
2	6
4	4
3	5
5	3

Total number of cases = 5. Number of favourable cases = 2. Hence the probability = 2/5.
Option (D) is the right answer choice.

31. The number of ways to arrange the red bushes in the desired fashion is as follows :
W1R1R2W2, W2R1R2W1, W1R2R1W2, W2R2R1W1.
Total number of ways in which 4 bushes can be arranged = 4! = 24.
Hence, probability that of the event occurring = 4/ 24 = 1/6.
Option (B) is the right answer choice.

32. To make a code number, we have 8 choices for the first digit, 7 choices for the second digit, and 6 choices for the third digit (subtracting one each time, since we cannot use the same digit more than once), and therefore 8*7*6 = 336 code numbers are possible in total.
Since 330 code numbers have been used already, there are 336-330 = 6 unused code numbers.
Option (A) is the right answer choice.

33. Let's interpret the question in the right fashion :
The question is basically: "what's the probability that it will rain on Monday and not on first two days."
Probability that it will not rain on the first two days = 0.8*0.8

Probability that it will rain on Monday = 0.2.
Hence overall probability = 0.8*0.8*0.2 = 0.128.
Option (B) is the right answer choice.

34. For all four digits of the number to be even, we have to only consider the digits {0,2,4,6,8} in our calculations.
    The first digit can be picked in 4 ways (we cannot consider 0).
    The second digit can be picked in 5 ways
    The third digit can be picked in 5 ways
    The fourth digit can be picked in 5 ways.
    Hence, total number of 4 digit positive integers = 4*5*5*5 = 500.
    Option (C) is the right answer choice.

35. Probability of one light bulb failing during time interval T = 0.06.
    Hence, probability of not failing is 0.94. We have 10 light bulbs in the string. Even if one light bulb fails, the entire string fails.
    Hence in order for the string to be successful, all the light bulbs need to pass. Lets find out the probability of not failing. P(10 light bulbs do not fail) = (0.94)^10. P(string of light bulbs failing) = 1-P(10 light bulbs not fail) = 1-(0.94)^10

36. Let the code be XYZ, X can take 8 values-2,3,4,5,6,7,8,9.
    Y can take 2 values- 0,1.
    Z can take 9 if y = 0 or if Y = 1 then 10 values, if y = 0 : we have 8*1*9 = 72 options; if y = 1 : we have 8*1*10 = 80 options; Total = 72+80 = 152.
    Hence option (B) is the right answer choice.

37. The easiest formula to remember for handshakes among n people is = nc2 = n(n-1)/2.
    The logic for the formula is that n people will shake hands with n-1 people. (Because a person won't shake hands with himself, therefore n-1 is used). So the total no. of handshakes is n(n-1). BUT we just double counted the handshakes, because we counted that person A shake hands with person B and ALSO counted person B shaking hands with A. We have to correct for this double counting by dividing by 2.
    Therefore the number of handshakes among n people = n(n-1)/2. There are a total of 18 reps. Total no. of handshakes among all reps (including own company) = (18 X 17)/2 = 153. No. of handshakes among one company's own reps = (3 X 2)/2 = 3. No. of handshakes among 6 company's own reps = 3 X 6 = 18. Total no. of handshakes among all reps excluding own company's reps = 153-18 = 135

38. Let the number of red cards be equal to x.
    Number of ways to pick 2 blue cards from a total of 9 blue cards = 9C2.
    Number of ways to pick 2 cards from a total of (9+x) cards = (9+x)C2.
    From the question prompt, 9C2/(9+x)C2 = 6/11.
    Solving for x, we get x= 3.
    Therefore, total number of cards in the stack = 9 + 3 = 12.
    Option (C) is the right answer choice.