

# EC5.402: Tim-Frequency Analysis (TFA)

## Mid-semester Examination, 2025

Answer all the questions. Marks for each question shown in the brackets. Max duration: 90 mins

- Please prove the following identity of TFD  $p(t, \omega)$  based on STFT:  $\langle \omega \rangle = \langle \omega \rangle^s + \langle \omega \rangle^h$  [5 Marks]
  - Compute  $\langle t \rangle$  and  $\langle \omega \rangle$  for the dilated wavelet, whose mother wavelet is  $\psi(t)$  with  $\langle t \rangle = 0$  and  $\langle \omega \rangle = \omega_\psi$ ? [5 Marks]
  - Why the window  $(h(t))$  used for computing Spectrogram symmetric? Show the necessary equation and prove the requirement? [2 Marks]
- Prove the basis  $\phi(t - n); n \in \mathbb{Z}$  is orthonormal  $\forall n$  when  $\sum_k |\Phi(\omega + 2\pi k)|^2 = 1$ ? [4 Marks]
  - Can the scaling function  $(\phi(t))$  and its variant  $\frac{1}{\sqrt{2}}\phi(\frac{t}{2})$  orthogonal, justify your answer? [4 Marks]
  - What is need for orthogonal dyadic wavelet and explain clearly? [2 Marks]
- Let  $g_1, g_2$  be an orthonormal basis of an 2 two-dimensional plane, then the 3 normalized vectors are  $\phi_1 = g_1, \phi_2 = -\frac{g_1}{2} + \frac{\sqrt{3}}{2}g_2, \phi_3 = -\frac{g_1}{2} - \frac{\sqrt{3}}{2}g_2$ . Find the basis type of  $\phi$ ? [3 Marks]
  - Deduce the basis for the continuous time Fourier transform (CTFT) and short-time Fourier transform (STFT)? [2 Marks]
- Calculate mean time, mean frequency, standard deviation in time and frequency for  $f(t) = e^{-at}u(t)$ , where  $u(t)$  is unit step function and  $a > 0$ ? [8 Marks]

$$\frac{1}{\sqrt{2}} \int \phi(t) \cdot \phi\left(\frac{t}{2}\right) \cdot dt$$

$$\text{Let } r = \frac{t}{2}$$

$$\frac{1}{\sqrt{2}} \int \phi(2r) \cdot \phi(r) \cdot dr$$