EC5.101 - Network, Signals and Systems
Mid Exam

Date: 20th December, 2022 Maximum marks: 100
Exam duration: 90 minutes

Instructions:

a) There are *7 questions* for a total of *100 marks*.

b) Mention any additional assumptions you make that is not given in the question.

c) Write your answers neatly and clearly show the steps used to arrive at the solutions.

d) Cellphones, calculators, etc. are not allowed.

e) Write answer in the final format shown (for circuits).

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1. [18 marks] Answer the following for the circuit shown in **Figure 1**.
   (a) [1+1] List the number of nodes & mesh.
   (b) [8] Write the KCL equations. Write the equations in the following format for each node:

$$Av_1 + Bv_2 + Cv_3 + \ldots + Dv_n = \text{constant},$$

   where $v_1, v_2, \ldots, v_n$ are node voltages.
   (c) [8] Write the KVL equation and write the equation $Pi_1 + Qi_2 + Ri_3 + \ldots + Xi_n = \text{constant}$.

2. [17 marks] Answer the following for the circuit shown in **Figure 2**.
   (a) [3+3] Write the KCL and KVL equations (format as above).
   (b) [8] State the superposition theorem and solve the Figure 2 circuit using superposition theorem. Show all the steps.
   (c) [3] Can you use superposition theorem for calculating the following?
      i. current
      ii. node voltage
      iii. power

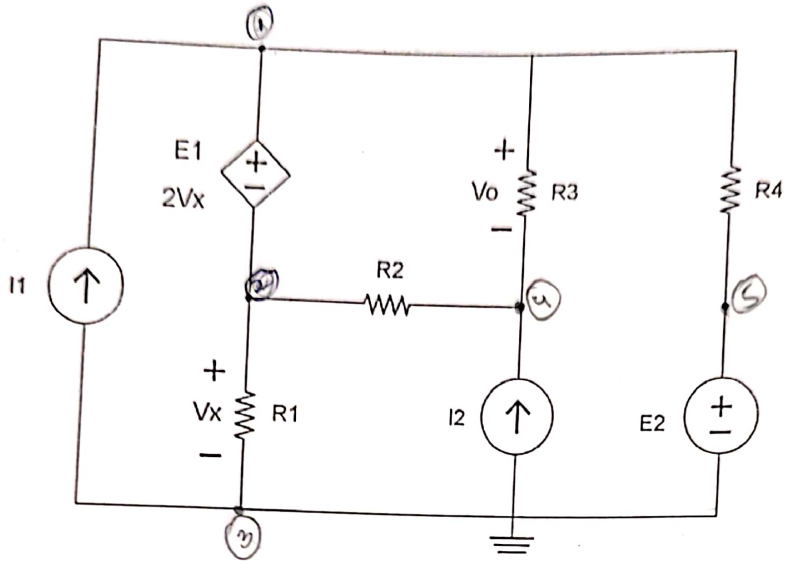


Figure 1

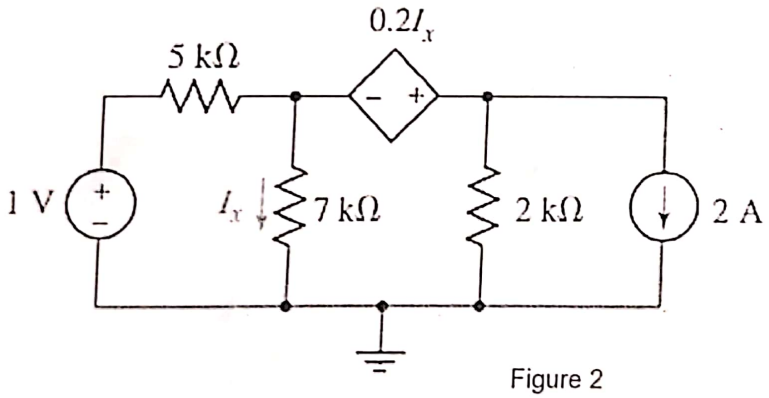
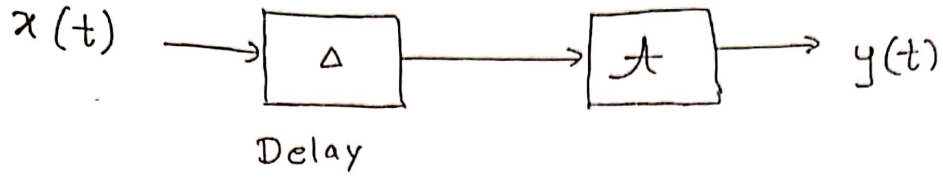


Figure 2

3. [15 marks] Consider the system with input signal $x(t)$ and output signal $y(t)$ shown below:



Let the amount of delay be $\Delta = 3$. Answer the following:

(a) [3] Find the impulse response of this system.

(b) [2] Is this system linear? Prove your answer.

(c) [2] Is this system time-invariant? Prove your answer.

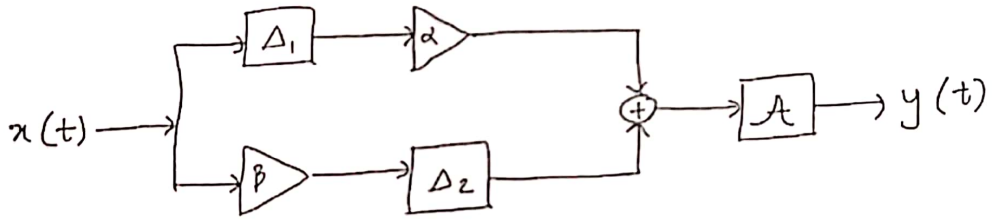
(d) [8] Find and sketch the output of this system for the following input signals:

i. $x(t) = \delta(t) - \delta(t-1)$

ii. $x(t) = u(t) + u(t-1)$

Here $\delta(t)$ and $u(t)$ denote the unit impulse and unit step signals respectively.

4. [10 marks] Consider the system with input signal $x(t)$ and output signal $y(t)$ shown below:



It consists of scaling blocks with parameters α and β and delay blocks with parameters Δ_1 and Δ_2. Answer the following:

(a) [5] Find the mathematical relation between input signal $x(t)$ and output signal $y(t)$.

(b) [5] Assuming this to be an LTI system, find $h(t)$ such that $y(t)$ can be expressed as $y(t) = x(t) * h(t)$ where the operator $*$ denotes the convolution.

5. [15 marks] Consider the signals $x(t)$ and $h(t)$ given by

$$x(t) = u(t) - u(t-2)$$

$$h(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 2-t, & 1 < t \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

(a) [3] Sketch $x(t)$ and $h(t)$.

(b) [12] Find the convolution between $x(t)$ and $h(t)$. Derive the expression and sketch it.

6. [15 marks] Shiva is investigating alternate representations which can be used instead of trigonometric Fourier series for real periodic signals $x(t)$ of period $T = \frac{2\pi}{\omega_0}$. He proposes to replace the original basis signals $\sin(k\omega_0 t)$ and $\cos(k\omega_0 t), k \ge 1$ with their modified (quantized) versions given below:

$$q_k^{\sin}(t) = \begin{cases} 1, & \text{if } \sin(k\omega_0 t) \ge 0 \\ -1, & \text{if } \sin(k\omega_0 t) < 0 \end{cases}$$

$$q_k^{\cos}(t) = \begin{cases} 1, & \text{if } \cos(k\omega_0 t) \ge 0 \\ -1, & \text{if } \cos(k\omega_0 t) < 0 \end{cases}$$

It can be shown that every pair of signals in the set of modified basis signals is orthogonal over the period T (you are not required to show this). The modified series for a periodic signal is given by

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k\, q_k^{\cos}(t) + \sum_{k=1}^{\infty} b_k\, q_k^{\sin}(t).$$

(a) [4] Sketch the modified basis signals $q_k^{\sin}(t)$ and $q_k^{\cos}(t)$ for $k = 1, 2$.

(b) [3] Of the four signals plotted in (a), identify all pairs of signals which are orthogonal over the period T. Prove your answers.

(c) [5] Assuming that a periodic signal $x(t)$ can be represented using the modified series, find the analysis equations, i.e., expressions for the coefficients a_k, b_k for $k \ge 1$ and a_0.

(d) [3] Find all the above coefficients for the following periodic signal with period T = 2,

$$x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & 1 < t \le 2. \end{cases}$$

7. [10 marks] A periodic signal with period $T = 1$ is given as follows:

$$x(t) = \delta(t - 0.5),\ 0 \le t \le 1.$$

Find all the complex Fourier series coefficients for this signal. Give simplified answers.