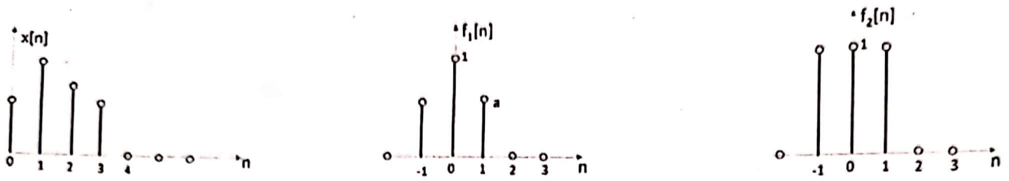
EC5.201: Signal Processing | Mid-semester Exam | Monsoon 2023

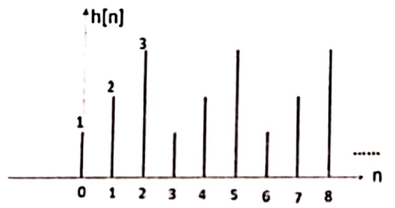
1. Upsampling a sequence x[n] by a factor of M is desired. A proposed method for this is :
   (i) create a sequence $x_e[n]$ by introducing M-1 zeros between successive samples of x[n].
   (ii) convolve $x_e[n]$ with a suitable sequence f[n] to obtain the final result.

   A sample x[n] and 2 possible candidates for f[n] are shown below. [2x5 +2 marks]



   a. Which of these sequences (f_1 or f_2) will give the best upsampled result? Why? What is the length of the final result with either of these sequences? Answer *without* doing any convolution.
   b. Sketch the appropriate f[n] for upsampling with M = 4.
   c. Identify the purpose of each of the steps in the proposed method.

2. A causal system with input x[n] and output y[n] has impulse response h[n] as given below. [6+6 marks]

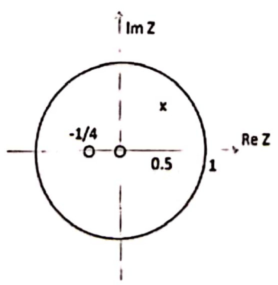


   a. Write h[n] in terms of δ[n] and use it to write the difference equation (part of which is given below) of this system.

   $$y[n] - y[n-3] = \_\_\_\_\_\_\_\_\_\_.$$

   b. Implement this equation using least number of these units: adders, delays and gains.

3. An *even* sequence is one which satisfies x[n] = x[-n]. Assuming it has a *rational* z-transform X(z) answer the following. [3+4+3 marks]
   a. What is the consequence of the *even* nature of the sequence for X(z)?
   b. Complete the partial pole-zero plot shown below for a *real, even* sequence and find X(z). What could be its ROC?



   c. Is the DTFT of the x[n] real and even, justify your answer? Can you find another signal as a function of x[n] such the that resultant signal DTFT is complex and even?

[PTO]

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4. Consider the $x_e[n]$, $f[n]$ and $x[n]$ defined in Question 1 and identify the following. [3+5+2 marks]
   a. Relate the DTFT of $x_e[n]$ with DTFT of $x[n]$.
   b. Let the $y[n]= x_e * f[n]$ and $y_1[n]= y[2n]$ then
      i. Express the DTFT of $y[n]$ in terms of DTFTs of $x[n]$ and $f[n]$.
      ii. Express the DTFT of $y_1[n]$ in terms of DTFTs of $x[n]$ and $f[n]$
      iii. Express the $y_1[n]$ in terms of $x[n]$ and $f[n]$
   c. Consider the relation obtained in b.iii and show that the relation holds for the sequence of $x[n]$ and $f[n]$ provided in Question 1.

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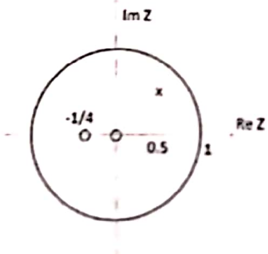
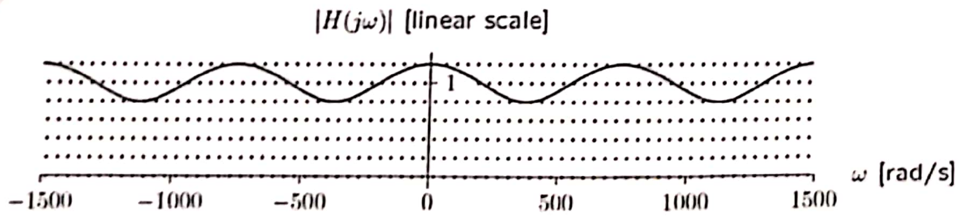
Signal Processing **Mid Sem Makeup exam** **October 2023**

1. A signal $x[n] = \delta(n+3) - \delta(n+1) + 2\delta(n) + 3\delta(n-2)$ with DTFT as $X(e^{j\omega}) = X_R(e^{j\omega}) + jX_I(e^{j\omega})$. [3+2+2+3] marks
   a. Compute $X_R(e^{j\omega})$ and $\int_{-\pi}^{\pi} X_I(e^{j\omega})\, d\omega$
   b. $DTFT\ (y[n]) = X_R(e^{j\omega})e^{j2\omega} + jX_I(e^{j\omega})$, find $y[n]$ without explicitly considering DTFT?
   c. Derive the DTFT of x[2n] in terms of DTFT of x[n], which is an arbitrary signal.
   d. For the given x[n] above, compute x[2n] and its DTFT and verify that the relation in part c holds.

2. An even sequence is one which satisfies x[n] = x[-n]. Assuming it has a rational z-transform X(z) answer the following. [3+4+3] marks
   a. What does even nature of signal imply for X(z)?
   b. Use your answer in part a to complete the partial pole-zero plot shown below for a real, even sequence and find X(z). What could be its ROC? Assume the pole is at radius r.
   c. If a signal g[n] = x[4-n]. How will the pole-zero plot of G(z) differ from that of X(z)? Explain.



3. An LTI system is represented by $h[n] = \delta[n - n_0] + \alpha\, \delta[n - n_1]$ with $n_1 > n_0$. The plot below shows the magnitude of the $H(z)$ when evaluated on the unit circle, i.e $|z| = 1, or\ z = e^{j\omega}$ where $\omega = \frac{2\pi}{T}$ is the angular frequency. [3+5+4] marks
   a. Assume $\alpha < 1$ and sketch $h[n]$. Find the system function $H(z)$.
   b. Justify the oscillatory pattern in $|H(e^{j\omega})|$ by evaluating $H(z)$on the unit circle. Relate the variables α, n_0 , n_1 in $h[n]$ to the oscillations. If $n_0 = 0$, find n_1 and α.
   c. Draw the block diagram of this system.



PTO for problem 4

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4. Upsampling a sequence x[n] by a factor of M is desired. A proposed method for this is :
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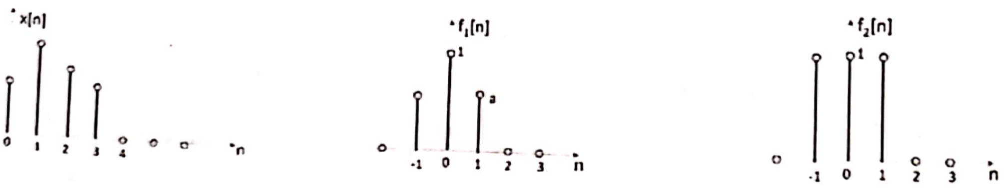
iii) A sample x[n] and 2 possible candidates for f[n] are shown below. [2x5 +2 marks]



a. Which of these sequences (f_1 or f_2) will give the best upsampled result? Why? What is the length of the final result with either of these sequences? Answer *without* doing any convolution.
b. Sketch the appropriate f[n] for upsampling with M = 4.
c. Identify the purpose of each of the steps in the proposed method.

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