

EC5.101 - Network, Signals and Systems  
Quiz 1

Date: 25th November, 2022  
Exam duration: 45 minutes

Maximum marks: 24

Instructions:

- There are 6 questions for a total of 24 marks.
- Mention any additional assumptions you make that is not given in the question.
- Clearly show the steps used to arrive at the solutions.

1. [4 marks] Consider the periodic signal given below:

$$x(t) = \sin(2t) + \sin^2(t).$$

- Find the trigonometric Fourier series (FS) coefficients of  $x(t)$ .
- Find the complex FS coefficients of  $x(t)$ .

2. [6 marks] The complex FS coefficients of a periodic signal  $x(t)$  are  $\{j, -1, 2, 0, 2, -1, -j\}$  where  $j = \sqrt{-1}$ . Find the FS coefficients of the following signals:

- $2x(2t)$
- $x(2 - t)$
- $x(t) - x(-t)$
- $2 - x(t)$

3. [2 marks] Identify whether the given signals are odd, even, or neither.

- $x(t) = t^2 \cos(3t)$
- $x(t) = t \cos(70t) + t \sqrt{1 + t^2}$
- $x(t) = \sin^2(7t) + \sin(t)$
- $x(t) = \log(\cos(t))$

4. [4 marks] Consider the operator representation of a system given by

$$Y = X + \mathcal{A}(X + \alpha Y)$$

- (a) Sketch the block diagram representation of this system.
- (b) Write down the differential equation representation of this system.

5. [4 marks] Let the trigonometric FS coefficients of a periodic signal  $x(t)$  be written in the format  $\{a_0, [a_1, a_2, \dots], [b_1, b_2, \dots]\}$ . FS coefficients of two periodic signals  $x_1(t)$  and  $x_2(t)$  having the same period are given below:

- (a)  $x_1(t) \longleftrightarrow \{1, [1, 0, 1], [2]\}$  and  $x_2(t) \longleftrightarrow \{0, [-1, 0, -1], [-2]\}$
- (b)  $x_1(t) \longleftrightarrow \{0, [1, 0, 1], [0, 2]\}$  and  $x_2(t) \longleftrightarrow \{1, [0, 1, 0], [2, 0]\}$

Assume that all other coefficients are zero. In each case, identify whether the pair of signals  $x_1(t)$  and  $x_2(t)$  are orthogonal or not. Justify your answer without performing explicit computations.

6. [4 marks] For a periodic signal with period  $T = 1$  it is given that,

$$\int_0^1 |x(t)|^2 dt = 4 = |a_{-1}|^2 + |a_0|^2 + |a_1|^2$$

where  $a_k$  denote the complex FS coefficients.

- (a) If it is known that the signal is even and  $a_{-1} = j$ , then compute  $a_0$ .
- (b) If it is known that the signal is even and has half-wave symmetry, compute  $a_{-1}$ .