Quiz 1
Information and Communication (Spring 2023)
Time : 40 mins,   Total Marks: 20

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Instructions:

- Reasons for all steps should be given, in general.

- This is a closed book, traditional, exam.

- Malpractice will directly result in 0 and further academic action will be initiated.

- You can request (not demand) for an additional hint from the course instructor (not from anyone else), who should be around. The discretion of providing (or not providing) the hint for a particular question will be left to the instructor. If the instructor is absent, no hint will be provided. *No debate or discussion will be there during the time of evaluation or post-evaluation regarding these hints.*

Questions:

1. (4+2+3+2=11 marks)

   Let $\text{rect}(t) \triangleq \begin{cases} 1, & \text{if } |t| \leq 0.5, \\ 0, & \text{otherwise} \end{cases}$. Let T_1, T_2 be two positive integers. **Define** the following signals mathematically (using equations, as we have defined $\text{rect}(t)$)

   - $\text{rect}(\frac{t}{T_2})$,
   - $\text{rect}(\frac{t-kT_1}{T_2})$ for $k = -2$.

   Further, sketch each of them. **Find** the Fourier spectrum of both these signals (The word 'Find' means you have to obtain the precise mathematical expressions). **Sketch** the real parts of these two spectrums separately. (When sketching, your curve need not be accurate, but should importantly make sure the shapes and the main points on the graph (points where curves cross X and Y axes) are correct).

2. (4 marks) Let T_1, T_2 be two positive integers such that $T_2 < T_1$. Recall that $\mathbb{Z}$ denotes the set of all integers. Consider a signal of the form $m(t) = \sum_{k \in \mathbb{Z}} \text{rect}\left(\frac{t-kT_1}{T_2}\right)$. Find the spectrum of this signal. Give a rough sketch of the real part of this spectrum.

3. (5 marks) Find the spectrum of the modulated signal $x(t) = m(t)\cos(2\pi f_c t)$, where $f_c \gg 1/T_1$ (this means, f_c is much greater than T_1). Give a rough sketch of the real part of this signal.