PES1201902409 LINEAR ALGEBRA ASSIGNMENT
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(1) Eq = $y = A + Bx + Cx^2$ passes through (1, 1)
(2, -1), (3, 1)

$\therefore$ we have,

$A + B + C = 1$
$A + 2B + 4C = -1$
$A + 3B + 9C = 1$

matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$ $x = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Augmented matrix $[A\ b] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & -1 \\ 1 & 3 & 9 & 1 \end{bmatrix}$

Using gaussian elimination

$R_2 \leftarrow R_2 - R_1$
$R_3 \leftarrow R_3 - R_1$
$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 2 & 8 & 0 \end{bmatrix}$

$R_3 \leftarrow R_3 - 2R_2$
$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$

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$\therefore 2C = 4$

$\Rightarrow \boxed{C = 2}$

$\Rightarrow B + 3C = -2$

$B + 3(2) = -2$

$\boxed{B = -8}$

$\Rightarrow A + B + C = 1$

$A - 8 + 2 = 1 \quad \Rightarrow \boxed{A = 7}$

$\therefore A = 7, B = -8, C = 2$

$\therefore$ eqn of parabola = $y = 7 - 8x + 2x^2$ //

(2) Given

$$A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 4 & 12 & 3 & 14 \\ -10 & -29 & -5 & 38 \\ 10 & 21 & 21 & -6 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$

$\therefore$ multiplier = 2

(L_{21}) $E_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_3 \leftarrow R_3 - (-5)R_1$

$\therefore$ Multiplier = -5

(L_{31}) $E_{31} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(L_{41}) $R_4 \leftarrow R_4 - 5R_1$

multiplier = 5

$$E_{41} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -5 & 0 & 0 & 1 \end{bmatrix}$$

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$$E_{41}E_{31}E_{21}A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & -4 & 5 & 13 \\ 0 & -4 & 11 & 19 \end{bmatrix}$$

$R_3 \leftarrow R_3 + 2R_2$

$R_4 \leftarrow R_4 + 2R_2$

$$E_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{42} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{bmatrix}$$

$$E_{42}E_{32}E_{41}E_{31}E_{21}A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 9 & 11 \end{bmatrix}$$

$R_4 \leftarrow R_4 - 3R_3$

$$E_{43} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{bmatrix}$$

$$E_{43}E_{42}E_{32}E_{41}E_{31}E_{21}A = \begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix} = U$$

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$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}$$

A = LU

$$\underbrace{\begin{bmatrix} 2 & 5 & 2 & -5 \\ 11 & 12 & 3 & -14 \\ -10 & -29 & -15 & 38 \\ 11 & 21 & 21 & -6 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ -5 & -2 & 1 & 0 \\ 5 & -2 & 3 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 2 & 5 & 2 & -5 \\ 0 & 2 & -1 & -4 \\ 0 & 0 & 3 & 5 \\ 0 & 0 & 0 & -4 \end{bmatrix}}_{U}$$

3. $T: R^3 \to R^3$

$$T(x, y, z) = (x + 2y - z,\ y + z,\ x + y - 2z)$$

(i) standard basis of R^3 are $(1,0,0)$, $(0,1,0)$ & $(0,0,1)$

using transformation given.

$T(1,0,0) = (1,0,1)$

$$= 1\begin{bmatrix}1\\0\\0\end{bmatrix} + 0\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$T(0,1,0) = (2,1,1)$

$$= 2\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} + 1\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$T(0,0,1) = (-1,1,-2)$

$$= -1\begin{bmatrix}1\\0\\0\end{bmatrix} + 1\begin{bmatrix}0\\1\\0\end{bmatrix} - 2\begin{bmatrix}0\\0\\1\end{bmatrix}$$

(ii.) for column space

$$T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

By G.E,

$R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$R_3 \rightarrow R_3 + R_2$

$$\begin{bmatrix} \boxed{1} & 2 & -1 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis for column space,

$C(T) = \{(1,0,1), (2,1,1)\}$

For null space,

solving for $Tx = 0$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

reducing to row reduced form

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$R_1 \leftarrow R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow x - 3z = 0$ $\quad\Rightarrow y + z = 0$

$x = 3z$ $\quad y = -z$

Let $z = Z$, some real no.

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$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

Basis for $N(T) = \{(3, -1, 1)\}$

for row space,

$$(T)^T = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 + R_1$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis for $C(T^T) = \{(1, 2, -1), (0, 1, 1)\}$

for left null space,

$$(T^T)X = 0$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + z = 0$

$x = -z$

$y - z = 0$

$y = z$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = z \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

let $z = z$ (some real no)

Basis for $N(T^T) = \{(-1, 1, 1)\}$

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(iii) $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$

finding eigen val :

$$|T - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & -2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)(-2-\lambda)-1\right] - 2(-1) - 1(-1+\lambda)$$

$$(1-\lambda)\left[(1-\lambda)(-2-\lambda)-1\right] + 2 + 1 - \lambda = 0$$

$\Rightarrow \lambda = 0, \sqrt{3}, -\sqrt{3}$ //

(Eigen values)

finding Eigen vectors.

for $\lambda_1 = 0$

$$(T - \lambda_1 I)x = 0$$

$$\left(\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

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$R_3 \to R_3 - R_1$ & $R_3 \leftarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$x + 2y - z = 0$

$y + z = 0$

$y = -z$

$$x = 2 \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

$$\lambda_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

For $\lambda_2 = \sqrt{3}$,

$$(T - \lambda_2) X = 0$$

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 1 & 1 & -2-\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$R_3 \to R_3 - \frac{1}{1-\sqrt{3}} R_1$$

$$\begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 1 - \frac{2}{1-\sqrt{3}} & \frac{-2\sqrt{3}+1}{1-\sqrt{3}} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$R_3 \leftarrow R_3 + \frac{\sqrt{3}+1}{(1-\sqrt{3})^2} R_2$$

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$$\Rightarrow \begin{bmatrix} 1-\sqrt{3} & 2 & -1 \\ 0 & 1-\sqrt{3} & 1 \\ 0 & 0 & -2-\sqrt{3}+\frac{1}{1-\sqrt{3}}+\frac{\sqrt{3}+1}{(1-\sqrt{3})^2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$z = Z$ some real value

$$(1-\sqrt{3})y = z = 0$$

$$\Rightarrow y = \frac{-1}{1-\sqrt{3}} z$$

$$(1-\sqrt{3})x + z\left(\frac{-1}{1-\sqrt{3}} z\right) - z = 0$$

$$\Rightarrow x = \frac{z(-\sqrt{3})}{1-\sqrt{3}}$$

$$\Rightarrow X_2 = \begin{bmatrix} -\sqrt{3}/(1-\sqrt{3}) \\ -1/(1-\sqrt{3}) \\ 1 \end{bmatrix}$$

For $\lambda_3 = -\sqrt{3}$

$$(T - \lambda_3 I)x = 0$$

$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ 1 & 1 & -2+\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$R_3 \leftarrow R_3 - \frac{1}{1+\sqrt{3}} R_1$$

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$$\begin{bmatrix} 1+\sqrt{3} & 2 & -1 \\ 0 & 1+\sqrt{3} & 1 \\ & & \end{bmatrix}$$

∴ Eigen values are,

$$X_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} -\sqrt{3}/(1-\sqrt{3}) \\ -1/(1-\sqrt{3}) \\ 1 \end{bmatrix}$$

$$X_3 = \begin{bmatrix} \sqrt{3}/(1+\sqrt{3}) \\ -1/(1+\sqrt{3}) \\ 1 \end{bmatrix}.$$

we got the eigen vectors by rationalising the denominator

$$X_1 = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \quad X_2 = \begin{bmatrix} \frac{\sqrt{3}+3}{2} \\ \frac{\sqrt{3}+1}{2} \\ 1 \end{bmatrix}, \quad X_3 = \begin{bmatrix} \frac{3-\sqrt{3}}{2} \\ \frac{1-\sqrt{3}}{2} \\ 1 \end{bmatrix}$$

$\lambda_1 = 0$, $\lambda_2 = \sqrt{3}$, $\lambda_3 = -\sqrt{3}$

(iv) $T = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$ $\quad a = (1, 0, 1)$, $b = (2, 1, 1)$, $c = (-1, 1, -2)$

Through Gram - Schimdt process.

$$q_1 = \frac{a}{\|a\|} = \frac{1}{\sqrt{2}}(1, 0, 1)$$

$$\|a\| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$$

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$q_2 = \frac{B}{\|B\|}$, where $B = b - (q_1^T b) q_1$

$q_1^T b = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \frac{3}{\sqrt{2}}$

$B = b - (q_1^T b) q_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} - \frac{3}{\sqrt{2}} \begin{bmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}$

$\Rightarrow \frac{1}{2} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1 \\ -1/2 \end{bmatrix}$

$q_2 = \frac{B}{\|B\|} = \frac{1}{\sqrt{6}} (1, 2, -1)$

$q_3 = \frac{C}{\|C\|}$, where $C = c - (q_2^T c) q_2 - (q_1^T c) q_1$

$q_1^T c = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} = \cancel{\phi}\ \frac{-3}{\sqrt{2}}$

$q_2^T c = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} = \frac{3}{\sqrt{6}}$

$c = (-1, 1, -2) - \frac{3}{\sqrt{6}} \times \frac{1}{\sqrt{6}} (1, 2, -1) - \left(\frac{-3}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(1, 0, 1)$

$c = (-1, 1, -2) + \left(-\frac{1}{2}, -1, \frac{1}{2}\right) + \left(\frac{3}{2}, 0, \frac{3}{2}\right) = (0, 0, 0)$

$q_3 = (0, 0, 0)$

QR factorization

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$$R = \begin{bmatrix} q_1^T a & q_1^T b & q_1^T c \\ 0 & q_2^T b & q_2^T c \\ 0 & 0 & q_3^T c \end{bmatrix}$$

$$q_1^T a = \frac{1}{\sqrt{2}} [1, 0\ 1] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \sqrt{2}$$

$$q_1^T b = \frac{1}{\sqrt{6}} [1\ \ 2\ \ -1] \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \frac{3}{\sqrt{6}}$$

$$q_3^T c = 0$$

$$R = \frac{1}{\sqrt{6}} \begin{bmatrix} \sqrt{12} & 3\sqrt{3} & -3\sqrt{3} \\ 0 & 3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$q_1^T b = \frac{3}{\sqrt{2}} = \frac{3\sqrt{3}}{\sqrt{6}}$$

$$q_3^T c = \frac{-3}{\sqrt{2}} = -\frac{3\sqrt{3}}{\sqrt{6}}$$

$$q_2^T c = \frac{3}{\sqrt{6}}$$

Q is $[q_1\ q_2\ q_3]$

$\Rightarrow T = QR$

$$\Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & 0 \\ 0 & 2/\sqrt{6} & 0 \\ 1/\sqrt{2} & -1/\sqrt{6} & 0 \end{bmatrix} \begin{bmatrix} \sqrt{2} & 3/\sqrt{2} & -3/\sqrt{2} \\ 0 & 3/\sqrt{6} & 3/\sqrt{6} \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$ //

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(4.) $y = c + dx$

$\therefore$ According to give info,

$c - 4d = 4$

$c + d = 6$

$c + 2d = 10$

$c + 3d = 8$

$$Ax = b \Rightarrow \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$A^T A \hat{x} = A^T b \qquad \Rightarrow \hat{x} = (A^T A)^{-1} A^T b$$

$$A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -4 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 2 & 30 \end{bmatrix}$$

$$(A^T A)^{-1} = \frac{1}{116} \begin{bmatrix} 30 & -2 \\ -2 & 4 \end{bmatrix}$$

$$(A^T A)^{-1} A^T = \frac{1}{116} \begin{bmatrix} 30 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -4 & 1 & 2 & 3 \end{bmatrix}$$

$$\Rightarrow \frac{1}{116} \begin{bmatrix} 38 & 28 & 26 & -24 \\ -18 & 2 & 6 & 10 \end{bmatrix}$$

$$\hat{x} = \begin{bmatrix} c \\ d \end{bmatrix} = (A^T A)^{-1} A^T b$$

$$= \frac{1}{116} \begin{bmatrix} 38 & 28 & 26 & -24 \\ -18 & 2 & 6 & 10 \end{bmatrix} \begin{bmatrix} 4 \\ 6 \\ 10 \\ 8 \end{bmatrix}$$

$$\Rightarrow \frac{1}{116}\begin{bmatrix} 772 \\ 80 \end{bmatrix} = \begin{bmatrix} \cancel{193/29} \\ \cancel{20} \end{bmatrix} \begin{bmatrix} 193/29 \\ 20/29 \end{bmatrix}$$

$\therefore$ The eqn is

$$y = \frac{193}{29} + \frac{20}{29}x \quad //$$

(5) $x_1 + x_2 + 3x_3 + 4x_4 = 0$

$$\begin{bmatrix} 1 & 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

$$x_1 = -x_2 - 3x_3 - 4x_4$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_2\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_3\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix} + x_4\begin{bmatrix} -4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Projection $P_1 = A(A^TA)^{-1}A^T$ & $Q = I - P$

$$A^TA = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ -4 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & -3 & -4 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 10 & 12 \\ 4 & 12 & 17 \end{bmatrix}$$

$$(A^TA)^{-1} = \begin{bmatrix} 26/27 & -1/9 & -4/27 \\ -1/9 & 2/3 & -4/9 \\ -4/27 & -4/9 & 11/29 \end{bmatrix}$$

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$$P = A(A^TA)^{-1}A^T = \begin{bmatrix} 26/27 & -1/27 & -1/9 & -4/27 \\ -1/27 & 26/27 & -1/9 & -4/27 \\ -1/9 & -3/27 & 6/9 & -12/27 \\ -4/27 & -4/27 & -4/9 & 11/27 \end{bmatrix}$$

$$= \frac{1}{29}\begin{bmatrix} 26 & -1 & -3 & -4 \\ -1 & 26 & -3 & -4 \\ -3 & -3 & 18 & -12 \\ -4 & -4 & -12 & +1 \end{bmatrix} //$$

$$Q = I - P = \frac{1}{27}\begin{bmatrix} 1 & 1 & 3 & 4 \\ 1 & 1 & 3 & 4 \\ 3 & 3 & 9 & 12 \\ 4 & 4 & 12 & 16 \end{bmatrix} //$$

6. $$A = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$$

will be +ve def. if:

pivots are +ve

reduced to Echelon form

$$R_2 \to R_2 - \frac{2}{a}R_1$$

$$R_3 \to R_3 - \frac{2}{a}R_1$$

$$\begin{bmatrix} a & 2 & 2 \\ 0 & a - \frac{4}{a} & 2 - \frac{4}{a} \\ 0 & 2 - \frac{4}{a} & a - \frac{4}{a} \end{bmatrix}$$

$$R_3 \to R_3 - \left(\frac{2a-4}{a^2-4}\right)R_2$$

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$$\begin{bmatrix} a & 2 & 2 \\ 0 & \frac{a^2-4}{a} & \frac{2a-4}{a} \\ 0 & 0 & \frac{a^2-4}{a} - \frac{(2a-4)(2a-4)}{a(a^2-4)} \end{bmatrix}$$

$\Rightarrow a > 0$

$\Rightarrow \frac{a^2-4}{a} > 0$

$\Rightarrow a^2 - 4 > 0$

$\Rightarrow (a-2)(a+2) > 0$

$\Rightarrow \underbrace{(-\infty, -2)}_{\text{as } (a>0)} \cup (2, \infty)$

$\Rightarrow \frac{a^2-4}{a} - \frac{(2a-4)(2a-4)}{a(a^2-4)}$

$\Rightarrow (a^2-4)^2 - (2a-4)^2 > 0$

$\Rightarrow (a^2 - 4 - 2a + 4)(a^2 - 4 + 2a - 4) > 0$

$\Rightarrow (a^2 - 2a)(a^2 + 2a - 8) > 0$

$\therefore (a^2 - 2a) > 0 \quad \& \quad (a^2 + 2a - 8) > 0$

$a(a-2) > 0 \quad a^2 + 4a - 2a - 8 > 0$

$a(a+4) - 2(a+4) > 0$

$(a+4)(a-2) > 0$

$\therefore$ we get $a > 0 \cup a > 2 \cup a > -4 \cup a > 2$

$\therefore a > 2$

$\therefore$ Range of a is $(2, \infty)$ //

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$f = x^T A x$

$f = 2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3$

let $x = (x_1, x_2, x_3)$

$A =$ req. 3×3 matrix

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$x^T A x = \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz$$

comparing with i),

$(x = x_1, \ y = x_2, \ z = x_3)$

$\Rightarrow a_{11} = 2 \quad a_{22} = 2 \quad a_{33} = 2$

$a_{12} = -1 \quad a_{13} = 0 \quad a_{23} = -1$

Req. 3×3 symmetric matrix $= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

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(7) $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}_{3\times 2}$ (Tall matrix of order 3×2,

$A_{3\times2} = U_{3\times3} \Sigma_{3\times2} V^T_{2\times2}$

SVD of A?)

$$A^T A = \begin{bmatrix} -3 & 6 & 6 \\ 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}_{2\times2}$$

To obtain eigen value solve for $|A - \lambda I| = 0$

$$\begin{vmatrix} 81-\lambda & -27 \\ -27 & 9-\lambda \end{vmatrix} = 0 \Rightarrow (81-\lambda)(9-\lambda) - (-27)(-27) = 0$$

$\Rightarrow (729 - 81\lambda - 9\lambda + \lambda^2 - 729) = 0$

$\Rightarrow -90\lambda + \lambda^2 = 0$

$\Rightarrow \lambda(\lambda - 90) = 0 \Rightarrow \lambda = 90, 0$

$\lambda_1 = 90, \ \lambda_2 = 0$.

To obtain, eigen vectors of $A^T A$,

→ From $\lambda_1 = 90$

$$\begin{bmatrix} -9 & -27 \\ -27 & -81 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$R_2 \leftarrow R_2 - 3R_1$

$$\begin{bmatrix} -9 & -27 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$\Rightarrow x = -3y$

$x_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$

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$\sqrt{(-3)^2+(1)^2} = \sqrt{10}$

=> From $\lambda_2 = 0$

$$\begin{bmatrix} 81 & -27 \\ -27 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$R_2 \leftarrow R_2 + \frac{1}{3}R_1$$

$$\begin{bmatrix} 81 & -27 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$81x - 27y = 0$$

$$x = \frac{1}{3}y$$

$x_2 = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$ $\qquad \sqrt{(1/3)^2+(1)^2} = \frac{\sqrt{10}}{3}$

Normalizing x_1 & x_2,

$V_1 = \begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$

matrix $V = \begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}$

Singular value of A are $\sigma_1 = \sqrt{90}$ & $\sigma_2 = 0$

Eigen value of $A \cdot A^T$ (order 3x3) are 90, 0, 0.

$$u_1 = \frac{Av_1}{\sigma_1} = \frac{\begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} -3/\sqrt{10} \\ 1/\sqrt{10} \end{bmatrix}}{\sqrt{10}} = \begin{bmatrix} 1/3 \\ -2/3 \\ -2/3 \end{bmatrix}$$

The other columns of U are found by finding vector sol^ns to $U_1^T x = 0$ as U_2 & U_3 are orthogonal to U_1.

$$U_1^T x = 0 \Rightarrow \frac{x_1}{3} - \frac{2x_2}{3} - \frac{2x_3}{3} = 0$$

$$\Rightarrow x_1 - 2x_2 - 2x_3 = 0$$

$$\Rightarrow \begin{bmatrix} 1 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\Rightarrow x_1 = 2x_2 + 2x_3$$

Basis $\Rightarrow$ $x_2 \underbrace{\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}}_{a_2} + x_3 \underbrace{\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}}_{a_3}$

Applying Gram-Schmidt process to a_2 & a_3 to obtain U_2 & U_3 (orthogonal vectors)

The vector $(2, 1, 0)$ & $(2, 0, 1)$

$$U_2 = \frac{a_2}{\|a_2\|} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 0\right)$$

$$e = a_2 - (U_2^T a_2) U_2$$

$$\Rightarrow \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} - \left(\begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right) \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \\ 0 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 8/5 \\ 4/5 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2/5 \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 8/5 \\ 4/5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2/5 \\ -4/5 \\ 1 \end{bmatrix} \qquad \|e\| = \sqrt{\frac{4}{25} + \frac{16}{25} + 1} = \frac{\sqrt{45}}{5}$$

$$= \frac{3}{\sqrt{5}}$$

$$u_3 = \frac{e}{\|e\|} = \left(\frac{2}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}, \frac{\sqrt{5}}{3}\right) \text{ or}$$

$$\left(\frac{2}{\sqrt{45}}, \frac{-4}{\sqrt{45}}, \frac{5}{\sqrt{45}}\right)$$

$$\Sigma = \begin{bmatrix} 90 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad A = U \Sigma V^T$$

$$\underset{A}{\begin{bmatrix} -3 & 1 \\ 6 & -2 \\ 6 & -2 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 1/3 & 2/\sqrt{5} & 2/\sqrt{45} \\ -2/3 & 1/\sqrt{5} & -4/\sqrt{45} \\ -2/3 & 0 & 5/\sqrt{45} \end{bmatrix}} \underset{\Sigma}{\begin{bmatrix} \sqrt{90} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}} \underset{V^T}{\begin{bmatrix} -3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & 3/\sqrt{10} \end{bmatrix}}$$ //