

# Vector and Matrix Calculations

## Vectors and Matrices in MATLAB

Matrices are fundamental to MATLAB; they can be visualized as tables of values. The size of a matrix is  $m \times n$ , where  $m$  is the number of rows and  $n$  is the number of columns.

You need to become familiar with matrix generation and manipulation. To type a matrix into MATLAB you must

- begin with a square bracket, [
- separate elements in a row with commas or spaces
- use semicolons to separate rows
- end the matrix with another square bracket, ]

# Vector and Matrix Calculations

## Vectors and Matrices in MATLAB

For example, to generate the matrix

$$A = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 12 & 2 \\ 32 & 16 & 8 \end{bmatrix}$$

in MATLAB, type

`A = [1 1 5; 7 12 2; 32 16 8]` or `A = [1,1,5;  
7,12,2; 32,16,8]`

A (column) vector can be entered in the same way as a matrix. For example, the vector

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

can be entered as `x = [1; 2; 3; 4]`

`linspace(a,b,n)` Generates a row vector of `n` equally spaced points between `a` and `b`.

# Vector and Matrix Calculations

## Vectors and Matrices in MATLAB

The transpose of a matrix  $A$ , denoted by  $A^T$ , is obtained by reversing the rows and columns of  $A$ . That is, if  $A = (a_{ij})$  then  $A^T = (a_{ji})$ . In MATLAB  $A^T$  is calculated by  $A'$  (i.e., a period followed by a single quote mark) or, simpler,  $A'$  (for matrices with real entries). Thus, the vector  $x$  is usually entered as

```
x=[1 2 3 4]'
```

### Special Matrices:

- `zeros(n)` Generates an  $n \times n$  matrix with all elements being 0.
- `zeros(m, n)` Generates an  $m \times n$  matrix with 0 entries.
- `zeros(size(A))` Generates a zero matrix of the size of  $A$ .
- `ones` Generates a matrix with all elements being 1. The arguments are the same as for `zeros`.
- `eye` Generates the identity matrix. The arguments are the same as for `zeros`.

# Vector and Matrix Calculations

## Vectors and Matrices in MATLAB

- `rand` Generates a matrix whose elements are uniformly distributed random numbers in the interval (0, 1). The arguments are the same as for `zeros`.
- `randi([MIN,MAX], n, m)` Generates an  $n \times m$  matrix with integer entries drawn from MIN:MAX.
- `magic(n)` Generates an  $n \times n$  matrix constructed from the integers 1 through  $n^2$  with equal row, column, and diagonal sums.
- The `gallery` function can return many different types of test matrices for applications.

# Vector and Matrix Calculations

## Elementary Vector and Matrix Operations

- $\text{dot}(x, y)$  Dot product of vectors  $x$  and  $y$ .
- $\text{cross}(x, y)$  Cross product of 3-D vectors  $x$  and  $y$ .
- $A + B$  Matrix addition.
- $A - B$  Matrix subtraction.
- $A * B$  Matrix multiplication.
- $A^n$  Matrix exponentiation.
- $A \setminus b$  The solution to  $Ax=b$  by Gaussian elimination when  $A$  is a square nonsingular matrix.
- $A . * B$  Elementwise multiplication.
- $A .^p$  Elementwise exponentiation.
- $A ./ B$  Elementwise division.

# Vector and Matrix Calculations

## Other Vector and Matrix Operations

- $\max(x)$  The maximum element of a real vector.  
 $[m, i] = \max(x)$  also returns the element which contains the maximum value in  $i$ .
- $\max(A)$  A row vector containing the maximum element in each column of a matrix.  
 $[m, i] = \max(A)$  also returns the element in each column which contains the maximum value in  $i$ .
- $\min(x)$  The minimum element of a real vector  $x$ .
- $\min(A)$  A row vector containing the minimum of the elements in each column in a matrix  $A$ .
- $\text{mean}(x)$  The mean, or average, of the elements of a vector.
- $\text{mean}(A)$  A row vector containing the mean of the elements in each column in a matrix.
- $\text{norm}(x)$  The Euclidean length of a vector.
- $\text{norm}(A)$  The matrix norm of  $A$ . Note: the norm of a matrix is not the Euclidean length of each column in the matrix.

# Vector and Matrix Calculations

## Other Vector and Matrix Operations

- `prod(x)` The product of the elements of a vector.
- `prod(A)` A row vector containing the product of the elements in each column in a matrix.
- `sort(x)` Sorts the elements in increasing order of a real vector.
- `sort(A)` Sorts the elements in increasing order in each column of a real matrix.
- `sum(x)` The sum of the elements of a vector.
- `sum(A)` A row vector containing the sums of the elements in each column in a matrix.
- `range(x)` Returns the range of the values in `x`. For a vector input, it is the difference between the maximum and minimum values.
- `range(A)` A row vector containing the range for each column.

# Vector and Matrix Calculations

## Matrix Manipulation

For a 4x3 matrix  $A$ :

`diag(A)` extracts the diagonal of matrix  $A$  as a vector.

`A(3:4, 2:3)` gets those elements of  $A$  that are located in rows 3 to 4 and columns 2 to 3.

`A(:, 4)=A(:, 1)` adds a fourth column to  $A$  and set it equal to the first column of  $A$ .

`A(2:4, 2:4)=eye(3)` replaces the last 3x3 submatrix of  $A$  by a 3x3 identity matrix.

`A([1 3], :)=[]` deletes the first and third rows of  $A$ .

`round(A)` rounds off all entries of  $A$ .

`A(:)` strings out all elements of  $A$  in a column.

`reshape(A, 2, 6)` transforms  $A$  into a 2x6 matrix.

`rot90(A)` rotates  $A$  by  $90^\circ$ .

`tril(A)` extracts the lower triangular part of  $A$ .

`triu(A)` extracts the upper triangular part of  $A$ .



# Vector and Matrix Calculations

## Eigenvalues and Eigenvectors

For an  $n \times n$  matrix  $A$ :

$\det(A)$  is the determinant of  $A$ .

$\text{inv}(A)$  is the inverse of  $A$ .

$\text{poly}(A)$  is a row vector with  $n+1$  elements which are the coefficients of the characteristic polynomial.

$[V, D] = \text{eig}(A)$  produces a diagonal matrix  $D$  of eigenvalues and a full matrix  $V$  whose columns are the corresponding eigenvectors.

### Appendix: Polynomials

$p = [5 \ -3 \ 0 \ 2 \ 1]$  specifies the polynomial

$$p(x) = 5x^4 - 3x^3 + 2x + 1.$$

$\text{polyval}(p, 3)$  gives  $p(3)$ .

$\text{roots}(p)$  finds the roots of  $p$ .

$\text{poly}(r)$  computes the coefficients of the polynomial whose roots are specified by the vector  $r$ .

$\text{conv}(p, q)$  computes the product of the polynomials  $p$  and  $q$ .

$[q, r] = \text{deconv}(\text{num}, \text{den})$  computes the result (i.e., *quotient* and *remainder*) of dividing polynomial  $\text{num}$  by polynomial  $\text{den}$ .