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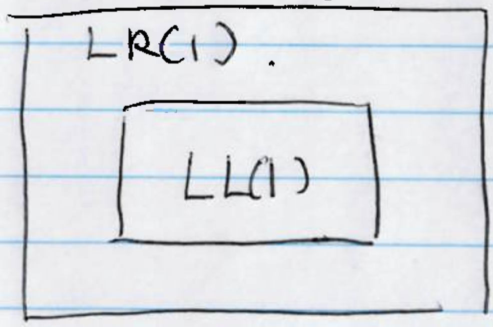
3.4. Bottom up parsing : build the perse tree from leaves to the root

$\downarrow$ right most derivation <u>in reverse</u>

LR(1) $\leftarrow$ look ahead. <u>one</u> token.

$\uparrow$ left to right scan.



LR(1) ① can recognize a larger set of grammars / languages.

$$L = \{ \underline{a^i b^j}, \quad i \geq j \geq 1 \}.$$

$$\begin{aligned} S &\rightarrow \overset{\sqcap}{a S b} \\ &\mid ab \\ &\mid TS \\ T &\rightarrow aT \\ &\mid \varepsilon \end{aligned}$$

aaab b b

$L \notin LL(k)$ for any k

$L \in LR(1)$

2. build ~~tree~~ a reversed right most derivation tree

3. at each step. the parser either

read/scan next token / shift

or ~~replace~~ reduction rhs of a prod. with its lhs. /

ex: $G \rightarrow E$

$E \rightarrow E + T$

$\quad E - T$

$\quad T$

$T \rightarrow T * F$

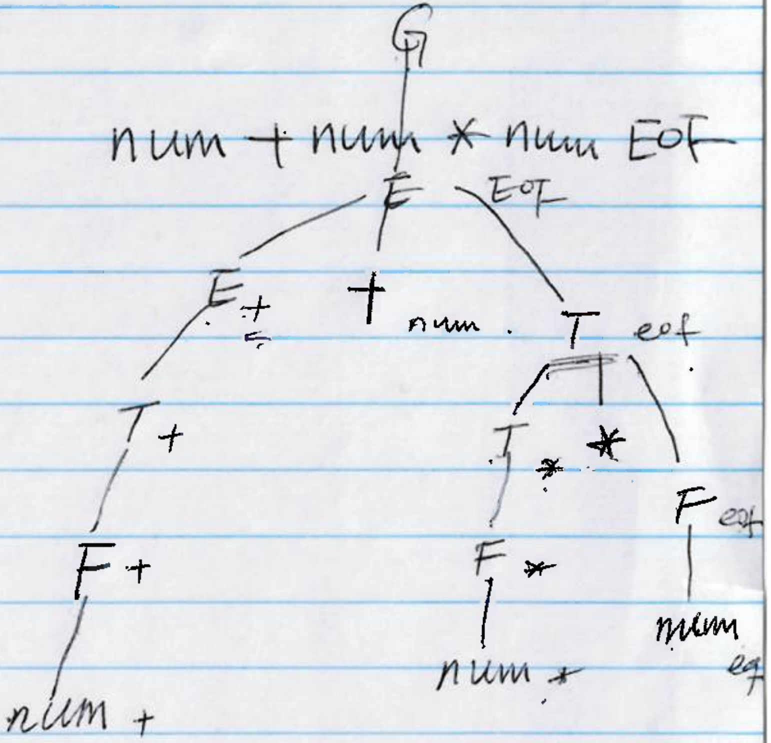
$\quad T / F$

$\quad F$

$F \rightarrow$ num

$\quad$ name

$\quad (E)$



shift:

reduction. $A \longrightarrow \underbrace{B_1 B_2 \cdots B_k}_{\text{handle}}$.

Q How to find a handle ?

ex: ① Goal → List

② List → List pair

③ | pair

④ pair → (List)

⑤ | ( )

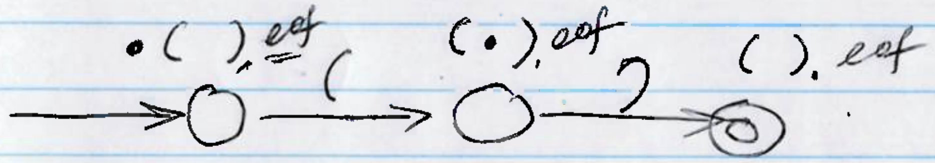
input ✗) EOF

✗ (( ) EOF

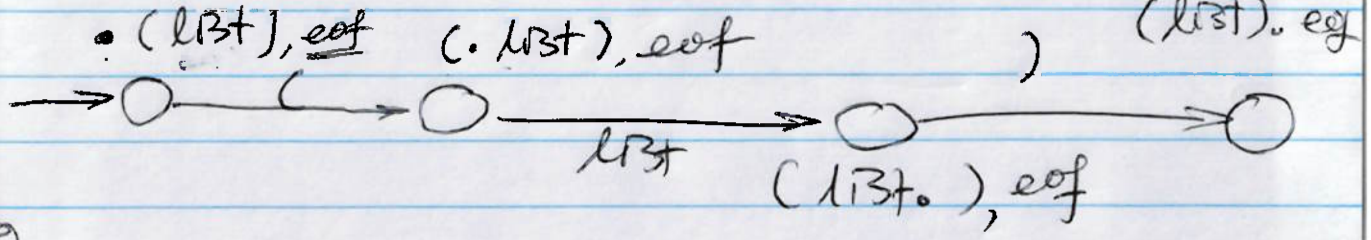
✓ ( ) ( ) EOF

✓ (( )) EOF

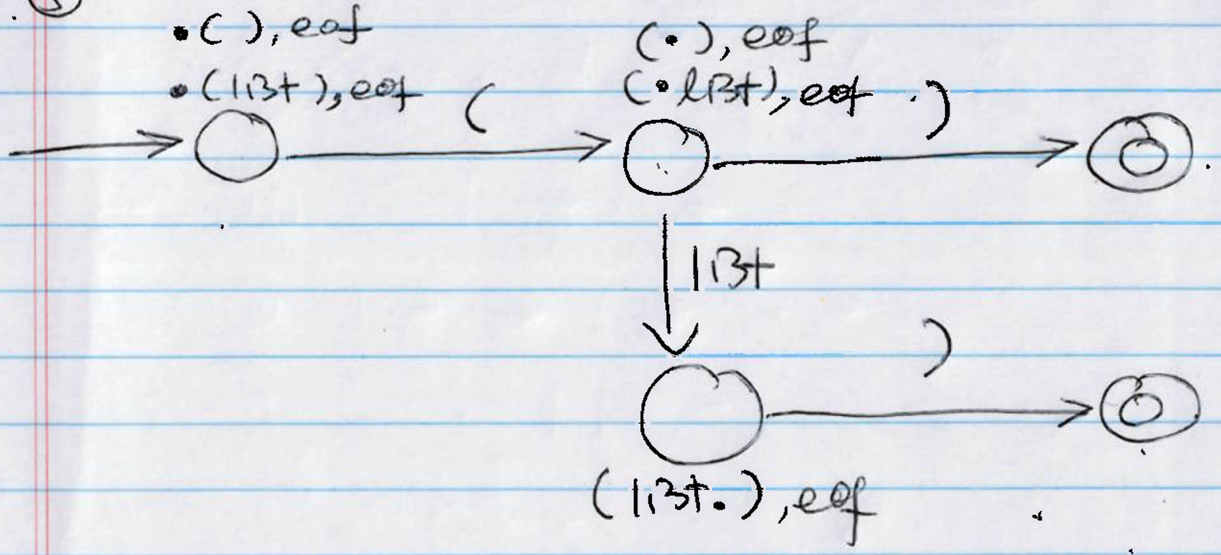
⑤. pair → $\underline{(\ \ )}$



pair → (list)



④ . ⑤



build DFA.

encode DFA into 2 tables

Action table

State	Terminal	EOF
S_0		
S_1		
⋮		

$$\text{Action}(\text{state}, a \in T) = \begin{cases} \text{accept} \\ s \quad \text{next state} \\ r \quad \text{rule\#} \\ \text{error / blank} \end{cases}$$

LR(1) Tables for Parenthesis Grammar

1	Goal	→	List
2	List	→	List Pair
3		|	Pair
4	Pair	→	(List)
5		|	( )

	ACTION Table			Goto Table	
State	EOF	(	)	List	Pair
s_0		s 3		1	2
s_1	Acc	s 3			4
s_2	r 3	r 3			
s_3		s 7	s 8	5	6
s_4	r 2	r 2			
s_5		s 7	s 10		9
s_6		r 3	r 3		
s_7		s 7	s 12	11	6
s_8	r 5	r 5			
s_9		r 2	r 2		
s_{10}	r 4	r 4			
s_{11}		s 7	s 13		9
s_{12}		r 5	r 5		
s_{13}		r 4	r 4		

$$\begin{tikzcd}
\arrow[r] & S_0 \arrow[r, "("] \arrow[d, "list"] \arrow[dr, "pair"] & S_3 \\
& \boxed{S_1} \arrow[ur, bend right=60, "("'] & S_2
\end{tikzcd}$$

Goto table .

state	NT
S_0	
S_1	
S_2	
⋮	

Goto(state, $A \in NT$)

= next state .

LR(1) skeleton parser . table driven .

//. makam a stack to keep the prefix of the upper frontrer of the parse tree

```
push ( $, $ )
push[(start symbol, S0)
word ← nextword( )
while ( true )
  (symbol, state) ← top( ) of the stack
  switch (Action (state, word))
      case "accept" word = eof
          then break
      case Si
          push( word, Si )
          word ← nextword( )
```

```
case r#   (A → B_1 B_2 --- B_k)
    pop  k times
    (symbol, state) ← top( )
    push(A, Goto(state, A))
otherwise
    report error
```

example: ( ) eof

State	lookahead	stack	handle	Action
	(	\$(Goal,0)		
0	(			S3
3	)	\$(Goal,0) ((,3)		S8
8	eof	\$(Goal,0)((,3),(),8)		r5 ~~A~~ ( )
2	eof	\$(Goal,0) (pair, 2)		r3
1	eof	\$(Goal,0)(List, 1)		Acc

( ))   (( )

( ) ( )   (( ))

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1	*Goal*	→	*List*	
2	*List*	→	*List Pair*	
3		|	*Pair*	
4	*Pair*	→	$\underline{(}$ *List* $\underline{)}$	
5		|	$\underline{(}$ $\underline{)}$	

State	eof	(	)	*List*	*Pair*
0		s 3		1	2
1	acc	s 3			4
2	r 3	r 3			
3		s 7	s 8	5	6
4	r 2	r 2			
5		s 7	s 10		9
6		r 3	r 3		
7		s 7	s 12	11	6
8	r 5	r 5			
9		r 2	r 2		
10	r 4	r 4			
11		s 7	s 13		9
12		r 5	r 5		
13		r 4	r 4		

) *Action* and *Goto* Tables for Parentheses Grammar

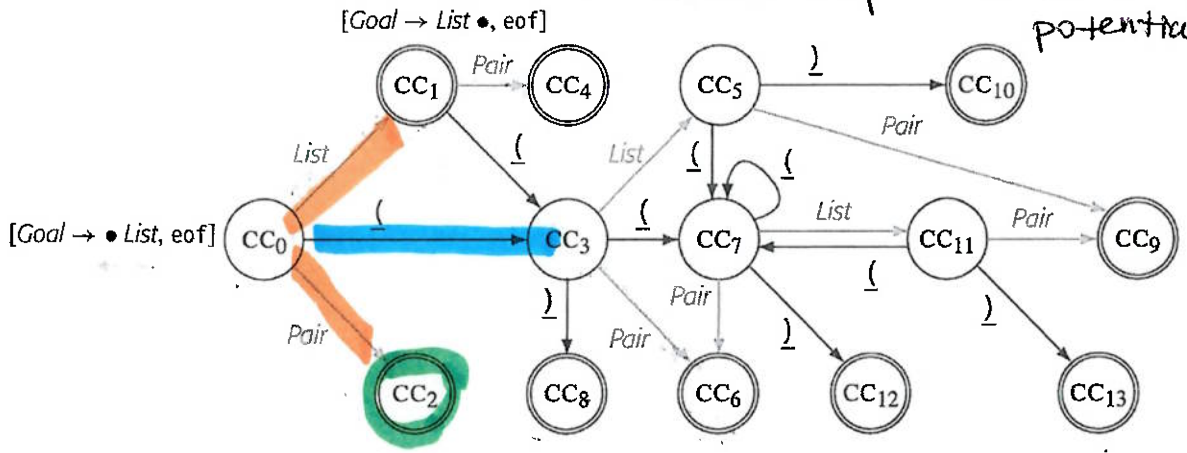
item: pair → $\underbrace{(\quad)}$, eof

•( ), eof

(•), eof

( )• eof

CCi: canonical collection of items a state of the parser that contains one or more items that represent ~~a~~ handles or potential handles



review subset construction

initial state   *: core item .

Goal → list
list → list pair
  | pair
pair → (list)
  ( )

CC0   * Goal → • list, eof

list → • list pair, eof   list → • pair, eof

$\beta \quad \circ \quad \delta \quad a$

list → • list pair, (list → • pair, (

pair → • (list), eof   pair → • ( ), eof

pair → • (list), (pair → • ( ), (

transition.

$$\begin{tikzcd}
CC0 \arrow[r, "list"] \arrow[d, "pair"'] \arrow[dr, "("] & CC1 \\
CC2 &
\end{tikzcd}$$

CC1   * Goal → list •, eof .

* list → list • pair, eof   * list → list • pair, (

pair → • (list), eof   pair → • ( ), eof   pair → • (list), (pair → • ( ), (

CC2   core   * list → pair • , (

* list → pair •, eof .

accepting state

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compute a state starting core states

β = a string left context

T ∪ NT string

```
closure(s)
    while (s is still changing) do
        for each item $[A \rightarrow \beta \bullet C\delta, a] \in s$ do
            lookahead ← $\delta a$
            for each production $C \rightarrow \gamma \in P$ do
                for each $b \in$ FIRST(lookahead) do
                    $s \leftarrow s \cup \{[C \rightarrow \bullet\gamma, b]\}$
    return s
```

```
goto(s, x)
    $t \leftarrow \emptyset$
    for each item $i \in s$ do
        if $i$ is $[\alpha \rightarrow \beta \bullet x\delta, a]$ then
            $t \leftarrow t \cup \{[\alpha \rightarrow \beta x \bullet \delta, a]\}$
    return closure(t)
```

computes a transition

1	Goal	→	List
2	List	→	List Pair
3		|	Pair
4	Pair	→	(List)
5		|	( )

$$CC_0 = \left\{\begin{array}{l} [Goal \rightarrow \bullet\, List, eof] \\ [List \rightarrow \bullet\, List\ Pair, eof]\ [List \rightarrow \bullet\, List\ Pair, (]\ [List \rightarrow \bullet\, Pair, eof]\ [List \rightarrow \bullet\, Pair, (] \\ [Pair \rightarrow \bullet\, (\ List\ ), eof]\ [Pair \rightarrow \bullet\, (\ List\ ), (]\ [Pair \rightarrow \bullet\, (\ ), eof]\ [Pair \rightarrow \bullet\, (\ ), (] \end{array}\right\}$$

$$CC_1 = \left\{\begin{array}{l} [Goal \rightarrow List \bullet, eof]\ [List \rightarrow List \bullet Pair, eof]\ [List \rightarrow List \bullet Pair, (] \\ [Pair \rightarrow \bullet\, (\ List\ ), eof]\ [Pair \rightarrow \bullet\, (\ List\ ), (]\ [Pair \rightarrow \bullet\, (\ ), eof]\ [Pair \rightarrow \bullet\, (\ ), (] \end{array}\right\}$$

$$CC_2 = \{ [List \rightarrow Pair \bullet, eof]\ [List \rightarrow Pair \bullet, (] \}$$

If $[A \rightarrow \beta; a] \in CC_i$, then $\boxed{CC_i}$

// build collection DFA

```
$CC_0 \leftarrow \emptyset$
for each production of the form $Goal \rightarrow \alpha$ do
    $CC_0 \leftarrow CC_0 \cup \{[Goal \rightarrow \bullet\alpha, eof]\}$
$CC_0 \leftarrow closure(CC_0)$
$CC \leftarrow \{CC_0\}$
while (new sets are still being added to CC) do
    for each unmarked set $CC_i \in CC$ do
        mark $CC_i$ as processed
        for each $x$ following a • in an item in $CC_i$ do
            temp ← goto($CC_i$, x)
            if temp ∉ CC then
                CC ← CC ∪ {temp}
            record transition from $CC_i$ to temp on x
```

// table filling Action Goto

```
for each $CC_i \in CC$ do
    for each item $I \in CC_i$ do        c ∈ T, and
    ① if I is $[A \rightarrow \beta \bullet c\gamma, a]$ and goto($CC_i$, c) = $CC_j$ then
            Action[i,c] ← "shift j"
    ② else if I is $[A \rightarrow \beta \bullet, a]$ then
            Action[i,a] ← "reduce $A \rightarrow \beta$"
    ③ else if I is $[Goal \rightarrow \beta \bullet, eof]$ then
            Action[i, eof] ← "accept"
    for each $n \in NT$ do
        if goto($CC_i$,n) = $CC_j$ then
            Goto[i,n] ← j
```

$$\begin{tikzcd} CC_i \arrow[r, "c"] & CC_j \end{tikzcd}$$

$\boxed{CC_i}$

$$\begin{tikzcd} CC_i \arrow[r, "n"] & CC_j \end{tikzcd}$$

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$$CC_0 = \left\{ \begin{array}{l} [Goal \rightarrow \bullet\ List,\ \text{eof}] \\ [List \rightarrow \bullet\ List\ Pair,\ \text{eof}]\ [List \rightarrow \bullet\ List\ Pair,\ \underline{(}]\ [List \rightarrow \bullet\ Pair,\ \text{eof}]\ [List \rightarrow \bullet\ Pair,\ \underline{(}] \\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \text{eof}]\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \text{eof}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{(}] \end{array} \right\}$$

$$CC_1 = \left\{ \begin{array}{l} [Goal \rightarrow List\ \bullet,\ \text{eof}]\ [List \rightarrow List\ \bullet\ Pair,\ \text{eof}]\ [List \rightarrow List\ \bullet\ Pair,\ \underline{(}] \\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \text{eof}]\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \text{eof}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{(}] \end{array} \right\}$$

$$CC_2 = \left\{ [List \rightarrow Pair\ \bullet,\ \text{eof}]\ [List \rightarrow Pair\ \bullet,\ \underline{(}] \right\}$$

$$CC_3 = \left\{ \begin{array}{l} [Pair \rightarrow \underline{(}\ \bullet\ List\ \underline{)},\ \text{eof}]\ [Pair \rightarrow \underline{(}\ \bullet\ List\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \underline{(}\ \bullet\ \underline{)},\ \text{eof}]\ [Pair \rightarrow \underline{(}\ \bullet\ \underline{)},\ \underline{(}] \\ [List \rightarrow \bullet\ List\ Pair,\ \underline{(}]\ [List \rightarrow \bullet\ List\ Pair,\ \underline{)}]\ [List \rightarrow \bullet\ Pair,\ \underline{(}]\ [List \rightarrow \bullet\ Pair,\ \underline{)}] \\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{)}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{)}] \end{array} \right\}$$

$$CC_4 = \left\{ [List \rightarrow List\ Pair\ \bullet,\ \text{eof}]\ [List \rightarrow List\ Pair\ \bullet,\ \underline{(}] \right\}$$

$$CC_5 = \left\{ \begin{array}{l} [List \rightarrow List\ \bullet\ Pair,\ \underline{(}]\ [List \rightarrow List\ \bullet\ Pair,\ \underline{)}]\ [Pair \rightarrow \underline{(}\ List\ \bullet\ \underline{)},\ \text{eof}]\ [Pair \rightarrow \underline{(}\ List\ \bullet\ \underline{)},\ \underline{(}] \\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{)}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{)}] \end{array} \right\}$$

$$CC_6 = \left\{ [List \rightarrow Pair\ \bullet,\ \underline{(}]\ [List \rightarrow Pair\ \bullet,\ \underline{)}] \right\}$$

$$CC_4 = \left\{ [List \rightarrow List\ Pair\ \bullet,\ \text{eof}]\ [List \rightarrow List\ Pair\ \bullet,\ \underline{(}] \right\}$$

$$CC_7 = \left\{ \begin{array}{l} [Pair \rightarrow \underline{(}\ \bullet\ List\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \underline{(}\ \bullet\ List\ \underline{)},\ \underline{)}]\ [Pair \rightarrow \underline{(}\ \bullet\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \underline{(}\ \bullet\ \underline{)},\ \underline{)}] \\ [List \rightarrow \bullet\ List\ Pair,\ \underline{(}]\ [List \rightarrow \bullet\ List\ Pair,\ \underline{)}]\ [List \rightarrow \bullet\ Pair,\ \underline{(}]\ [List \rightarrow \bullet\ Pair,\ \underline{)}] \\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{)}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{)}] \end{array} \right\}$$

$$CC_8 = \left\{ [Pair \rightarrow \underline{(}\ \underline{)}\ \bullet,\ \text{eof}]\ [Pair \rightarrow \underline{(}\ \underline{)}\ \bullet,\ \underline{(}] \right\}$$

$$CC_9 = \left\{ [List \rightarrow List\ Pair\ \bullet,\ \underline{(}]\ [List \rightarrow List\ Pair\ \bullet,\ \underline{)}] \right\}$$

$$CC_{10} = \left\{ [Pair \rightarrow \underline{(}\ List\ \underline{)}\ \bullet,\ \text{eof}]\ [Pair \rightarrow \underline{(}\ List\ \underline{)}\ \bullet,\ \underline{(}] \right\}$$

$$CC_{11} = \left\{ \begin{array}{l} [List \rightarrow List\ \bullet\ Pair,\ \underline{(}]\ [List \rightarrow List\ \bullet\ Pair,\ \underline{)}]\ [Pair \rightarrow \underline{(}\ List\ \bullet\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \underline{(}\ List\ \bullet\ \underline{)},\ \underline{)}] \\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ List\ \underline{)},\ \underline{)}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{(}]\ [Pair \rightarrow \bullet\ \underline{(}\ \underline{)},\ \underline{)}] \end{array} \right\}$$

$$CC_{12} = \left\{ [Pair \rightarrow \underline{(}\ \underline{)}\ \bullet,\ \underline{(}]\ [Pair \rightarrow \underline{(}\ \underline{)}\ \bullet,\ \underline{)}] \right\}$$

$$CC_{13} = \left\{ [Pair \rightarrow (\ List\ \underline{)}\ \bullet,\ \underline{(}]\ [Pair \rightarrow (\ List\ \underline{)}\ \bullet,\ \underline{)}] \right\}$$

Is the grammar LR(1) ?

reduce reduce conflict
reduce- shift conflict

when filling the table