Workshop on image processing, computer vision and graphics

Registration

class:7-8, 30.06.2011, time: 11.15am to 1.15 pm

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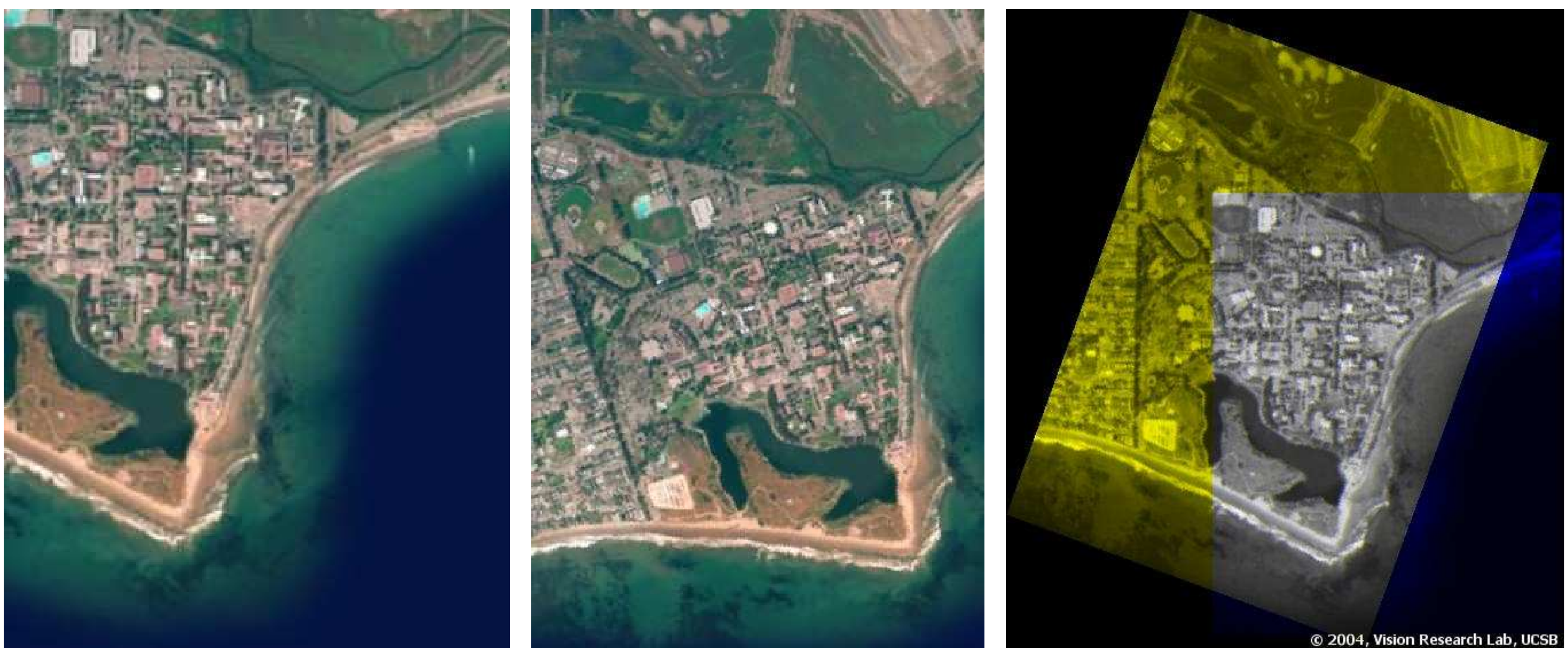
Motion models and Registration

- motion models
- Translation
- Rotation
- Affine motion model
- Registration

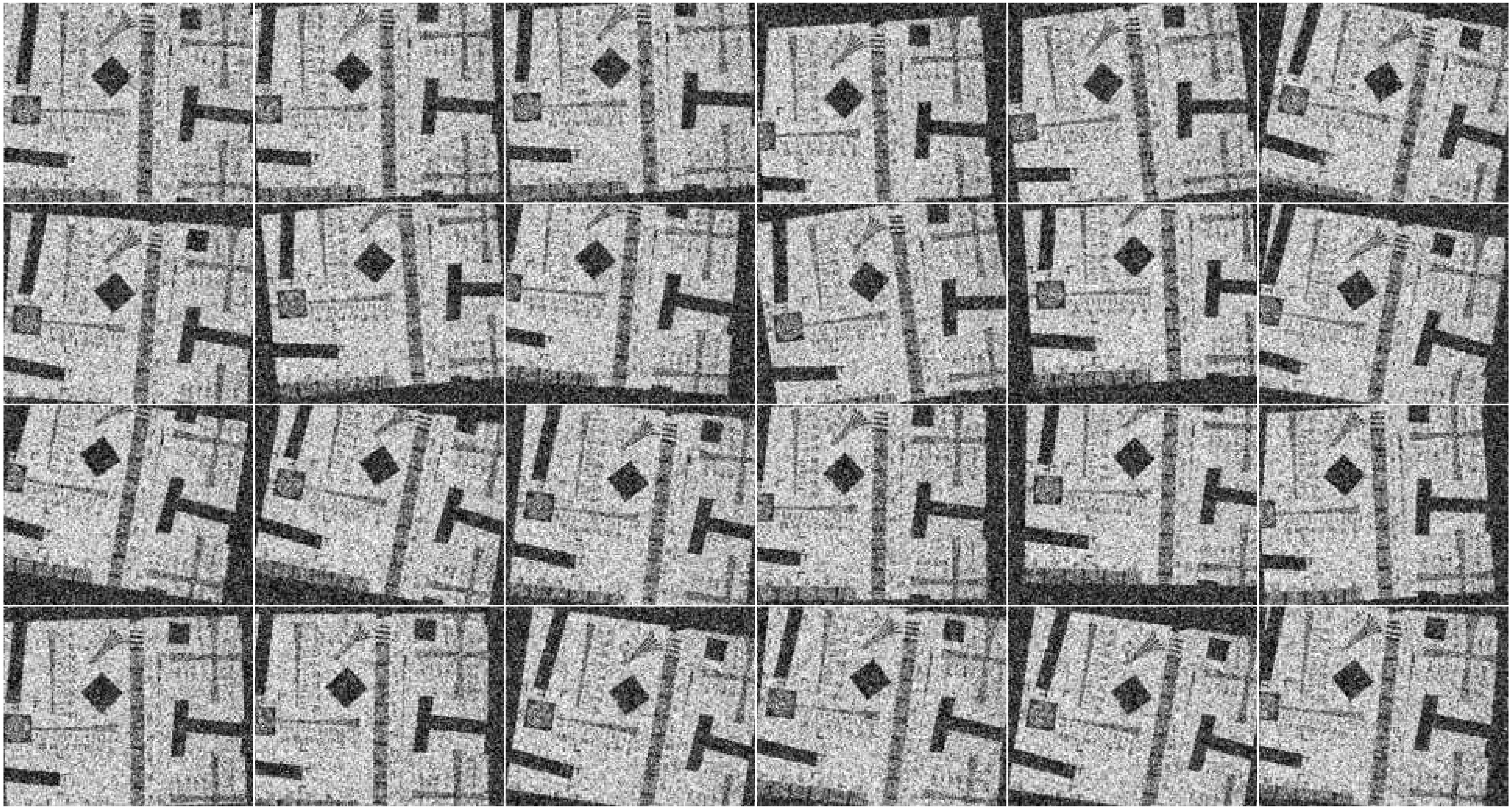
Registration example 1

- a process of establishing a point-point correspondence between two images of a scene



Registration example 2

- Input registered images



Motivation

- First step in many methods
  - Super resolution
  - Mosaicking
  - Restoration using many images
  - Noise removal
- Needs co-ordinate transformation
- Many methods

Registration methods

- Extrinsic methods:
  - uses external well detectable objects
  - shapes, different colours, markings,
  - widely used in image guided surgery because of ease
  - no need of complex optimization algorithms

Registration methods

- Intrinsic methods
  - Feature based: use of higher level structures like edges, lines, shapes etc
  - Intensity based
    - pixel based
    - local
    - global
    - Course to fine refinement strategy

Pyramid based registration method

- Pyramid construciton: multi resolution framework
- Motion estimation: main step in the registraton
- image warping: use current model parameters to warp second image to the reference image
- course-to-fine refinement: propogate the current estimates from lower level to higher

Main Features of Pyramid

- Large dispalcements become smaller and can be computed at low resolution images
- Improve the estimates by using high resolution images
- It is efficient to ignore higher frequecies when computing larger distances
- It is necessary because of aliasing

Motion estimation

- Main step in the registration
- Vary according to motion model
- In all cases in involves the minimization of sum of squared differences (SSD) betweem reference image and warped images using motion parameters
- We use affine motion model: only translations and rotations are involved

Image warping

- Warping uses bilinear interpolation
- Uses current parameters of motion estimation
- Warped image is used to compute the error with reference image at that level
- This step is repeated for all the levels

Course to fine refinement

- Propogates the current motion estimates to the next level
- In the next level the motion parameters are used as initial conditions and are refined in that level
- Parameters of the model are trasmitted to the next level
- Motion estimation and warping are used to estimates the parameters at each level

Motiopn estimation

Intensity conservation/constancy

- Image intensity as a function of space and time

$$f(x(t), y(t), t) = C$$

Then

$$\frac{d}{dt}(f(x,y,t)) = 0$$

Using total derivative and chain rule, we get

$$\frac{d}{dt}(f(x,y,t)) = \frac{df(x,y,t)}{dx}\frac{dx}{dt} + \frac{df(x,y,t)}{dy}\frac{dy}{dt} + \frac{df(x,y,t)}{dt}\frac{dt}{dt}$$

Intensity conservation

We also have

$$\frac{d}{dt}(f(x,y,t)) = 0$$

This gives

$$f_x\theta_x + f_y\theta_y + f_t = 0$$

where $\theta_x = \frac{dx}{dt}$ and $\theta_y = \frac{dy}{dt}$

We can also write

$$f_x\theta_x + f_y\theta_y = -f_t$$

$$\begin{bmatrix} f_x & f_y \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} = -f_t$$

Intensity derivatives and optical flow

where f_x and f_y are the spatial derivatives in the x and y direction, and θ_x and θ_y are the velocities in the x and y direction

- When 3D point moves in space it generates a 2D path
- Otical flow:Temporal derivatives of dense 2D path is equal to vector field of 2D derivatives

Gradient constraint

Time varying intensity can be approximated by first order Taylor series expansion

- Let f_x, f_y and f_t are the spatial and temporal derivatives of image intensity, then

$$f(x+\theta_x, y+\theta_y, t+1) = f(x,y,t) + \theta_x f_x(x,y,t)$$
$$+ \theta_y f_y(x,y,t) + f_t(x,y,t)$$

- We have $\theta_x f_x(x,y,t) + \theta_y f(x,y,t) + f_t(x,y,t) = 0$
- Velocity at one location in the image is spatial and temporal derivatives of image intensity at that location

Motion estimation

- $f(x,y,t)$: image intensity at a point $\mathbf{x} = (x,y)$ in the reference image
- (θ_x, θ_y): the image velocity at point $\mathbf{x}$
- $(x+\theta_x, y+\theta_y)$: the corresponding point in the second image
- Then we have

$$f(x,y,t) = f(x+\theta_x, y+\theta_y, t+1)$$

- Minimize the following SSD error

$$\Big(f(x,y,t) - f(x+\theta_x, y+\theta_y, t+1)\Big)^2$$

- This can be approximated to $(f_x\theta_x + f_y\theta_y + f_t)^2$
- Estimation problem can be reformulated using motion parametrs

Motion estimation contd..

- Let $P = (p_1, p_2, p_3, p_4, p_5, p_6)^T$ be the motion parameters, then the velocity are

$$\theta_x = p_1 x + p_2 y + p3$$

$$\theta_y = p_4 x + p_5 y + p6$$

- Then $\theta = \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} = A\,P$

- where A is give by

$$A = \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix}$$

Motion estimation contd..

- We have

$$\begin{bmatrix} f_x & f_y \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} = -f_t$$

- We can write

$$\begin{bmatrix} f_x & f_y \end{bmatrix} AP = -f_t$$

$$\begin{bmatrix} f_x & f_y \end{bmatrix} \begin{bmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix} = -f_t$$

Motion estimation contd..

θ: image velocity at each pixel

A: a 2×6 matrix (depends on image position)

P: the vector of motion parameters

$[f_x \;\; f_y]$: the vector of spatial derivatives

f_t: the temporal derivative

Solve the following equation for P

$$\begin{bmatrix} f_x & f_y \end{bmatrix} AP = -f_t$$

Motion estimation contd..

- We can write

$$\begin{bmatrix} xf_x & yf_x & f_x & xf_y & yf_y & f_y \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{bmatrix} = -f_t$$

$$XP = b$$

Iterative refinement of P

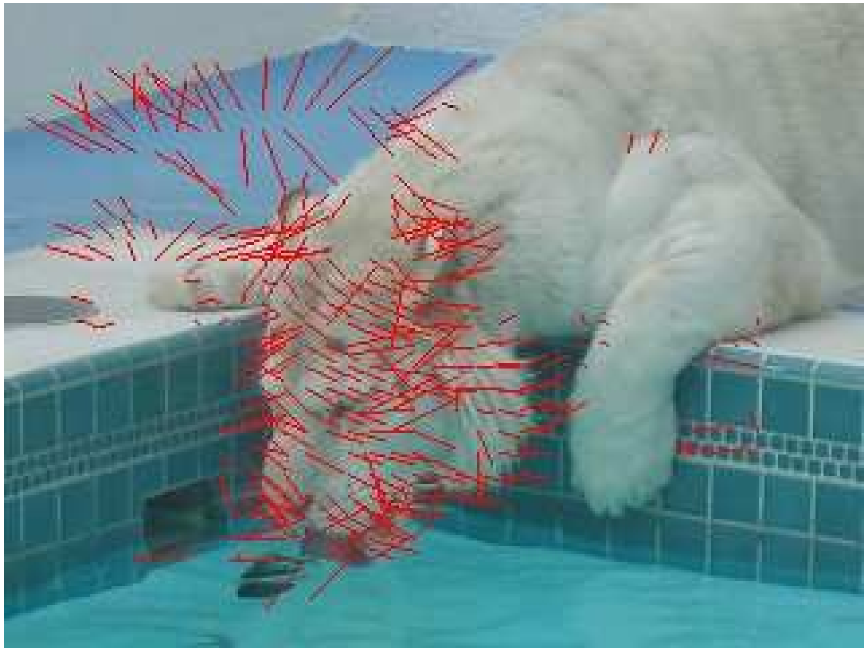
1. Estimate the flow by solving $XP = b$
2. Update motion parameters
3. Warp the second image towards the reference image using current motion parameter
4. Repeat steps 1 to 3 until the process converges

Coarse-to-fine-refinement of P

1. Start from lowest level up to highest level in the image pyramid loop
2. Warp the second image towards reference image using current motion parameters.
3. Estimate the motion between reference image and warped second image by using iterative refinement algorithm
4. Update the motion parameters for that level

Optical flow

- It is the pattern of apparent motion of objects surfaces and edges in a visual scene
- Occurs due to relative motion between observer and the scene
- The optical flow methods try to calculate the motion between two image frames which are taken at times t and $t+\delta t$ at every voxel position. These methods are called differential since they are based on local Taylor series approximations of the image signal; that is, they use partial derivatives with respect to the spatial and temporal coordinates.
- We have $\theta_x f_x(x,y,t) + \theta_y f(x,y,t) + f_t(x,y,t) = 0$
- First given by Horn



Optical flow methods

- Phase correlation: inverse of normalized cross-power spectrum
- Block based methods: minimizing sum of squared differences or sum of absolute differences, or maximizing normalized cross-correlation
- Differential methods of estimating optical flow, based on partial derivatives of the image signal and/or the sought flow field and higher-order partial derivatives.
- Discrete optimization methods: the search space is quantized, and then image matching is addressed through label assignment at every pixel, such that the corresponding deformation minimizes the distance between the source and the target image. The optimal solution is often recovered through min-cut max-flow algorithms, linear programming or belief propagation methods.

Optical flow methods: derivative based

- Lucas Kanade Optical Flow Method: regarding image patches and an affine model for the flow field
- Horn Schunck method: optimizing a functional based on residuals from the brightness constancy constraint, and a particular regularization term expressing the expected smoothness of the flow field
- Buxton Buxton method: based on a model of the motion of edges in image sequences
- Black Jepson method: coarse optical flow via correlation
- General variational methods Ð a range of modifications/extensions of HornÐSchunck, using other data terms and other smoothness terms.

Need for motion models

- The relationship between the optical flow in the image plane and the velocities of objects in the three dimensional world is not necessarily obvious
- We perceive motion when a changing picture is projected onto a stationary screen
- Conversely, a moving object may give rise to a constant brightness pattern: for example, a uniform sphere which exhibits shading because its surface elements are oriented in many different directions. Yet, when it is rotated, the optical flow is zero at all points in the image, since the shading does not move with the surface.
- We solve a particularly simple world where the apparent velocity of brightness patterns can be directly identified with the movement of surfaces in the scene.

Restrictions

- To avoid variations in brightness due to shading effects we initially assume that the surface being imaged is flat.
- We further assume that the incident illumination is uniform across the surface.
- The brightness at a point in the image is then proportional to the reflectance of the surface at the corresponding point on the object.
- Also, we assume at first that reflectance varies smoothly and has no spatial discontinuities: This condition assures that the image brightness is differentiable.
- We exclude situations where objects occlude one another, in part, because discontinuities in reflectance are found at object boundaries.
- WIth above conditions, the motion of the brightness patterns in the image is determined directly by the motions of corresponding points on the surface of the object

Optical flow calculation

- Let the image brightness at the point (x, y) in the image plane at time t be denoted by $f(x, y, t)$. Now consider what happens when the pattern moves. The brightness of a particular point in the pattern is constant, so that $\frac{df}{dt} = 0$ (Intensity conservation/constancy)
- Using total derivative and chain rule, we get

$$\frac{d}{dt}(f(x,y,t)) = \frac{df(x,y,t)}{dx}\frac{dx}{dt} + \frac{df(x,y,t)}{dy}\frac{dy}{dt} + \frac{df(x,y,t)}{dt}\frac{dt}{dt}$$

Optical flow calculation

- This gives

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We can also write

$$f_x\theta_x + f_y\theta_y = -f_t$$

$$\begin{bmatrix} f_x & f_y \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix} = -f_t$$

- Thus the component of the movement in the direction of the brightness gradient (f_x, f_y) equals

$$-\frac{f_t}{\sqrt{f_x^2 + f_y^2}}$$

Smoothness constraint

- If every point of the brightness pattern can move independently, there is little hope of recovering the velocities. In case of opaque objects, neighboring points on the objects have similar velocities and the velocity field of the brightness patterns in the image varies smoothly almost everywhere.
- Discontinuities in flow can be expected where one object occludes another.
- An algorithm based on a smoothness constraint is likely to have difficulties with occluding edges as a result.
- One way to express the additional constraint is to minimize the square of the magnitude of the gradient of the optical flow velocity:

$$\left(\frac{d\theta_x}{dx}\right)^2+\left(\frac{d\theta_x}{dy}\right)^2 \quad \text{and} \quad \left(\frac{d\theta_y}{dx}\right)^2+\left(\frac{d\theta_y}{dy}\right)^2$$

Smoothness constraint

- Another measure of the smoothness of the optical flow field is the sum of the squares of the Laplacians of the x- and y-components of the flow.

$$(\frac{d^2\theta_x}{dx^2}) + (\frac{d^2\theta_x}{dy^2}) \quad \text{and} \quad (\frac{d^2\theta_y}{dx^2}) + (\frac{d^2\theta_y}{dy^2})$$

- In simple situations, both Laplacians are zero. If the viewer translates parallel to a flat object, rotates about a line perpendicular to the surface or travels orthogonally to the surface, then the second partial derivatives of both θ_x and θ_y vanish (assuming perspective projection in the image formation).

THANK YOU