

**CBA: Practice Problem Set 2**  
**Topics: Sampling Distributions and Central Limit Theorem**

1. Examine the following normal Quantile plots carefully. Which of these plots indicates that the data ...

I. Are nearly normal?

**Sol:** C the normal

II. Have a bimodal distribution? (One way to recognize a bimodal shape is a “gap” in the spacing of adjacent data values.)

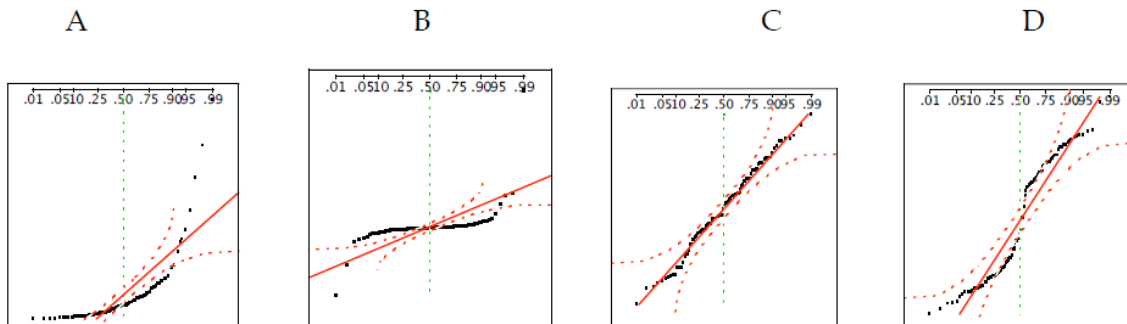
**Sol:-** B is the option which recognize a bimodal shape is a gap in the data value.

III. Are skewed (i.e. not symmetric) ?

**Sol:-** A, C and D are skewed.

IV. Have outliers on both sides of the center?

**Sol:** A



2. For each of the following statements, indicate whether it is True/False. If false, explain why.

The manager of a warehouse monitors the volume of shipments made by the delivery team. The automated tracking system tracks every package as it moves through the facility. A sample of 25 packages is selected and weighed every day. Based on current contracts with customers, the weights should have  $\mu = 22$  lbs. and  $\sigma = 5$  lbs.

- (i) Before using a normal model for the sampling distribution of the average package weights, the manager must confirm that weights of individual packages are normally distributed.

**Sol:- TRUE** - In this case 30 samples are packages selected and weighed everyday. Based on the central limit theorem, the sampling distribution of the sample mean approach normal distribution as the sample size become bigger.

- (ii) The standard error of the daily average  $SE(\bar{x}) = 1$ .

**Sol:- TRUE** – Standard error equal to the deviation divided by square root of sample size.

$$= 5/\sqrt{25}$$

$$= 1$$

3. Auditors at a small community bank randomly sample 100 withdrawal transactions made during the week at an ATM machine located near the bank's main branch. Over the past 2 years, the average withdrawal amount has been \$50 with a standard deviation of \$40. Since audit investigations are typically expensive, the auditors decide to not initiate further investigations if the mean transaction amount of the sample is between \$45 and \$55. What is the probability that in any given week, there will be an investigation?

- A. 1.25%
- B. 2.5%
- C. 10.55%

- D. 21.1%  
E. 50%

The screenshot shows the Spyder Python IDE with a script file open. The script performs a hypothesis test for a normal distribution with unknown variance. It calculates the test statistic  $z$  for two different sample means (45 and 55) and compares them to a critical value of 1.25. The console output shows the results of these calculations.

```
1 Created on Thu Dec 29 16:48:55 2022
2
3 @author: vainsav
4
5
6
7
8 import numpy as np
9 from scipy import stats
10 from scipy.stats import norm
11 #n=100, Pop Mean=50, Pop SD=40 As no. of samples is more than 30, we can consider it normal di
12
13 # For No Investigation P(45
14 # For Investigation 1-P(45
15 # find z-scores at x=45; z=(s_mean-P_mean)/(p_SD/sqrt(n))
16
17 z=(45-50)/(40/100**0.5)
18 z
19
20
21
22 # find z-scores at x=55; z=(s_mean-P_mean)/(p_SD/sqrt(n))
23 z=(55-50)/(40/100**0.5)
24 z
25 #1.25
26
27
28 # For No Investigation P(45
29 stats.norm.cdf(1.25)-stats.norm.cdf(-1.25)
30 # 0.7887004526662893
31
32
33 stats.norm.interval(0.7887,loc=50,scale=40/(100**0.5))
34 #(45.00000495667348, 54.99999504332652)
35
36
37 # For Investigation 1-P(45
38 1-0.7887
39 #211
40
```

Console Output:

```
Out[2]: -0.820782681668124
In [3]: p_value=1-stats.norm.cdf(abs(z_score))
Out[3]: 0.20588503245107104
In [4]: import numpy as np
Out[4]:
In [5]: z=(45-50)/(40/100**0.5)
Out[5]: -1.25
In [6]: z=(55-50)/(40/100**0.5)
Out[6]: 1.25
In [7]: stats.norm.cdf(1.25)-stats.norm.cdf(-1.25)
Out[7]: 0.7887004526662893
In [8]: stats.norm.interval(0.7887,loc=50,scale=40/(100**0.5))
Out[8]: (45.00000495667348, 54.99999504332652)
In [9]: 1-0.7887
Out[9]: 0.21130000000000004
In [10]:
```

**Sol:** D is the answer 21%

4. The auditors from the above example would like to maintain the probability of investigation to 5%. Which of the following represents the minimum number transactions that they should sample if they do not want to change the thresholds of 45 and 55? Assume that the sample statistics remain unchanged.

- A. 144  
B. 150  
C. 196  
D. 250  
E. Not enough information

**Sol:** C - 196

5. An educational startup that helps MBA aspirants write their essays is targeting individuals who have taken GMAT in 2012 and have expressed interest in applying to FT top 20 b-schools. There are 40000 such individuals with an average GMAT score of 720 and a standard deviation of 120. The scores are distributed between 650 and 790 with a very long and thin tail towards the higher end resulting in substantial skewness. Which of the following is likely to be true for randomly chosen samples of aspirants?

A. The standard deviation of the scores within any sample will be 120.

**Sol:-** True

B. The standard deviation of the mean of across several samples will be 120.

**Sol:-** D of mean across several samples will also not be 120. It will be less. False

C. The mean score in any sample will be 720.

**Sol:-** True

D. The average of the mean across several samples will be 720.

**Sol:-** Data is distributed with leptokurtic kurtosis mean more information at the center and lesser information at the tail. This mean there is higher chance that average of mean of a aspirant that randomly chosen will be 720 that fall in between 650 and 790 at the center.

E. The standard deviation of the mean across several samples will be 0.60

**Sol:-** The SEM will be 0.60. This is likely, given the sample size, which even with a lot of skewness will tend towards normality given the sample size. I would use this in calculations. The mean would have an expected value of 720, but in calculations, the SEM is 0.6.

