Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

    A. 0.3875
    B. 0.2676
    C. 0.5
    D. 0.6987

Sol:- Let X be amount of time takes to complete the repair on customer's car.

P(X > 50) = 1 – P(X ≤50)

Z = (X - μ)/ σ

= (X - 45)/8.0

The question can be in the normal distribution table

P (X ≤ 50) = P (Z ≤ (50 – 45)/8.0)

P (Z ≤ 0.625) = 73.4%

The service manager will not meet his demand = 100 – 73.4 = 26.6%

= 0.2676

The Answer is "B"

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
    A. More employees at the processing center are older than 44 than between 38 and 44.
    B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Sol:- The Normal Distribution with μ = 38 and σ=6.

Let X be the number of employees

A Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 -Pr(X ≤ 44).

Z = (X - μ)/σ = (X -38)/6

By using the normal table

Pr(X ≤ 44) = Pr(Z ≤ (44 -38)/6)

= Pr(Z ≤ 1)=84.1345%

Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

So the probability of number of employees between 38-44 years of age

$Pr(X<44)-0.5=84.1345-0.5= 34.1345\%$
Therefore, the statement that "More employees at the processing centre are older than 44 than between 38 and 44" is TRUE

B Probability of employees less than age of

30 = $Pr(X<30).Z = (X - \mu)/\sigma$

$= (30 - 38)/6$

By using the normal table

$Pr(X \le 30) = Pr(Z \le (30 - 38)/6)$

$= Pr(Z \le -1.333)=9.12\%$

So the number of employees with probability 0.912 of them being under age

30 = 0.0912*400=36.48(or 36 employees)

Therefore, the statement B of the question is also TRUE

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *id* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Sol:- The Normal Distribution is defined by two parameters , Then Mean μ, and Variance σ^2
And written as $X \sim N(\mu, \sigma^2)$ **.**
Given $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are the independent identically distributed random variables.
From the properties of normal random variables
If $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\mu, \sigma^2)$
Are two independent identically distributed random variables
Sum of Random variables
$X + Y \sim N(\mu1 + \mu2, \sigma_1^2 + \sigma_2^2)$,
The difference of normal random variable is
$X - Y \sim N(\mu1 - \mu2, \sigma_1^2 + \sigma_2^2)$
When Z= aX, The Product of X is given by
$Z \sim N(a\mu1, a^2 \sigma_1^2)$
When Z = aX + bY , the Linear combination of X and Y is given by
$Z \sim N(a\mu1 - b\mu2, a^2 \sigma_c^2 + b^2 \sigma_2^2)$
Given to find $2X_1$
$2X_1 \sim N(2\mu, 2^2 \sigma^2) \rightarrow 2X_1 \sim N(2\mu, 4 \sigma^2)$
$X_1 + X_2 \sim N(\mu + \mu , \sigma^2 + \sigma^2) \sim N(2\mu, 2 \sigma^2)$
The difference between two is
$2X_1 - (X_1 + X_2) \sim N(2\mu - 2\mu , 2\sigma_1{}^2 + 4\sigma_2{}^2) \sim N(0, 6 \sigma^2)$
Then mean of $2X_1$ and $X_1 + X_2$ is same but the var(σ^2) of $2X_1$ is 2 times more than the variance of $X_1 + X_2$.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
   A. 90.5, 105.9
   B. 80.2, 119.8

C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9

Sol:- Given

Mean = 100

Standard Deviation = 20

The standard normal distribution such the area enclosed is 99.

$$\frac{x-\mu}{\sigma} \text{ says that } X = \sigma[Z] + \mu$$

"a" = 0.5th percentile for X = 20[-2.57] + 100 = 48.5

"b" = 99.5th percentile for X = 20[+2.57] + 100 = 151.4

The mean for the given standard normal distribution are [48.5,151.4]. Then the option "D " is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Sol:-

Given $1 = Rs. 45

Profit 1 ~ N $(5, 3^2)$

Profit 2 ~ N $(7, 4^2)$

Profit ~ $N(5+7, 3^2+4^2) = N(12, 5^2)$

A) 95% of the Probability lies between 1.96 Standard Deviations of the mean.

Thus Range is:

= (12 – 1.96 × 5, 12 + 1.96 × 5)

= $(2.2, 22.8)

= Rs (99M, 1026M)

B) 5^{th} % of Profit

$$P(Z \leq \frac{P-12}{5}) = 0.05$$

From p values of Z score table

$$\frac{P-12}{5} = -1.644$$

$$p = 12 - 8.22 = 3.78$$

= $ 3.78 × 45

= Rs. 170.1

The 5th % of profit lies is Rs. 170.1 Million

C) Loss in Profit < 0

Then

The 1st division of company have larger probability of making a loss in a given year.