Matlab exercises Ch0 and Ch1

For every .m file, first read the corresponding course slide(s). The question(s) marked by * are non-trivial.

Ch0a.m

1. Run the code several times with $n = 8$. Note that the sample mean and variance of 10^6 draws are relatively stable across different runs. This is the law of large numbers at work.
2. We can also draw from the same F-distribution by making only use of a standard normal random number generator. Implement this in MATLAB.
3. Let $X \sim F(m, n)$. What is the requirement on (m, n) for the existence of $E(X)$? Of $\mathrm{var}(X)$? (Hint: inspect the formulas and look it up to confirm.)
4. Set $n = 5$ and run the code several times. What do you observe? Why?
5. Set $n = 3$ and run the code several times. What do you observe? Why?
6. *Where does the requirement on (m, n) for the existence of $E(X)$ arise from?

Ch0b.m

1. Increase n from 5 to 10 and run the code. Is there anything that changes in the distribution of $X'AX$? Why (not)?
2. Decrease the number of draws of $X'AX$ from $R = 10^6$ to 10^5 to $R = 10^4$ to get an idea of how fast the histogram (always with 1000 bins) converges.
3. Add a few lines of code to generate X according to

$$X \sim N(\mu, \Sigma), \qquad \mu = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \qquad \Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix},$$

   and show that (cf. slide 63)

$$(X - \mu)'\Sigma^{-1}(X - \mu) \sim \chi^2(2)$$

   Hint: `Sigma=[1,1;1,2]; X=randn(1,2)*chol(Sigma);`

Ch0c.m

1. Since $AB = 0$, $X'AX$ and $X'BX$ are independent, and therefore their sample covariance should be close to zero. Modify the code to compute the sample covariance.

Ch0d.m

1. Modify the code to draw from the unit-exponential distribution (slide 69). (In the figure, change values on the x-axis.)
2. The Thm has an interesting converse: if X is a continuous r.v. with cdf F, then $F(X) \sim U(0,1)$. Show this, by simulations, for $X \sim N(0,1)$. Hint: the standard normal cdf is `normcdf()`.

Ch0e.m

1. Explain the formula in last line of the code.
2. Generate X according to

$$X \sim N(0,\Sigma), \qquad \Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix},$$

   and compute the Monte Carlo estimate of $\Pr[0 < X < 1]$ for any chosen ρ. Compute the limits

$$\lim_{\rho\to 1} \Pr[0 < X < 1], \qquad \lim_{\rho\to -1} \Pr[0 < X < 1],$$

   numerically (by Monte Carlo) and derive them analytically.
3. Calculate the integral

$$I = \int_0^1 e^x dx = e - 1 = 1.7183$$

   by simulation.

Ch1a.m

1. Check that $X'e = 0$ (slide 88).
2. Check that $y'y = \widehat{y}'\widehat{y} + e'e$.
3. Compute R^2_{uc} (the uncentred R^2). What happens to R^2_{uc} as σ^2 increases from 1 to 10000? (Hint: implement this by multiplying `epsilon` by 100.) And as β increases from 1 to 100 (with $\sigma^2 = 1$)? Why?

Ch1b.m

1. Draw x from $N(10, 1)$ instead of $N(0, 1)$ (hint: `X=10+randn(n,1)`). What happens to the variance of the OLS estimator? Why?

2. Draw x from $N(0, 100)$ (hint: `X=10*randn(n,1)`). What happens to the variance of the OLS estimator? Why?

Ch1c.m

1. What happens when you increase n from 15 to 30? To 50? Why?

2. What happens when you set $n = 50$ and test the false null $H_0 : \beta = 0.8$ instead of $H_0 : \beta = 0.2$? Why

Ch1d.m

1. (This is more a pure matlab exercise.) Compute the F statistic as defined in the first displayed equation of slide 102 and check that the result is identical to the expression in the second equation (which is implemented in the code).

Ch1e.m

1. The OLS estimator $b(X, y)$ has the following equivariance properties. Regression equivariance:

$$b(X, y + X\alpha) = b(X, y) + \alpha \qquad \text{for all } K \times 1 \text{ vectors } \alpha.$$

Scale equivariance:

$$b(X, \lambda y) = \lambda b(X, y) \qquad \text{for all scalars } \lambda.$$

Affine equivariance:

$$b(XA, y) = A^{-1} b(X, y) \qquad \text{for all nonsinuglar } K \times K \text{ matrices } A.$$

The code illustrates regression equivariance. Add code to illustrate scale and affine equivariance. Then prove all three equivariances theoretically.