**Topics: Normal distribution, Functions of Random Variables**

* 1) The time required for servicing transmissions is normally distributed with mean= 45 minutes and standard deviation = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
* 0.3875
* **0.2676**
* 0.5
* 0.6987

**ANS**:- Probability that the service manager cannot meet his commitment: **0.2676**

2) The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean = 38 and Standard deviation =6. For each statement below, please specify True/False. If false, briefly explain why.

* a) More employees at the processing center are older than 44 than between 38 and 44.

**ANS**:- Probability of employees **older than 44**:

**0.15865525393145707**

Probability of employees **between 38 and 44:**

**0.3413447460685429**

More employees are older than 44 than between 38 and 44:

**False**

* b) A training program for employees under the age of 30 at the

center would be expected to attract about 36 employees.

**ANS**:- Expected number of employees under the age of 30:

**36.484487890347154** approximately **36 employees**.

**SO THE ABOVE STATEMENT IS TRUE**.

* 3) If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS :-** **X ~ N(µ1 , σ12 ),** and **Y ~ N(µ2, σ22 )** are two independent

random variables then

X + Y ~ N(µ1 + µ2, σ12 + σ22 )

X − Y ~ N(µ1 − µ2, σ12 + σ22 )

When Z=aX , the product of X is given by

Z ~ N(aµ1, a2σ12 )

**2X1 ~ N(2μ , 4 σ2)** and **X1+X2** ~ **N(2 μ , 2σ2 )**

2 *X*1 - (*X*1 + *X*2) ~ *N*(2μ - 2μ , 4 σ2 + 2σ2 ) ~ *N*(0, 6 σ2)

**2 *X*1 - (*X*1 + *X*2) ~ *N*(0, 6 σ2)**

The distribution remains the same for every sample subset of

similar source, it tends to fall under Normal distribution and

slight deviations in parameters.

The **mean of 2 *X*1 and *X*1 + *X*2** is **same** but the **variance of 2X1** is

**two times more** than the **variance of *X*1 + *X*2** .

* 4) Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
* 90.5, 105.9
* 80.2, 119.8
* 22, 78
* **48.5, 151.5**
* 90.1, 109.9

**ANS**:- Two values symmetric about mean , such that the probability

of the random variable taking a value betweem them is 0.99 are

**48.5 , 151.5**

* 5) Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
* a) Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**ANS:-** **Mean Profit** is **Rs 540 Million**

**Standard Deviation** is **Rs 225.0 Million**

**Rupee range**: **(99.00810347848784, 980.9918965215122**) in

Millions.

* b) Specify the 5th percentile of profit (in Rupees) for the company

**ANS**:- **5th percentile of profit (in Million Rupees) is 170.0**

* c) Which of the two divisions has a larger probability of making a loss in a given year?

**ANS:-**

Probability of making a loss for **Division1: 0.0477903522728147**

Probability of making a loss for **Division2 : 0.040056863817086**

**Division 1 has a higher probability i.e. 0.05** of making a loss in a

given year.