AI1103-Assignment 3

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Download all python codes from

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https://github.com/vaishnavi-w/AI1103/blob/main/Assignment3/code3.py
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and latex-tikz codes from

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https://github.com/vaishnavi-w/AI1103/blob/main/Assignment3/latex3.tex
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Question

Probability density function $p(x)$ of random variable x is as shown below. The value of a is

A) $\frac{2}{c}$
B) $\frac{1}{c}$
C) $\frac{2}{(b+c)}$
D) $\frac{1}{(b+c)}$

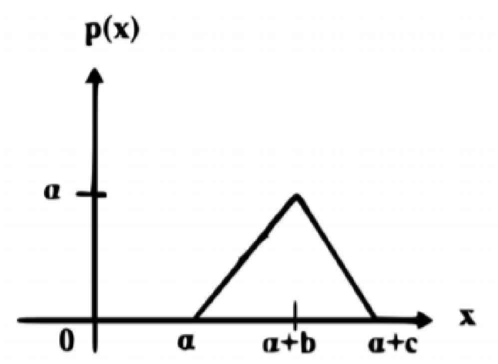

Fig. 4: PDF

Solution

Let Y_1 and Y_2 be two independent and identically distributed (IID) random variables.
Let X be a random variable such that

$$X = Y_1 + Y_2 \tag{0.0.1}$$

Let

$$p_{Y_1}(y) = \Pr(Y_1 = y) \tag{0.0.2}$$
$$p_{Y_2}(y) = \Pr(Y_2 = y) \tag{0.0.3}$$
$$p_X(x) = \Pr(X = x) \tag{0.0.4}$$

be the probability densities of random variables Y_1, Y_2 and X.
Y_1 and Y_2 lie in the range $\left(\frac{-c}{4}, \frac{c}{4}\right)$, therefore,

$$\int_{\frac{-c}{4}}^{\frac{c}{4}} p_{Y_1}(y)\, dy = 1 \tag{0.0.5}$$
$$\frac{c}{2} \times p_{Y_1}(y) = 1 \tag{0.0.6}$$
$$p_{Y_1}(y) = \frac{2}{c} \tag{0.0.7}$$

The PDF for Y_1 and Y_2,

$$p_{Y_1}(y) = p_{Y_2}(y) = \begin{cases} \frac{2}{c} & \frac{-c}{4} \le y \le \frac{c}{4} \\ 0 & \text{otherwise} \end{cases} \tag{0.0.8}$$

The density of X is obtained by convolution of Y_1 and Y_2

$$p_X(x) = p_{Y_1}(x) * p_{Y_2}(x) \tag{0.0.9}$$

where $*$ denotes the convolution operation. Since convolution operation is time invariant,

$$\begin{aligned} p_X(x-t) &= p_{Y_1}(x-t) * p_{Y_2}(x) \\ &= p_{Y_1}(x) * p_{Y_2}(x-t) \end{aligned} \tag{0.0.10}$$

On time shifting Y_1 by shifting factor $t = a + \frac{c}{2}$,

$$p_X\left(x - \left(a + \frac{c}{2}\right)\right) = p_{Y_1}\left(x - \left(a + \frac{c}{2}\right)\right) * p_{Y_2}(x) \tag{0.0.11}$$

Thus, the PDF of time shifted X obtained by convolution is,

$$p_x = \begin{cases} \frac{4}{c^2}(x-a) & a \le x \le a + \frac{c}{2} \\ \frac{4}{c^2}(a+c-x) & a + \frac{c}{2} \le x \le a + c \\ 0 & \text{otherwise} \end{cases} \tag{0.0.12}$$

On comparing the parameters of PDF of time shifted X with that in the question, we have

$$b = \frac{c}{2} \tag{0.0.13}$$

$$a = \frac{2}{c} \tag{0.0.14}$$

Answer : Option A

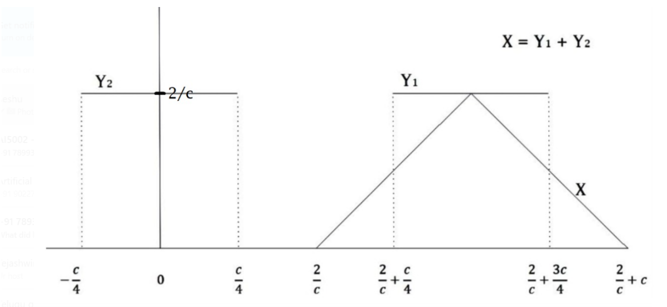

Fig. 4: PDF of time shifted X

1) The sum of two equally distributed random variables will lead to a triangular probability density
2) The two uniformly distributed random variables lie in the range $\left(\frac{-c}{4}, \frac{c}{4}\right)$ and $\left(\frac{2}{c} + \frac{c}{4}, \frac{2}{c} + \frac{3c}{4}\right)$. $\because$X = Y$_1 + Y_2$ the range of X is thus $\left(\frac{2}{c}, \frac{2}{c} + c\right)$
3) On time shifting Y_1 to the right by a factor $a+\frac{c}{2}$, the convoluted PDF of X also shifts by the same factor without any change in it's width.

Fig 3 and Fig 3 are the simulated plots of PDF and CDF obtained by taking c=2

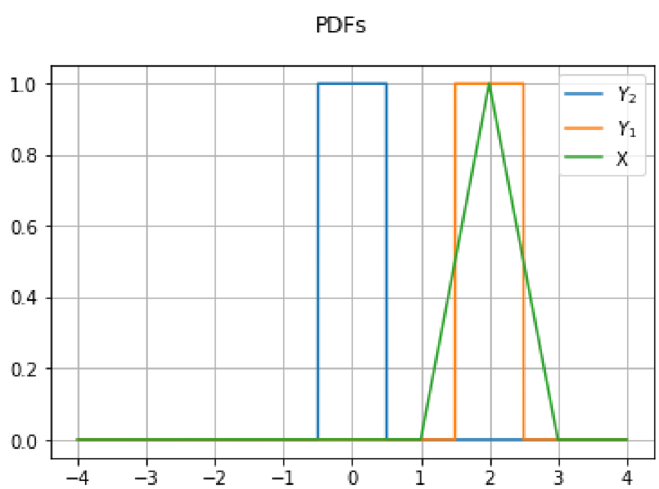

Fig. 3: PDF of Y_1, Y_2 and X

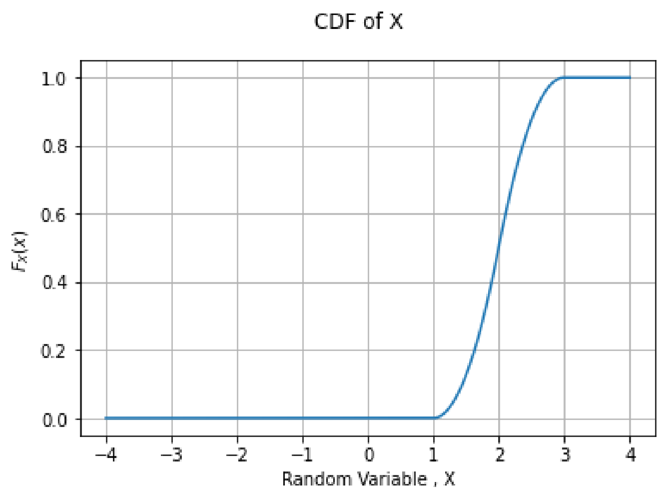

Fig. 3: CDF of X