AI1103-Assignment 2

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Download all python codes from

https://github.com/vaishnavi−w/AI1103/blob/main/Assignment2/code2.py

and latex-tikz codes from

https://github.com/vaishnavi−w/AI1103/blob/main/Assignment2/latex2.tex

Question

A binary symmetric channel (BSC) has a transition probability of 1/8. If the binary transmit symbol X is such that $\Pr(X = 0) = 9/10$, then the probability of error for an optimum receiver

A) $\frac{7}{80}$
B) $\frac{63}{80}$
C) $\frac{9}{10}$
D) $\frac{1}{10}$

Solution

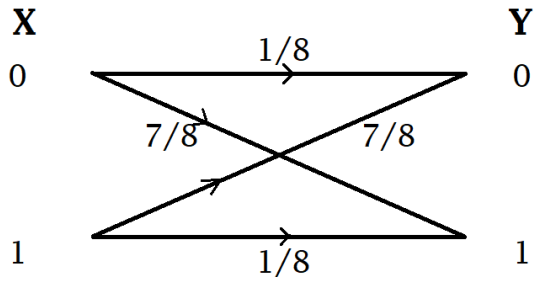

Fig. 4: Binary symmetric channel

Let random variables, $X \in \{0, 1\}$ denote the bit transmitted and $Y \in \{0, 1\}$ denote the output bit received.
From the given information,

$$\Pr(X = 0) = \frac{9}{10} \tag{0.0.1}$$

$$\Pr(X = 1) = 1 - \Pr(X = 0) = \frac{1}{10} \tag{0.0.2}$$

Also given, transition probability = $\frac{1}{8}$. Transition probability is the probability with which the bit is transmitted correctly. That gives,

$$\Pr(Y = 1|X = 1) = \Pr(Y = 0|X = 0) = \frac{1}{8} \tag{0.0.3}$$

Probability that the bit is not transmitted correctly
= 1 − transition probability

$$= 1 - \frac{1}{8} = \frac{7}{8} \tag{0.0.4}$$

That gives,

$$\Pr(Y = 0|X = 1) = \Pr(Y = 1|X = 0) = \frac{7}{8} \tag{0.0.5}$$

Let E denote the event that bit is transmitted incorrectly. Probability of error, $\Pr(E)$

$$\Pr(E) = \Pr(X = 0)\Pr(Y = 1|X = 0) + \Pr(X = 1)\Pr(Y = 0|X = 1) \tag{0.0.6}$$

On substituting the values,

$$\Pr(E) = \frac{9}{10} \times \frac{7}{8} + \frac{1}{10} \times \frac{7}{8} \tag{0.0.7}$$

$$= \frac{63}{80} + \frac{7}{80} \tag{0.0.8}$$

$$= \frac{7}{8} \tag{0.0.9}$$

Answer: No option matches