

# AI1103-Assignment 3

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Download all python codes from

<https://github.com/vaishnavi-w/AI1103/blob/main/Assignment3/code3.py>

and latex-tikz codes from

<https://github.com/vaishnavi-w/AI1103/blob/main/Assignment3/latex3.tex>

## QUESTION

Probability density function  $p(x)$  of random variable  $x$  is as shown below. The value of  $a$  is

- A)  $\frac{2}{c}$
- B)  $\frac{1}{c}$
- C)  $\frac{2}{(b+c)}$
- D)  $\frac{1}{(b+c)}$

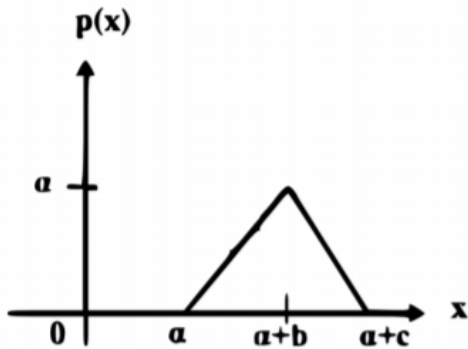


Fig. 4: PDF

## SOLUTION

Let  $Y_1$  and  $Y_2$  be two independent and identically distributed (IID) uniform random variables.

Let  $X$  be a random variable such that

$$X = Y_1 + Y_2 \quad (0.0.1)$$

Let

$$p_{Y_1}(y) = \Pr(Y_1 = y) \quad (0.0.2)$$

$$p_{Y_2}(y) = \Pr(Y_2 = y) \quad (0.0.3)$$

$$p_X(x) = \Pr(X = x) \quad (0.0.4)$$

be the probability densities of random variables  $Y_1, Y_2$  and  $X$ .

$Y_1$  and  $Y_2$  lie in the range  $\left(-\frac{c}{4}, \frac{c}{4}\right)$ , therefore, the PDF for  $Y_1$  and  $Y_2$ ,

$$p_{Y_1}(y) = p_{Y_2}(y) = \begin{cases} \frac{2}{c} & -\frac{c}{4} \leq y \leq \frac{c}{4} \\ 0 & \text{otherwise} \end{cases} \quad (0.0.5)$$

The density of  $X$  is obtained by convolution of  $Y_1$  and  $Y_2$

$$p_X(x) = p_{Y_1}(x) * p_{Y_2}(x) \quad (0.0.6)$$

where  $*$  denotes the convolution operation. Since convolution operation is time invariant,

$$\begin{aligned} p_X(x-t) &= p_{Y_1}(x-t) * p_{Y_2}(x) \\ &= p_{Y_1}(x) * p_{Y_2}(x-t) \end{aligned} \quad (0.0.7)$$

On time shifting  $Y_1$  by shifting factor  $t = a + \frac{c}{2}$ ,

$$p_X\left(x - \left(a + \frac{c}{2}\right)\right) = p_{Y_1}\left(x - \left(a + \frac{c}{2}\right)\right) * p_{Y_2}(x) \quad (0.0.8)$$

Thus, the PDF of time shifted  $X$  obtained by convolution is,

$$p_x = \begin{cases} \frac{4}{c^2}(x-a) & a \leq x \leq a + \frac{c}{2} \\ \frac{4}{c^2}(a+c-x) & a + \frac{c}{2} \leq x \leq a+c \\ 0 & \text{otherwise} \end{cases} \quad (0.0.9)$$

On comparing the parameters of PDF of time shifted  $X$  with that in the question, we have

$$b = \frac{c}{2} \quad (0.0.10)$$

$$a = \frac{2}{c} \quad (0.0.11)$$

Answer : Option A

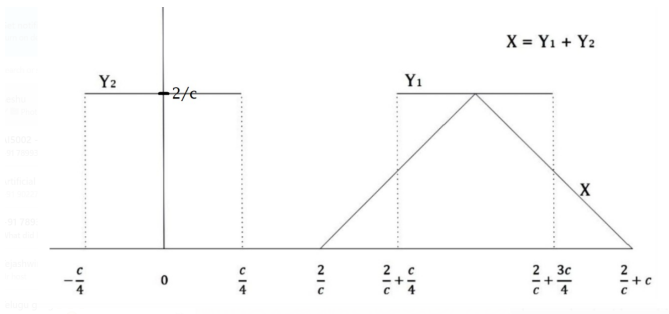


Fig. 4: PDF of time shifted X

The following are some observations:

- 1) The sum of two equally distributed random variables will lead to a triangular probability density
- 2) The two uniformly distributed random variables lie in the range  $(-\frac{c}{4}, \frac{c}{4})$  and  $(\frac{2}{c} + \frac{c}{4}, \frac{2}{c} + \frac{3c}{4})$ .  
 $\therefore X = Y_1 + Y_2$  the range of X is thus  $(\frac{2}{c}, \frac{2}{c} + c)$
- 3) On time shifting  $Y_1$  to the right by a factor  $a + \frac{c}{2}$ , the convoluted PDF of X also shifts by the same factor without any change in it's width.

Fig 3 and Fig 3 are the plots of PDF and CDF obtained by taking  $c=2$

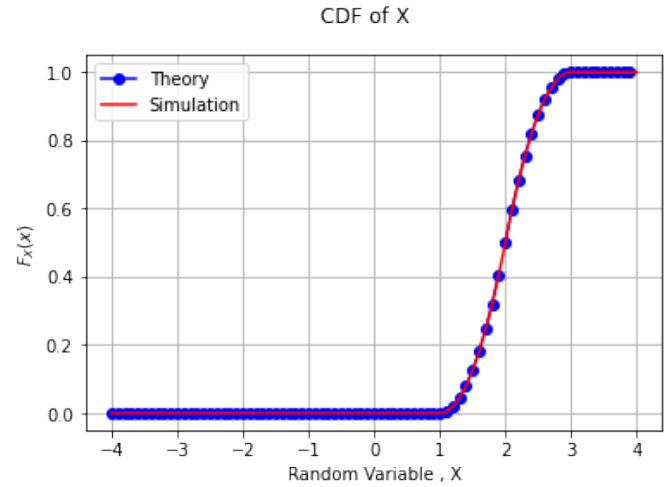


Fig. 3: CDF of X

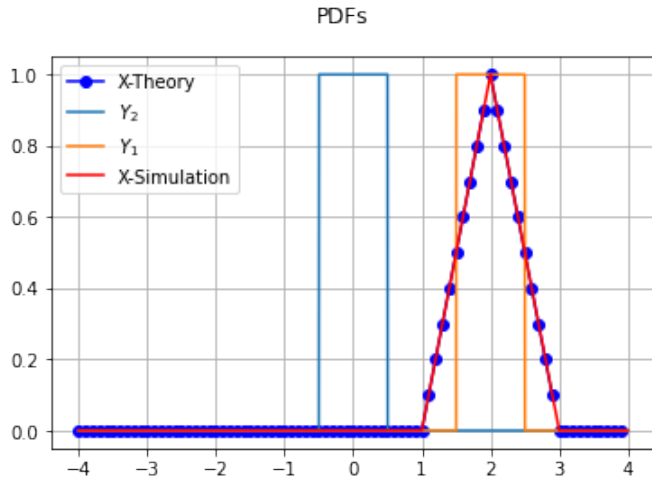


Fig. 3: PDF of  $Y_1, Y_2$  and X