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AI1103-Assignment 2

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Download all python codes from

https://github.com/vaishnavi-w/AI1103/blob/main/ Assignment2/code2.py

and latex-tikz codes from

https://github.com/vaishnavi-w/AI1103/blob/main/ Assignment2/latex2.tex

QUESTION

A binary symmetric channel (BSC) has a transition probability of 1/8. If the binary transmit symbol X is such that Pr(X = 0) = 9/10, then the probability of error for an optimum receiver

- A) $\frac{1}{6}$
- B) $\frac{6}{8}$
- C) $\frac{9}{10}$
- D) $\frac{10}{10}$

Solution

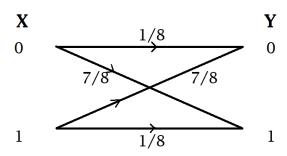


Fig. 4: Binary symmetric channel

Let random variables, $X \in \{0, 1\}$ denote the bit transmitted and $Y \in \{0, 1\}$ denote the output bit received.

From the given information,

$$\Pr(X=0) = \frac{9}{10} \qquad (0.0.1)$$

$$Pr(X = 1) = 1 - Pr(X = 0) = \frac{1}{10}$$
 (0.0.2)

Also given, transition probability = $\frac{1}{8}$. Transition probability is the probability with which the bit is transmitted correctly. That gives,

$$Pr(Y = 1|X = 1) = Pr(Y = 0|X = 0) = \frac{1}{8}$$
 (0.0.3)

Probability that the bit is not transmitted correctly
= 1-transition probability

$$=1-\frac{1}{8}=\frac{7}{8}$$
 (0.0.4)

That gives,

$$Pr(Y = 0|X = 1) = Pr(Y = 1|X = 0) = \frac{7}{8}$$
 (0.0.5)

Let E denote the event that bit is transmitted incorrectly. Probability of error, Pr(E)

$$Pr(E) = Pr(X = 0) Pr(Y = 1|X = 0) + Pr(X = 1) Pr(Y = 0|X = 1) (0.0.6)$$

On substituting the values,

$$Pr(E) = \frac{9}{10} \times \frac{7}{8} + \frac{1}{10} \times \frac{7}{8}$$
 (0.0.7)
= $\frac{63}{10} + \frac{7}{10}$ (0.0.8)

$$= \frac{63}{80} + \frac{7}{80}$$
 (0.0.8)
= $\frac{7}{80}$ (0.0.9)

Answer: No option matches