AI1103-Assignment 4

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Download all python codes from

https://github.com/vaishnavi−w/AI1103/blob/main/Assignment4/code4.py

and latex-tikz codes from

https://github.com/vaishnavi−w/AI1103/blob/main/Assignment4/latex4.tex

Question

Let $\Omega = (0, 1]$ be the sample space and let P(.) be a probability distribution given by

$$\mathrm{P}((0, x]) = \begin{cases} \frac{x}{2} & 0 \le x < \frac{1}{2} \\ x & \frac{1}{2} \le x \le 1 \end{cases}$$

Find $\mathrm{P}\left(\frac{1}{2}\right)$

Solution

CDF of X is defined as,

$$F(x) = \Pr(X \le x) \tag{0.0.1}$$

When $x < \frac{1}{2}$

$$\Pr(X \le x) = \int_{-\infty}^{x} p_X(x)\, dx \tag{0.0.2}$$
$$= \int_{0}^{x} p_X(x)\, dx \tag{0.0.3}$$
$$= P((0, x]) \tag{0.0.4}$$
$$= \frac{x}{2} \tag{0.0.5}$$

Similarly, when $\frac{1}{2} \le x \le 1$

$$\Pr(X \le x) = \int_{-\infty}^{x} p_X(x)\, dx \tag{0.0.6}$$
$$= \int_{0}^{x} p_X(x)\, dx \tag{0.0.7}$$
$$= P((0, x]) \tag{0.0.8}$$
$$= x \tag{0.0.9}$$

Thus, CDF of X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \le x < \frac{1}{2} \\ x & \frac{1}{2} \le x \le 1 \\ 1 & x \ge 1 \end{cases} \tag{0.0.10}$$

$$\Pr\left(\frac{1}{2}\right) = F\left(\frac{1}{2}\right) - F\left(\frac{1}{2}^{-}\right) \tag{0.0.11}$$
$$= \frac{1}{2} - \frac{1/2}{2} \tag{0.0.12}$$
$$= \frac{1}{4} \tag{0.0.13}$$

The plot of CDF is given in the Figure 0

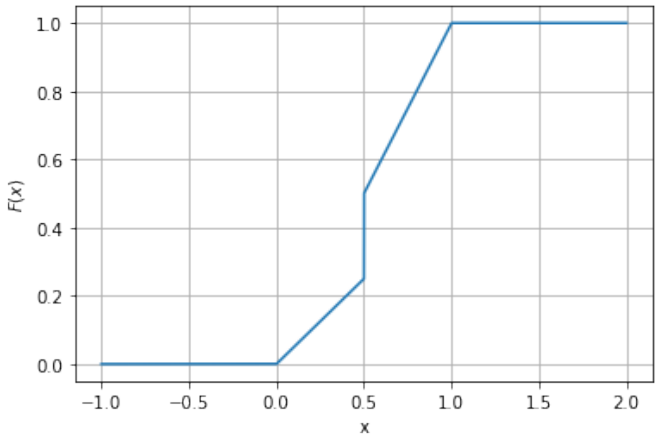

Fig. 0: CDF of X