

# EE3900-Gate Assignment

W Vaishnavi  
AI20BTECH11025

Download all latex-tikz codes from

<https://github.com/vaishnavi-w/EE3900/blob/main/Gate1/gatelatex.tex>

and python codes from

<https://github.com/vaishnavi-w/EE3900/blob/main/Gate1/codes/fourier.py>

## 1 GATE EC 2016 Q.10

Find energy of the signal  $x(t) = \frac{\sin(4\pi t)}{4\pi t}$

## 2 SOLUTION

**Lemma 2.1.** Parseval's theorem states that there is no loss of information in Fourier transform and the amount of energy remains the same in time and frequency domains.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \quad (2.0.1)$$

Let,

$$x(t) = \begin{cases} \frac{1}{4} & \text{if } |t| \leq 2 \\ 0 & \text{if } |t| > 2 \end{cases} \quad (2.0.2)$$

$$x(t) \xrightarrow{\mathcal{F}} X(f) \quad (2.0.3)$$

Finding the Fourier transform of  $x(t)$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{i2\pi f t} dt \quad (2.0.4)$$

$$= \int_{-2}^2 \frac{1}{4} e^{i2\pi f t} dt \quad (2.0.5)$$

$$= \frac{e^{i4\pi f} - e^{-i4\pi f}}{i8\pi f} \quad (2.0.6)$$

$$= \text{sinc}(4f) \quad (2.0.7)$$

From Duality of Fourier transform, we have

$$x(t) \xrightarrow{\mathcal{F}} X(f) \quad (2.0.8)$$

$$X(t) \xrightarrow{\mathcal{F}} x(-f) \quad (2.0.9)$$

$$\Rightarrow \text{sinc}(4t) \xrightarrow{\mathcal{F}} x(-f) \quad (2.0.10)$$

$$x(-f) = x(f) = \begin{cases} \frac{1}{4} & \text{if } |f| \leq 2 \\ 0 & \text{if } |f| > 2 \end{cases} \quad (2.0.11)$$

Energy of the signal is given by,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \text{sinc}^2(4t) dt \quad (2.0.12)$$

From Parseval's theorem, we have

$$\int_{-\infty}^{\infty} \text{sinc}^2(4t) dt = \int_{-2}^2 \frac{1}{4^2} df \quad (2.0.13)$$

$$= \frac{1}{4} \quad (2.0.14)$$

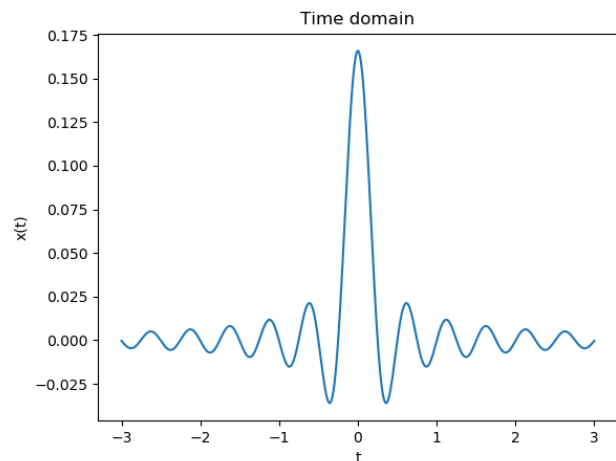


Fig. 0: Plot of signal in Time domain

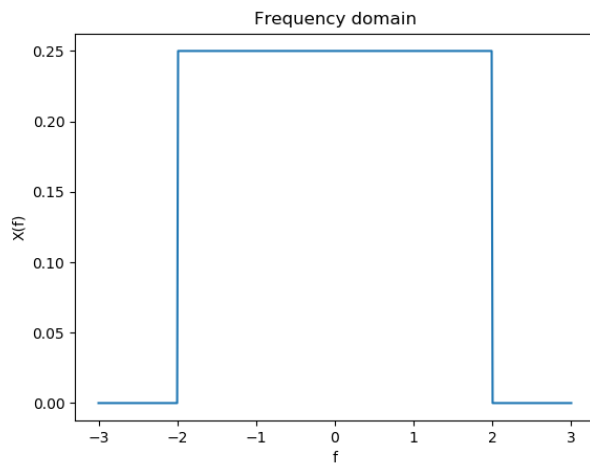


Fig. 0: Plot of signal in Frequency domain