

EE3900-Assignment 5

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Download all latex-tikz codes from

<https://github.com/vaishnavi-w/EE3900/blob/main/Assignment5/latex5.tex>

and python codes from

<https://github.com/vaishnavi-w/EE3900/blob/main/Assignment5/hyperbola.py>

The directrix of the hyperbola passes through the point $\begin{pmatrix} 0 \\ \frac{b}{e} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{11}{12} \end{pmatrix}$ and perpendicular to the y-axis. It is given as,

$$(0 \ 1) \left(\mathbf{x} - \begin{pmatrix} 0 \\ \frac{11}{12} \end{pmatrix} \right) = 0 \quad (2.0.9)$$

$$\Rightarrow (0 \ 1) \mathbf{x} = \frac{11}{12} \quad (2.0.10)$$

Comparing it with $\mathbf{n}^T \mathbf{x} = c$

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, c = \frac{11}{12} \quad (2.0.11)$$

$$\Rightarrow \|\mathbf{n}\| = 1 \quad (2.0.12)$$

Calculating \mathbf{V}, \mathbf{u} and f ,

$$\begin{aligned} \mathbf{V} &= 1^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \left(\frac{6}{\sqrt{11}} \right)^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & -\frac{25}{11} \end{pmatrix} \end{aligned} \quad (2.0.13)$$

$$\mathbf{u} = \frac{11}{12} \left(\frac{6}{\sqrt{11}} \right)^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} - 1^2 \begin{pmatrix} 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.14)$$

$$f = 3^2 - \left(\frac{11}{12} \times \frac{6}{\sqrt{11}} \right)^2 = \frac{25}{4} \quad (2.0.15)$$

Equation of the hyperbola,

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & -\frac{25}{11} \end{pmatrix} \mathbf{x} + \frac{25}{4} = 0 \quad (2.0.16)$$

1 QUADRATIC FORMS Q.30

Find the equation of a hyperbola with the vertices $\begin{pmatrix} 0 \\ \pm \frac{\sqrt{11}}{2} \end{pmatrix}$ and foci $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$

2 SOLUTION

Lemma 2.1. The equation of a conic with directrix $\mathbf{n}^T \mathbf{x} = c$, eccentricity e and focus \mathbf{F} is given by

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

where

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^T, \quad (2.0.2)$$

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F}, \quad (2.0.3)$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \quad (2.0.4)$$

Solution : Let the focus and vertex be,

$$\mathbf{F} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ \frac{\sqrt{11}}{2} \end{pmatrix} \quad (2.0.5)$$

$$\Rightarrow \|\mathbf{F}\| = 3 \quad (2.0.6)$$

Let e be the eccentricity of the hyperbola and b the length of the semi-major axis. From observation,

$$b = \frac{\sqrt{11}}{2}, be = 3 \quad (2.0.7)$$

$$\Rightarrow e = \frac{3}{\sqrt{11}/2} = \frac{6}{\sqrt{11}} \quad (2.0.8)$$

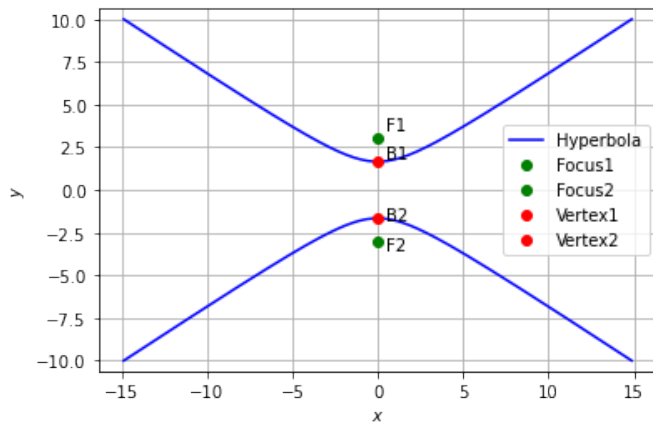


Fig. 0: Plot of Hyperbola