EE3900-Gate Assignment

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Download all latex-tikz codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate3/latex3.tex

Gate EC - 2001 Q1.21

If a signal $f(t)$ has energy E, the energy of the signal $f(2t)$ is equal to

A) E
B) $\frac{E}{2}$
C) $2E$
D) $4E$

Solution

The energy of the signal $f(t)$ is given as.

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt \tag{0.0.1}$$

The energy of signal $f(2t)$,

$$E' = \int_{-\infty}^{\infty} |f(2t)|^2 dt \tag{0.0.2}$$

Putting $u = 2t$,

$$du = 2dt \tag{0.0.3}$$

$$E' = \int_{-\infty}^{\infty} |f(u)|^2 \frac{du}{2} = \frac{E}{2} \tag{0.0.4}$$

Answer: Option B

Example

Lemma 0.1. *Parseval's theorem states that there is no loss of information in Fourier transform and the amount of energy remains the same in time and frequency domains.*

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \tag{0.0.5}$$

Consider a signal

$$f(t) = sinc(t) \tag{0.0.6}$$

$$sinc(t) \overset{\mathcal{F}}{\rightleftharpoons} rect(f) \tag{0.0.7}$$

where

$$sinc(t) = \begin{cases} 1 & t = 0 \\ \frac{\sin \pi t}{\pi t} & otherwise \end{cases} \tag{0.0.8}$$

$$rect(t) = \begin{cases} 1 & \text{if } |t| \leq \frac{1}{2} \\ 0 & \text{if } otherwise \end{cases} \tag{0.0.9}$$

Energy of the signal using Parseval's thoerem,

$$\int_{-\infty}^{\infty} sinc^2(t) dt = \int_{-\infty}^{\infty} (rect(f))^2 df \tag{0.0.10}$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} df = 1 \tag{0.0.11}$$

Consider a signal,

$$f(2t) = sinc(2t) \tag{0.0.12}$$

When a time signal $g(t)$ is time scaled by α, the resulting Fourier transform is given by:

$$g(\alpha t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{|\alpha|} G\left(\frac{f}{\alpha}\right) \tag{0.0.13}$$

$$\implies sinc(2t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{2} rect\left(\frac{f}{2}\right) \tag{0.0.14}$$

Energy of signal,

$$E' = \int_{-\infty}^{\infty} sinc^2(2t) dt \tag{0.0.15}$$

$$= \int_{-\infty}^{\infty} \left(\frac{1}{2} rect\left(\frac{f}{2}\right)\right)^2 df \tag{0.0.16}$$

$$= \frac{1}{4} \int_{-1}^{1} df = \frac{1}{2} = \frac{E}{2} \tag{0.0.17}$$

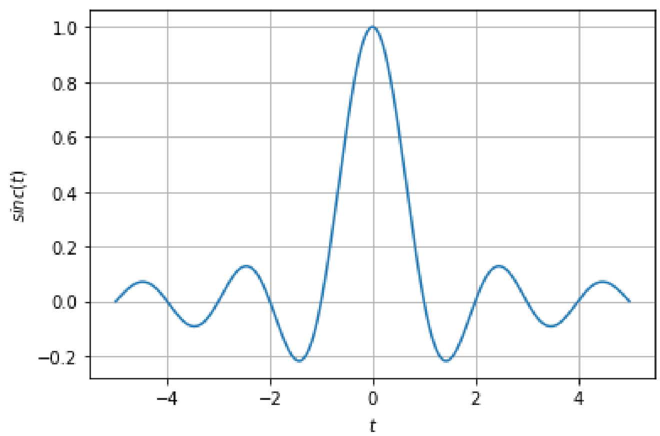


Fig. 4: Plot of signal $f(t)$

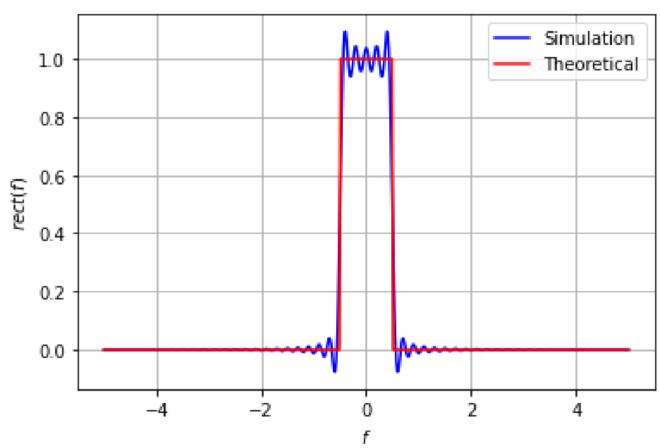


Fig. 4: Fourier transform of $f(t)$

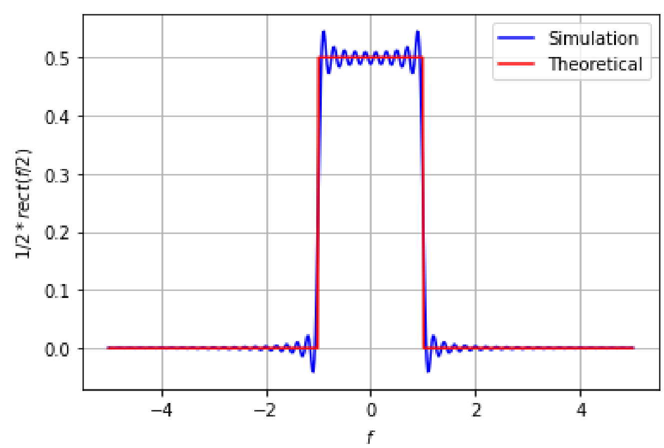
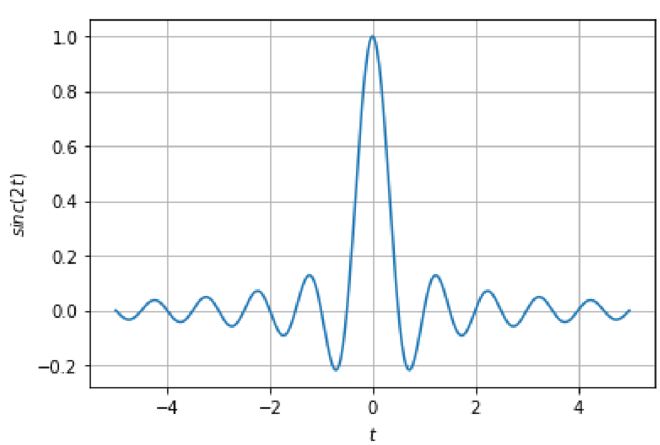


Fig. 4: Fourier transform of $f(2t)$



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