EE3900 Quiz2

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Download all latex-tikz codes from

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https://github.com/vaishnavi−w/EE3900/blob/main/
Quiz2
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1 3.18 a

A causal LTI system has the system function

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)(1 - z^{-1})} \tag{1.0.1}$$

Find the impulse response of the system, $h[n]$

2 Solution

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)(1 - z^{-1})} \tag{2.0.1}$$

$$= \frac{2\left(z^2 + 2z + 1\right)}{(2z + 1)(z - 1)} \tag{2.0.2}$$

$$= \frac{2(3z + 4/3)}{2z + 1} - \frac{2(z - 7/3)}{z - 1} \tag{2.0.3}$$

$$= H_1(z) + H_2(z) + H_3(z) + H_4(z) \tag{2.0.4}$$

where

$$H_1(z) = \frac{6z}{2z + 1} = 3\left(\frac{1}{1 - (-2z)^{-1}}\right), ROC_1 = |z| > \frac{1}{2} \tag{2.0.5}$$

$$H_2(z) = \frac{-2z}{z - 1} = -2\left(\frac{1}{1 - z^{-1}}\right), ROC_2 = |z| > 1 \tag{2.0.6}$$

$$H_3(z) = \frac{8}{6z + 3} = \frac{4}{3}\left(\frac{z^{-1}}{1 - (-2z)^{-1}}\right) ROC_3 = |z| > \frac{1}{2}\backslash\{0\} \tag{2.0.7}$$

$$H_4(z) = \frac{14}{3z - 3} = \frac{14}{3}\left(\frac{z^{-1}}{1 - z^{-1}}\right), ROC_4 = |z| > 1\backslash\{0\} \tag{2.0.8}$$

Thus, ROC of $H(z) = |z| > \frac{1}{2}\backslash\{0\}$

The $\mathcal{Z}$ transform of a sequence of the form $a^n u[n]$ is given as,

$$\mathcal{Z}(a^n u[n]) = \sum_{n=-\infty}^{\infty} a^n u[n] z^{-n} = \frac{1}{1 - az^{-1}} \tag{2.0.9}$$

$$a^n u[n] \overset{Z}{\longleftrightarrow} \frac{1}{1 - az^{-1}} \tag{2.0.10}$$

with ROC $|az^{-1}| < 1$
For a discrete signal $x[n]$ and it's $\mathcal{Z}$ transform $X(z)$ from time shifting property, we have

$$x[n] \overset{Z}{\longleftrightarrow} X(z) \tag{2.0.11}$$

$$x[n - n_0] \overset{Z}{\longleftrightarrow} z^{-n_0} X(z) \tag{2.0.12}$$

That gives,

$$a^{n-n_0} u[n - n_0] \overset{Z}{\longleftrightarrow} \frac{z^{-n_0}}{1 - az^{-1}} \tag{2.0.13}$$

with ROC for $n_0 > 0, |az^{-1}| < 1$ except $z = 0$
Taking the inverse $\mathcal{Z}$ transforms,

$$h_1[n] = \mathcal{Z}^{-1}(H_1(z)) = 3\left(-2^{-1}\right)^n u[n] \tag{2.0.14}$$

$$h_2[n] = \mathcal{Z}^{-1}(H_2(z)) = -2u[n] \tag{2.0.15}$$

$$h_3[n] = \mathcal{Z}^{-1}(H_3(z)) = \frac{4\left(-2^{-1}\right)^{n-1} u[n - 1]}{3} \tag{2.0.16}$$

$$h_4[n] = \mathcal{Z}^{-1}(H_4(z)) = \frac{14}{3} u[n - 1] \tag{2.0.17}$$

Impulse response of the system

$$h[n] = h_1[n] + h_1[n] + h_1[n] + h_1[n]$$
$$= u[n]\left(3\left(-2^{-1}\right)^n - 2\right) + \frac{u[n - 1]}{3}\left(4\left(-2^{-1}\right)^{n-1} + 14\right) \tag{2.0.18}$$

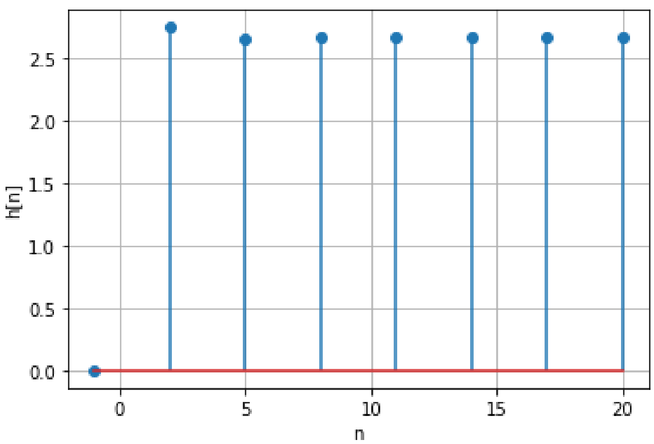

Fig. 0: Plot of impulse response $g(f)$