EE3900 - Gate Assignment 4

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Download all latex-tikz codes from

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https://github.com/vaishnavi−w/EE3900/blob/main/
    Gate4/latex4.tex
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Gate EC - 1997 Q1.4

The function $f(t)$ has the fourier transform $g(\omega)$. The fourier transform of $g(t)\left(\int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt\right) =$

A) $\frac{1}{2\pi} f(\omega)$
B) $\frac{1}{2\pi} f(-\omega)$
C) $2\pi f(-\omega)$
D) None of the above

Solution

Lemma 0.1. ***Duality Property*** *: Given a function $f(x)$ and it's fourier transform $g(\omega)$, the duality property of fourier transform says*

$$f(t) \overset{\mathcal{F}}{\rightleftharpoons} g(\omega) \tag{0.0.1}$$

$$g(t) \overset{\mathcal{F}}{\rightleftharpoons} 2\pi f(-\omega) \tag{0.0.2}$$

Proof. Given, the fourier transform of $f(t)$ is $g(\omega)$

$$g(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt \tag{0.0.3}$$

The inverse fourier transform can be given as

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} g(\omega)\, e^{j\omega t} d\omega \tag{0.0.4}$$

$$\implies 2\pi f(t) = \int_{-\infty}^{\infty} g(\omega)\, e^{j\omega t} d\omega \tag{0.0.5}$$

Putting $t = -t$

$$2\pi f(-t) = \int_{-\infty}^{\infty} g(\omega)\, e^{-j\omega t} d\omega \tag{0.0.6}$$

Interchanging t and ω

$$2\pi f(-\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t} dt \tag{0.0.7}$$

Thus, the fourier transform of $g(t)$ is $2\pi f(-\omega)$ □

Answer: Option C

Example :
The exponential representation of Dirac-Delta function is given as

$$\delta(x - x_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j(x-x_0)p} dp \tag{0.0.8}$$

Also, for any function $f(x)$ that is continuous at $x = x_0$,

$$\int_{-\infty}^{\infty} f(x)\, \delta(x - x_0)\, dx = f(x_0) \tag{0.0.9}$$

Consider,

$$f(t) = \delta(t) \tag{0.0.10}$$

Fourier transform of $f(t)$,

$$\int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t} dt = e^{-j\omega t}\Big|_{t=0} = 1 \tag{0.0.11}$$

Consider,

$$g(t) = 1 \tag{0.0.12}$$

It's fourier transform,

$$\int_{-\infty}^{\infty} 1 \times e^{-j\omega t} dt = 2\pi\delta(-\omega) \tag{0.0.13}$$

Thus, we have

$$\delta(t) \overset{\mathcal{F}}{\rightleftharpoons} 1 \tag{0.0.14}$$

$$1 \overset{\mathcal{F}}{\rightleftharpoons} 2\pi\delta(-\omega) \tag{0.0.15}$$