EE3900-Assignment 5

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Download all latex-tikz codes from

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https://github.com/vaishnavi-w/EE3900/blob/main/
    Assignment5/latex5.tex
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and python codes from

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https://github.com/vaishnavi-w/EE3900/blob/main/
    Assignment5/hyperbola.py
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1 Quadratic Forms Q.30

Find the equation of a hyperbola with the vertices $\begin{pmatrix} 0 \\ \pm\frac{\sqrt{11}}{2} \end{pmatrix}$ and foci $\begin{pmatrix} 0 \\ \pm 3 \end{pmatrix}$

2 Solution

Let the foci be

$$\mathbf{C_1} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \mathbf{C_2} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{2.0.1}$$

and vertices

$$\mathbf{V_1} = \begin{pmatrix} 0 \\ \frac{\sqrt{11}}{2} \end{pmatrix}, \mathbf{V_2} = \begin{pmatrix} 0 \\ \frac{-\sqrt{11}}{2} \end{pmatrix} \tag{2.0.2}$$

Let $\mathbf{x}$ be a point on the hyperbola. Then,

$$xC_1 = \|\mathbf{x} - \mathbf{C_1}\| \tag{2.0.3}$$

$$xC_2 = \|\mathbf{x} - \mathbf{C_2}\| \tag{2.0.4}$$

$$V_1V_2 = \|\mathbf{V_1} - \mathbf{V_2}\| = \sqrt{11} \tag{2.0.5}$$

Hyperbola is a set of points whose absolute difference of distances from two foci is a constant - the distance between vertices.

$$|xC_1 - xC_2| = V_1V_2 \tag{2.0.6}$$

$$\implies xC_1 = xC_2 \pm V_1V_2 \tag{2.0.7}$$

$$\implies \|\mathbf{x} - \mathbf{C_1}\| = \|\mathbf{x} - \mathbf{C_2}\| \pm \|\mathbf{V_1} - \mathbf{V_2}\| \tag{2.0.8}$$

Squaring on both sides,

$$\|\mathbf{x}\|^2 + \|\mathbf{C_1}\|^2 - 2\mathbf{x}^\top\mathbf{C_1} = \|\mathbf{x}\|^2 + \|\mathbf{C_2}\|^2 - 2\mathbf{x}^\top\mathbf{C_2} + 11 \pm 2\sqrt{11}\|\mathbf{x} - \mathbf{C_2}\| \tag{2.0.9}$$

Substituting $\mathbf{C_2} = -\mathbf{C_1}$

$$4\mathbf{x}^\top\mathbf{C_1} + 11 = \mp 2\sqrt{11}\|\mathbf{x} + \mathbf{C_1}\| \tag{2.0.10}$$

Squaring on both sides again and simplifing,

$$16\mathbf{x}^\top \begin{pmatrix} 0 \\ 3 \end{pmatrix} \begin{pmatrix} 0 & 3 \end{pmatrix} \mathbf{x} + 88\mathbf{x}^\top\mathbf{C_1} + 121 = 44\mathbf{x}^\top\mathbf{x} + 88\mathbf{x}^\top\mathbf{C_1} + 396 \tag{2.0.11}$$

$$16\mathbf{x}^\top \begin{pmatrix} 0 & 0 \\ 0 & 9 \end{pmatrix} \mathbf{x} = 44\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 275 \tag{2.0.12}$$

$$\mathbf{x}^\top \begin{pmatrix} 44 & 0 \\ 0 & -100 \end{pmatrix} \mathbf{x} + 275 = 0 \tag{2.0.13}$$

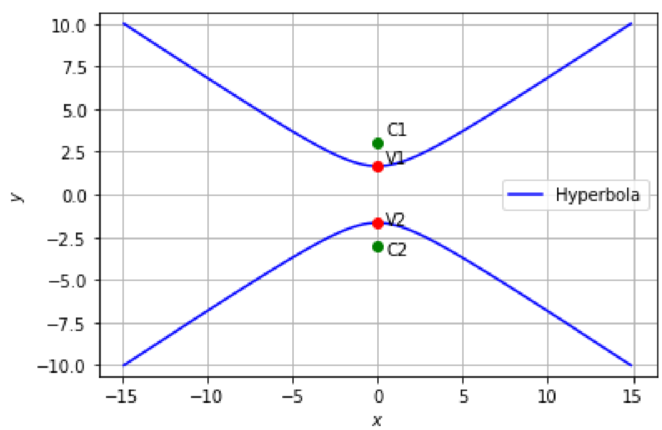

Fig. 0: Plot of Hyperbola