EE3900 - Gate Assignment 4

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Download all latex-tikz codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate4/latex4.tex

and python codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate4/codes/duality.py

Gate EC - 1997 Q1.4

The function $h(t)$ has the fourier transform $g(f)$. The fourier transform of $g(t)\left(\int_{-\infty}^{\infty} g(t)e^{-j2\pi ft}dt\right) =$

Solution

Lemma 0.1. ***Duality Property*** *: Given a function $h(x)$ and it's fourier transform $g(t)$, the duality property of fourier transform says*

$$h(t) \overset{\mathcal{F}}{\rightleftharpoons} g(f) \tag{0.0.1}$$

$$g(t) \overset{\mathcal{F}}{\rightleftharpoons} h(-f) \tag{0.0.2}$$

Proof. Given, the fourier transform of $h(t)$ is $g(f)$

$$g(f) = \int_{-\infty}^{\infty} h(t)\, e^{-j2\pi ft}dt \tag{0.0.3}$$

The inverse fourier transform can be given as

$$h(t) = \int_{-\infty}^{\infty} g(f)\, e^{j2\pi ft}df \tag{0.0.4}$$

Putting $t = -t$

$$h(-t) = \int_{-\infty}^{\infty} g(f)\, e^{-j2\pi ft}df \tag{0.0.5}$$

Interchanging t and f

$$h(-f) = \int_{-\infty}^{\infty} g(t)\, e^{-j2\pi ft}dt \tag{0.0.6}$$

Thus, the fourier transform of $g(t)$ is $h(-f)$ □

Answer: Option C

Example

Consider,

$$h(t) = rect(t+1) \tag{0.0.7}$$

where

$$sinc(x) = \begin{cases} 1 & x = 0 \\ \frac{\sin \pi x}{\pi x} & otherwise \end{cases} \tag{0.0.8}$$

$$rect(x) = \begin{cases} 1 & \text{if } |x| \le \frac{1}{2} \\ 0 & \text{if } otherwise \end{cases} \tag{0.0.9}$$

Finding the Fourier transform of $h(t)$,

$$g(f) = \int_{-\infty}^{\infty} rect(t+1)e^{-j2\pi ft}dt \tag{0.0.10}$$

$$= \int_{\frac{-3}{2}}^{\frac{-1}{2}} e^{-j2\pi ft}dt \tag{0.0.11}$$

$$= \frac{e^{j\pi f} - e^{3j\pi f}}{-j2\pi f} \tag{0.0.12}$$

$$= e^{j2\pi f} sinc(f) \tag{0.0.13}$$

Finding the inverse fourier transform of $h(-f)$,

$$G(t) = \int_{-\infty}^{\infty} rect(-f+1)e^{j2\pi ft}df \tag{0.0.14}$$

$$= \int_{\frac{1}{2}}^{\frac{3}{2}} e^{j2\pi ft}df \tag{0.0.15}$$

$$= \frac{e^{3j\pi t} - e^{j\pi t}}{j2\pi t} \tag{0.0.16}$$

$$= e^{j2\pi t} sinc(t) = g(t) \tag{0.0.17}$$

We have,

$$rect(t+1) \overset{\mathcal{F}}{\rightleftharpoons} e^{j2\pi f} sinc(f) \tag{0.0.18}$$

$$e^{j2\pi t} sinc(t) \overset{\mathcal{F}}{\rightleftharpoons} rect(-t+1) \tag{0.0.19}$$

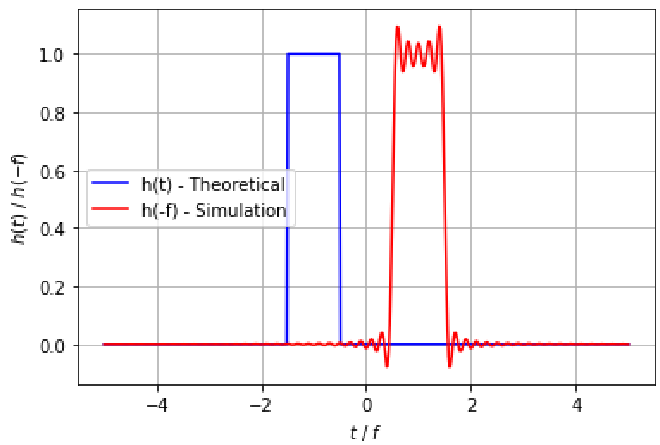

Fig. 0: Plot of signals $h(t)$ and $h(-f)$

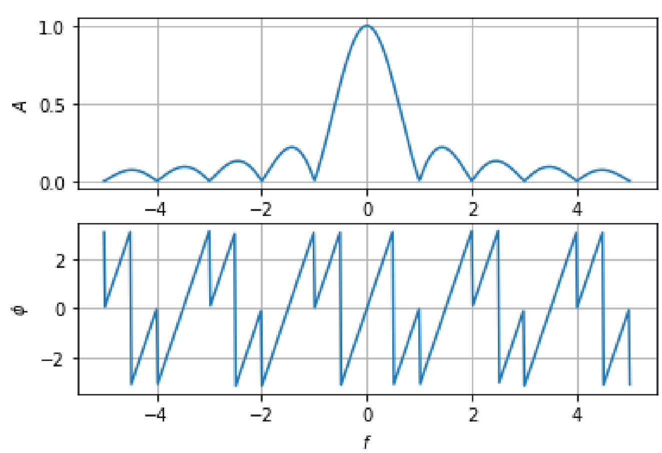

Fig. 0: Plot of amplitude and phase of signal $g(f)$