EE3900-Gate Assignment

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Download all latex-tikz codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate1/gatelatex.tex

and python codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate1/codes/fourier.py

1 Gate EC 2016 Q.10

Find energy of the signal $x(t) = \frac{sin(4\pi t)}{4\pi t} = sinc(4t)$

2 Solution

Lemma 2.1. *Parseval's theorem states that there is no loss of information in Fourier transform and the amount of energy remains the same in time and frequency domains.*

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \tag{2.0.1}$$

Consider a rectangular function

$$rect(t) = \begin{cases} \frac{1}{4} & \text{if } |t| \le 2 \\ 0 & \text{if } |t| > 2 \end{cases} \tag{2.0.2}$$

Let the Fourier transform of $rect(t)$ be given as $Y(f)$

$$rect(t) \overset{\mathcal{F}}{\rightleftharpoons} Y(f) \tag{2.0.3}$$

Finding the Fourier transform,

$$Y(f) = \int_{-\infty}^{\infty} rect(t) e^{i2\pi f t} dt \tag{2.0.4}$$

$$= \int_{-2}^{2} \frac{1}{4} e^{i2\pi f t} dt \tag{2.0.5}$$

$$= \frac{e^{i4\pi f} - e^{-i4\pi f}}{i8\pi f} \tag{2.0.6}$$

$$= sinc(4f) \tag{2.0.7}$$

where $sinc(t)$ is defined as

$$sinc(t) = \begin{cases} 1 & t = 0 \\ \frac{\sin \pi t}{\pi t} & otherwise \end{cases} \tag{2.0.8}$$

For any signal $g(t)$ and it's Fourier transform $G(t)$, from Duality of Fourier transform, we have

$$g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f) \tag{2.0.9}$$

$$G(t) \overset{\mathcal{F}}{\rightleftharpoons} g(-f) \tag{2.0.10}$$

$$\implies sinc(4t) \overset{\mathcal{F}}{\rightleftharpoons} rect(-f) \tag{2.0.11}$$

$$rect(-f) = rect(f) = \begin{cases} \frac{1}{4} & \text{if } |f| \le 2 \\ 0 & \text{if } |f| > 2 \end{cases} \tag{2.0.12}$$

Energy of the signal is given by,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} sinc^2(4t) dt \tag{2.0.13}$$

From Parseval's theorem, we have

$$\int_{-\infty}^{\infty} sinc^2(4t) dt = \int_{-\infty}^{\infty} rect^2(f) df \tag{2.0.14}$$

which is the area under the graph 0

$$Area = 4 \times \frac{1}{4^2} = \frac{1}{4} \tag{2.0.15}$$

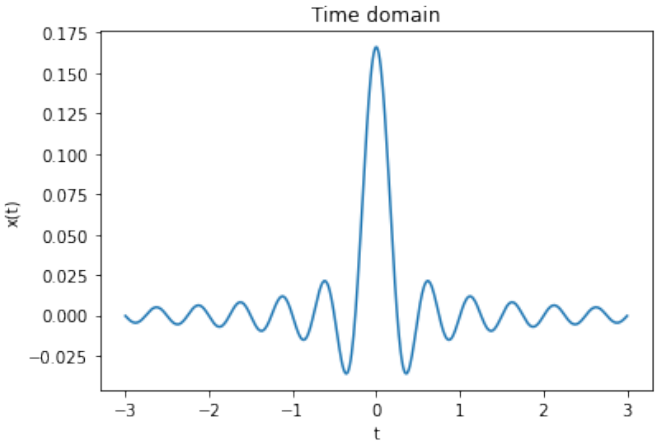

Fig. 0: Plot of signal in Time domain

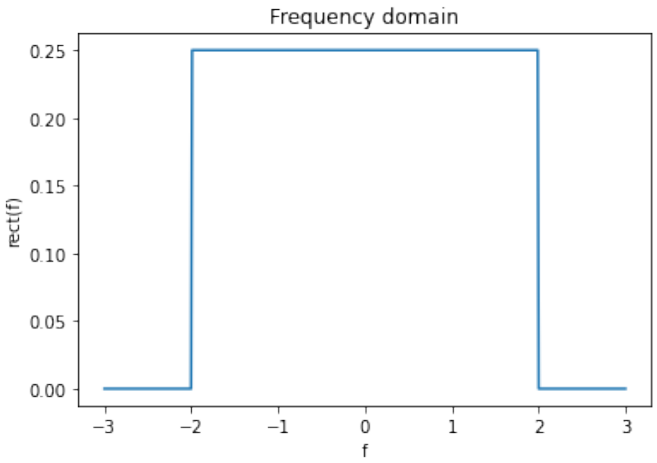

Fig. 0: Plot of signal in Frequency domain

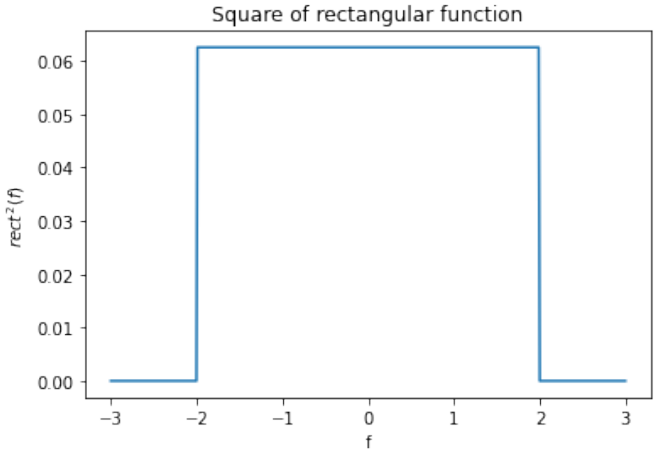

Fig. 0: Plot of square of rectangular function