EE3900-Gate Assignment

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Download all latex-tikz codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate1/gatelatex.tex

and python codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate1/codes/fourier.py

1 Matrices 2.8

Find energy of the signal $x(t) = \frac{sin(4\pi t)}{4\pi t}$

2 Solution

Lemma 2.1. *Parseval's theorem states that there is no loss of information in Fourier transform and the amount of energy remains the same in time and frequency domains.*

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega \tag{2.0.1}$$

Let,

$$f(t) = \begin{cases} \frac{1}{4} & \text{if } |t| \le 4\pi \\ 0 & \text{if } |t| > 4\pi \end{cases} \tag{2.0.2}$$

$$f(t) \overset{\mathcal{F}}{\rightleftharpoons} F(\omega) \tag{2.0.3}$$

Finding the Fourier transform of $f(t)$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt \tag{2.0.4}$$

$$= \int_{-4\pi}^{4\pi} \frac{1}{4} e^{i\omega t} dt \tag{2.0.5}$$

$$= \frac{e^{i4\pi\omega} - e^{-i4\pi\omega}}{4i\omega} \tag{2.0.6}$$

$$= \frac{\sin 4\pi\omega}{2\omega} \tag{2.0.7}$$

From Duality of Fourier transform, we have

$$f(t) \overset{\mathcal{F}}{\rightleftharpoons} F(\omega) \tag{2.0.8}$$

$$F(t) \overset{\mathcal{F}}{\rightleftharpoons} 2\pi f(-\omega) \tag{2.0.9}$$

$$\implies \frac{\sin 4\pi t}{2t} \overset{\mathcal{F}}{\rightleftharpoons} 2\pi f(-\omega) \tag{2.0.10}$$

$$\implies \frac{\sin 4\pi t}{4\pi t} \overset{\mathcal{F}}{\rightleftharpoons} f(-\omega) \tag{2.0.11}$$

$$f(-\omega) = f(\omega) = \begin{cases} \frac{1}{4} & \text{if } |\omega| \le 4\pi \\ 0 & \text{if } |\omega| > 4\pi \end{cases} \tag{2.0.12}$$

Energy of the signal is given by,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left(\frac{\sin 4\pi t}{4\pi t}\right)^2 dt \tag{2.0.13}$$

From Parseval's theorem, we have

$$\int_{-\infty}^{\infty} \left(\frac{\sin 4\pi t}{4\pi t}\right)^2 dt = \frac{1}{2\pi}\int_{-4\pi}^{4\pi} \frac{1}{4^2} d\omega \tag{2.0.14}$$

$$= \frac{1}{4} \tag{2.0.15}$$

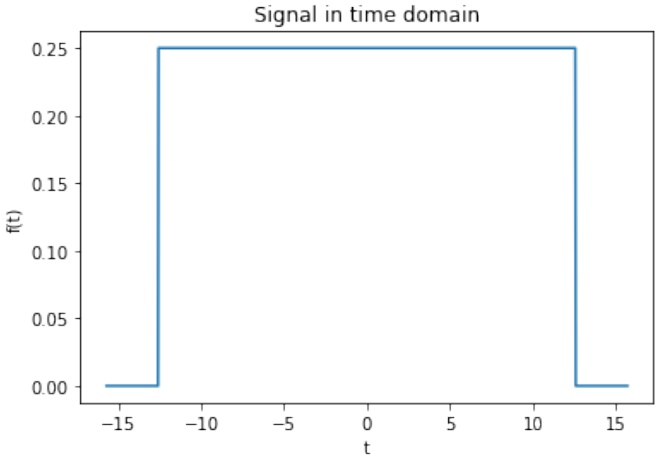

Fig. 0: Plot of signal in Time domain

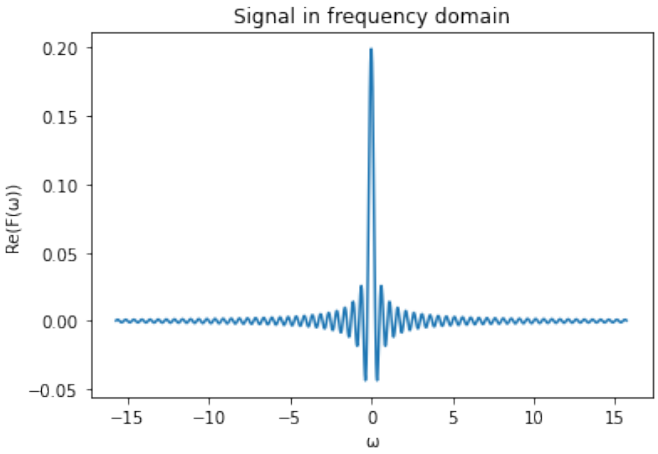

Fig. 0: Plot of signal in Frequency domain