EE3900-Assignment 4

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Download all latex-tikz codes from

https://github.com/vaishnavi-w/EE3900/blob/main/Assignment4/latex4.tex

and python codes from

https://github.com/vaishnavi-w/EE3900/blob/main/Assignment4/codes/linesplot.tex

1 Linear Forms Q.2.32

If the co-ordinates of the points A,B,C and D be $\begin{pmatrix}1\\2\\3\end{pmatrix}$, $\begin{pmatrix}4\\5\\7\end{pmatrix}$, $\begin{pmatrix}-4\\3\\-6\end{pmatrix}$, $\begin{pmatrix}2\\9\\2\end{pmatrix}$. Then find the angle between lines AB and CD

2 Solution

The direction vector for the line AB is

$$\mathbf{m_1} = \mathbf{B} - \mathbf{A} \tag{2.0.1}$$

$$\implies \mathbf{m_1} = \begin{pmatrix}4\\5\\7\end{pmatrix} - \begin{pmatrix}1\\2\\3\end{pmatrix} \tag{2.0.2}$$

$$\implies \mathbf{m_1} = \begin{pmatrix}3\\3\\4\end{pmatrix} \tag{2.0.3}$$

The direction vector for the line CD is

$$\mathbf{m_2} = \mathbf{D} - \mathbf{C} \tag{2.0.4}$$

$$\implies \mathbf{m_2} = \begin{pmatrix}2\\9\\2\end{pmatrix} - \begin{pmatrix}-4\\3\\-6\end{pmatrix} \tag{2.0.5}$$

$$\implies \mathbf{m_2} = \begin{pmatrix}6\\6\\8\end{pmatrix} = 2\begin{pmatrix}3\\3\\4\end{pmatrix} = 2\mathbf{m_1} \tag{2.0.6}$$

The lines are scalar multiples of one another. Hence, they are parallel.
Angle between two lines is given as

$$\cos\theta = \frac{\mathbf{m_1}^\top \mathbf{m_2}}{\|\mathbf{m_1}\|\|\mathbf{m_2}\|} \tag{2.0.7}$$

$$\mathbf{m_1}^\top \mathbf{m_2} = \begin{pmatrix}3 & 3 & 4\end{pmatrix}\begin{pmatrix}6\\6\\8\end{pmatrix} \tag{2.0.8}$$

$$= 68 \tag{2.0.9}$$

$$\|\mathbf{m_1}\| = \sqrt{(3)^2 + (3)^2 + (4)^2} = \sqrt{34} \tag{2.0.10}$$

$$\|\mathbf{m_2}\| = \sqrt{(6)^2 + (6)^2 + (8)^2} = 2\sqrt{34} \tag{2.0.11}$$

$$\implies \cos\theta = \frac{68}{2\sqrt{34} \times \sqrt{34}} \tag{2.0.12}$$

$$\theta = \cos^{-1}(1) \tag{2.0.13}$$

$$= 0^\circ \tag{2.0.14}$$

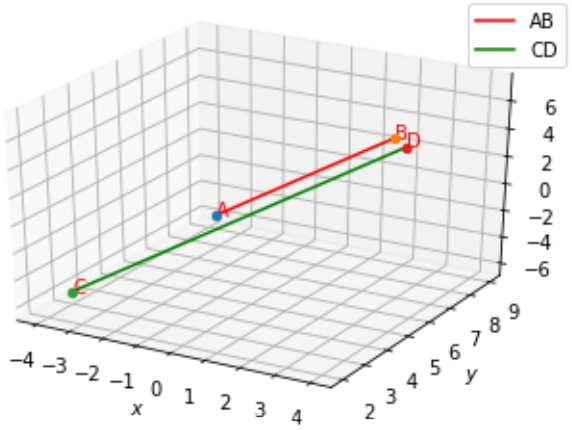

Fig. 0: Plot of lines AB and CD