

# EE3900-Gate Assignment

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Download all latex-tikz codes from

<https://github.com/vaishnavi-w/EE3900/blob/main/Gate1/gatelatex.tex>

and python codes from

<https://github.com/vaishnavi-w/EE3900/blob/main/Gate1/codes/fourier.py>

## 1 GATE EC 2016 Q.10

Find energy of the signal  $x(t) = \frac{\sin(4\pi t)}{4\pi t} = \text{sinc}(4t)$

## 2 SOLUTION

**Lemma 2.1.** Parseval's theorem states that there is no loss of information in Fourier transform and the amount of energy remains the same in time and frequency domains.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \quad (2.0.1)$$

Consider a unit rectangular function

$$\text{rect}(t) = \begin{cases} 1 & \text{if } |t| \leq \frac{1}{2} \\ 0 & \text{if otherwise} \end{cases} \quad (2.0.2)$$

Let the Fourier transform of  $\text{rect}(t)$  be given as  $Y(f)$

$$\text{rect}(t) \xrightarrow{\mathcal{F}} Y(f) \quad (2.0.3)$$

Finding the Fourier transform,

$$Y(f) = \int_{-\infty}^{\infty} \text{rect}(t) e^{j2\pi ft} dt \quad (2.0.4)$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} e^{j2\pi ft} dt \quad (2.0.5)$$

$$= \frac{e^{j\pi f} - e^{-j\pi f}}{j2\pi f} \quad (2.0.6)$$

$$= \text{sinc}(f) \quad (2.0.7)$$

where  $\text{sinc}(f)$  is defined as

$$\text{sinc}(t) = \begin{cases} 1 & f = 0 \\ \frac{\sin \pi f}{\pi f} & \text{otherwise} \end{cases} \quad (2.0.8)$$

For any signal  $g(t)$  and its Fourier transform  $G(f)$ , from Duality of Fourier transform, we have

$$g(t) \xrightarrow{\mathcal{F}} G(f) \quad (2.0.9)$$

$$G(t) \xrightarrow{\mathcal{F}} g(-f) \quad (2.0.10)$$

$$\Rightarrow \text{sinc}(t) \xrightarrow{\mathcal{F}} \text{rect}(-f) \quad (2.0.11)$$

$$\text{rect}(-f) = \text{rect}(f) = \begin{cases} 1 & \text{if } |f| \leq \frac{1}{2} \\ 0 & \text{if otherwise} \end{cases} \quad (2.0.12)$$

When a time signal  $g(t)$  is time scaled by  $\alpha$ , the resulting Fourier transform is given by:

$$g(\alpha t) \xrightarrow{\mathcal{F}} \frac{1}{|\alpha|} G\left(\frac{f}{\alpha}\right) \quad (2.0.13)$$

$$\Rightarrow \text{sinc}(4t) \xrightarrow{\mathcal{F}} \frac{1}{4} \text{rect}\left(\frac{f}{4}\right) \quad (2.0.14)$$

Fourier transform of  $\text{sinc}(4t)$  is given as,

$$\frac{1}{4} \text{rect}\left(\frac{f}{4}\right) = \begin{cases} \frac{1}{4} & \left|\frac{f}{4}\right| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} = \begin{cases} \frac{1}{4} & |f| \leq 2 \\ 0 & \text{otherwise} \end{cases} \quad (2.0.15)$$

Energy of the signal using Parseval's theorem,

$$\begin{aligned} \int_{-\infty}^{\infty} |x(t)|^2 dt &= \int_{-\infty}^{\infty} \text{sinc}^2(4t) dt \\ &= \int_{-\infty}^{\infty} \left(\frac{1}{4} \text{rect}\left(\frac{f}{4}\right)\right)^2 df \end{aligned} \quad (2.0.16)$$

which is the area under the graph 0

$$\text{Area} = 4 \times \frac{1}{4^2} = \frac{1}{4} \quad (2.0.17)$$

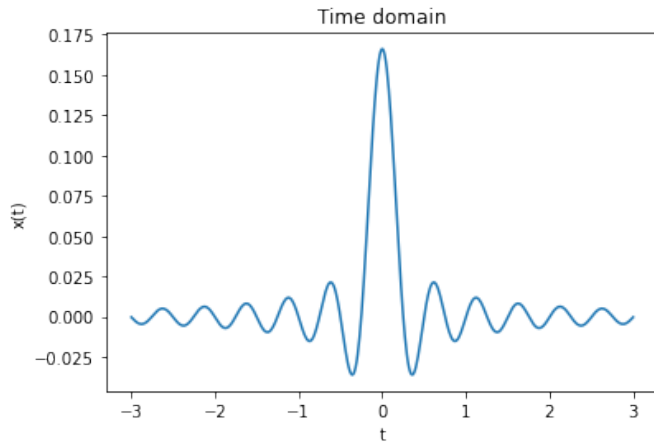


Fig. 0: Plot of signal in Time domain

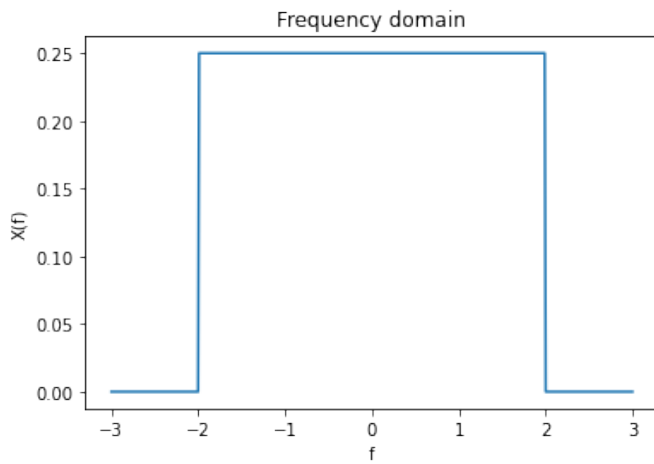


Fig. 0: Plot of signal in Frequency domain

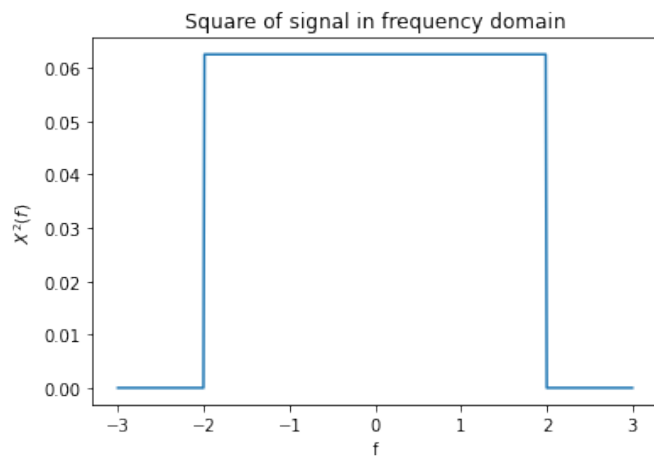


Fig. 0: Plot of square of signal in frequency domain