

EE3900 - Gate Assignment 4

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Download all latex-tikz codes from

<https://github.com/vaishnavi-w/EE3900/blob/main/Gate4/latex4.tex>

where

$$\text{sinc}(x) = \begin{cases} 1 & x = 0 \\ \frac{\sin \pi x}{\pi x} & \text{otherwise} \end{cases} \quad (0.0.8)$$

$$\text{rect}(x) = \begin{cases} 1 & \text{if } |x| \leq \frac{1}{2} \\ 0 & \text{if otherwise} \end{cases} \quad (0.0.9)$$

GATE EC - 1997 Q1.4

The function $h(t)$ has the fourier transform $g(f)$. The fourier transform of $g(t) \left(\int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt \right) =$

Finding the Fourier transform of $h(t)$,

SOLUTION

Lemma 0.1. Duality Property : Given a function $h(x)$ and it's fourier transform $g(t)$, the duality property of fourier transform says

$$h(t) \xrightarrow{\mathcal{F}} g(f) \quad (0.0.1)$$

$$g(t) \xrightarrow{\mathcal{F}} h(-f) \quad (0.0.2)$$

Proof. Given, the fourier transform of $h(t)$ is $g(f)$

$$g(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt \quad (0.0.3)$$

The inverse fourier transform can be given as

$$h(t) = \int_{-\infty}^{\infty} g(f) e^{j2\pi ft} df \quad (0.0.4)$$

Putting $t = -t$

$$h(-t) = \int_{-\infty}^{\infty} g(f) e^{-j2\pi ft} df \quad (0.0.5)$$

Interchanging t and f

$$h(-f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt \quad (0.0.6)$$

Thus, the fourier transform of $g(t)$ is $h(-f)$ \square

Answer: Option C

EXAMPLE

Consider,

$$h(t) = \text{rect}(t + 1) \quad (0.0.7)$$

$$g(f) = \int_{-\infty}^{\infty} \text{rect}(t + 1) e^{-j2\pi ft} dt \quad (0.0.10)$$

$$= \int_{-\frac{3}{2}}^{\frac{1}{2}} e^{-j2\pi ft} dt \quad (0.0.11)$$

$$= \frac{e^{j\pi f} - e^{3j\pi f}}{-j2\pi f} \quad (0.0.12)$$

$$= e^{j2\pi f} \text{sinc}(f) \quad (0.0.13)$$

Finding the inverse fourier transform of $h(-f)$,

$$G(t) = \int_{-\infty}^{\infty} \text{rect}(-f + 1) e^{j2\pi ft} df \quad (0.0.14)$$

$$= \int_{\frac{1}{2}}^{\frac{3}{2}} e^{j2\pi ft} df \quad (0.0.15)$$

$$= \frac{e^{3j\pi t} - e^{j\pi t}}{j2\pi t} \quad (0.0.16)$$

$$= e^{j2\pi t} \text{sinc}(t) = g(t) \quad (0.0.17)$$

We have,

$$\text{rect}(t + 1) \xrightarrow{\mathcal{F}} e^{j2\pi f} \text{sinc}(f) \quad (0.0.18)$$

$$e^{j2\pi t} \text{sinc}(t) \xrightarrow{\mathcal{F}} \text{rect}(-t + 1) \quad (0.0.19)$$

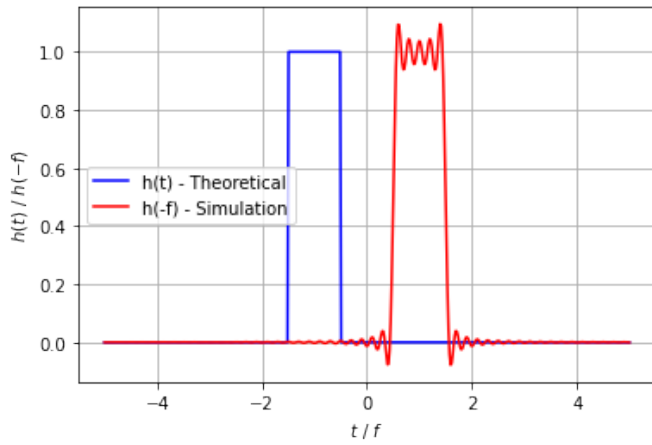


Fig. 0: Plot of signals $h(t)$ and $h(-f)$

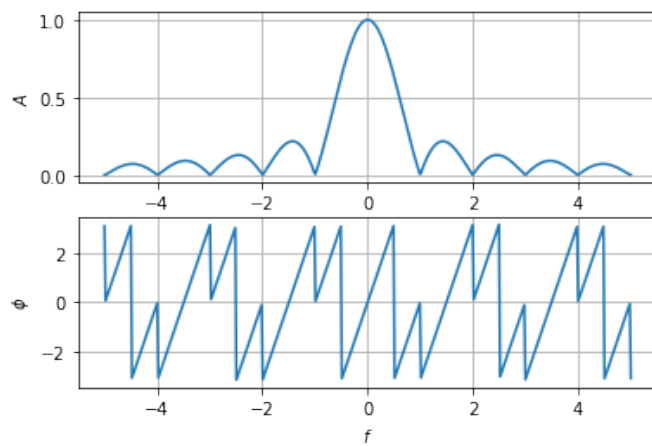


Fig. 0: Plot of amplitude and phase of signal $g(f)$