EE3900-Gate Assignment

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Download all latex-tikz codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate1/gatelatex.tex

and python codes from

https://github.com/vaishnavi−w/EE3900/blob/main/Gate1/codes/fourier.py

1 Gate EC 2016 Q.10

Find energy of the signal $x(t) = \frac{sin(4\pi t)}{4\pi t} = sinc(4t)$

2 Solution

Lemma 2.1. *Parseval's theorem states that there is no loss of information in Fourier transform and the amount of energy remains the same in time and frequency domains.*

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \tag{2.0.1}$$

Consider a unit rectangular function

$$rect(t) = \begin{cases} 1 & \text{if } |t| \le \frac{1}{2} \\ 0 & \text{if } otherwise \end{cases} \tag{2.0.2}$$

Let the Fourier transform of $rect(t)$ be given as $Y(f)$

$$rect(t) \overset{\mathcal{F}}{\rightleftharpoons} Y(f) \tag{2.0.3}$$

Finding the Fourier transform,

$$Y(f) = \int_{-\infty}^{\infty} rect(t) e^{j2\pi ft} dt \tag{2.0.4}$$

$$= \int_{\frac{-1}{2}}^{\frac{1}{2}} e^{j2\pi ft} dt \tag{2.0.5}$$

$$= \frac{e^{j\pi f} - e^{-j\pi f}}{j2\pi f} \tag{2.0.6}$$

$$= sinc(f) \tag{2.0.7}$$

where $sinc(f)$ is defined as

$$sinc(t) = \begin{cases} 1 & f = 0 \\ \frac{\sin \pi f}{\pi f} & otherwise \end{cases} \tag{2.0.8}$$

For any signal $g(t)$ and it's Fourier transform $G(f)$, from Duality of Fourier transform, we have

$$g(t) \overset{\mathcal{F}}{\rightleftharpoons} G(f) \tag{2.0.9}$$

$$G(t) \overset{\mathcal{F}}{\rightleftharpoons} g(-f) \tag{2.0.10}$$

$$\implies sinc(t) \overset{\mathcal{F}}{\rightleftharpoons} rect(-f) \tag{2.0.11}$$

$$rect(-f) = rect(f) = \begin{cases} 1 & \text{if } |f| \le \frac{1}{2} \\ 0 & \text{if } otherwise \end{cases} \tag{2.0.12}$$

When a time signal $g(t)$ is time scaled by α, the resulting Fourier transform is given by:

$$g(\alpha t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{|\alpha|} G\left(\frac{f}{\alpha}\right) \tag{2.0.13}$$

$$\implies sinc(4t) \overset{\mathcal{F}}{\rightleftharpoons} \frac{1}{4} rect\left(\frac{f}{4}\right) \tag{2.0.14}$$

Fourier transform of $sinc(4t)$ is given as,

$$\frac{1}{4} rect\left(\frac{f}{4}\right) = \begin{cases} \frac{1}{4} & \left|\frac{f}{4}\right| \le \frac{1}{2} \\ 0 & otherwise \end{cases} = \begin{cases} \frac{1}{4} & |f| \le 2 \\ 0 & otherwise \end{cases} \tag{2.0.15}$$

Energy of the signal using Parseval's thoerem,

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} sinc^2(4t) dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{1}{4} rect\left(\frac{f}{4}\right)\right)^2 df \tag{2.0.16}$$

which is the area under the graph 0

$$Area = 4 \times 0.0625 = \frac{1}{4} \tag{2.0.17}$$

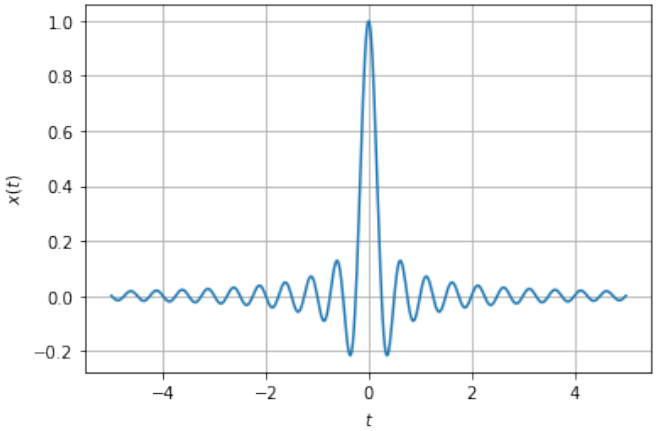

Fig. 0: Plot of signal in Time domain

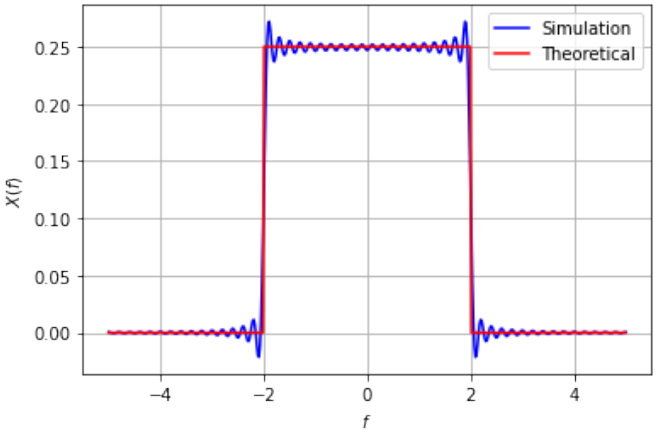

Fig. 0: Plot of signal in Frequency domain

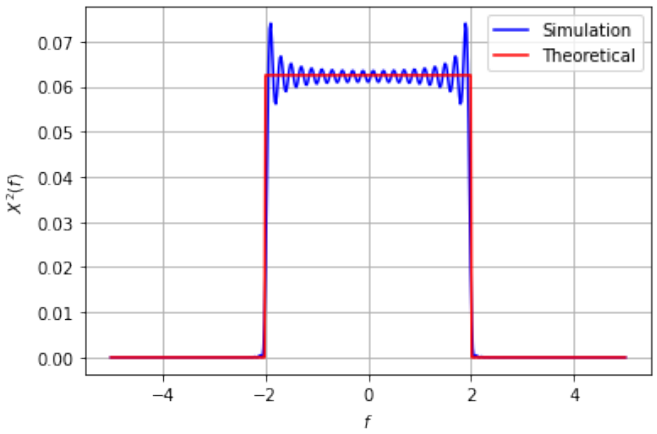

Fig. 0: Plot of square of signal in frequency domain