NCERT Maths 11.9.2 Q9

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Question: The sum of the first n terms of two arithmetic progressions (AP) is in the ratio $5n+4 : 9n+6$. Find the ratio of their 18th terms.

solution:

TABLE I
INPUT PARAMETERS

Parameter	Description
$x_1(0)$	First term of the first arithmetic progression (AP).
$x_2(0)$	First term of the second arithmetic progression (AP).
d_1	Common difference of the first AP.
d_2	Common difference of the second AP.
n	Index of the term in the sequences.
$x_1(n)$	nth term of the first arithmetic progression (AP).
$x_2(n)$	nth term of the second arithmetic progression (AP).

$$x_1(n) = (x_1(0) + nd_1)u(n) \tag{1}$$

$$x_2(n) = (x_2(0) + nd_2)u(n) \tag{2}$$

Applying Z transform:

$$x_1(z) = \frac{x_1(0)}{1-z^{-1}} + \frac{d_1 z^{-1}}{(1-z^{-1})^2} \tag{3}$$

$$x_2(z) = \frac{x_2(0)}{1-z^{-1}} + \frac{d_2 z^{-1}}{(1-z^{-1})^2} \tag{4}$$

Region of Convergence or R.O.C :

$$|z| > 1 \tag{5}$$

For AP, the sum of first n+1 terms can be written as :

$$y(n) = x(n) * u(n) \tag{6}$$

Applying Z transform on both sides

$$x_1(z) = \frac{x_1(0)}{(1-z^{-1})^2} + \frac{d_1 z^{-1}}{(1-z^{-1})^3} \tag{7}$$

$$x_2(z) = \frac{x_2(0)}{(1-z^{-1})^2} + \frac{d_2 z^{-1}}{(1-z^{-1})^3} \tag{8}$$

Using contour integration to find inverse Z transform:

$$y_1(n) = \frac{1}{2\pi j}\oint_C Y(z) z^{n-1} dz \tag{9}$$

$$= \frac{1}{2\pi j}\oint_C \left[\frac{x_1(0)}{(1-z^{-1})^2} - \frac{d_1 z^{-1}}{(1-z^{-1})^3}\right] z^{n-1}\, dz \tag{10}$$

$$y_2(n) = \frac{1}{2\pi j}\oint_C Y(z) z^{n-1} dz \tag{11}$$

$$= \frac{1}{2\pi j}\oint_C \left[\frac{x_2(0)}{(1-z^{-1})^2} - \frac{d_2 z^{-1}}{(1-z^{-1})^3}\right] z^{n-1}\, dz \tag{12}$$

The sum of the terms of the sequence is computed using the residue theorem, expressed as R_i, which represents the residue of the Z-transform at $z = 1$ for the expression $Y(z)$.

$$R_i = R_1 + R_2 \tag{13}$$

R_1 and R_2 are residues calculated at the poles of the Z-transform.

$$fory_1(n)R_1 = \frac{1}{(2-1)!}\left.\frac{d(x_1(0)z^{n+1})}{dz}\right|_{z=1} \tag{14}$$

$$R_2 = \frac{1}{(3-1)!}\left.\frac{d^2(d_1 z^{n+1})}{dz^2}\right|_{z=1} \tag{15}$$

The sum of terms is given by R_i:

$$y_1(n) = x_1(0)(n+1) + \frac{d_1}{2}n(n+1) \tag{16}$$

$$fory_2(n)R_1 = \frac{1}{(2-1)!}\left.\frac{d(x_2(0)z^{n+1})}{dz}\right|_{z=1} \tag{17}$$

$$R_2 = \frac{1}{(3-1)!}\left.\frac{d^2(d_2 z^{n+1})}{dz^2}\right|_{z=1} \tag{18}$$

The sum of terms is given by R_i:

$$y_2(n) = x_2(0)(n+1) + \frac{d_2}{2}n(n+1) \tag{19}$$

taking ratio of $y_1(n){:}y_2(n)$ and equating to $5(n+1)+4{:}9(n+1)+6$

$$\frac{y_1(n)}{y_2(n)} = \frac{x_1(0)(n+1) + \frac{d_1}{2}n(n+1)}{x_2(0)(n+1) + \frac{d_2}{2}n(n+1)} \tag{20}$$

$$= \frac{x_1(0) + \frac{d_1}{2}n}{x_2(0) + \frac{d_2}{2}n} \tag{21}$$

we need ratio of 18th terms $\frac{n}{2}$ is 17 and n=34 substitute it in $5(n+1)+4{:}9(n+1)+6$ then we get ratio as $\frac{179}{321}$