EE23BTECH11014- DEVARAKONDA GUNA VAISHNAVI*

Question: The sum of the first n terms of two arithmetic progressions (AP) is in the ratio $5n+4 : 9n+6$. Find the ratio of their 18th terms.

Solution: :

Parameter	value	Description
$x_1(0)$	$\frac{9}{2}$	First term of the first arithmetic progression (AP).
$x_2(0)$	$\frac{15}{2}$	First term of the second arithmetic progression (AP).
d_1	5	Common difference of the first AP.
d_2	9	Common difference of the second AP.
n	-	Index of the term in the sequences.

TABLE I
INPUT PARAMETERS

$$x_1(n) = (x_1(0) + nd_1)u(n) \tag{1}$$

$$x_2(n) = (x_2(0) + nd_2)u(n) \tag{2}$$

Applying Z transform: Taking Z-Transform:

1) $\mathcal{Z}\{u(n)\}$

$$u(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{1}{1-z^{-1}}\{|z| > 1\} \tag{3}$$

2) $\mathcal{Z}\{nu(n)\}$

$$nu(n) \overset{\mathcal{Z}}{\leftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2}\{|z| > 1\} \tag{4}$$

$$\frac{\frac{n}{2}[2x_1(0) + (n-1)d_1]}{\frac{n}{2}[2x_2(0) + (n-1)d_2]} = \frac{5n+4}{9n+6} \tag{5}$$

$$\frac{2x_1(0) + nd_1 - d_1}{2x_2(0) + nd_2 - d_2} = \frac{5n+4}{9n+6} \tag{6}$$

On comparing we get, $d_1 = 5$ and $d_2 = 9$

$$2x_1(0) - d_1 = 4 \tag{7}$$

$$x_1 = \frac{9}{2} \tag{8}$$

$$2x_2(0) - d_2 = 6 \tag{9}$$

$$x_2(0) = \frac{15}{2} \tag{10}$$

$$x_1(z) = \frac{9/2}{1-z^{-1}} + \frac{5z^{-1}}{(1-z^{-1})^2} \tag{11}$$

$$x_2(z) = \frac{15/2}{1-z^{-1}} + \frac{9z^{-1}}{(1-z^{-1})^2} \tag{12}$$

$$x_1(n) = \{9/2, 19/2, 29/2, ...\} \tag{13}$$

$$x_2(n) = \{15/2, 33/2, 51/2, ...\} \tag{14}$$

$$\frac{x_1(18)}{x_2(18)} = \frac{179}{321} \tag{15}$$

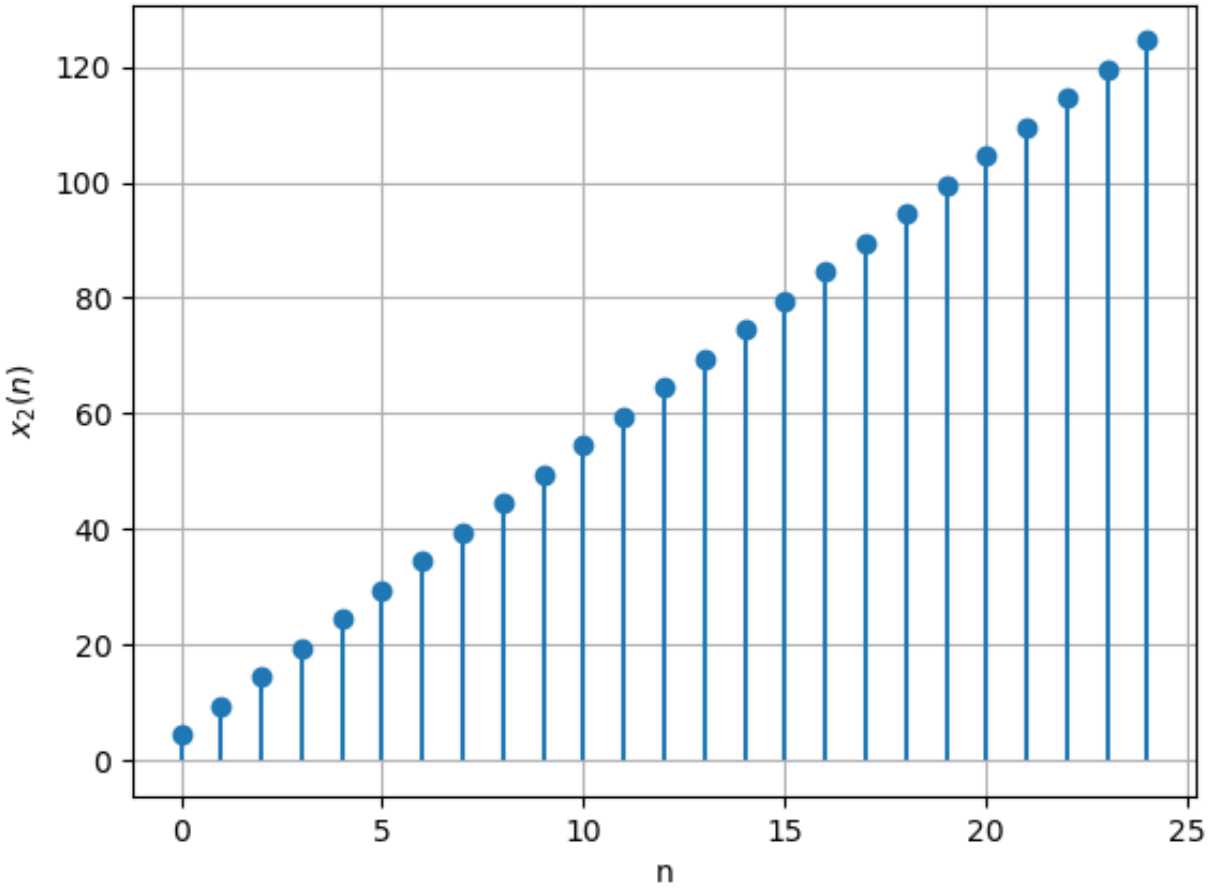

Fig. 1. stem plot of $x_1(n)$

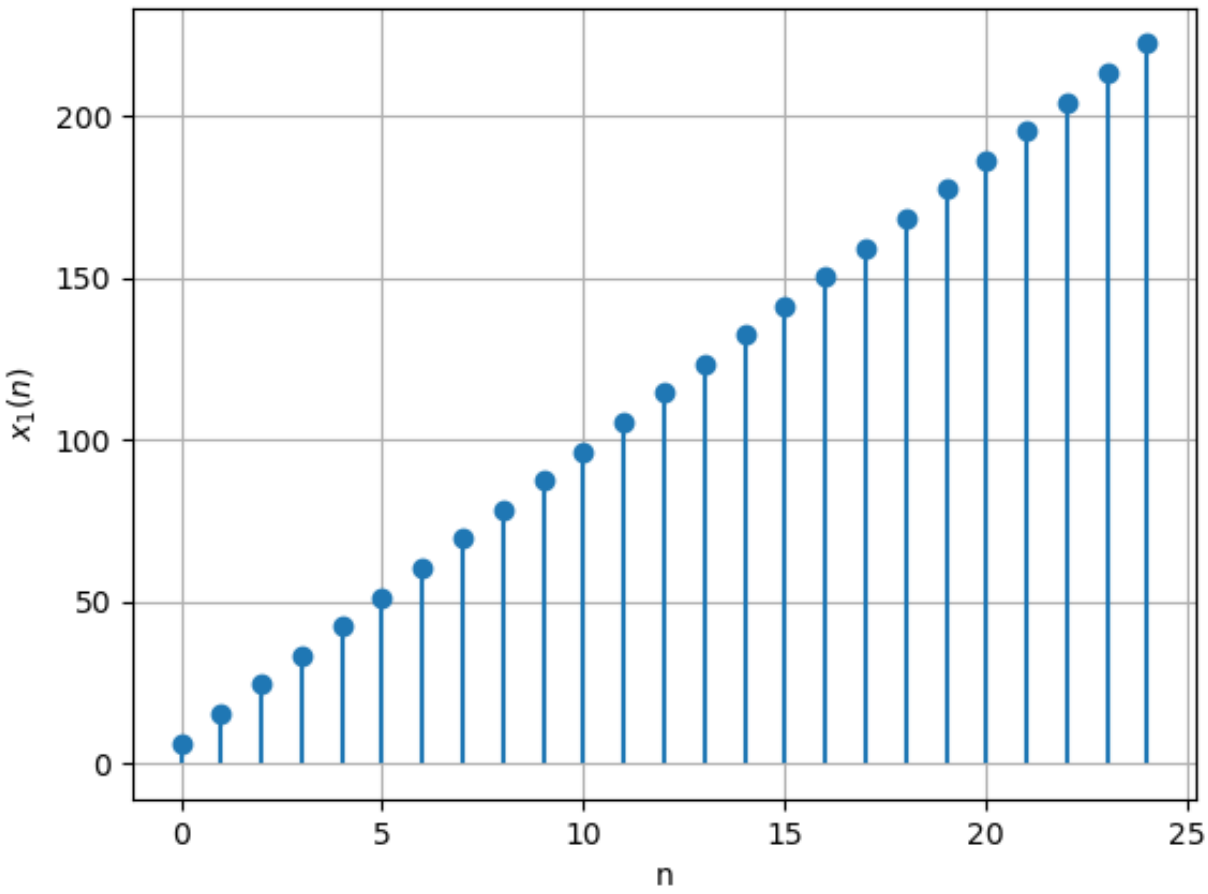

Fig. 2. stem plot of $x_2(n)$