

NCERT 11.9.2 Q9

EE23BTECH11014- Devarakonda Guna vaishnavi

Question:

Question: The sum of the first n terms of two arithmetic progressions (AP) is in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18th terms.

For the First AP:

Given Information:

There are two arithmetic progressions (AP) with different first terms and common differences.

For the First AP:

Let the first term be a and the common difference be d . The sum of n terms is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

The n -th term is given by:

$$a_n = a + (n-1)d$$

For the Second AP:

Let the first term be A and the common difference be D . The sum of n terms is given by:

$$S_n = \frac{n}{2}[2A + (n-1)D]$$

The n -th term is given by:

$$A_n = A + (n-1)D$$

Given that the sum of n terms of the first AP to the sum of n terms of the second AP is in the ratio $5n + 4 : 9n + 6$, we have the equation:

$$a + \left(\frac{n-1}{2}\right)d \div A + \left(\frac{n-1}{2}\right)D = \frac{5n+4}{9n+6}$$

Introducing Z-transforms:

Let $X(z)$ be the Z-transform of the sequence x_n :

$$X(z) = \sum_{n=0}^{\infty} x_n z^{-n}$$

Apply Z-transform to the equations for a_n and A_n :

$$A(z) = a + zd$$

$$B(z) = A + zD$$

Now, express the given ratio in terms of Z-transforms:

$$\frac{A(z)}{B(z)} = \frac{5z+4}{9z+6}$$

To Find the Ratio of their 18th Terms in Z-transforms:

The Z-transform of the 18th term of the first AP is $a + 17d$, and the Z-transform of the 18th term of the second AP is $A + 17D$. The ratio is:

$$\text{Ratio} = \frac{a + 17d}{A + 17D}$$

Solving for n :

Since $\frac{n-1}{2} = 17$, solving for n gives $n = 35$.

Substituting $n = 35$ into the equation, we get:

$$a + \left(\frac{35-1}{2}\right)d \cdot A + \left(\frac{35-1}{2}\right)D = \frac{5(35)+4}{9(35)+6}$$

Solving the equation yields $a + 17d \div A + 17D = \frac{179}{321}$.

Hence, the ratio of the 18th term of the first AP to the 18th term of the second AP is $179 : 321$.