Signals and Systems - Gate2023-ee-Q46

EE23BTECH11014- Devarakonda Guna vaishnavi

February 18, 2024

Consider the state-space description of an LTI system with matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 3 & -2 \end{bmatrix}, D = \begin{bmatrix} 1 \end{bmatrix}.$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is __________ (Round off to the nearest integer).

Solution:

Parameter	**Value**
System Matrix, A	$\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$
Input Matrix, B	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Output Matrix, C	$\begin{bmatrix} 3 & -2 \end{bmatrix}$
Feedthrough Matrix, D	$\begin{bmatrix} 1 \end{bmatrix}$
Input Signal, $u(t)$	$\sin(\omega t)$, $\omega > 0$

Table 1: Input Parameters

Transfer function given by:

$$T.F = C\left[sI - A\right]^{-1} B + D \tag{1}$$

$$\left[sI - A\right] = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \tag{2}$$

$$\left[sI - A\right]^{-1} = \frac{1}{s(s+2)+1} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \tag{3}$$

Referencing from equation (3), equation (1) becomes

$$\begin{aligned} T.F &= \begin{bmatrix} \frac{3}{s^2+2s+1} & \frac{-2}{s^2+2s+1} \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 && (4) \\ &= \begin{bmatrix} \frac{3}{s^2+2s+1} & \frac{-2}{s^2+2s+1} \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} + 1 && (5) \\ &= \frac{s^2+4}{s^2+2s+1} && (6) \\ H(s) &= T.F && (7) \\ H(s) &= \frac{s^2+4}{s^2+2s+1} && (8) \end{aligned}$$

Substituting $s = j\omega$ in equation (8),

$$H(j\omega) = \frac{4-(\omega)^2}{1+2j\omega-(\omega)^2} \tag{9}$$

Steady state output of system is zero:

$$\begin{aligned} 4-(\omega)^2 &= 0 && (10) \\ \omega &= 2 \text{ rad/sec} && (11) \end{aligned}$$