signals and systems

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Consider the state-space description of an LTI system with matrices

$$A = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & -2 \end{bmatrix}, \quad D = \begin{bmatrix} 1 \end{bmatrix}$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is _____ (Round off to the nearest integer).
solution:

Parameter	**Value**
System Matrix, A	$\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$
Input Matrix, B	$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$
Output Matrix, C	$\begin{bmatrix} 3 & -2 \end{bmatrix}$
Feedthrough Matrix, D	1
Input Signal, $u(t)$	$\sin(\omega t)$, $\omega > 0$

Table 1: Input Parameters

Transfer function given by:

$$T.F = C\left[sI - A\right]^{-1} B + D \tag{1}$$

$$\left[sI - A\right] = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \tag{2}$$

$$\left[sI - A\right]^{-1} = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}^{-1} \tag{3}$$

$$\left[sI - A\right]^{-1} = \frac{1}{s(s+2)+1} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{4}$$

Referencing from equation (4), equation (1) becomes

$$T.F = \begin{bmatrix} 3/s^2+2s+1 & -2/s^2+2s+1 \end{bmatrix} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \tag{5}$$

$$= \begin{bmatrix} 3/s^2+2s+1 & -2/s^2+2s+1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \tag{6}$$

$$= \frac{s^2+4}{s^2+2s+1} \tag{7}$$

$$H(S) = T.F \tag{8}$$

$$H(S) = \frac{s^2+4}{s^2+2s+1} \tag{9}$$

Substituting $s = j\omega$ in equation (9).

$$H(j\omega) = \frac{4 - (\omega)^2}{1 + 2j\omega - (\omega)^2} \tag{10}$$

Steady state output of system is zero.

$$4 - (\omega)^2 = 0 \tag{11}$$

$$\omega = 2rad/sec \tag{12}$$