signals and systems

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Question:
Consider the state-space description of an LTI system with matrices

$$\text{A} = \begin{pmatrix} \begin{pmatrix}0\end{pmatrix} & \begin{pmatrix}1\end{pmatrix} \\ \begin{pmatrix}-1\end{pmatrix} & \begin{pmatrix}-2\end{pmatrix} \end{pmatrix}, \quad \text{B} = \begin{pmatrix} \begin{pmatrix}0\end{pmatrix} \\ \begin{pmatrix}1\end{pmatrix} \end{pmatrix}, \quad \text{C} = \begin{pmatrix} \begin{pmatrix}3\end{pmatrix} & \begin{pmatrix}-2\end{pmatrix} \end{pmatrix}, \quad \text{D} = \begin{pmatrix} \begin{pmatrix}1\end{pmatrix} \end{pmatrix}$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is _____ (Round off to the nearest integer).
solution:

TABLE I
INPUT PARAMETERS

Parameter	Value
System Matrix, A	$\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$
Input Matrix, B	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
Output Matrix, C	$\begin{pmatrix} 3 & -2 \end{pmatrix}$
Feedthrough Matrix, D	1
Input Signal, $u(t)$	$\sin(\omega t)$, $\omega > 0$

$$\text{A} = \begin{pmatrix} \begin{pmatrix}0\end{pmatrix} & \begin{pmatrix}1\end{pmatrix} \\ \begin{pmatrix}-1\end{pmatrix} & \begin{pmatrix}-2\end{pmatrix} \end{pmatrix} \text{B} = \begin{pmatrix} \begin{pmatrix}0\end{pmatrix} \\ \begin{pmatrix}1\end{pmatrix} \end{pmatrix} \text{C=} \begin{pmatrix} \begin{pmatrix}3\end{pmatrix} & \begin{pmatrix}-2\end{pmatrix} \end{pmatrix} \text{D=} \begin{pmatrix} \begin{pmatrix}1\end{pmatrix} \end{pmatrix}$$

transfer function given by

$$T.F = C(sI - A)^{-1}B + D \tag{1}$$

$$\left(\left(sI-A\right)\right)=\begin{pmatrix}\left(s\right) & \left(-1\right)\\ \left(1\right) & \left(s+2\right)\end{pmatrix}\left(\left(sI-A\right)\right)^{-1}=\begin{pmatrix}s & \left(-1\right)\\ \left(1\right) & \left(s+2\right)\end{pmatrix}^{-1} \tag{2}$$

$$\left(\left(sI-A\right)\right)^{-1}=\frac{1}{s(s+2)+1}\begin{pmatrix}\left(s+2\right) & \left(1\right)\\ \left(-1\right) & \left(s\right)\end{pmatrix}\begin{pmatrix}\left(0\right)\\ \left(1\right)\end{pmatrix} \tag{3}$$

from equation 3

$$T.F=\left(\left(3/s^2+2s+1\right) \quad \left(-2/s^2+2s+1\right)\right)\begin{pmatrix}\left(s+2\right) & \left(1\right)\\ \left(-1\right) & \left(s\right)\end{pmatrix}\begin{pmatrix}\left(0\right)\\ \left(1\right)\end{pmatrix}+1 \tag{4}$$

solving equation 4 results

$$T.F=\left(\left(\left(\right)3/s^2+2s+1\right) \quad \left(\left(\right)-2/s^2+2s+1\right)\right)\begin{pmatrix}\left(0\right)\\ \left(1\right)\end{pmatrix}+1 \tag{5}$$

$$T.F=s^2+4*(1/s^2+2s+1) \tag{6}$$

$$H(S)=T.F=s^2+4*(\frac{1}{s^2+2s+1}) \tag{7}$$

s=jω

substitute s=jω

$$H(j\omega)=4-(\omega)^2*(\frac{1}{1+2j\omega-(\omega)^2}) \tag{8}$$

Steady state output of system is zero

$$4 - (\omega)^2 = 0 \tag{9}$$

from equation 9

$\omega = 2rad/sec$