Signals and Systems - Gate2023-EE-Q46

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Question Consider the state-space description of an LTI system with matrices

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 3 & -2 \end{pmatrix}, D = \begin{pmatrix} 1 \end{pmatrix}.$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is _____ (Round off to the nearest integer).
(GATE EE 2023)

Solution:

The state-space representation of the system is given by:

Parameter	Value
System Matrix, A	$\begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix}$
Input Matrix, B	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
Output Matrix, C	$\begin{pmatrix} 3 & -2 \end{pmatrix}$
Feedthrough Matrix, D	$\begin{pmatrix} 1 \end{pmatrix}$
Input Signal, $u(t)$	$\sin(\omega t)$, $\omega > 0$

TABLE I
INPUT PARAMETERS

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$
$$y(t) = Cx(t) + Du(t) \tag{2}$$

Transfer function given by:

$$T.F = C\left(sI - A\right)^{-1} B + D \tag{3}$$
$$\left(sI - A\right) = \begin{pmatrix} s & -1 \\ 1 & s+2 \end{pmatrix} \tag{4}$$
$$\left(sI - A\right)^{-1} = \frac{1}{s(s+2)+1} \begin{pmatrix} s+2 & 1 \\ -1 & s \end{pmatrix} \tag{5}$$

Referencing from equation (5), equation (3) becomes

$$T.F = \begin{pmatrix} \frac{3}{s^2+2s+1} & \frac{-2}{s^2+2s+1} \end{pmatrix} \begin{pmatrix} s+2 & 1 \\ -1 & s \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 1 \tag{6}$$
$$= \begin{pmatrix} \frac{3}{s^2+2s+1} & \frac{-2}{s^2+2s+1} \end{pmatrix} \begin{pmatrix} 1 \\ s \end{pmatrix} + 1 \tag{7}$$
$$= \frac{s^2+4}{s^2+2s+1} \tag{8}$$
$$H(s) = T.F \tag{9}$$
$$H(s) = \frac{s^2+4}{s^2+2s+1} \tag{10}$$

Substituting $s = j\omega$ in equation (10),

$$H(j\omega) = \frac{4 - (\omega)^2}{1 + 2j\omega - (\omega)^2} \tag{11}$$

Steady state output of system is zero:

$$4 - (\omega)^2 = 0 \tag{12}$$

$$\omega = 2 \text{ rad/sec} \tag{13}$$