signals and systems

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Consider the state-space description of an LTI system with matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & -2 \end{bmatrix}, \quad D = 1$$

For the input, $\sin(\omega t)$, $\omega > 0$, the value of ω for which the steady-state output of the system will be zero, is _____ (Round off to the nearest integer).

solution:

$$\text{A} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \text{ B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ C= } \begin{bmatrix} 3 & -2 \end{bmatrix} \text{ D= } \begin{bmatrix} 1 \end{bmatrix}$$

transfer function given by T.F = C(sI - A)$^{-1}B + D$

$$\begin{bmatrix} sI - A \end{bmatrix} = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \quad \begin{bmatrix} sI - A \end{bmatrix}^{-1} = \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} sI - A \end{bmatrix}^{-1} = \frac{1}{s(s+2)+1} \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\text{T.F}=\begin{bmatrix}3/s^2+2s+1 & -2/s^2+2s+1\end{bmatrix}\begin{bmatrix}s+2 & 1\\ -1 & s\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}+1$$

$$\text{T.F}=\begin{bmatrix}3/s^2+2s+1 & -2/s^2+2s+1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}+1$$

$$\text{T.F}=\text{s}^2+4*1/s^2+2s+1$$

$$\text{H(S)=T.F=s}^2+4*(\tfrac{1}{s^2+2s+1})$$

$$\text{s=j}\omega$$

$$\text{H(j}\omega)=4-(\omega)^2*(\tfrac{1}{1+2j\omega-(\omega)^2})$$

Steady state output of system is zero

$$4\text{-}(\omega)^2=0$$

$$\omega=2rad/sec$$