signals and systems

Gate2021-ec-Q52

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Question:

A message signal having peak-to-peak value of 2 V, root mean square value of 0.1 V, and bandwidth of 5 kHz is sampled and fed to a pulse code modulation (PCM) system that uses a uniform quantizer. The PCM output is transmitted over a channel that can support a maximum transmission rate of 50 kbps. Assuming that the quantization error is uniformly distributed, calculate the maximum signal-to-quantization noise ratio (rounded off to two decimal places).

solution:

Symbols and Descriptions

The given problem requires finding the maximum signal-to-quantization noise ratio (SQNR) for a PCM system with uniform quantization. We'll follow the steps to calculate it.

term	Description
Peak-to-Peak	Peak-to-peak value of the message signal (in volts).
RMS	Root mean square value of the message signal (in volts).
Bandwidth	Bandwidth of the message signal (in kilohertz).
N	Number of bits per sample used in quantization.
Quantization range	Range of values in which the quantized signal falls (in volts).
Step size	The size of each quantization interval (in volts).
Sampling rate	Rate at which the message signal is sampled (in kilohertz).
Maximum transmission rate	Maximum transmission rate supported by the channel (in kilobits per second).
SQNR	Signal-to-quantization noise ratio.

TABLE I
Input Parameters

1) **Calculate Peak and RMS values:**

$$\text{RMS} = \frac{\text{Peak-to-Peak}}{2\sqrt{2}} \tag{1}$$

2) **Calculate Peak Value:**

$$\text{Peak} = \sqrt{2} \times \text{RMS} \tag{2}$$

3) **Calculate Quantization Range:**

$$\text{Quantization range} = 2 \times \text{Peak} \tag{3}$$

4) **Determine Number of Bits (N):**

$$N = \log_2(\text{quantization levels}) \tag{4}$$

5) **Calculate Step Size:**

$$\text{Step size} = \frac{\text{Quantization range}}{2^N} \tag{5}$$

6) **Calculate SQNR:**

$$\text{SQNR} = 6.02 \times N + 1.76 \tag{6}$$

Now, let's substitute the values and calculate SQNR:

$$\text{RMS} = \frac{2}{2\sqrt{2}} = 0.707\,\text{V}$$
$$\text{Peak} = \sqrt{2} \times 0.707 = 1.414\,\text{V}$$
$$\text{Quantization range} = 2 \times 1.414 = 2.828\,\text{V}$$
$$\text{Sampling rate} = 10\,\text{kHz}$$
$$\text{Maximum transmission rate} = 50\,\text{kbps}$$
$$N \leq 5$$
$$\text{SQNR} = 6.02 \times 5 + 1.76$$
$$\text{SQNR} = 30.1 + 1.76 = 31.86$$

$\implies$ SQNR = 31.86

So, the maximum signal to quantization noise ratio that can be obtained by the PCM system is approximately 31.86, rounded off to two decimal places.