Recitation Covering

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Problem

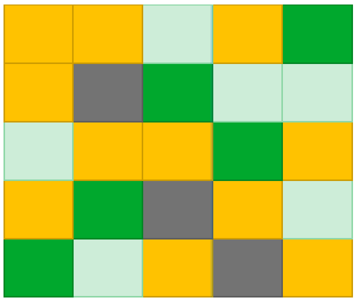

Figure 1: Yellow is flat, deep green is forest, light green is hill, black is the cave.

The initialization status is:

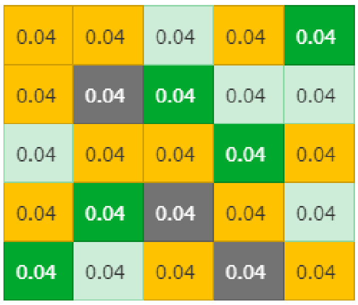

Figure 2: Belief state initialization.

Assume we check this cell, (3, 3), and find nothing. Then how will the probability change?

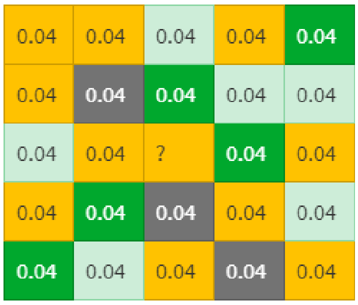

Figure 3: The problem.

Solution

Before we start, we want to first give a description of the events. Define $X_{i,j}$ as the existence of target at (i, j). Define $Y_{i,j}$ as the search result of this time at (i, j).

The problem of this problem becomes: You need to know $P(X_{i,j}|Y_{3,3} = 0)$. Then there are two different cases here. First $i = 3, j = 3$. Second the other cases. We need to discuss them separately.

(a). $P(X_{3,3}|Y_{3,3} = 0)$

For $P(X_{3,3}|Y_{3,3} = 0)$, we can use Bayesian method to compute it:

$$P(X_{3,3} = 1|Y_{3,3} = 0) = \frac{P(Y_{3,3} = 0|X_{3,3} = 1)P(X_{3,3} = 1)}{P(Y_{3,3} = 0)} \tag{0.1}$$

What is $P(Y_{3,3} = 0|X_{3,3} = 1)$? This is called likelihood. It is defined in the text. Since the terrain is flat. We know $P(Y_{3,3} = 0|X_{3,3} = 1) = 0.1$.

What is $P(X_{3,3} = 1)$? This is called prior probability. We defined them during initialization. $P(X_{3,3} = 1) = 0.04$.

The final problem is what is $P(Y_{3,3} = 0)$? We can compute it from the marginalization.

$$P(Y_{3,3} = 0) = P(Y_{3,3} = 0|X_{3,3} = 0)P(X_{3,3} = 0) + P(Y_{3,3} = 0|X_{3,3} = 1)P(X_{3,3} = 1) \tag{0.2}$$

$P(Y_{3,3} = 0|X_{3,3} = 0) = 1$, $P(X_{3,3} = 0) = 0.96$, $P(Y_{3,3} = 0|X_{3,3} = 1) = 0.1$, $P(X_{3,3} = 1) = 0.04$. Thus, $P(Y_{3,3} = 0) = 0.964$.

Finally, we get:

$$P(X_{3,3} = 1|Y_{3,3} = 0) = \frac{0.004}{0.964} = 0.004149, P(X_{3,3} = 0|Y_{3,3} = 0) = 0.99585 \tag{0.3}$$

(b). $P(X_{i,j}|Y_{3,3} = 0), i, j \neq (3, 3)$

Take $(i, j) = (1, 1)$ for example. Again, we can use Bayesian formula:

$$P(X_{1,1} = 1|Y_{3,3} = 0) = \frac{P(Y_{3,3} = 0|X_{1,1} = 1)P(X_{1,1} = 1)}{P(Y_{3,3} = 0)} \tag{0.4}$$

We know $P(X_{1,1} = 1)$. It is prior probability, right? And we know $P(Y_{3,3} = 0)$, we computed it from marginalization. But what is $P(Y_{3,3} = 0|X_{1,1} = 1)$? Yeah, it is a likelihood. But we never defined it.

The truth here is although we never define it, we know how to compute it. The meaning of $P(Y_{3,3} = 0|X_{1,1} = 1)$ is if the target is in $(1,1)$, what is the possibility of checking $(3,3)$ and finding nothing? It is 100%. $P(Y_{3,3} = 0|X_{1,1} = 1) = 1$. Thus we have

$$P(X_{1,1} = 1|Y_{3,3} = 0) = \frac{0.04}{0.964} = 0.04149, P(X_{1,1} = 0|Y_{3,3} = 0) = 0.95851 \tag{0.5}$$

As for (i, j) equals to others. You can treat them using the same way.

This is a tiny example for how to compute the posterior distribution. In the next step, you posterior distribution of this step will be your prior distribution for next step.