Nonlinear Observer Design for ECG signals

VAISHNAVI C K

1 Introduction

The Electrocardiogram (ECG) signal represents the electrical activity of the heart and can be modeled using a non-linear dynamical system. ECG signals are complex and non linear in nature, making the use of non linear observers for estimating the state variables beneficial for their analysis. Some of the commonly used non linear observers for ECG signals are Extended Kalman Filter and Sliding mode observer.

The estimation of state variables with the help of these non linear observers can provide valuable information about the underlying physiological processes of the heart and can be used for diagnosis and treatment of cardiovascular diseases. Moreover, ECG signals are subject to various types of noise and artifacts, which can affect the accuracy of the signal analysis. Non linear observers can handle these uncertainties and disturbances better than other methods, providing more accurate estimates of the state variables.

An important state variable that is commonly used to describe the current state of the heart is "phase". Fig1 depicts the phase of an ECG signal. By estimating the phase angle of the heart rate signal, it is possible to determine the timing of specific events in the cardiac cycle. This can be useful for a range of applications, including monitoring heart rate variability, assessing the effects of interventions on heart rate and detecting abnormal heart rhythms.

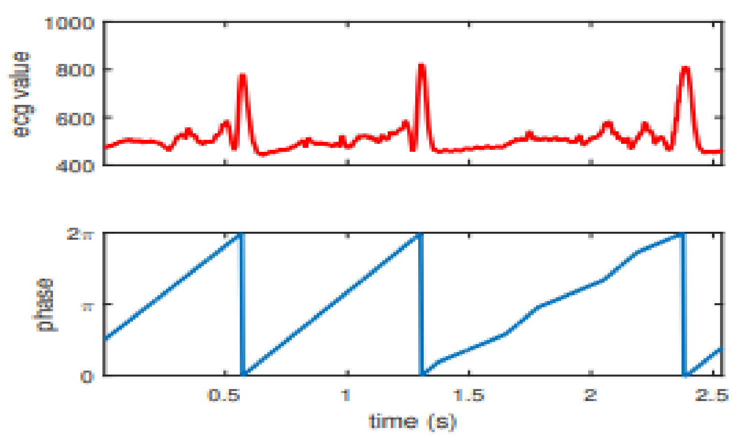

Figure 1: ECG signal phase plot.

2 Analysis and Simulations

2.1 Phase estimation of ECG signal using Extended Kalman Filter as the Non linear observer

The Extended Kalman Filter (EKF) is designed to handle non-linear systems. It is an algorithm that estimates the state of a system with uncertain or incomplete observations by recursively updating a probability distribution over the state of the system.

General state-space model for an EKF can be expressed in terms of a state function f, which is differentiable and the measurement function h as follows,

$$x_{k+1} = f(x_k, u_k) + w_k \tag{1}$$

$$z_k = h(x_k) + v_k \tag{2}$$

Here w_k and v_k are the respective state and measurement noise variables. EKF estimates the posterior of the state x_k through a prediction and update step which are each based on a Taylor-series linearization of the nonlinear state and measurement functions. Since this is a non-linear model, EKF relies on linearization, F_k and H_k are set as the Jacobian matrices of the state and measurement function, evaluated at the most recently estimated state value.

Prediction step formula in EKF is as follows,

$$x'_{k|k-1} = f(x'_{k-1|k-1}, u_k) \tag{3}$$

And the Updation step formula in EKF is given as,

$$x'_{k|k} = x'_{k|k-1} + K_k(z_k - h(x'_{k|k-1})) \tag{4}$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} \tag{5}$$

Here K_k is the optimal kalman gain matrix, $x'_{k|k-1}$ is the predicted state estimate with covariance $P_{k|k-1}$, and $x'_{k|k}$ is the updated state estimate with covariance $P_{k|k}$

The observed ECG signal is modelled as double beat sinusoid with non linear phase. This is a simple form of the state dynamics based on the instantaneous frequency. Considering this signal along with its derivative, the raw value of the phase at a discrete set of observed time points is to be estimated.

The state variable at time t_k is defined as follows, (note: $\phi'(t) = \omega(t)$)

$$\phi(t) = \phi(t_k) + (t - t_k)\omega(t_k) + \frac{(t - t_k)^2}{2}\omega'(t_k) + ... \tag{6}$$

Since here the signal is nearly periodic, it can be assumed that the angular frequency $\omega(t)$ is approximately constant for time difference much smaller than the time period of the signal, thereby allowing the higher order terms to be neglected.

Thus the state model is as follows,

$$x_{k+1} = f(x_k) + w_k = F_k x_k + w_k = \phi_k + \Delta t_k \omega_k + \omega_k + constant \tag{7}$$

Here F_k is the jacobian matrix. In order to avoid estimating frequencies which would be subject to aliasing, the constraint that angular frequency $\omega(t)$ should be less than nyquist frequency $\frac{f_s}{2}$ for sampling rate $f_s = \frac{1}{\Delta t_k}$ is imposed.

2.2 Simulation Results

The simulation of the phase estimation of ECG (modelled as double beat sinusoidal signal) using extended kalman filter has been implemented in MATLAB.

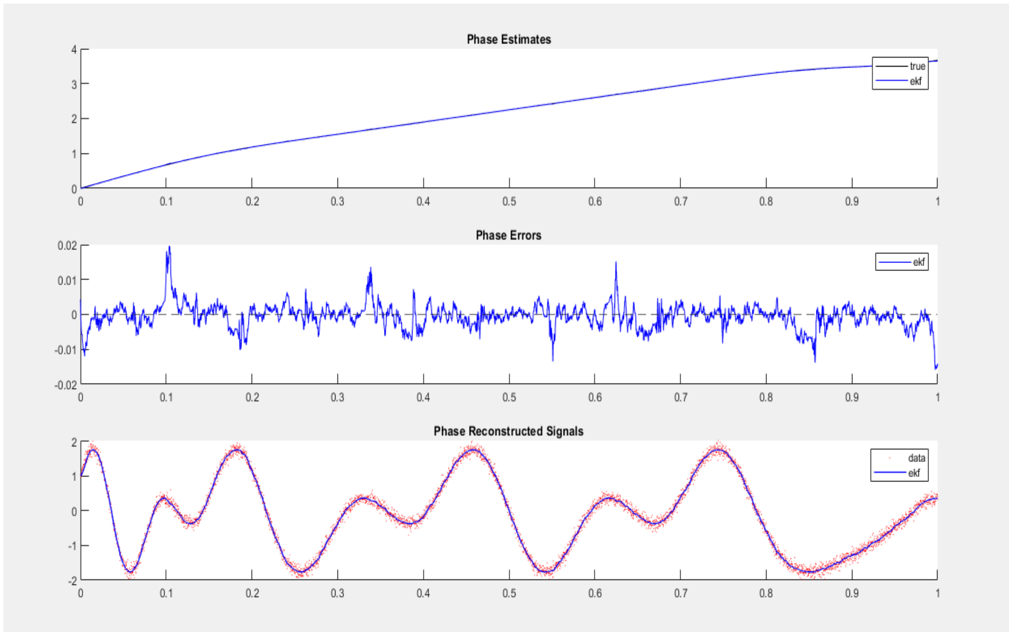

Figure 2: Phase estimation of ECG signal using extended kalman filter.

From Fig 2, it can be seen that the error between the actual phase and estimated phase using extended kalman filter is very minimal (in the range: [-0.02, 0.02]).

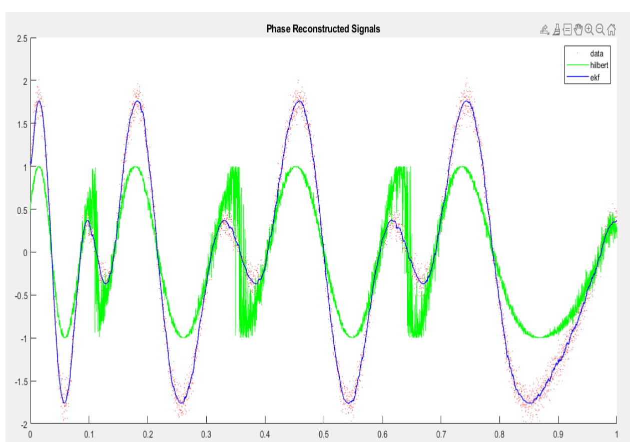

Figure 3: Comparing extended kalman filter and hilbert transform approach

Using Hilbert transform approach for the same, and comparing it with extended kalman filter as depicted in Fig 3, it can be seen that the error generated is more than extended kalman filter approach, and the reconstructed phase using hilbert transform deviates too much from the true phase of the signal.

Mean squared error obtained using Extended Kalman filter approach is 1.1931e-05, and that of Hilbert transform approach is 5.1005. This goes to show that the Extended Kalman filter as the non linear observer for phase estimation of ECG signals is more reliable and robust.

3 Conclusion

From the literature survey and simulations done, its seen that the use of Extended Kalman filter (EKF) as the non linear observer for the phase estimation of ECG signals is more efficient and robust when compared to standard approaches such as hilbert transform. EKF method is more profound when there is uncertainty about amplitude variation.

EKF method also comes with a lot of limitations. Since EKF approximates systems which are nonlinear in nature using linearization, it cannot be generalized across wide ranges of non linear systems. To overcome this, additional fine tuninhg of parameters will be required (Machine learinng can be used), and more higher order terms will need to considered.

4 References

[1] Holt, N. (2019). Parameter Estimation of a Cardiac Model Using the Local Ensemble Transform Kalman Filter.

[2] Sameni, Reza and Shamsollahi, Mohammad and Jutten, Christian. (2005). Filtering Electrocardiogram Signals Using the Extended Kalman Filter. Conference proceedings : ... Annual International Conference of the IEEE Engineering in Medicine and Biology Society. IEEE Engineering in Medicine and Biology Society. Conference. 6. 5639-42. 10.1109/IEMBS.2005.1615765.

[3] Li Q, Mark RG, Clifford GD. Robust heart rate estimation from multiple asynchronous noisy sources using signal quality indices and a Kalman filter. Physiol Meas. 2008 Jan;29(1):15-32. doi: 10.1088/0967-3334/29/1/002. Epub 2007 Dec 10. PMID: 18175857; PMCID: PMC2259026.

[4] Kurz, Gerhard and Hanebeck, Uwe. (2015). Heart Phase Estimation Using Directional Statistics for Robotic Beating Heart Surgery.

[5] Ting, Chee-Ming and Salleh, Shussain. (2010). ECG based personal identification using extended Kalman filter. 774 - 777. 10.1109/ISSPA.2010.5605516.