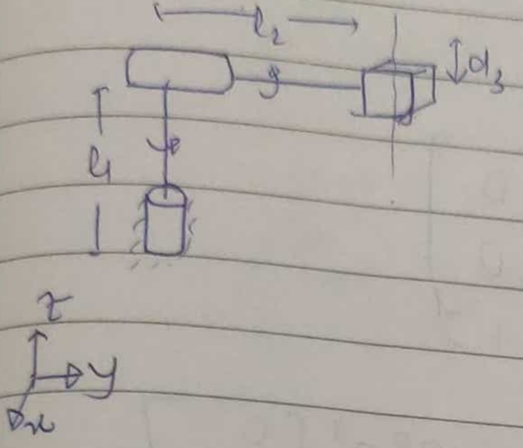
Assignment 5

Task 2 : Stanford Manipulator



$$\begin{bmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1/2 \end{bmatrix}$$

$$\begin{bmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} \end{bmatrix} = \begin{bmatrix} l_2/2 \sin q_1 \\ l_2/2 \cos q_1 \\ l_1 \end{bmatrix}$$

$$\begin{bmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} \end{bmatrix} = \begin{bmatrix} l_2 \sin q_1 + d_3/2 \sin q_2 \\ l_2 \cos q_1 \\ l_1 + d_3/2 \cos q_2 \end{bmatrix}$$

So the J_vs are ,

$$J_{v_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad J_{v_2} = \begin{bmatrix} l_2/2 \cos q_1 & 0 & 0 \\ -l_2/2 \sin q_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J_{v_3} = \begin{bmatrix} l_2 \cos q_1 & d_3/2 \cos q_2 & (\sin q_2)/2 \\ -l_2 \sin q_1 & 0 & 0 \\ 0 & -d_3/2 \sin q_2 & (\cos q_2)/2 \end{bmatrix}$$

The angular velocity vectors are:

$$\vec{\omega}_1 = \dot{q}_1 \hat{k} \quad , \quad \vec{\omega}_2 = \dot{q}_1 \hat{k} + \dot{q}_2 \hat{j}$$

$$D(q) = m_1 J_{v_1}^T J_{v_1} + m_2 J_{v_2}^T J_{v_2} + m_3 J_{v_3}^T J_{v_3} + \begin{bmatrix} I_2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D(q) = \begin{bmatrix} m_2 l_2^2/4\, c_{q_1}^2 + m_3 [l_2^2 c_{q_1}^2 + d_3^2/4\, c_{q_2}^2 + s_{q_2}^2/4] + I_2 & -m_2 l_2^2/4\, c_{q_1} s_{q_1} + m_3 [-l_2^2 c_{q_1} s_{q_1}] & m_3 [-d_3^2/4\, c_{q_2} s_{q_2} + c_{q_2} s_{q_2}/4] \\ -m_2 l_2^2/4\, c_{q_1} s_{q_1} - m_3 l_2^2 s_{q_1} c_{q_1} & m_2 l_2^2/4\, s_{q_1}^2 + m_3 [l_2^2 s_{q_1}^2] & 0 \\ m_3 [-d_3^2/4\, s_{q_1} c_{q_1} + s_{q_2} c_{q_2}/4] & 0 & m_3 [d_3^2/4\, s_{q_2}^2 + c_{q_2} s_{q_2}/4] \end{bmatrix}$$

$$\phi_1 = m_1 \frac{l_1}{2} g$$

$$\phi_2 = m_2 l_1 g$$

$$\phi_3 = m_3 (l_1 + d_3 \cos q_2)$$

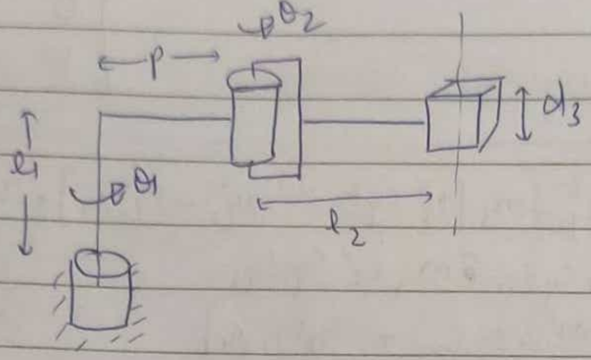
Now, the dynamic equations:-

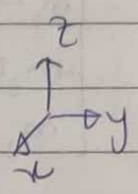
$$\tau_1 = [(d_{11})\ddot{q}_1 + d_{12}\ddot{q}_2 + d_{13}\ddot{d}_3] + [c_{111}\dot{q}_1\dot{q}_1 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2\dot{q}_2] + \phi_1 + [c_{311}\dot{q}_1\dot{d}_3 + c_{131}\dot{q}_1\dot{d}_3 + c_{321}\dot{q}_2\dot{d}_3 + c_{231}\dot{q}_2\dot{d}_3]$$

$$\tau_2 = [d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + \overset{0}{\cancel{d_{23}}}\ddot{d}_3] + [c_{112}\dot{q}_1\dot{q}_2 + c_{122}\dot{q}_1\dot{q}_2 + c_{212}\dot{q}_2\dot{q}_1 + c_{222}\dot{q}_2\dot{q}_2 + c_{312}\dot{q}_1\dot{d}_3 + c_{132}\dot{q}_1\dot{d}_3 + c_{322}\dot{q}_2\dot{d}_3 + c_{232}\dot{d}_3\dot{q}_2 + c_{332}\dot{d}_3\dot{d}_3] + \overset{0}{\phi_2}$$

$$\tau_3 = [d_{31}\ddot{q}_1 + \overset{0}{\cancel{d_{32}}}\ddot{q}_2 + d_{33}\ddot{d}_3] + [c_{113}\dot{q}_1\dot{q}_2 + c_{123}\dot{q}_1\dot{q}_2 + c_{213}\dot{q}_2\dot{q}_1 + c_{223}\dot{q}_2\dot{q}_2 + c_{313}\dot{q}_1\dot{d}_3 + c_{133}\dot{q}_1\dot{d}_3 + c_{323}\dot{q}_2\dot{d}_3 + c_{233}\dot{d}_3\dot{q}_2 + c_{333}\dot{d}_3\dot{d}_3] + \phi_3$$

Task 3: SCARA





$$\begin{bmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} \end{bmatrix} = \begin{bmatrix} -S_1 \, p/2 \\ p/2 \, C_1 \\ l_1/2 \end{bmatrix}$$

$$\begin{bmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} \end{bmatrix} = \begin{bmatrix} -pS_1 - S_2 \, l_2/2 \\ pC_1 + l_2/2 \, C_2 \\ l_1 \end{bmatrix}$$

$$\begin{bmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} \end{bmatrix} = \begin{bmatrix} -pS_1 - S_2 l_2 \\ pC_1 + l_2 C_2 \\ l_1 + d_3 \end{bmatrix}$$

So, the velocity Jacobians are:-

$$Jv_{c_1} = \begin{bmatrix} -p/2 \, C_1 & 0 & 0 \\ -p/2 \, S_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \quad Jv_{c_2} = \begin{bmatrix} -pC_1 & -l_2/2 \, C_2 & 0 \\ -pS_1 & -l_2/2 \, S_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Jv_{c_3} = \begin{bmatrix} -pC_1 & -l_2 C_2 & 0 \\ -pS_1 & -l_2 S_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$\omega_1 = \dot{q}_1 \hat{k} \qquad \& \qquad \omega_2 = (\dot{q}_2 + \dot{q}_1)\hat{k}$$

$$D(q) = m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} + m_3 J_{v_{c_3}}^T J_{v_{c_3}} + \begin{bmatrix} I_1+I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

column 1 ↓

$$D(q) = \begin{bmatrix} m_1 p^2/4\, c_1^2 + m_2\left[p^2 c_1^2 + l_2^2 c_2^2/4\right] + m_3\left[p^2 c_1^2 + l_2^2 c_2 s_2\right] + I_1 + I_2 \\ m_1 p^2/4\, c_1 s_1 + m_2\left[p^2 s_1 c_1 + l_2^2/4\, c_1 s_1\right] + m_3\left[p^2 s_1 c_1 + l_2^2 c_2 s_2\right] + I_2 \\ 0 \end{bmatrix}$$

column 2 ↓   column 3 ↓

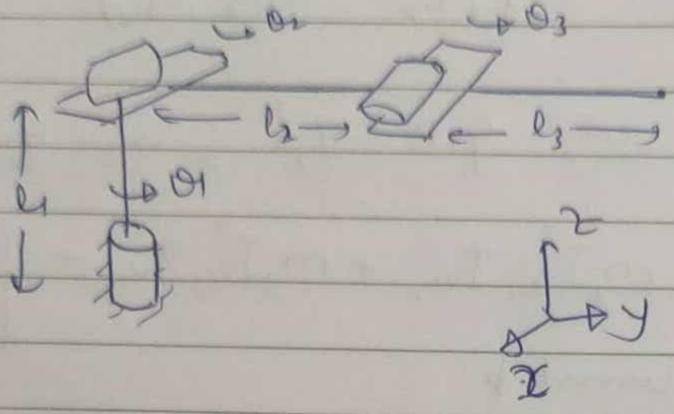
$$\begin{bmatrix} m_1 p^2/4\, c_1 s_1 + m_2\left[p^2 c_1 s_1 + l_2^2/4\, s_2 c_2\right] + m_3\left[p^2 c_1 s_1 + l_2^2 c_2 s_2\right] + I_2 & 0 \\ m_1 p^2/4\, s_1^2 + m_2\left[p^2 s_1^2 + l_2^2/4\, s_2^2\right] + m_3\left[p^2 s_1^2 + l_2^2 s_2^2\right] + I_2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\phi_1 = m_1 \frac{l_1}{2} g$$

$$\phi_2 = m_2 l_1 g$$

$$\phi_3 = m_3 (l_1 + d_3) g$$

Task 4: PUMA



$$\begin{bmatrix} x_{c_1} \\ y_{c_1} \\ z_{c_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l_1/2 \end{bmatrix}$$

$$\begin{bmatrix} x_{c_2} \\ y_{c_2} \\ z_{c_2} \end{bmatrix} = \begin{bmatrix} -l_2/2\, S_1 \\ l_2/2\, C_1 + l_2/2\, C_2 \\ l_1 + l_2/2\, S_2 \end{bmatrix}$$

$$\begin{bmatrix} x_{c_3} \\ y_{c_3} \\ z_{c_3} \end{bmatrix} = \begin{bmatrix} -l_2 S_1 \\ l_2 C_1 + l_2 C_2 + l_3/2\, C_3 \\ l_1 + l_2 S_2 + l_3/2\, S_3 \end{bmatrix}$$

Now,

$$Jv_{c_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Jv_{c_2} = \begin{bmatrix} -l_2/2\, C_1 & 0 & 0 \\ -l_2/2\, S_1 & -l_2/2\, S_2 & 0 \\ 0 & l_2/2\, C_2 & 0 \end{bmatrix}$$

$$J_{v_{c_3}} = \begin{bmatrix} -l_2 c_1 & 0 & 0 \\ -l_2 s_1 & -l_2 s_2 & -l_3/2\, s_3 \\ 0 & l_2 c_2 & l_3/2\, c_3 \end{bmatrix}$$

$\omega_1 = \dot{q}_1 \hat{k}$, $\omega_2 = \dot{q}_1 \hat{k} + \dot{q}_2 \hat{i}$; $\omega_3 = \dot{q}_1 \hat{k} + (\dot{q}_2 + \dot{q}_3)\hat{i}$

$$D(q) = m_1 J_{v_{c_1}}^T J_{v_{c_1}} + m_2 J_{v_{c_2}}^T J_{v_{c_2}} + m_3 J_{v_{c_3}}^T J_{v_{c_3}} + \begin{bmatrix} I_2 + I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

column 1 ↓

$$D(q) = \begin{bmatrix} m_2 l_2^2/4\, c_1^2 + m_3 l_2^2 c_1^2 + I_2 + I_3 \\ m_2 l_2^2/4\, s_1 c_1 + m_3 l_2^2 s_1 c_1 \\ 0 \end{bmatrix}$$

column 2 ↓

$m_2 l_2^2/4\, s_1 c_1 + m_3 l_2^2 c_1 s_1$

$m_2 [l_2^2/4\, s_1^2 + l_2^2/4\, s_2^2] + m_3 [l_2^2 s_1^2 + l_2^2 s_2^2 + l_3^2/4\, s_3^2]$

$-m_2 l_2^2/4\, s_1 c_2 + m_3 [-l_2^2 s_2 c_2 - l_3^2/4\, s_3 c_3]$

column 3 ↓

0

$-m_2 l_2^2/4\, s_2 c_2 + m_3 [-l_2^2 s_2 c_2 - l_3^2/4\, s_3 c_3]$

$m_2 l_2^2/4\, c_2^2 + m_3 [l_2^2 c_2^2 + l_3^2/4\, c_3^2]$

$$\phi_1 = m_1 \frac{l_1}{2} g$$

$$\phi_2 = m_2 \left[l_1 + \frac{l_2}{2} \sin(\theta_2) \right] g$$

$$\phi_3 = m_3 \left[l_1 + l_2 \sin(\theta_2) + \frac{l_3}{2} \sin(\theta_3) \right] g$$