Part 1:

1) **Bottom up version:**

```
int** bottomupSW(char* X, char* Y, int n, int m, int** H, int** P)
{
   int i, j;
   int p1, p2, p3;

   for(i=0; i<n; i++)
   {
      H[i][0] = P[i][0] = 0;
   }
   for(j=0; j<m; j++)
   {
      H[0][j] = P[0][j] = 0;
   }
   for(i=0; i<n; i++)
   {
      for(j=0; j<m; j++)
      {
         if(X[i] == Y[j])
            p1 = H[i][j] + 2;
            else
               p1 = H[i][j] -1;
         p2 = H[i][j+1] -1;
         p3 = H[i+1][j] -1;

         //H[i+1][j+1] = max(p1,p2,p3);

         if(p1>=p2 && p1>=p3)
                              H[i][j]=p1;
                        else if(p2>=p1 && p2>=p3)
                              H[i][j]=p2;
                        else if(p3>=p1 && p3>=p2)
                              H[i][j]=p3;

         if(H[i+1][j+1] == p1)
            P[i+1][j+1] = '@';
            else
            {
               if(H[i+1][j+1] == p2)
                  P[i+1][j+1] = '|';
                  else
                  {
                     P[i+1][j+1] = '#';
                  }
            }
```

```
        }

    }
    return H;
}
```

2) **Top down with memorization:**

```
int topdownSW(char* X, char* Y, int m, int n, int** ops)
{
    if (m == 0 || n == 0)
        return 0;

    if (ops[m-1][n-1] != INT_MIN)
        return ops[m-1][n-1];

    if (X[m-1] == Y[n-1]) {

        ops[m-1][n-1] = 2 + topdownSW(X, Y, m - 1, n - 1,ops);
        return ops[m-1][n-1];
    }
    else {
       int t = max((topdownSW(X, Y, m, n - 1,ops)-1),
                        (topdownSW(X, Y, m - 1, n,ops)-1));
       ops[m-1][n-1]=max(t,topdownSW(X, Y, m - 1, n - 1,ops)-1);
        return ops[m-1][n-1];
    }
}
```

3) **Print-Seq-Align-X and Print-Seq-Align-Y:**

```
void printSeqAlignX(char* X,int** P,int n,int m)
{
    if(P[n][m] == '@')
    {
    printSeqAlignX(X,P,n-1,m-1);
    cout << X[n-1];
    }
        else
        {
            if (P[n][m] == '#')
            {
                printSeqAlignX(X,P,n,m-1);
                cout << "-";
            }
                else
                {
                    printSeqAlignX(X,P,n-1,m);
```

```
            cout << X[n-1];
          }
       }
}

void printSeqAlignY(char* Y,int** P,int n,int m)
{
   if(P[n][m] == '@')
   {
   printSeqAlignY(Y,P,n-1,m-1);
   cout << Y[n];
   }
      else
      {
         if (P[n][m] == '#')
         {
         printSeqAlignY(Y,P,n,m-1);
         cout << "-";
         }
            else
            {
               printSeqAlignY(Y,P,n-1,m);
               cout << Y[n];
            }
      }
}
```

4) Find the maximum alignment for **X=dcdcbacbbb and Y=acdccabdbb** by using Smith-Waterman algorithm. Execute the pseudocode algorithm and fill the necessary tables H and P in a bottom-up fashion. Reconstruct the strings X' and Y' using the tables H and P.

X/Y		A	C	D	C	C	A	B	D	B	B
	0	0	0	0	0	0	0	0	0	0	0
D	0	-1	-1	2	1	0	-1	-1	2	1	0
C	0	-1	1	1	4	3	2	1	1	1	0
D	0	-1	0	3	3	3	2	1	3	2	1
C	0	-1	1	2	5	5	4	3	2	2	1
B	0	-1	0	1	4	4	4	6	5	4	4
A	0	2	1	0	3	3	6	5	5	4	3
C	0	1	4	3	2	5	5	5	4	4	3
B	0	0	3	3	2	4	4	7	6	6	6
B	0	-1	2	2	2	3	3	6	6	8	8
B	0	-1	1	1	1	2	2	5	5	8	10
			C	**D**	**C**	**C**		**B**		**B**	**B**

Solution: <CDCCBBB>.

Part 2:

1) Show, by means of a counter example, that the following “greedy” strategy does not always determine an optimal way to cut rods. Define the **density** of a rod of length *i* to be *pi/i,* that is, its value per inch. The greedy strategy for a rod of length *n* cuts off a first piece of length *i,* where 1 <= I <= n, having maximum density. It then continues by applying the greedy strategy to be remaining piece of length *n-i.*

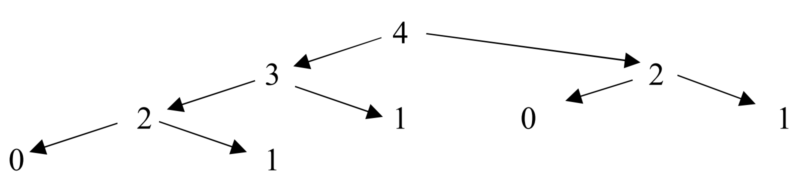
   Counter example for greedy strategy; Let n be 5 which will be the length of the rod.

i → number of cuts	**1**	**2**	**3**	**4**	**5**
pi → price per inch	2	20	33	36	40
pi/i → density	2	10	11	9	8
Solution	5(1) * 2 – 10	2(2) +1 – 42	3(1) + 1+1 - 37 3(1) + 2(1) - 53	4(1)+1-38	5 – 40 4+1 – 38 3+2 – 53 2+2+1 – 42
Optimal Solution					**3+2=53**

According to greedy strategy, without cutting the rod, the total value would be 40. If we cut the rod into two, 4 inch and 1 inch, the value would be 38. If we cut the rod, 3 inch and 2 inches, the value would be 53. If it is cut into 3, 2-inch, 2 inch and 1 inch, value would be 42. **Hence the optimal solution is 53.**

2) The Fibonacci numbers are defined by recurrence(3.22). Give an *O(n)* time dynamic-programming algorithm to compute the n-th Fibonacci number. Draw the subproblem graph. How many vertices and edges are in the graph?

```
Fibonacci(n)
    Let fib(0….n) be a new array
    fib(0) = fib(1) =1
    For i = 2 to n
            fib(i) = fib(i-1) + fib(i-2)
    Return fib(n)
```



The number of vertices in the tree will follow the recurrences. Each number in the sequence is sum of two previous numbers in the sequence.
V(n) = 1+ v(n-2) +v(n-1).
The initial conditions are v(0) = v(1) =1.
The above graph shows that
V(n) = 1+ (2 * fib(n-2) – 1) + (2* fib(n-1)-1) = 2 * fib(n) -1
Thus, subproblem graph consists of n+1 vertex.

The number of edges will satisfy the recurrence;
E(n) = 2 + E(n-1) + E(n-2)
And the base cases are E(0) = E(1) =0.

By induction,
E(n) = 2*fib(n) - 2

Thus, the subproblem graph has 2n – 2 edges.

3) Determine an LCS of (1,0,0,1,0,1,0,1) and (0,1,0,1,1,0,1,1,0).
The selected cell is shaded with grey colour. The LCS is <010101>.

S		**0**	**1**	**0**	**1**	**1**	**0**	**1**	**1**	**0**
	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	**0**	1	1	2	2	2	2	2	2	2
0	0	**1**	1	2	2	2	3	3	3	3
1	0	1	**2**	2	3	3	3	4	4	4
0	0	1	2	**3**	**3**	3	4	4	4	5
1	0	1	2	3	4	**4**	4	5	5	5
0	0	1	2	3	4	4	**5**	**5**	5	6
1	0	1	2	3	4	5	5	6	**6**	**6**
LCS		**0**	**1**	**0**		**1**	**0**		**1**	