CS 105: Department Introductory Course on Discrete Structures

Instructor : S. Akshay

Jul 29, 2025

Lecture 01 – Introduction

Welcome to CSE@IIT Bombay!

Logistics

Course hours: Slot 10;

Tue 14:00-15:25, Thu 14:00-15:25

Office hours: To be announced.

Problem Solving/Help Session (Optional): One hour per week, run by teaching assistants. (Time and Venue to be decided)

Logistics

Course hours: Slot 10;

Tue 14:00-15:25, Thu 14:00-15:25

Office hours: To be announced.

Problem Solving/Help Session (Optional): One hour per week, run by teaching assistants. (Time and Venue to be decided)

Course material, references will be posted at

- `http://www.cse.iitb.ac.in/~akshayss/teaching.html`

Logistics

Course hours: Slot 10;

Tue 14:00-15:25, Thu 14:00-15:25

Office hours: To be announced.

Problem Solving/Help Session (Optional): One hour per week, run by teaching assistants. (Time and Venue to be decided)

Course material, references will be posted at

- http://www.cse.iitb.ac.in/~akshayss/teaching.html
- Announcements, Problem sheets and Online Discussion: Piazza
  - https://piazza.com/iit_bombay/fall2025/cs105

Logistics

Course hours: Slot 10;

Tue 14:00-15:25, Thu 14:00-15:25

Office hours: To be announced.

Problem Solving/Help Session (Optional): One hour per week, run by teaching assistants. (Time and Venue to be decided)

Course material, references will be posted at

- http://www.cse.iitb.ac.in/~akshayss/teaching.html
- Announcements, Problem sheets and Online Discussion: Piazza
  - https://piazza.com/iit_bombay/fall2025/cs105
- One problem sheet will be posted (almost) every week!

Logistics

Course hours: Slot 10;

Tue 14:00-15:25, Thu 14:00-15:25

Office hours: To be announced.

Problem Solving/Help Session (Optional): One hour per week, run by teaching assistants. (Time and Venue to be decided)

Course material, references will be posted at

- `http://www.cse.iitb.ac.in/~akshayss/teaching.html`
- Announcements, Problem sheets and Online Discussion: Piazza
  - `https://piazza.com/iit_bombay/fall2025/cs105`
- One problem sheet will be posted (almost) every week!

Attendance

As per Institute rules: SAFE

More Logistics

Evaluation

- Quizzes: 30%
- Midsem: 25%
- Endsem: 40%
- Other {participation, pop quizzes, assignments}: 5%

More Logistics

Evaluation

- Quizzes: 30%
- Midsem: 25%
- Endsem: 40%
- Other {participation, pop quizzes, assignments}: 5%

Minimum requirements (Tentative)

10/40 in endsem + 15/60 in remaining.

More Logistics

Evaluation

- Quizzes: 30%
- Midsem: 25%
- Endsem: 40%
- Other {participation, pop quizzes, assignments}: 5%

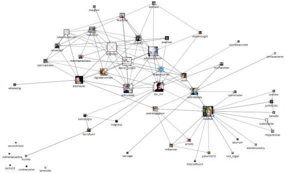
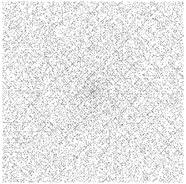
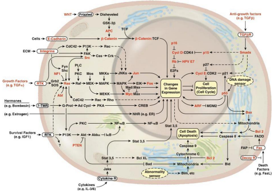
Minimum requirements (Tentative)

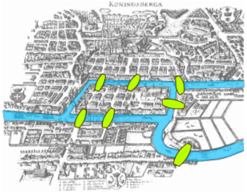
10/40 in endsem + 15/60 in remaining.

How to reach me after class?

- Send a message on piazza
- Drop by my office...
  - CS 507 (5th floor of New CSE/CC building)
  - Temporarily CC 313 (3rd floor!)

Goal




First things first...

- What are discrete structures?
- Why are we interested in them?

Course Outline

What we will broadly cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory
4. If time permits: Selected topics: e.g, abstract algebra and/or number theory

Course Outline

What we will broadly cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory
4. If time permits: Selected topics: e.g, abstract algebra and/or number theory

What we don't cover

1. Logic : predicate, first-order logic– CS228
2. Discrete probability – CS215
3. Algorithms – CS218
4. Data structures – CS213 and CS293
5. Automata theory – CS310
6. Details and applications of everything above – rest of your (academic) life!

Course Outline

What we will cover in this course

1. Mathematical reasoning: proofs and structures
2. Counting and combinatorics
3. Elements of graph theory
4. If time permits, Selected topics e.g., on abstract algebra and/or number theory

Textbooks

- Discrete Mathematics and its Applications with Combinatorics and Graph Theory, by Kenneth H Rosen.
- Discrete Mathematics by Norman Biggs.
- More will be listed on webpage as we go along.

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.
2. (New and old) techniques for problem solving: how to attack problems that you have never seen before.

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.
2. (New and old) techniques for problem solving: how to attack problems that you have never seen before.
3. To convey your ideas clearly: argue logically and write proofs formally.

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.
2. (New and old) techniques for problem solving: how to attack problems that you have never seen before.
3. To convey your ideas clearly: argue logically and write proofs formally.
4. To learn abstractions and abstract reasoning.

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.
2. (New and old) techniques for problem solving: how to attack problems that you have never seen before.
3. To convey your ideas clearly: argue logically and write proofs formally.
4. To learn abstractions and abstract reasoning.

Prerequisities

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.
2. (New and old) techniques for problem solving: how to attack problems that you have never seen before.
3. To convey your ideas clearly: argue logically and write proofs formally.
4. To learn abstractions and abstract reasoning.

Prerequisities

- Nothing!

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.
2. (New and old) techniques for problem solving: how to attack problems that you have never seen before.
3. To convey your ideas clearly: argue logically and write proofs formally.
4. To learn abstractions and abstract reasoning.

Prerequisities

- Nothing! ... Well, high school mathematics

More lofty aims of the course

1. Introduce mathematical background needed in various branches of computer science.
2. (New and old) techniques for problem solving: how to attack problems that you have never seen before.
3. To convey your ideas clearly: argue logically and write proofs formally.
4. To learn abstractions and abstract reasoning.

Prerequisities

- Nothing! ... Well, high school mathematics
- Logical mind and critical thinking

Chapter 1: Proofs and Logical reasoning

Outline of next few classes

- Propositions, statements
- What/why of proofs and some generic proof strategies
- Mathematical induction

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n + 1)(n - 1) = (n^2 - 1)$ for any integer n

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n + 1)(n - 1) = (n^2 - 1)$ for any integer n

What is common between these statements?

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n + 1)(n - 1) = (n^2 - 1)$ for any integer n

A proposition is a statement that is either true or false (but not both).

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n + 1)(n - 1) = (n^2 - 1)$ for any integer n

A proposition is a statement that is either true or false (but not both).

Give an example of a statement that is not a proposition.

Propositions

What is a proposition?

- It is raining
- $1 + 1 = 2$
- every odd number is a prime
- $2^{67} - 1$ is a prime
- $(n+1)(n-1) = (n^2 - 1)$ for any integer n

A proposition is a statement that is either true or false (but not both).

Give an example of a statement that is not a proposition.

- $x + 1 = 8$

Propositional calculus

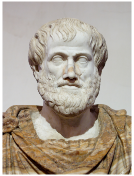


Figure: Aristotle (384 – 322 BCE)

- propositions are statements that are either true or false.
- Just as we use variables $x, y, \ldots$ for numbers, we will use variables $p, q, \ldots$ for propositions.

Propositional calculus

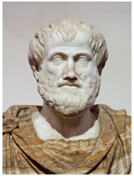


Figure: Aristotle (384 – 322 BCE)

- propositions are statements that are either true or false.
- Just as we use variables $x, y, \ldots$ for numbers, we will use variables $p, q, \ldots$ for propositions.
- "if it is raining, it will be wet" : $p \to q$

Propositional calculus

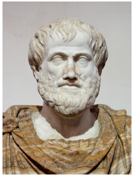


Figure: Aristotle (384 – 322 BCE)

- propositions are statements that are either true or false.
- Just as we use variables $x, y, \ldots$ for numbers, we will use variables $p, q, \ldots$ for propositions.
- "if it is raining, it will be wet" : $p \rightarrow q$
- This is one way to combine propositions!

Propositional calculus and Boolean algebra

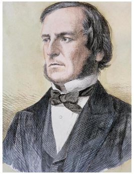


Figure: George Boole (1815 – 1864)

Combining propositions

- $\neg p$: It is not raining

Propositional calculus and Boolean algebra

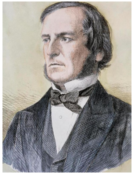


Figure: George Boole (1815 – 1864)

Combining propositions

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.

Propositional calculus and Boolean algebra

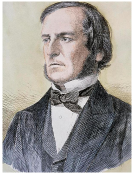


Figure: George Boole (1815 – 1864)

Combining propositions

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella

Propositional calculus and Boolean algebra

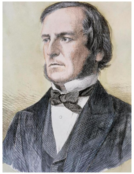


Figure: George Boole (1815 – 1864)

Combining propositions

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \rightarrow q$: If it is raining then it will be wet.

Propositional calculus and Boolean algebra

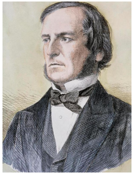


Figure: George Boole (1815 – 1864)

Combining propositions

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \rightarrow q$: If it is raining then it will be wet.
- $p \leftrightarrow q$: p if and only if q (also written p iff q)

Propositional calculus and Boolean algebra

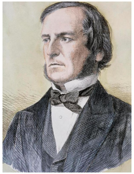


Figure: George Boole (1815 – 1864)

Combining propositions

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \to q$: If it is raining then it will be wet.
- $p \leftrightarrow q$: p if and only if q (also written p iff q)
- p, q are called atomic propositions and $\vee, \neg, \wedge, \to, \leftrightarrow$ are Boolean connectives.

Propositional calculus and Boolean algebra

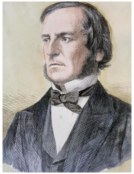


Figure: George Boole (1815 – 1864)

Combining propositions

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \to q$: If it is raining then it will be wet.
- $p \leftrightarrow q$: p if and only if q (also written p iff q)
- p, q are called atomic propositions and $\vee, \neg, \wedge, \to, \leftrightarrow$ are Boolean connectives.
- Ex: If it is raining or there is a sprinkler overhead and I dont have an umbrella, then I will get wet

Propositional calculus and Boolean algebra

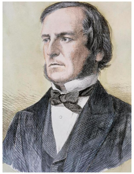


Figure: George Boole (1815 – 1864)

Combining propositions

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \to q$: If it is raining then it will be wet.
- $p \leftrightarrow q$: p if and only if q (also written p iff q)
- p, q are called atomic propositions and $\vee, \neg, \wedge, \to, \leftrightarrow$ are Boolean connectives.
- Ex: If it is raining or there is a sprinkler overhead and I dont have an umbrella, then I will get wet: $((p \vee q) \wedge r) \to s$.

Truth Tables and Logical Equivalence

p	$\neg p$
T	F
F	T

- $\neg p$: It is not raining

Truth Tables and Logical Equivalence

p	$\neg p$
T	F
F	T

p	q	$p \vee q$	$p \wedge q$
F	F	F	F
F	T	T	F
T	F	T	F
T	T	T	T

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella

Truth Tables and Logical Equivalence

p	$\neg p$
T	F
F	T

p	q	$p \vee q$	$p \wedge q$	$p \rightarrow q$
F	F	F	F	T
F	T	T	F	T
T	F	T	F	F
T	T	T	T	T

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \rightarrow q$: If I am elected then I will lower taxes.

Truth Tables and Logical Equivalence

p	$\neg p$
T	F
F	T

p	q	$p \vee q$	$p \wedge q$	$p \to q$
F	F	F	F	T
F	T	T	F	T
T	F	T	F	F
T	T	T	T	T

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \to q$: If I am elected then I will lower taxes.

Logical Equivalence: Truth tables are identical!

- $p \to q$ is "same as" or logically equivalent to $\neg p \vee q$

Truth Tables and Logical Equivalence

p	$\neg p$
T	F
F	T

p	q	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$
F	F	F	F	T	T
F	T	T	F	T	F
T	F	T	F	F	F
T	T	T	T	T	T

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \rightarrow q$: If I am elected then I will lower taxes.

Logical Equivalence: Truth tables are identical!

- $p \rightarrow q$ is "same as" or logically equivalent to $\neg p \vee q$

Truth Tables and Logical Equivalence

p	$\neg p$
T	F
F	T

p	q	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$
F	F	F	F	T	T
F	T	T	F	T	F
T	F	T	F	F	F
T	T	T	T	T	T

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \rightarrow q$: If I am elected then I will lower taxes.

Logical Equivalence: Truth tables are identical!

- $p \rightarrow q$ is "same as" or logically equivalent to $\neg p \vee q$
- $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$

Truth Tables and Logical Equivalence

p	$\neg p$
T	F
F	T

p	q	$p \vee q$	$p \wedge q$	$p \rightarrow q$	$p \leftrightarrow q$
F	F	F	F	T	T
F	T	T	F	T	F
T	F	T	F	F	F
T	T	T	T	T	T

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \rightarrow q$: If I am elected then I will lower taxes.

Logical Equivalence: Truth tables are identical!

- $p \rightarrow q$ is "same as" or logically equivalent to $\neg p \vee q$
- $p \leftrightarrow q$ is equivalent to $(p \rightarrow q) \wedge (q \rightarrow p)$
- You will pass this course iff you score $\geq 10/40$ in endsem and $\geq 15/60$ in rest.

Truth Tables and Logical Equivalence

p	$\neg p$
T	F
F	T

p	q	$p \vee q$	$p \wedge q$	$p \to q$	$p \leftrightarrow q$
F	F	F	F	T	T
F	T	T	F	T	F
T	F	T	F	F	F
T	T	T	T	T	T

- $\neg p$: It is not raining
- $p \vee q$: It is raining or there is a sprinkler overhead.
- $p \wedge q$: It is raining and I don't have an umbrella
- $p \to q$: If I am elected then I will lower taxes.

Logical Equivalence: Truth tables are identical!

- $p \to q$ is "same as" or logically equivalent to $\neg p \vee q$
- $p \leftrightarrow q$ is equivalent to $(p \to q) \wedge (q \to p)$
- You will pass this course iff you score $\geq 10/40$ in endsem and $\geq 15/60$ in rest.

Warning: English can be imprecise, but logic is precise!

Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled.

1. What is its converse?
2. What is its contra-positive?
3. What is its negation?

Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled. $(p \to q)$

1. What is its converse?
2. What is its contra-positive?
3. What is its negation?

Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled. $(p \to q)$

1. What is its converse? $(q \to p)$
2. What is its contra-positive?
3. What is its negation?

Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled. $(p \rightarrow q)$

1. What is its converse? $(q \rightarrow p)$
2. What is its contra-positive? $(\neg q \rightarrow \neg p)$
3. What is its negation?

Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled. $(p \to q)$

1. What is its converse? $(q \to p)$
2. What is its contra-positive? $(\neg q \to \neg p)$
3. What is its negation? $\neg(p \to q)$

Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled. $(p \to q)$

1. What is its converse? $(q \to p)$
2. What is its contra-positive? $(\neg q \to \neg p)$
3. What is its negation? $\neg(p \to q)$

Which of these are equivalent to the original proposition? Why?

Negation, Converse and Contrapositive

Consider the proposition: If it rains today, the match is cancelled. $(p \to q)$

1. What is its converse? $(q \to p)$
2. What is its contra-positive? $(\neg q \to \neg p)$
3. What is its negation? $\neg(p \to q)$

Which of these are equivalent to the original proposition? Why?

Exercise

Consider the proposition: If I will eat samosa or bhel puri, then I will not eat rice.

1. Write it as a Boolean combination of atomic propositions.
2. Write its converse, negation and contra-positive, both in plain english and formally.