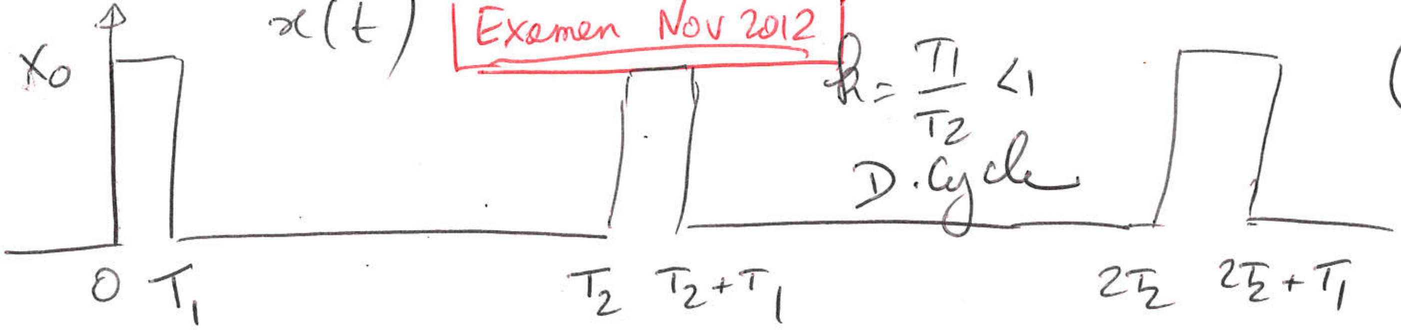
Examen Nov 2012 ①



$k = \frac{T_1}{T_2} < 1$

D. cycle

Ici $k = \frac{1}{4}$

① Draw

② F.T de $x(t) = \underline{X}(f)$

$$\cancel{\underline{X}(f) = \int_0^{T_1}}$$

$$x(t) = \cancel{\frac{X_0 T_1}{T_2}} + \sum_{n=-\infty}^{+\infty} C_n \, e^{2j\pi \frac{nt}{T_2}}$$

$$C_n = \frac{1}{T_2} \int_0^{T_1} X_0 \, e^{-2j\pi \frac{nt}{T_2}} \, dt$$

$$= \frac{X_0}{\cancel{T_2}} \left[\frac{e^{-2j\pi n t/T_2}}{-2j\frac{\pi n}{\cancel{T_2}}} \right]_0^{T_1}$$

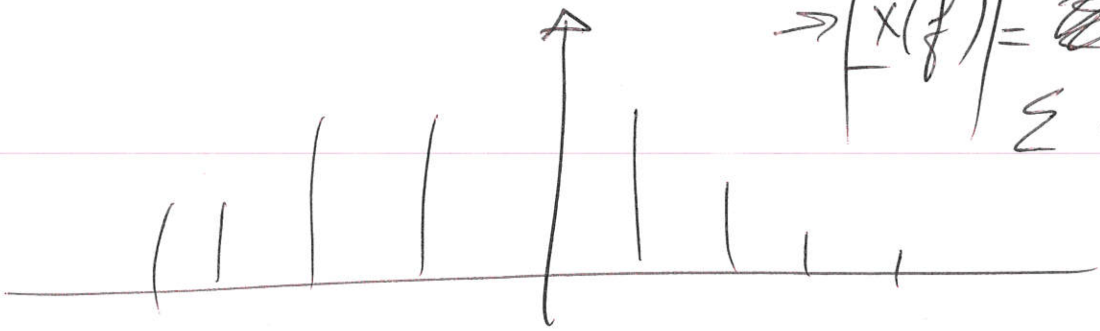
$$= \frac{X_0}{\pi n} \, \frac{e^{-2j\pi n \frac{T_1}{T_2}} - 1}{-2j}$$

$$\frac{X_0}{\pi n} \, e^{-j\pi n \frac{T_1}{T_2}} \left[\frac{e^{-j\pi n \frac{T_1}{T_2}} - e^{j\pi n \frac{T_1}{T_2}}}{2j} \right]$$

Module pair

$$\frac{T_1}{T_2} \, \frac{X_0}{\pi n} \, \frac{\sin \pi n \, T_1/T_2}{T_1/T_2} = \frac{X_0 T_1}{T_2} \, \mathrm{Sinc}\left[n k \right]$$

module $= \frac{X_0 T_1}{T_2} \, \mathrm{Sinc}\left[n k\right]$



$\rightarrow |X(f)| = \cancel{\Sigma} X_0 k \sum \mathrm{Sinc}(nk)\,\delta(f - n f_2)$

$$\boxed{\varphi = - n \pi k}$$

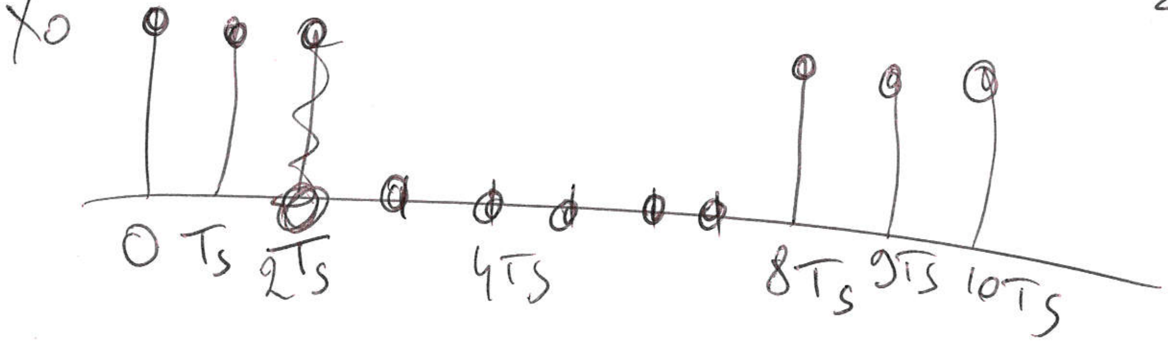
phase imposée —

- Mean Power of the signal $x(t)$:

$$\frac{1}{T_2} X_0^2 T_1 = X_0^2 k$$

Sampling with $T_s = \frac{T_1}{2} \rightarrow T_1 = 2T_s$

$T_2 = 8T_s$



Draw $x_s(t) \equiv x_n$: $n: 0 \rightarrow 10$

Note that $x_n(t)$ being periodic $N = 8$ samples are enough for study.

Mean power of a discrete signal...

$$\frac{1}{N}\sum_{n=0}^{N-1} x_n^{2} = \frac{2\,x_0^{2}}{8} = \frac{x_0^{2}}{4}$$

$$W_8 = e^{-2j\frac{\pi}{8}} = e^{-j\pi/4}$$

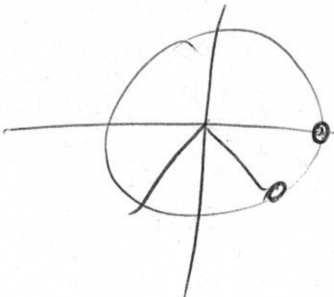
$$\begin{pmatrix} X_s(0) \\ X_s(1) \\ \vdots \\ \\ \\ \\ \\ X_s(7) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_8^{1} & & & & & & \\ 1 & W_8^{2} & & & & & & \\ 1 & W_8^{3} & & & & & & \\ 1 & W_8^{4} & & & & & & \\ 1 & W_8^{5} & & & & & & \\ 1 & W_8^{6} & & & & & & \\ 1 & W_8^{7} & & & & & & \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$X_s(0) = 2$

$X_s(1) = 1 + e^{-j\pi/4} = 1 + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$X_s(2) = 1 - j$

$X_s(3) = 1 - \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$



$$x_s(4) = 1 - 1 = 0$$

$$X_s(5) = 1 - \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

$$X_s(6) = 1 + j$$

$$X_s(7) = 1 + \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

$$|X_s(0)|^2 = 4$$

$$|X_s(1)|^2 = \left(1 + \frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2} = 1 + \frac{1}{2} + \sqrt{2} + \frac{1}{2}$$
$$= 2 + \sqrt{2}$$

$$|X_s(2)|^2 = 2$$

$$|X_s(3)|^2 = \left(1 - \frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2} = 2 - \sqrt{2}$$

$$|X_s(4)|^2 = 0$$

$$|X_s(5)|^2 = 2 - \sqrt{2}$$

$$|X_s(6)|^2 = 2$$

$$|X_s(7)|^2 = 2 + \sqrt{2}$$

$$\sum |X_s(k)|^2 = 4 + 2 + 2 + 2 + \cancel{\sqrt{2}} + 2 - \cancel{\sqrt{2}} + 2 - \cancel{\sqrt{2}} + 2 + \cancel{\sqrt{2}} = 16$$

$$\frac{\sum |X_s(k)|^2}{8^2} = \frac{16}{8 \cdot 8} = \frac{1}{4} \cancel{= 16}$$