Lectures for semester 1
Course Module : Digital Signal Processing
Term exam, November 2012
1.5 hour, documents not allowed
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In this problem, we will first process an analog and periodic signal x(t) such as
$x(t) = X_0$ if t belongs to $[0; T_1[$, and $x(t) = 0$ if t belongs to $[T_1; T_2[$.
In addition $x(t + T_2) = x(t)$ for any t. X_0, $T_1 = 1 / F_1$ and $T_2 = 1/ F_2$ are real and positive numbers.
Remember that the constant $\alpha = T_1/T_2$, which is less than 1, is called the duty cycle. Here, this duty cycle will be fixed to 1/4.

1. Draw x(t). Is this signal even or odd?

2. As a consequence, is the Fourier transform $\underline{X}(f)$ of x(t) complex or real? Is it an even or an odd function of frequency f ? Is it a continuous function of f, or a discrete one?
Give the modulus and the phase of X(f).
Draw these results versus $f = n * F_2$, n being the number of harmonic frequency.

3. Remember that the mean power of a continuous-time signal is given by the integral over the period T_2 of the squared modulus of the signal, divided by the time period. Calculate this mean power, and give, without calculations, a second formula using Parseval relation (applied to continuous-time signals).

We sample now x(t) with a sampling frequency $F_s = 1/T_s$.

4. How must be chosen F_s according to F_2, in order to have a successful sampling? Which theorem is then applied to say that? We choose indeed $Ts = T_1/2$. How many samples are then involved in a period of new signal $x_S(t)$?

5. Give the analytical expression of the new signal, $x_S(t)$, involving x(t) and the Dirac Comb distribution $\sum_{k=-\infty}^{k=+\infty} \delta(t-kT_s)$. Draw $x_S(t)$. Be careful with the 2 discontinuities of x(t) inside the period interval: so, take $x_S(0) = x_S(1) = X_0$, but $x_S(2) = 0$, …

6. For this question and the following ones, we restrict the study of $x_S(t)$ to the first period $[0; T_2]$, and consider that $x_S(t)$ is definitively zero before 0 and after T_2. You know then a discrete signal $x_S(n)$ for N = 8 samples, and you know that the Discrete Fourier Transform will involve N coefficients X(n'). Give the matrix giving the relation between the array $[X_S(n')]$ and the array $[x_S(n)]$. Simplify it knowing that $8 = 2^3$. Use the *Twiddle factor* $W_8 = \exp(- j \pi /4)$.

7. Knowing explicitly that some samples $x_S(n)$ are indeed zero, give the N = 8 coefficients $X_S(n')$ of the Fast Fourier Transform of $x_S(n)$.

8. Give the mean power of signal $x_S(n)$. Calculate it also by Parseval relation (applied to discrete signals) using the squared modulus of the 8 coefficients $X_S(n')$ calculated in question 7).