Lectures for semester 1
Course Module : Digital Signal Processing
Term exam, December 17^{th} 2013
1.5 hour, documents allowed
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The students can begin either by Section I, or by Section II. But, in Section III, they are invited to comment how similar the results of both first Sections are and why.

I. In this problem, we will first process an analog and periodic signal x(t) such as $x(t) = \cos(2\pi t / T) + \cos(4\pi t / T)$. For numerical calculations, T will be taken equal to unity.

I.a) What is the period of this signal?

I.b) Is this signal even or odd? Is it real-valued? Deduce two important properties concerning its Fourier transform $\underline{X}(f)$.

I.c) Calculate the Fourier Transform X(f), by using Euler's theorem
$\cos(\alpha) = 0.5\,[\exp(j\alpha) + \exp(-j\alpha)]$,
and using several times the Dirac distribution $\delta(f - f_0)$, where f_0 is any fixed frequency.

I.d) Draw X(f) versus the frequency f in the relevant interval. Show that this spectrum contains exactly 4 different Dirac peaks.

I.e) Calculate by 2 differents ways the mean power of the signal x(t).

I.f) We sample now the signal x(t) with a sampling frequency $f_s = 8/T$. Is it conform to Shannon-Nyquist's theorem? If yes, calculate the different samples $x_s(k)$ and draw the numeric signal $x_s(k)$ versus time in the interval $[0 ; 3T/2]$.

I.g) What effect this sample process has on the spectrum $X_s(f)$ of $x_s(t)$? Draw this new spectrum versus frequency in the interval $[-f_s ; 2f_s]$.

II. In this section, we will look for a numerical signal y(k) for k = 0, 1, …, 7. That is, the signal y(k) is to be determined for N = 8 samples.

II.a) Knowing that the Discrete Fourier Transform will involve N coefficients Y(n), give the relation giving the array [y(k)] as a product of the array[Y(n)] and a 8x8 matrix. Simplify it knowing that $8 = 2^3$. Use the *Twiddle factor* $W_8 = \exp(-j\pi/4)$.

II.b) What sufficient condition on the 8 values Y(0) to Y(7) (supposed all real-valued) implies that the 8 values of y(k) are real-valued?

II.c) We take now $Y(n) = {}^t[0; 4; 4; 0; 0; 0; 4; 4]$, where the superscript ${}^t[Y(n)]$ corresponds to the transpose array of array[Y(n)] and is only used to save place in the text. We recall that

$W_8^{-1} = \frac{1}{2}(\sqrt{2} + j\sqrt{2})$ $W_8^{-3} = \frac{1}{2}(-\sqrt{2} + j\sqrt{2})$ $W_8^{-5} = \frac{1}{2}(-\sqrt{2} - j\sqrt{2})$ $W_8^{-7} = \frac{1}{2}(\sqrt{2} - j\sqrt{2})$

While the 4 others are simple to get. Calculate the 8 values of y(k).

II.d) Calculate the energy of signal y(k) such as : Energy = $\sum_0^{N-1} |y(k)|^2$.

Compare with : $\frac{1}{N}\sum_0^{N-1} |Y(n)|^2$. What theorem is then highlighted ?

III. Compare the signal $x_s(t)$ studied in Section I with the signal y(k) studied in Section II. Namely, what is required for any numerical signal s(k) known for N samples to have the following property?

The Discrete Fourier Transform on a signal for N samples corresponds to the restriction to the N central values of the Fourier Transform of $s_s(t)$ from – infinity to + infinity.