Create account or Sign in

Mathonline
LEARN MATHEMATICS

The LU Decomposition of a Matrix Examples 1

Fold

Table of Contents

The LU Decomposition of a Matrix Examples 1

Recall from The LU Decomposition of a Matrix page that if we have an $n \times n$ matrix A, then provided that under Gaussian Elimination, an upper triangular matrix U can be produced without pivoting, then there exists another matrix L that is lower triangular such that $A = LU$.

We will now look at some concrete examples of finding an LU decomposition of a matrix.

Example 1

Find an LU decomposition for the matrix $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$.

We will start by applying Gaussian Elimination to get a row equivalent form of A that is upper triangular. We do this by the elementary row operation $R_2 - \frac{4}{3} R_1 \to R_2$ to immediately obtain an upper triangular matrix, U:

$$U = \begin{bmatrix} 3 & 1 \\ 0 & \frac{2}{3} \end{bmatrix} \tag{1}$$

Now our corresponding lower triangular matrix L is going to have 1's along its main diagonal.

$$L = \begin{bmatrix} 1 & 0 \\ * & 1 \end{bmatrix} \tag{2}$$

The entry below the main diagonal is obtained as the inverse row operations applied to U. In this case, we have $R_2 + \frac{4}{3} R_1 \to R_2$ to obtain:

$$L = \begin{bmatrix} 1 & 0 \\ \frac{4}{3} & 1 \end{bmatrix} \tag{3}$$

Therefore our LU decomposition of A is:

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{4}{3} & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & \frac{2}{3} \end{bmatrix} = LU \tag{4}$$

Note that we will only be using the elementary row operations of addition/subtraction of a multiple of one row to another, and so the inverse operations will always be the negative of the multipliers used in performing Gaussian Elimination to get A to U.

Example 2

Find an LU decomposition for the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$.

Once again, we begin by using Gaussian Elimination. We take $R_2 - 4R_1 \to R_2$ to get:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 9 \end{bmatrix} \tag{5}$$

We now take $R_3 - 7R_1 \to R_3$ to get:

(6)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix}$$

Lastly we take $R_3 - 2R_2 \to R_3$ to obtain our upper triangular matrix U:

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} \quad (7)$$

Our corresponding lower triangular matrix L will once again have 1's along the main diagonal, and the entries underneath the main diagonal are obtained from the corresponding inverse operations. Thus:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \quad (8)$$

Therefore an LU decomposition for A is:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{bmatrix} = LU \quad (9)$$

Note in this particular example that the third row of U is all zeroes. This implies that A itself is noninvertible.