B31YS Robotics Systems Science

Bayesian Filtering and Robot Localisation
Probabilistic Robotics

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Schedule – Week 3 & 4

	Monday			Thursday	Friday
	10.15 – 11.15	13.15 – 15.15	16.15 – 17.15	16.15 – 17.15	13.15 – 15.15
Week 3	**Lecture** - Bayes filters - Recursive Bayes filters - Robot motion models - Odometry and velocity models	**Lecture** - Observation model - Kalman filter - Extended Kalman Filter - filter consistency	**Lab – Matlab** - understand KF & EKF - robot localisation - filter tuning and consistency	**Lab - Fusion** - EKF fusion of odometry + GPS using real data for localisation	**Lab - Fusion** - EKF fusion of odometry + GPS - prepare presentations
Week 4	**Lecture** - Importance Sampling - Particle filter - comparison with KF/EKF	**Lecture** -Monte Carlo Localisation **Lab – ROS** AMCL in simulation/real robot using Laser (Kinect) + odometry	**Lab – ROS** AMCL in simulation/real robot using Laser (Kinect) + odometry	**Lab – ROS** (Continued) AMCL in simulation/ robot **Lab - Fusion(optional)** - fusion of odometry + GPS using PF	**Lecture** - Student presentations of sensor fusion - Discussion / Q&A

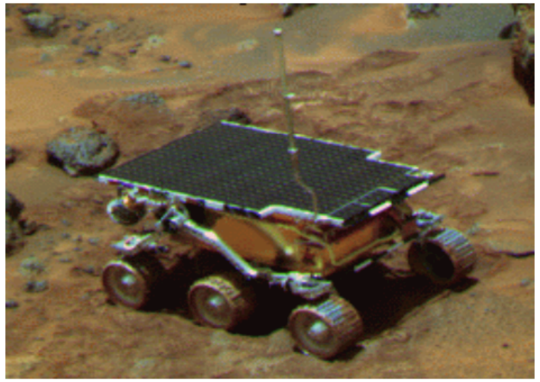
Textbooks:

- Sebastian Thrun, Wolfram Burgard, Dieter Fox. Probabilistic robotics. Ch.2 – Ch. 8
- Timothy D. Barfoot. State Estimation for Robotics.
- Yaakov Bar-Shalom, X.-Rong Li, Thiagalingam Kirubarajan. Estimation with Applications to Tracking and Navigation.

Previous

Autonomous Robots

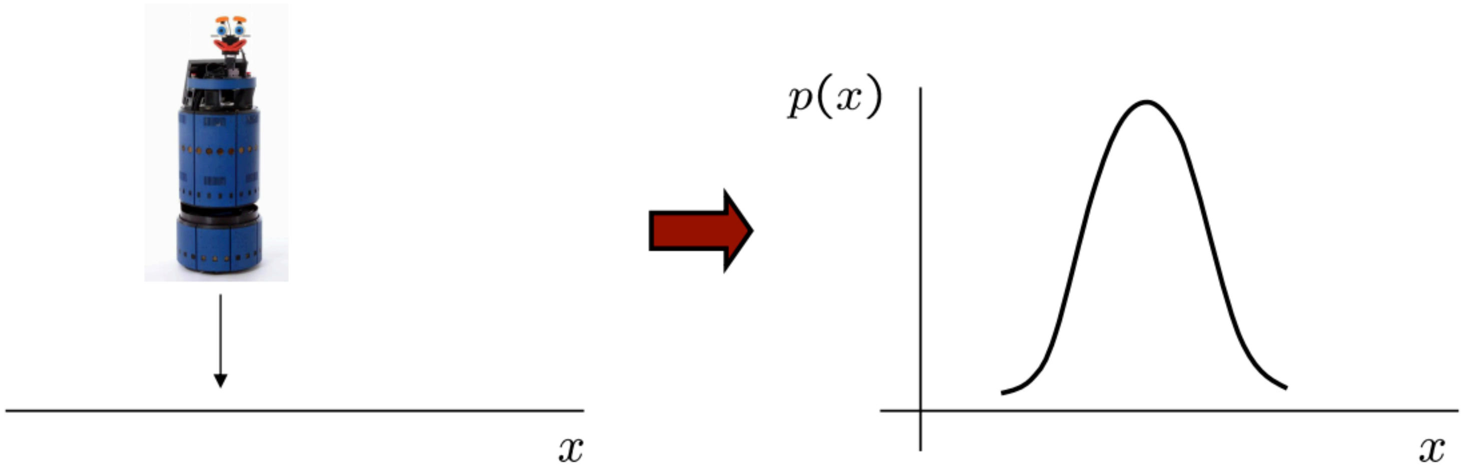
- The control of autonomous robots involves a number of subtasks
  - Understanding and modeling of the mechanism
    - Kinematics, Dynamics, and Odometry
  - Reliable control of the actuators
    - Closed-loop control
  - Generation of task-specific motions
    - Path planning
  - Integration of sensors
    - Selection and interfacing of various types of sensors
  - Coping with noise and uncertainty
    - Filtering of sensor noise and actuator uncertainty
  - Creation of flexible control policies
    - Control has to deal with new situations



Why Probabilistic Robotics

- Noise and uncertainty are everywhere in real world
  - actuators: failure, drift
  - sensors: observations, imprecise, unreliable
  - non-observability: system state(s), environment initially unknown

- Use probability theory to explicitly represent the uncertainty



"The robot is exactly here"

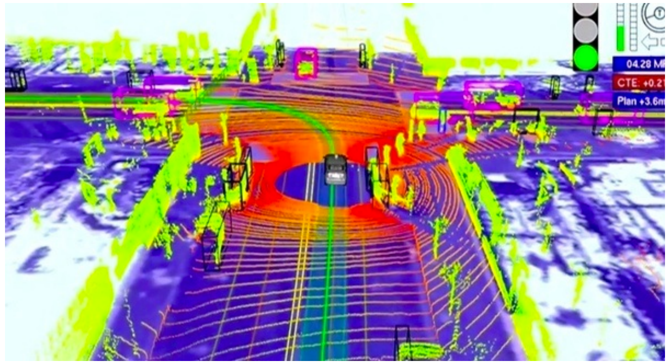
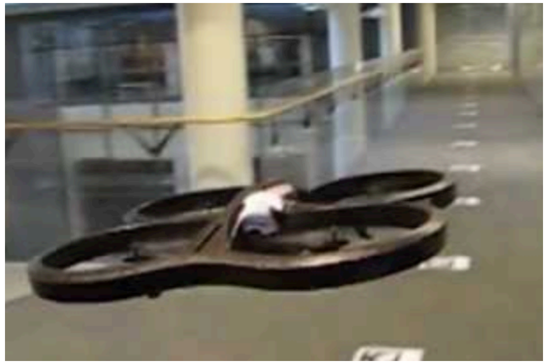
"The robot is somewhere here"

Related Terms

- State Estimation
- Multi-Sensor Fusion
- Localisation
- Mapping
- SLAM
- Navigation

Robot Localisation

- how a robot figures out globally where it is
- fundamental for autonomous mobile robots


Three Main Paradigms:

Kalman Filter (W3)

Particle Filter (W4)

Graph based Methods (W7)

Formulation of Robot Localisation

- **Bayes' Theorem/Rule**

posterior $\boxed{P(x \mid y) = \dfrac{P(y \mid x)\, P(x)}{P(y)} = \dfrac{\text{likelihood} \cdot \text{prior}}{\text{evidence}}}$

robot localisation/SLAM:

$$p(x_{0:T}, m \mid z_{1:T}, u_{1:T})$$

distribution path map given observations controls

Motion Model - Prior

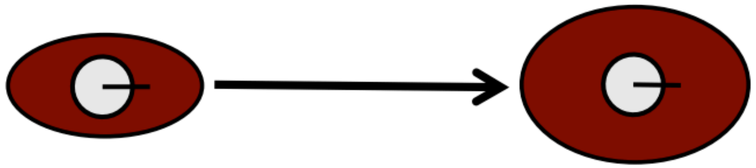
- The motion model describes the relative motion of the robot

$$p(x_t \mid x_{t-1}, u_t)$$

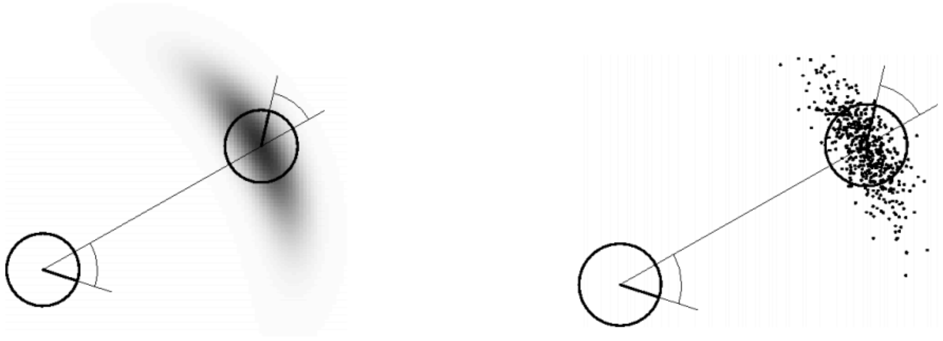
distribution new pose given old pose control

- Examples

▪ Gaussian model



▪ Non-Gaussian model



Observation/Sensor Model - Likelihood

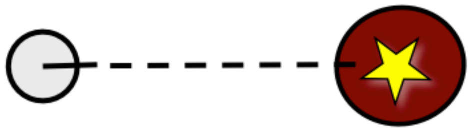
- The observation or sensor model relates measurements with the robot's pose

$$p(z_t \mid x_t)$$

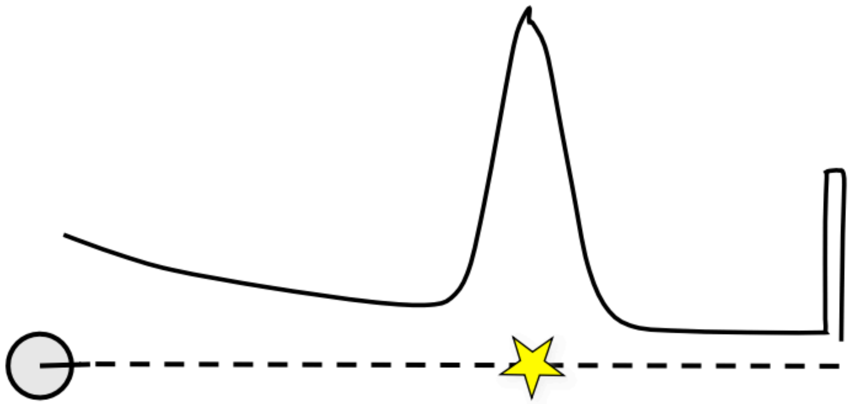
distribution observation given pose

- Examples

- Gaussian model



- Non-Gaussian model



(here $x_r = x$)

$$\overline{bel}_0(x) = \frac{1}{L}$$

z_0: door detected

$$p(z_0|x)$$

$$bel_0(x) = \eta\, p(z_0|x_0)\overline{bel}_0(x) = \eta' p(z_0|x_0)$$

u_0: the robot moves

$$\overline{bel}_1(x) = \int_0^L p(x|x_0 = y, u_0) bel_0(y) dy$$

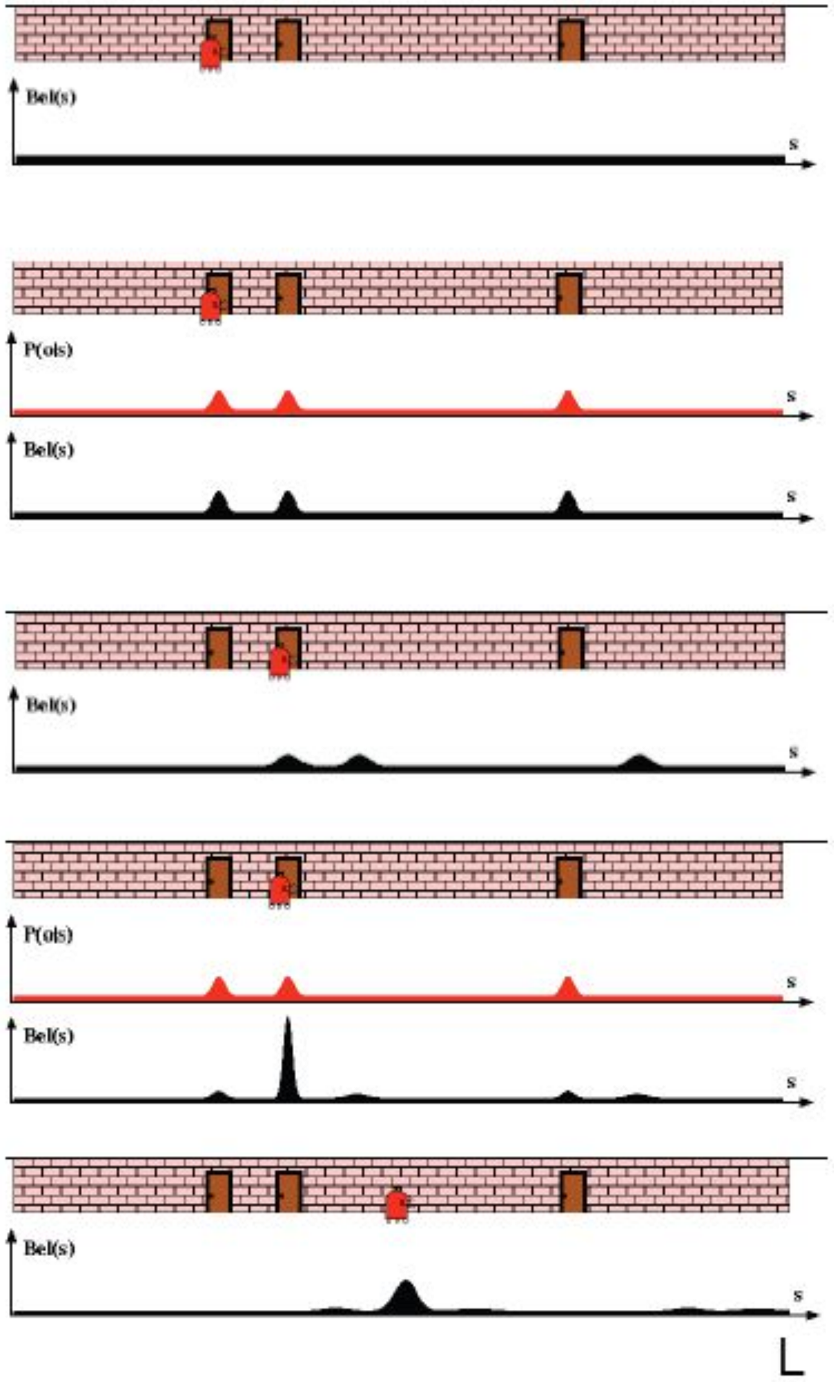
z_1: door detected again

$$p(z_1|x)$$

$$bel_1(x) = \eta\, p(z_1|x_1)\overline{bel}_1(x)$$

u_1: the robot moves again

$$\overline{bel}_2(x) = \int_0^L p(x|x_1 = y, u_1) bel_1(y) dy$$



From Thrun Burgard Fox, Probabilistic Robotics, MIT Press 2006

Filter Tuning

How to decide if the estimates produced are a "reasonably good" reflection of the true state? And filter parameters?

- filter tuning: process of choosing a suitable Q and R
- Innovation: difference between true and predicted observations

$$\mu_t = \bar{\mu}_t + K_t(\underline{z_t - h(\bar{\mu}_t)})$$ innovation

$$K_t = \bar{\Sigma}_t \, H_t^T (\underline{H_t \, \bar{\Sigma}_t \, H_t^T + Q_t})^{-1}$$ innovation covariance

- most important analysis: using innovation sequence
- innovation sequence: white and unbiased process
- Normalised Innovation Squared, etc.

Filter Tuning

- 4 cases:
  - under-estimate of process noise
  - under-estimate of observation noise
  - over-estimate of process noise
  - over-estimate of observation noise