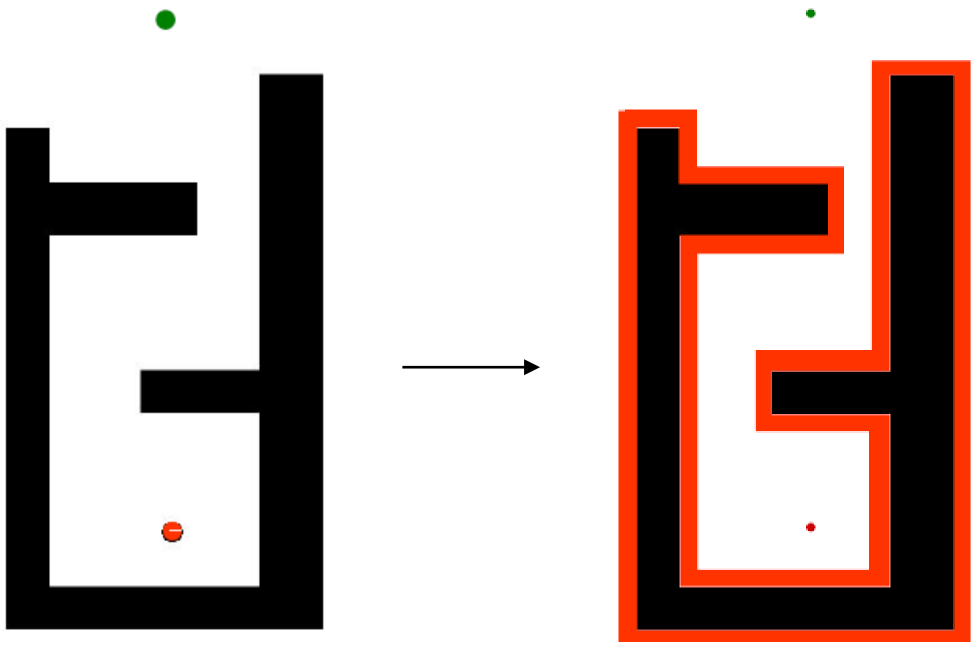
Configuration space

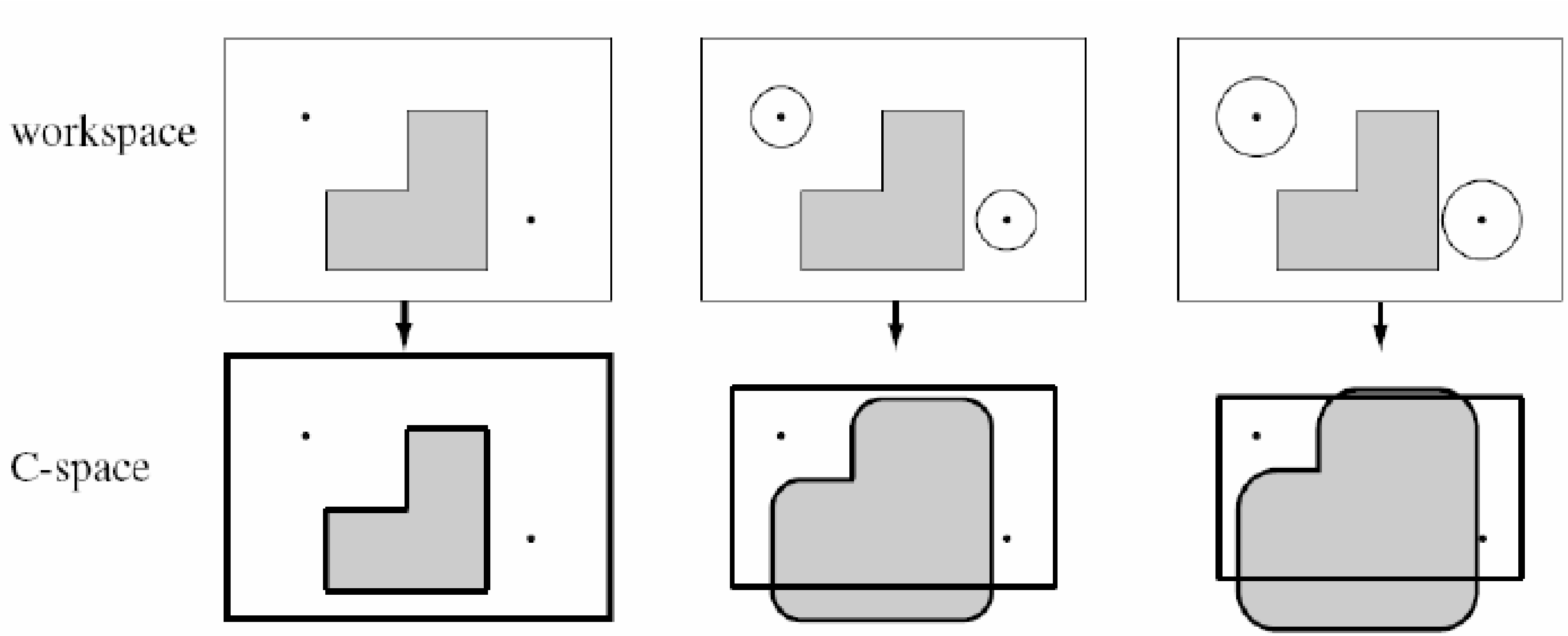
- Bug algorithms do not plan the path
- In order to plan, it is necessary to know the environment
- The environment also includes the robot, which has a body
- The configuration space simplifies the planning process by treating the robot as a point



Configuration space

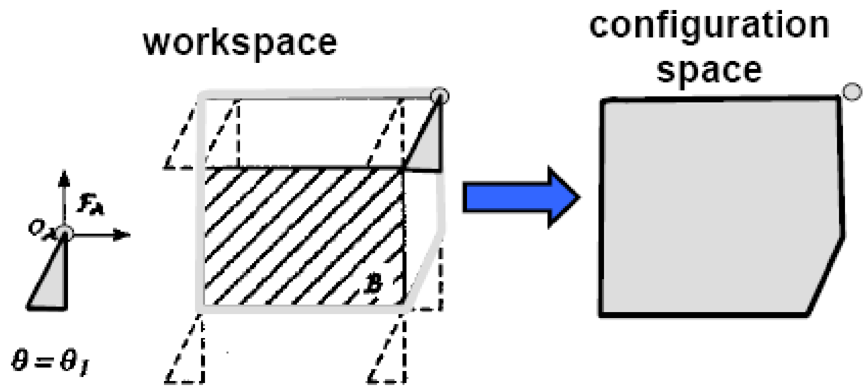
Configuration of an object: set of parameters that define its position.

Configuration space: set of all configurations.



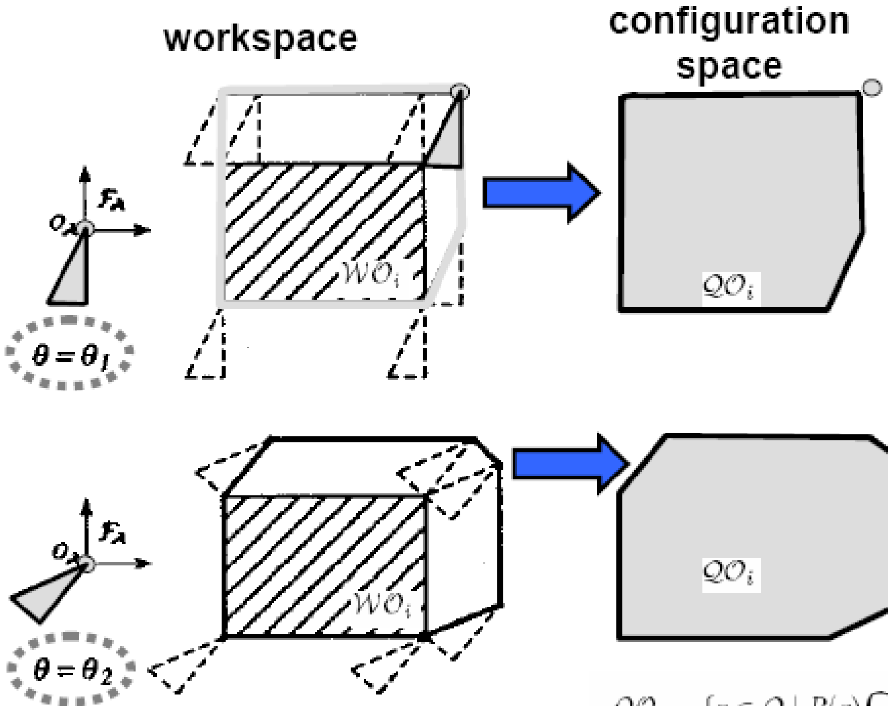
Configuration space

Non circular objects:



Configuration space

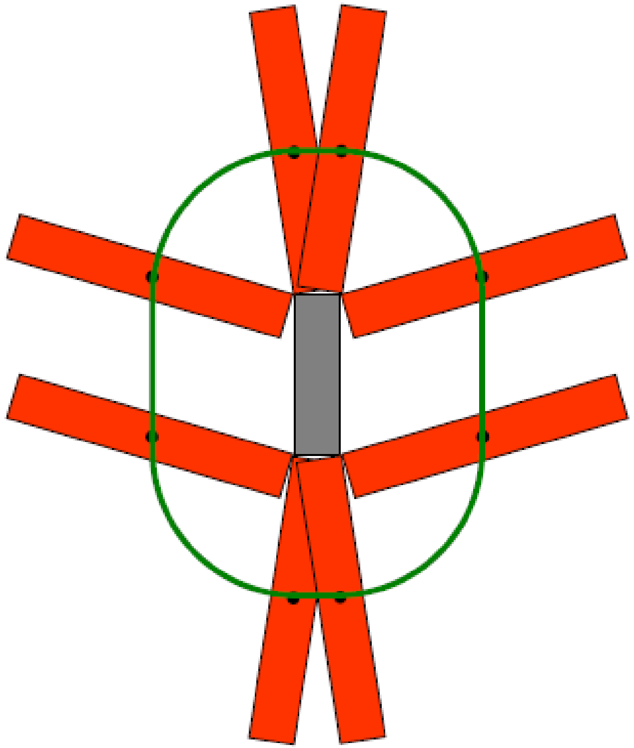
Non circular objects + rotation

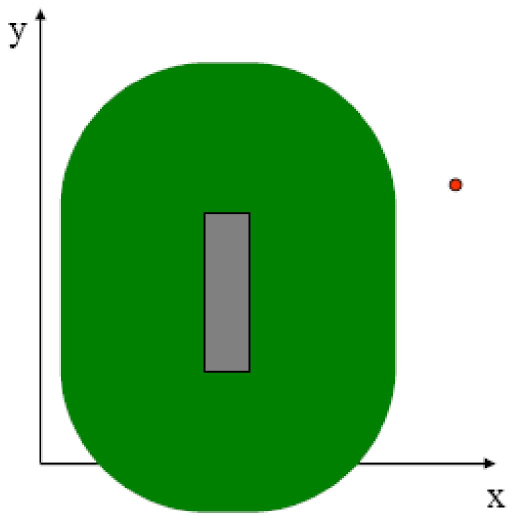


$$\mathcal{QO}_i = \{q \in \mathcal{Q} \mid R(q) \bigcap \mathcal{WO}_i \neq \emptyset\}.$$

Configuration space

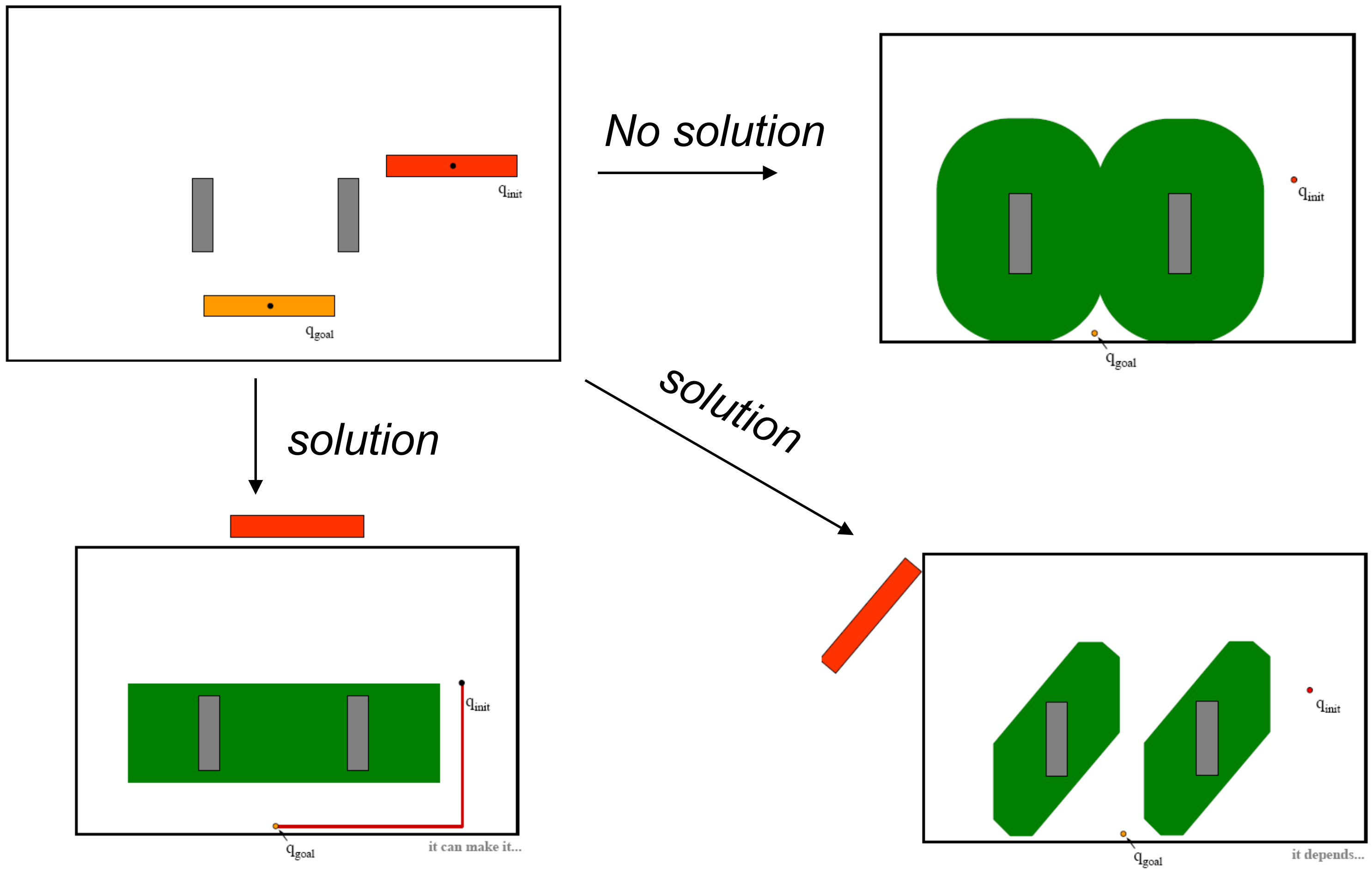
Conservative approach:





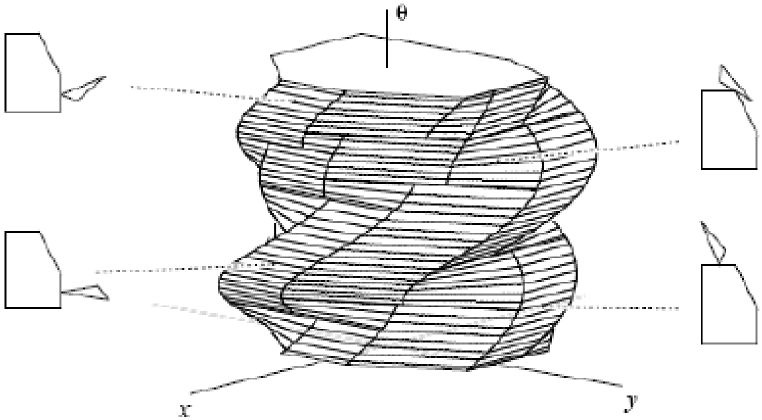
Configuration space

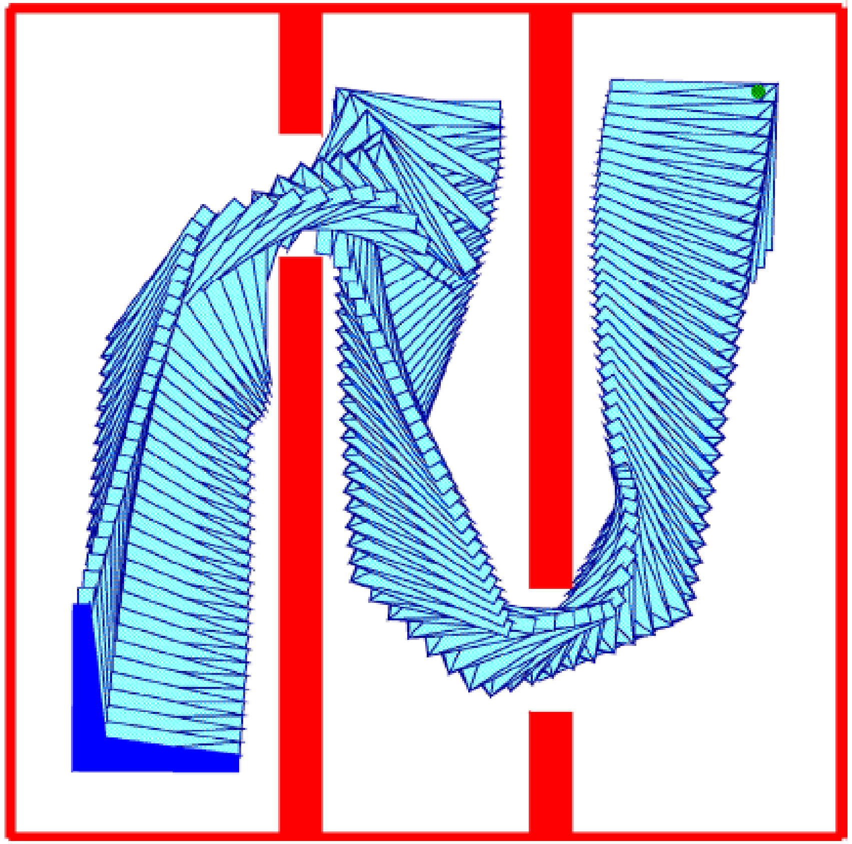
Solution in the configuration space



Configuration space

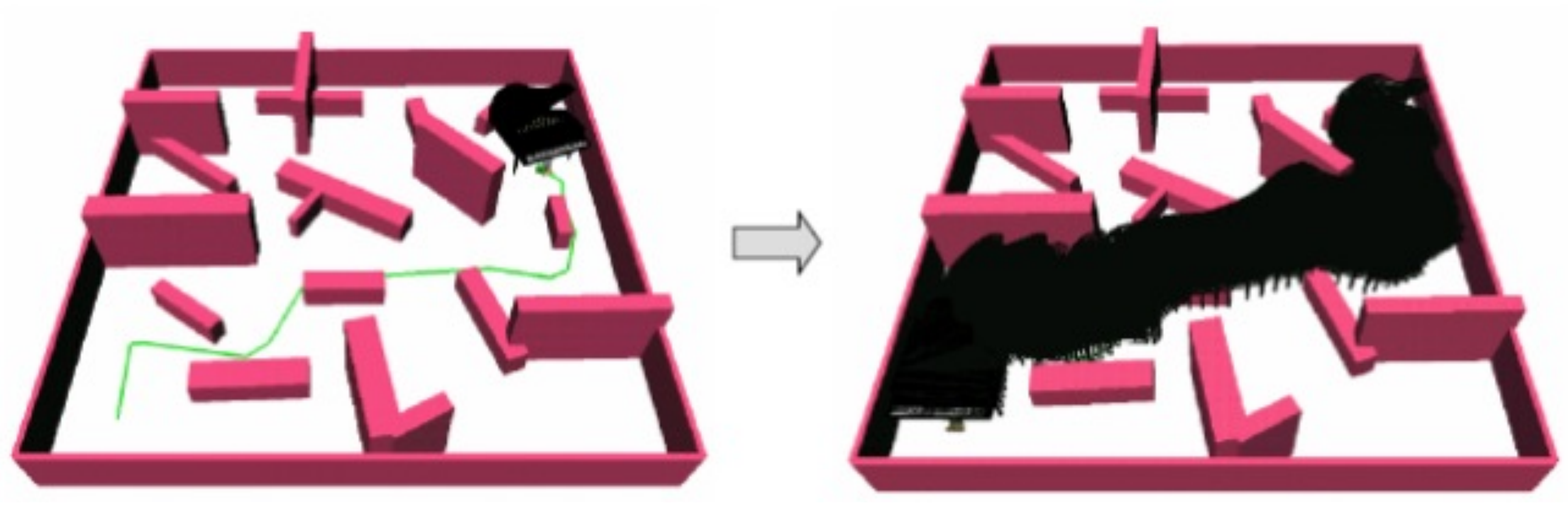
Orientation in the configuration space





Configuration space

Orientation in the configuration space



Potential functions

- A potential function is used to move the robot from high potential (starting point) to low potential (goal)
- The potential function is build using:

$$U(q) = U_{\mathrm{att}}(q) + U_{\mathrm{rep}}(q)$$

  - U_{att} is the "attractive" potential --- move to the goal
  - U_{rep} is the "repulsive" potential --- avoid obstacles

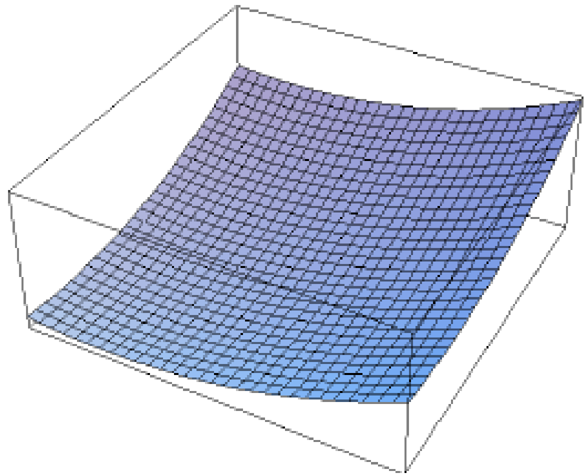
- Similar than behaviour-based robotics, but having the knowledge of the world and, therefore, assuring the finding of the path

Potential functions

- Attractive potential

$$U_{\text{att}}(q) = \begin{cases} \frac{1}{2}\zeta d^2(q, q_{\text{goal}}), & d(q, q_{\text{goal}}) \le d^*_{\text{goal}}, \\ d^*_{\text{goal}}\zeta d(q, q_{\text{goal}}) - \frac{1}{2}\zeta (d^*_{\text{goal}})^2, & d(q, q_{\text{goal}}) > d^*_{\text{goal}}. \end{cases}$$

$$\nabla U_{\text{att}}(q) = \begin{cases} \zeta(q - q_{\text{goal}}), & d(q, q_{\text{goal}}) \le d^*_{\text{goal}}, \\ \frac{d^*_{\text{goal}}\zeta(q - q_{\text{goal}})}{d(q, q_{\text{goal}})}, & d(q, q_{\text{goal}}) > d^*_{\text{goal}}, \end{cases}$$

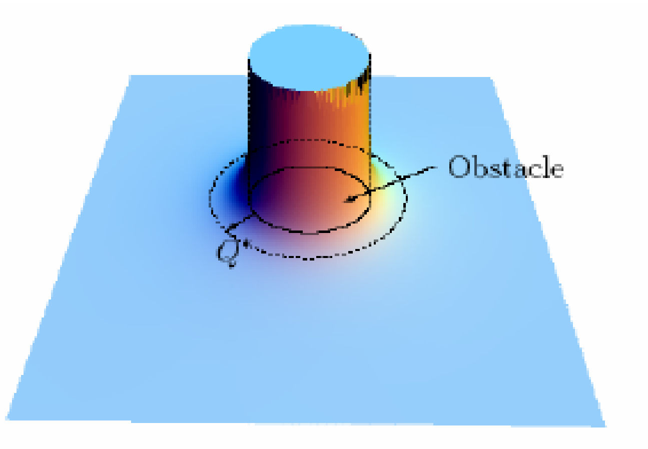


Quadratic near the goal and conic farther away

The gradient is used to know the direction to follow

Potential functions

- Repulsive potential



$$U_{\text{rep}}(q) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{D(q)} - \frac{1}{Q^*}\right)^2, & D(q) \le Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

whose gradient is

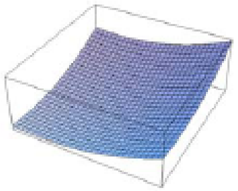
$$\nabla U_{\text{rep}}(q) = \begin{cases} \eta\left(\frac{1}{Q^*} - \frac{1}{D(q)}\right)\frac{1}{D^2(q)}\nabla D(q), & D(q) \le Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

Potential functions

- Total potential functions

$$U(q) = U_{att}(q) + U_{rep}(q)$$

$$F(q) = -\nabla U(q)$$



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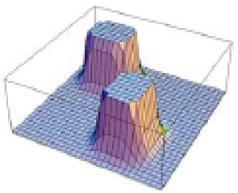

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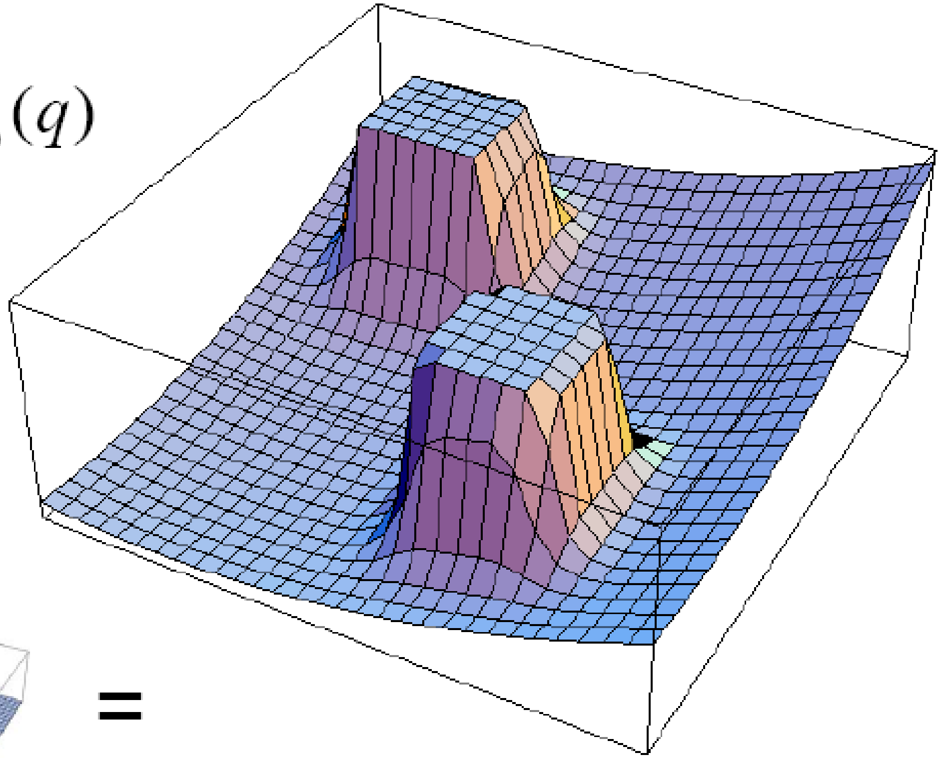
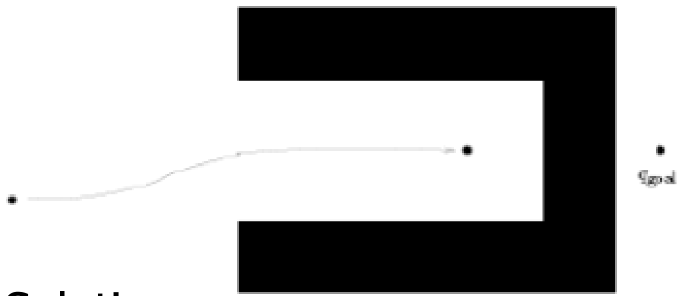


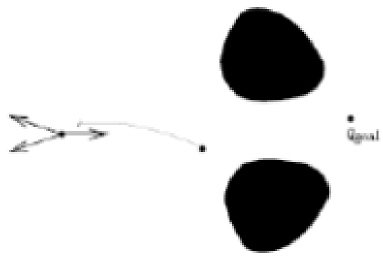
Potential functions

- Finding the minimum:
  - The gradient of the total potential function indicates the way to the goal:

$$\dot{c}(t) = -\nabla U(c(t)).$$

  - since the total potential function depends on the number, position and shape of the obstacles, there can be local minimums!!

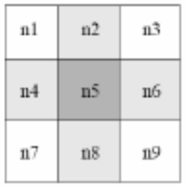




  - Solutions:
    - to operate mathematically the functions to eliminate local minimums → navigation functions
    - to divide the space into a grid → brushfire algorithm and wavefront planner

Potential functions

- Brushfire algorithm:
  - To compute the gradient of the repulsive functions
  - Define a grid on the space
  - Choose 4 or 8 point connectivity



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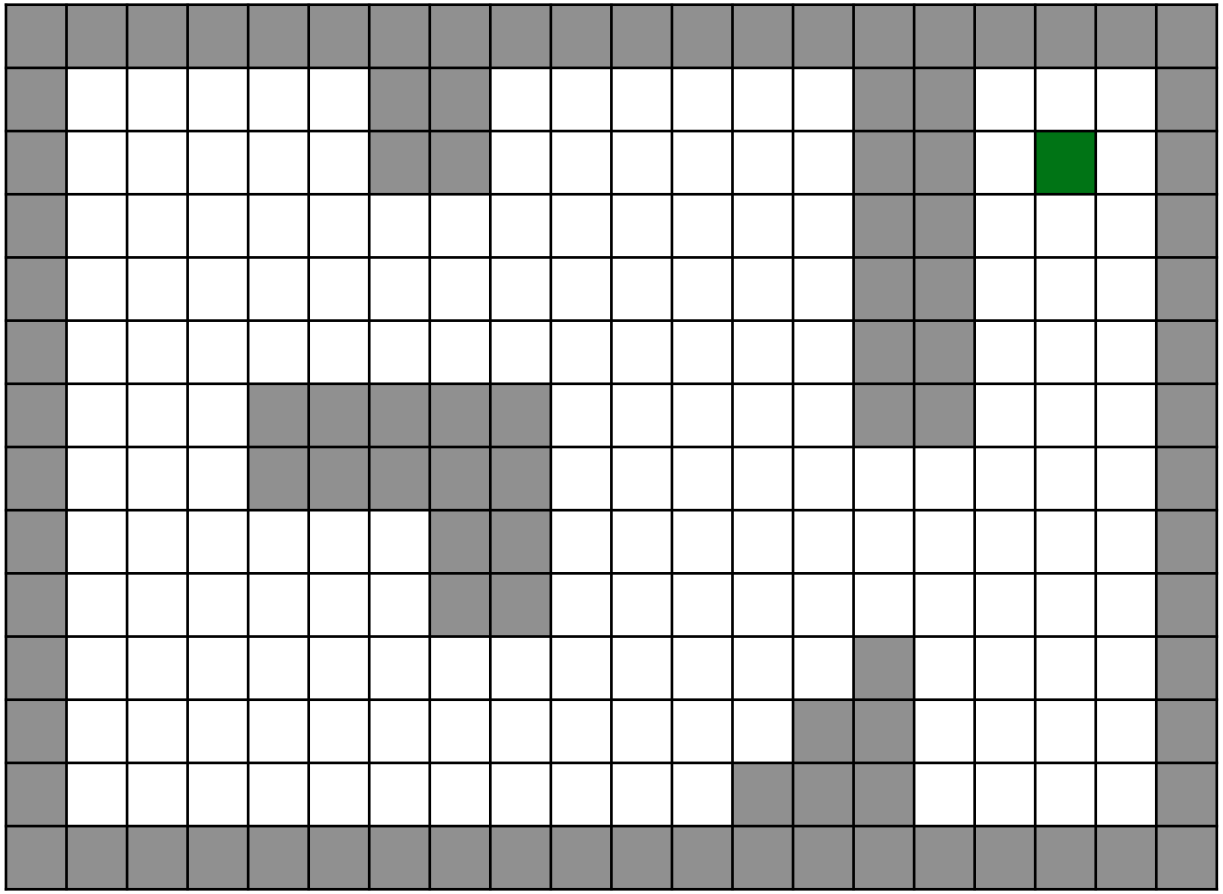
  - Obstacles start with a 1; free space zero
  - Until all cells >0; assign to all **connected cells** the minimum non-zero value plus 1
  - The result is a map where each cell holds the minimum distance to an obstacle
  - The gradient of distance is easily found by taking differences with all neighbouring cells

Potential functions

- Brushfire algorithm:

  2D finite environment, 20x14 cells



Potential functions

- Brushfire algorithm:

  with 4-point connectivity, 1st iteration

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Potential functions

- Brushfire algorithm:

  with 4-point connectivity, 2nd iteration

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	1	1	2	2	2	2	2	2	1	1	2	2	2	1
1	2	0	0	0	2	1	1	2	0	0	0	0	2	1	1	2	0	2	1
1	2	0	0	0	0	2	2	0	0	0	0	0	2	1	1	2	0	2	1
1	2	0	0	0	0	0	0	0	0	0	0	0	2	1	1	2	0	2	1
1	2	0	0	2	2	2	2	2	0	0	0	0	2	1	1	2	0	2	1
1	2	0	2	1	1	1	1	1	2	0	0	0	2	1	1	2	0	2	1
1	2	0	2	1	1	1	1	1	2	0	0	0	0	2	2	0	0	2	1
1	2	0	0	2	2	2	1	1	2	0	0	0	0	0	0	0	0	2	1
1	2	0	0	0	0	2	1	1	2	0	0	0	0	2	0	0	0	2	1
1	2	0	0	0	0	0	2	2	0	0	0	0	2	1	2	0	0	2	1
1	2	0	0	0	0	0	0	0	0	0	0	2	1	1	2	0	0	2	1
1	2	2	2	2	2	2	2	2	2	2	2	1	1	1	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Potential functions

- Brushfire algorithm:

  with 4-point connectivity, 5th iteration

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	1	1	2	2	2	2	2	2	1	1	2	2	2	1
1	2	3	3	3	2	1	1	2	3	3	3	3	2	1	1	2	3	2	1
1	2	3	4	4	3	2	2	3	4	4	4	3	2	1	1	2	3	2	1
1	2	3	4	3	3	3	3	3	4	5	4	3	2	1	1	2	3	2	1
1	2	3	3	2	2	2	2	2	3	4	4	3	2	1	1	2	3	2	1
1	2	3	2	1	1	1	1	1	2	3	4	3	2	1	1	2	3	2	1
1	2	3	2	1	1	1	1	1	2	3	4	4	3	2	2	3	3	2	1
1	2	3	3	2	2	2	1	1	2	3	4	5	4	3	3	4	3	2	1
1	2	3	4	3	3	2	1	1	2	3	4	4	3	2	3	4	3	2	1
1	2	3	4	4	4	3	2	2	3	4	4	3	2	1	2	3	3	2	1
1	2	3	3	3	3	3	3	3	3	3	3	2	1	1	2	3	3	2	1
1	2	2	2	2	2	2	2	2	2	2	2	1	1	1	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Potential functions

- Brushfire algorithm:

  with 8-point connectivity, 4th iteration

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	2	2	2	2	1	1	2	2	2	2	2	2	1	1	2	2	2	1
1	2	3	3	3	2	1	1	2	3	3	3	3	2	1	1	2	3	2	1
1	2	3	4	3	2	2	2	2	3	4	4	3	2	1	1	2	3	2	1
1	2	3	3	3	3	3	3	3	3	3	4	3	2	1	1	2	3	2	1
1	2	3	2	2	2	2	2	2	2	3	4	3	2	1	1	2	3	2	1
1	2	3	2	1	1	1	1	1	2	3	4	3	2	1	1	2	3	2	1
1	2	3	2	1	1	1	1	1	2	3	4	3	2	2	2	2	3	2	1
1	2	3	2	2	2	2	1	1	2	3	4	3	3	3	3	3	3	2	1
1	2	3	3	3	3	2	1	1	2	3	3	3	2	2	2	3	3	2	1
1	2	3	4	4	3	2	2	2	2	3	3	2	2	1	2	3	3	2	1
1	2	3	3	3	3	3	3	3	3	3	2	2	1	1	2	3	3	2	1
1	2	2	2	2	2	2	2	2	2	2	2	1	1	1	2	2	2	2	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Potential functions

- Wavefront planner:
  - Planner based on the brushfire algorithm
  - The algorithm starts from the goal position (labelled with a 2)
  - The "1" cells are not considered
  - The result is the distance to the goal (-2)
  - Gradient descent indicates the direction to go
  - Drawbacks
    - The planner has to search the entire space
    - Does not scale well in higher dimensions or big spaces!! Computationally intractable. In 3D,

      4-point connectivity → 6-point connectivity

      8-point connectivity → 26-point connectivity

Potential functions

- Wavefront planner:

  with 4-point connectivity, 1st iteration

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	2	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	1	1	1	1	1	0	0	0	0	0	1	1	0	0	0	1
1	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Potential functions

- Wavefront planner:

  with 4-point connectivity, 10th iteration

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	4	3	4	1
1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	3	2	3	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	4	3	4	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	5	4	5	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	6	5	6	1
1	0	0	0	1	1	1	1	1	0	0	0	0	0	1	1	7	6	7	1
1	0	0	0	1	1	1	1	1	0	0	0	0	11	10	9	8	7	8	1
1	0	0	0	0	0	0	1	1	0	0	0	0	0	11	10	9	8	9	1
1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	11	10	9	10	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	11	10	11	1
1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	11	0	1
1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Potential functions

- Wavefront planner:

  with 4-point connectivity, 27th iteration

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	29	28	27	26	25	1	1	22	21	20	19	18	17	1	1	4	3	4	1
1	28	27	26	25	24	1	1	21	20	19	18	17	16	1	1	3	2	3	1
1	27	26	25	24	23	22	21	20	19	18	17	16	15	1	1	4	3	4	1
1	26	25	24	23	22	21	20	19	18	17	16	15	14	1	1	5	4	5	1
1	25	24	23	22	21	20	19	18	17	16	15	14	13	1	1	6	5	6	1
1	26	25	24	1	1	1	1	1	16	15	14	13	12	1	1	7	6	7	1
1	27	26	25	1	1	1	1	1	15	14	13	12	11	10	9	8	7	8	1
1	28	27	26	25	24	23	1	1	16	15	14	13	12	11	10	9	8	9	1
1	27	26	25	24	23	22	1	1	17	16	15	14	13	12	11	10	9	10	1
1	26	25	24	23	22	21	20	19	18	17	16	15	14	1	12	11	10	11	1
1	27	26	25	24	23	22	21	20	19	18	17	16	1	1	13	12	11	12	1
1	28	27	26	25	24	23	22	21	20	19	18	1	1	1	14	13	12	13	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Potential functions

- Wavefront planner:

  with 4-point connectivity, one shortest trajectory

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	29	28	27	26	25	1	1	22	21	20	19	18	17	1	1	4	3	4	1
1	28	27	26	25	24	1	1	21	20	19	18	17	16	1	1	3	2	3	1
1	27	26	25	24	23	22	21	20	19	18	17	16	15	1	1	4	3	4	1
1	26	25	24	23	22	21	20	19	18	17	16	15	14	1	1	5	4	5	1
1	25	24	23	22	21	20	19	18	17	16	15	14	13	1	1	6	5	6	1
1	26	25	24	1	1	1	1	1	16	15	14	13	12	1	1	7	6	7	1
1	27	26	25	1	1	1	1	1	15	14	13	12	11	10	9	8	7	8	1
1	28	27	26	25	24	23	1	1	16	15	14	13	12	11	10	9	8	9	1
1	27	26	25	24	23	22	1	1	17	16	15	14	13	12	11	10	9	10	1
1	26	25	24	23	22	21	20	19	18	17	16	15	14	1	12	11	10	11	1
1	27	26	25	24	23	22	21	20	19	18	17	16	1	1	13	12	11	12	1
1	28	27	26	25	24	23	22	21	20	19	18	1	1	1	14	13	12	13	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

From starting point, gradient descent indicates direction to goal.

Potential functions

- Wavefront planner:

  with 8-point connectivity, 20th iteration

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	21	20	19	18	18	1	1	14	14	14	14	14	14	1	1	3	3	3	1
1	21	20	19	18	17	1	1	14	13	13	13	13	13	1	1	3	2	3	1
1	21	20	19	18	17	16	15	14	13	12	12	12	12	1	1	3	3	3	1
1	21	20	19	18	17	16	15	14	13	12	11	11	11	1	1	4	4	4	1
1	21	20	19	18	17	16	15	14	13	12	11	10	10	1	1	5	5	5	1
1	21	20	19	1	1	1	1	1	13	12	11	10	9	1	1	6	6	6	1
1	21	20	19	1	1	1	1	1	13	12	11	10	9	8	7	7	7	7	1
1	21	20	19	18	17	17	1	1	13	12	11	10	9	8	8	8	8	8	1
1	21	20	19	18	17	16	1	1	13	12	11	10	9	9	9	9	9	9	1
1	21	20	19	18	17	16	15	14	13	12	11	10	10	1	10	10	10	10	1
1	21	20	19	18	17	16	15	14	13	12	11	11	1	1	11	11	11	11	1
1	21	20	19	18	17	16	15	14	13	12	12	1	1	1	12	12	12	12	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Potential functions

- Wavefront planner:

  with 8-point connectivity, one shortest trajectory

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	21	20	19	18	18	1	1	14	14	14	14	14	14	1	1	3	3	3	1
1	21	20	19	18	17	1	1	14	13	13	13	13	13	1	1	3	2	3	1
1	21	20	19	18	17	16	15	14	13	12	12	12	12	1	1	3	3	3	1
1	21	20	19	18	17	16	15	14	13	12	11	11	11	1	1	4	4	4	1
1	21	20	19	18	17	16	15	14	13	12	11	10	10	1	1	5	5	5	1
1	21	20	19	1	1	1	1	1	13	12	11	10	9	1	1	6	6	6	1
1	21	20	19	1	1	1	1	1	13	12	11	10	9	8	7	7	7	7	1
1	21	20	19	18	17	17	1	1	13	12	11	10	9	8	8	8	8	8	1
1	21	20	19	18	17	16	1	1	13	12	11	10	9	9	9	9	9	9	1
1	21	20	19	18	17	16	15	14	13	12	11	10	10	1	10	10	10	10	1
1	21	20	19	18	17	16	15	14	13	12	11	11	1	1	11	11	11	11	1
1	21	20	19	18	17	16	15	14	13	12	12	1	1	1	12	12	12	12	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

From starting point, gradient descent indicates direction to goal.