F20DL / F21DL Data Mining and Machine Learning Lecture 9 Attribute (feature) selection

Diana Bental
Ekaterina Komendantskaya
(with material from David Corne)

09/10/2018

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Attribute selection

- · You should be able to
 - Decide when to apply attribute selection
 - Choose suitable methods
 - Evaluate the results
- Coursework 1

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Attribute (feature) selection – what?

- You have some data, and you want to use it to build a classifier so that you can predict something (e.g. likelihood of cancer)
- The data has 10 000 attributes (features)
- You need to cut it down to 1 000 fields before you try machine learning. Which 1,000?
- The process of choosing the 1,000 fields to use is called Attribute Selection (or Feature Selection)

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Attribute selection – why?

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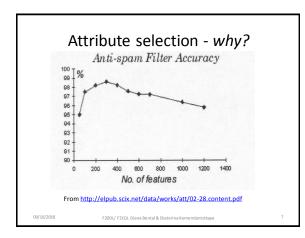
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Attribute selection – why?

- 1. Datasets with many attributes
 - Gene expression datasets (~10,000 attributes)
 - http://www.ncbi.nlm.nih.gov/sites/entrez?db=gds
 - Proteomics (proteins and peptides) datasets (~20,000 attributes)
 - http://www.ebi.ac.uk/pride/
 - Satellite data
 - Weather data etc
- · Computationally expensive

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Attribute selection - why?

 Quite easy to find lots more cases from papers, where experiments show that accuracy reduces when you use more attributes

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Attribute selection - why?

- Why does accuracy reduce with more attributes?
- Suppose the best attribute set has 20 attributes. If you add another 5 attributes, typically the accuracy of machine learning may reduce. But you still have the original 20 attributes!! Why does this happen???

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Noise / Spurious Correlations / Explosion

- The additional features typically add noise.
- Machine learning will pick up on spurious correlations, that might be true in the training set, but not in the test set.
- For some ML methods, more attributes means more parameters to learn (more NN weights, more decision tree nodes, etc...) – the increased space of possibilities is more difficult to search.

Recent survey shows that 100% of people who drink water, die.

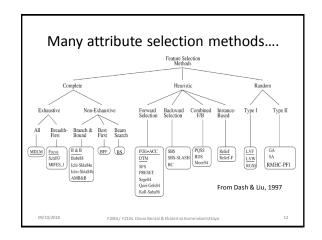
That's a fact.

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Attribute selection - why?

 So: Aim to remove attributes that are redundant, irrelevant, only weakly relevant

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Method types: Filter Methods and Wrapper Methods

- Filter methods
 - Use correlation, or information gain, or some other method, to pick some attributes
 - Then run the machine learning method on just those attributes
- · Wrapper methods
 - Choose some attributes
 - Run the machine learning method
 - Is it good enough?
 - If so then stop.
 - If not then change the set of attributes
 - And repeat.

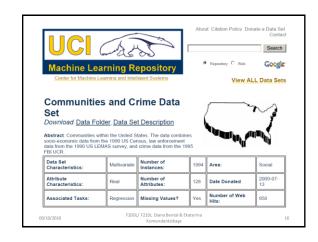
Filter Method: Correlation based ranking

- · It is used often by practitioners
- · It is fine for certain datasets
- · (Not considered in Dash and Liu's survey at all)

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e.g. An interesting data set...

• The Communities and Crime dataset (C&C)



- state: US state (by number) county: numeric code for county community: numeric code for community --- communityname: community name –
 -- fold: fold number for non-random 10 fold cross validation, population: population for community: (numeric - decimal)
 householdsize: mean people per household (numeric - decimal)
 agePct12t21: percentage of population that is 12-21 in age (numeric - decimal) agePct12t29: percentage of population that is 12-29 in age (numeric - decimal) agePct16t24: percentage of population that is 16-24 in age (numeric - decimal) agerct6sup: percentage of population that is 65 and over in age (numeric - decimal)
 numbUrban: number of people living in areas classified as urban (numeric - decimal)
 pctUrban: percentage of people living in areas classified as urban (numeric - decimal) - medincome: median household income (numeric - decimal) -- pctWWage: percentage of households with wage or salary income in 1989 (numeric - decimal)
 - pctWFarmSelf: percentage of households with farm or self employment income in 1989 [etc etc etc --- 128 fields altogether] ViolentCrimesPerPop: total number of violent crimes per 100K population (numeric - decimal) class attribute (to be predicted) F20DL/ F21DL Diana Bental & Ekaterina Komendantstkava
- .. About 2000 instances F20DL/ F21DL Diana Bental & Ekaterina Komendantstkava

Recap: Correlation

- Remember how to calculate r
- If we have pairs of values (x, y) Pearson's r is

$$\frac{1}{(n-1)} \cdot \sum_{(x,y) \in Sample} \frac{(x - \mu_x)}{std_x} \cdot \frac{(y - \mu_y)}{std_y}$$

• r between -1 and 1, 0 = no correlation

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	The names f	ile in	the C&	cC data	set has con	rrelation	
	values (with	the c	lass att	ribute)	for each a	ttribute	
	min	Max	mean	std	correlation	median	mode
Population	0	1	0.06	0.13	0.37	0.02	0.01
Householdsize	0	1	0.46	0.16	-0.03	0.44	0.41
agePct12t21	0	1	0.42	0.16	0.06	0.4	0.38
agePct12t29	0	1	0.49	0.14	0.15	0.48	0.49
agePct16t24	0	1	0.34	0.17	0.1	0.29	0.29
agePct65up	0	1	0.42	0.18	0.07	0.42	0.47
numbUrban	0	1	0.06	0.13	0.36	0.03	0
pctUrban	0	1	0.7	0.44	0.08	1	1
medincome	0	1	0.36	0.21	-0.42	0.32	0.23
pctWWage	0	1	0.56	0.18	-0.31	0.56	0.58
pctWFarmSelf	0	1	0.29	0.2	-0.15	0.23	0.16
pctWInvinc	0	1	0.5	0.18	-0.58	0.48	0.41
pctWSocSec	0	1	0.47	0.17	0.12	0.475	0.56
pctWPubAsst	0	1	0.32	0.22	0.57	0.26	0.1
pctWRetire	0	1	0.48	0.17	-0.1	0.47	0.44
medFamInc	0	1	0.38	0.2	-0.44	0.33	0.25
perCapInc	0	1	0.35	0.19	-0.35	0.3	0.23
NumUnderPov	0	1	0.06	0.13	0.45	0.02	0.01
PctPopUnderPo	, 0	1	0.3	0.23	0.52	0.25	0.08
PctLess9thGrad	е 0	1	0.32	0.21	0.41	0.27	0.19

ere						1	
	min	Max	mean	std	correlation i	nedian	mode
Population	0	1	0.06	0.13	0.37	0.02	0.01
Householdsize	0	1	0.46	0.16	-0.03	0.44	0.41
agePct12t21	0	1	0.42	0.16	0.06	0.4	0.38
agePct12t29	0	1	0.49	0.14	0.15	0.48	0.49
agePct16t24	0	1	0.34	0.17	0.1	0.29	0.29
agePct65up	0	1	0.42	0.18	0.07	0.42	0.47
numbUrban	0	1	0.06	0.13	0.36	0.03	0
pctUrban	0	1	0.7	0.44	0.08	- 1	1
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NumUnderPov	0	1	0.06	0.13	0.45	0.02	0.01
PctPopUnderPov	0	1	0.3	0.23	0.52	0.25	0.08
PctLess9thGrade	0	1	0.32	0.21	0.41	0.27	0.19

But...

- Can anyone see a potential problem with choosing only (for example) the 10 attributes that correlate best with the target class?
- So .. Look for attributes which correlate highly with the class attribute and do not correlate with each other
- · Other filter methods:
 - Entropy/Information gain methods.
 - Build a decision tree and reject the unused attributes, then use nearest neighbour
 - Relief method see end slides if interested

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Filter methods

- First choose the attributes, and *then* apply machine learning
- Use a heuristic (clever guessing method) to decide which attributes are *likely* to be best
- Can use different kinds of machine learning methods afterwards
- Fast not iterative, chooses attributes without repeatedly running the machine learning method
- May overfit to the data
- Tend to select big subsets may select the full attribute set as the "best" set
- · Filter methods generally look at attributes individually

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A made-up dataset

f1	f2	f3	f4	 class
0.4	0.6	0.4	0.6	1
0.2	0.4	1.6	-0.6	1
0.5	0.7	1.8	-0.8	1
0.7	0.8	0.2	0.9	2
0.9	0.8	1.8	-0.7	2
0.5	0.5	0.6	0.5	2

Correlated with the class

		1		
f1	f2	f3	f4	 class
0.4	0.6	0.4	0.6	1
0.2	0.4	1.6	-0.6	1
0.5	0.7	1.8	-0.8	1
0.7	0.8	0.2	0.9	2
0.9	0.8	1.8	-0.7	2
0.5	0.5	0.6	0.5	2

uncorrelated with the class / seemingly random

			_			
f1	f2	f3		f4		class
0.4	0.6	0.4		0.6		1
0.2	0.4	1.6		-0.6		1
0.5	0.7	1.8		-0.8		1
0.7	0.8	0.2		0.9		2
0.9	0.8	1.8		-0.7		2
0.5	0.5	0.6		0.5		2

David Corne, and Nick Taylor, Heriot-Watt University - dwcorne@gmail.com

So correlation based attribute selection reduces the dataset to this.

f1	f2		 class
0.4	0.6		1
0.2	0.4		1
0.5	0.7		1
0.7	0.8		2
0.9	0.8		2
0.5	0.5		2

But, col 5 shows us f3 + f4 – which is perfectly correlated with the class!

			_			
f1	f2	f3		f4	f5	class
0.4	0.6	0.4	I	0.6	1	1
0.2	0.4	1.6		-0.6	1	1
0.5	0.7	1.8		-0.8	1	1
0.7	0.8	0.2		0.9	1.1	2
0.9	0.8	1.8		-0.7	1.1	2
0.5	0.5	0.6		0.5	1.1	2

That example is cheating a bit...

- Adding a new attribute based on existing attributes is feature extraction
- It would be very hard to guess that we should add those two attributes together
- But feature extraction is a common operation in e.g. image processing
 - Look for edges, surfaces, eyes... Before interpreting the image as a whole

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 So... Need to consider how well attributes work together

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`Complete' methods

- · Original dataset has N attributes
- You want to use a subset of k attributes
- A complete attribute selection method means: try every subset of k attributes, and choose the best!
 - the number of subsets is N! / k!(N-k)!
 - what is this when N is 100 and k is 5?
 - 75.287.520 -- almost nothing

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'Complete' methods

- · Original dataset has N attributes
- You want to use a subset of k attributes
- A complete attribute selection method means: try every subset of k attributes, and choose the best!
 - the number of subsets is N! / k!(N-k)!
 - what is this when N is 10 000 and k is 100?

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'Complete' methods

- · Original dataset has N attributes
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- A complete attribute selection method means: try every subset of k attributes, and choose the best!
 - the number of subsets is N! / k!(N-k)!
 - what is this when N is 10 000 and k is 100?
 - -5,000,000,000,000,000,000,000,000,000,

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Continued 2

Continued 3

Continued 4

... continued for another 114 slides.

Actually it is around $5 \times 10^{35,101}$ (there are around 10^{80} atoms in the universe)

'Complete' methods

 Can you see a problem with complete methods?

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Stochastic filter methods

- One way to try to find a good subset is to run a stochastic search algorithm
 - e.g. Hillclimbing, simulated annealing, genetic algorithm, particle swarm optimisation, ...

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Wrapper methods Choose the attributes alternately with running the machine learning method Change attributes Change attributes Change attributes Change attributes Change attributes Factor (F210): Change Bental & Exterina Salada Section (Saladarerina) Factor (F210): Change Bental & Exterina Salada Section (Saladarerina) Factor (F210): Change Bental & Exterina Salada Section (Saladarerina) Salada Section (Saladarerina)

`Forward' wrapper methods

These methods 'grow' a set S of attributes –

- 1. S starts empty
- Find the best attribute to add (by checking each attribute in turn to see which one gives best performance on a test set when combined with S).
- 3. If overall performance has improved, return to step 2; else stop

Forward selection illustrated

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	

Selected attribute set {}

Run ML with each attribute in turn to find out which works best with current attribute set ...

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	

Selected attribute set {}

Test each attribute in turn to find out which works best with current attribute set ...

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	ļ
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	
65%		•	•		•	•			

Selected attribute set {}

Test each attribute in turn to find out which works best with current attribute set ...

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	
	58%								

Selected attribute set {}

Test each attribute in turn to find out which works best with current attribute set ...

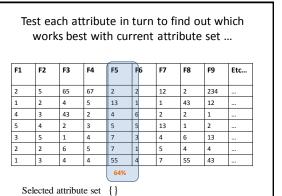
F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	

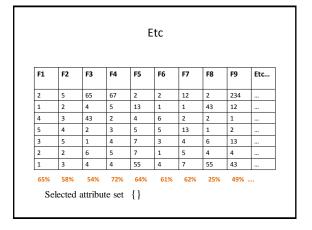
Selected attribute set {}

Test each attribute in turn to find out which works best with current attribute set ...

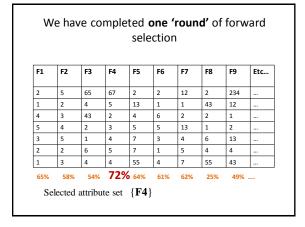
F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	

Selected attribute set {}





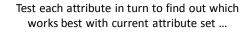
			ŭ		ute s				
F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	
65%	58%	54%	72%	6 64%	61%	62%	25%	49%	



works best with current attribute set ... F1 F2 F3 F4 F5 F6 F7 F8 F9 Etc... 234 65 67 2 2 12 13 43 12 43 2 13 2 13 6 55 43 Selected attribute set $\{F4\}$

Test each attribute in turn to find out which

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
 2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	В	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	В	4	4	55	4	7	55	43	



F1		F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	l	5	65	67	2	2	12	2	234	
1		2	4	5	13	1	1	43	12	
4		3	43	2	4	6	2	2	1	
5		4	2	3	5	5	13	1	2	
3		5	1	4	7	3	4	6	13	
2		2	6	5	7	1	5	4	4	
1		3	4	4	55	4	7	55	43	
		59%		•						

Selected attribute set $\ \{F4\}$

Test each attribute in turn to find out which works best with current attribute set ...

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	
		58%							

Selected attribute set $\{F4\}$

Test each attribute in turn to find out which works best with current attribute set ...

F1	F2	F3	F4	F5	F	6	F7	F8	F9	Etc
2	5	65	67	2	2		12	2	234	
1	2	4	5	13	1		1	43	12	
4	3	43	2	4	6		2	2	1	
5	4	2	3	5	5		13	1	2	
3	5	1	4	7	3		4	6	13	
2	2	6	5	7	1		5	4	4	
1	3	4	4	55	4		7	55	43	
	_	•		66%	_		•	'		

Selected attribute set $\{F4\}$

Etc

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	
C40/	=00/	=00/		660/	C00/	250/	470/	400/	

Selected attribute set $\ \{F4\}$

Add the winning attribute to the selected attribute set

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	
61%	59%	58%		66%	68%	75%	47%	49%	

Selected attribute set {F4, F7}

We have completed the second '**round'** of forward selection

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	
619/	EQ9/	58%		66%	68%	75%	47%	40%	•

Selected attribute set {F4, F7}

Continue...

adding one attribute after each round, until overall accuracy starts to reduce

F1	F2	F3	F4	F5	F6	F7	F8	F9	Etc
2	5	65	67	2	2	12	2	234	
1	2	4	5	13	1	1	43	12	
4	3	43	2	4	6	2	2	1	
5	4	2	3	5	5	13	1	2	
3	5	1	4	7	3	4	6	13	
2	2	6	5	7	1	5	4	4	
1	3	4	4	55	4	7	55	43	
61%	59%	58%		66%	68%	75%	47%	49% .	

Selected attribute set $\{F4, F7\}$

'Backward' methods

These methods **remove** attributes one by one.

- 1. S starts with the full attribute set
- 2. Find the best feature to *remove* (by checking which removal from S gives best performance on a test set).
- 3. If overall performance has improved, return to step 2; else stop

Forward and backward are heuristic (clever guess) methods

- Neither forward nor backward are guaranteed to give the best set of attributes
- · When to choose forward instead of backward?

About wrapper methods

- Accurate give good accuracy with a specific classification method (but may be the wrong set for a different method)
- Avoid over-fitting by running machine learning with cross-validation
- Slow need to build and test the model for every subset of features

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Take away...

- Too many attributes may mean less accurate ML
 - Even if the attributes are relevant
- Attribute selection is difficult, with many different methods and algorithms
- Can use machine learning methods to select attributes
 - Can *filter* the attributes beforehand and then apply machine learning
 - Or wrap attribute selection in with the learning

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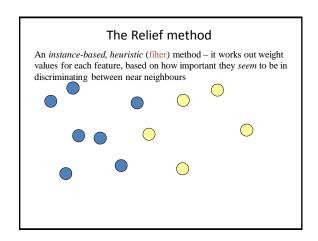
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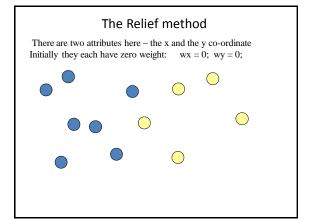
For more...

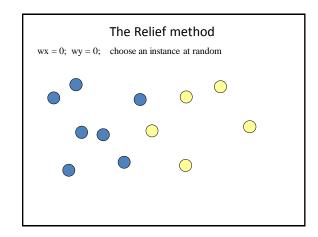
- · Data Mining and Machine Learning
 - Section 7.1 Attribute Selection
- Practical overview
 - https://machinelearningmastery.com/performfeature-selection-machine-learning-data-weka/
- Papers
 - Kira and Rendell AAAI 1992 "The Feature Selection Problem: Traditional Methods and a New Algorithm"
 - Dash and Liu IDA 1997 "Feature selection for classification"

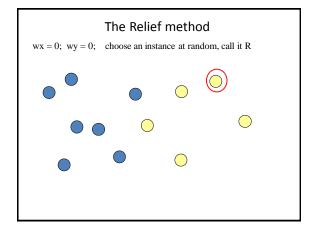
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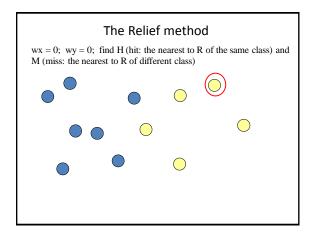
Relief is the classic example of an instance-based heuristic filter method

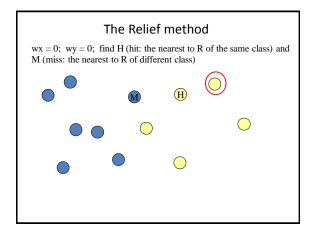


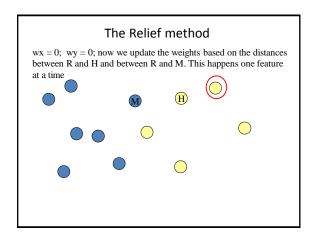


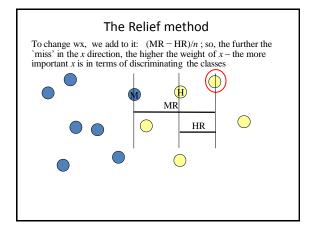


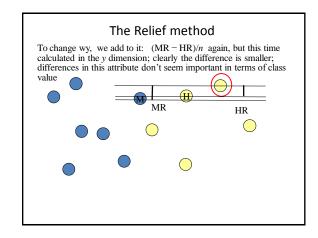


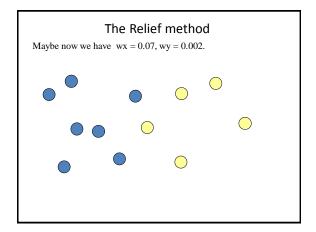


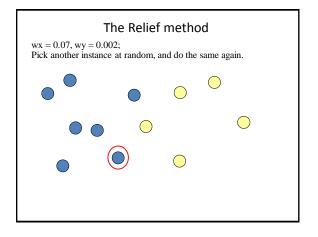


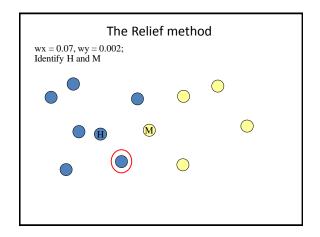


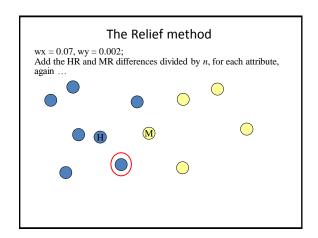












The Relief method

- In the end, there is a weight value for each attribute. The higher the value, the more relevant the attribute.
- We can use these weights for attribute selection, simply by choosing the attributes with the S highest weights (if we want to use S attributes)
- Once we have chosen S attributes, we can use a classifier with them
- Notes:
 - Relief works for numeric min-max normalized data with two classes, and for nominal data (using a Hamming distance)
 - Need to extend it for multiple classes... how?
 - Why divide by n? Then the weight values can be interpreted as a difference in probabilities.
 - Doesn't work well if there isn't enough training data

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The Relief method, plucked directly from the original paper (Kira and Rendell 1992) Relief(S, m, t) Separate S into S* = (positive instances) and S*= (negative instances)

Separate S into S⁺ = {positive instances} and S⁻ = {negative instances} W = (0, 0, 0)
For i = 1 to m
Pick at random an instance X ∈ S
Pick at random one of the positive instances
closest to X, Z⁺ ∈ S⁺
Pick at random one of the negative instances
closest to X, Z⁺ ∈ S⁺
if (X is a positive instances
then Near-hit = Z⁺, Near-miss = Z⁺
else Near-hit = Z⁺, Near-miss = Z⁺
update-weight(W, X, Near-hit, Near-miss)
Relevance = (1/m)W
For i = 1 to p
if (relevance; ≥ t)
then f_i is a relevant feature
else f_i is an irrelevant feature

 $\begin{aligned} & \text{update-weight}(W, \ X, \ Near-hit, \ Near-miss) \\ & For \ i = 1 \ \text{to} \ p \\ & W_i = W_i \cdot \text{diff}(x_i, \ near-hit_i)^2 + \text{diff}(x_i, \ near-miss_i)^2 \end{aligned}$