Lecture 3

Image Primitives
and
Correspondence

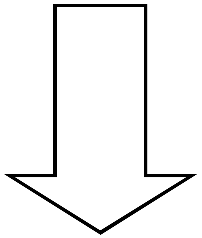
Some slides from Frolova&Simakov, David Jacobs and Cordelia Schmid

Class Objectives

- Understand the concept of interest point
  - Corner detectors
- Understand the “aperture” problem
- Know some basic similarity measures that allow to match interest points in different images
  - SSDs
  - Cross-correlation

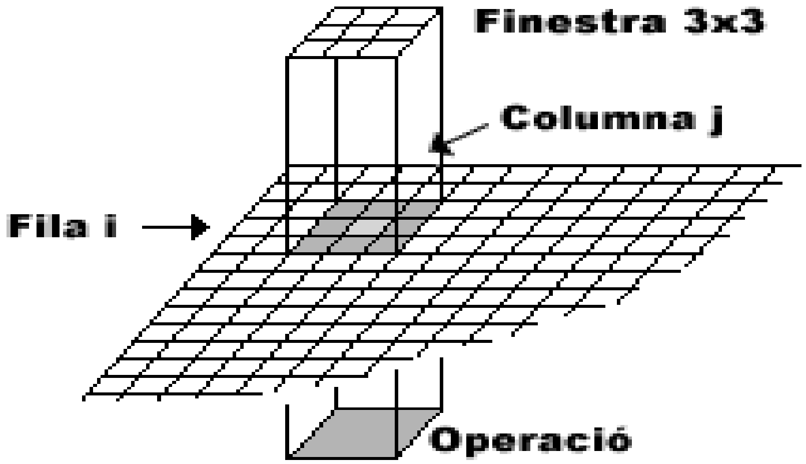
Where do we begin…

- Algorithms for extracting interest points start with an edge segmented image
- Let’s review the edge detection process

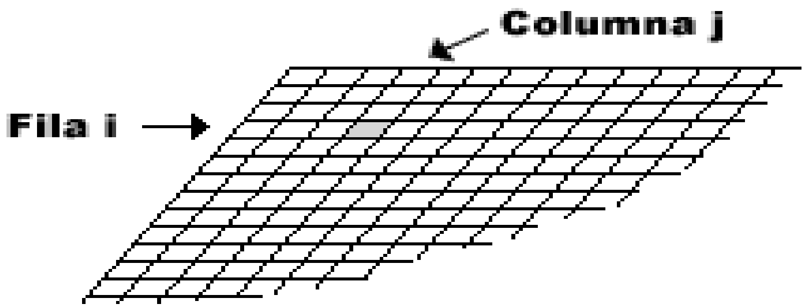


Convolution

Image filtering and enhancing



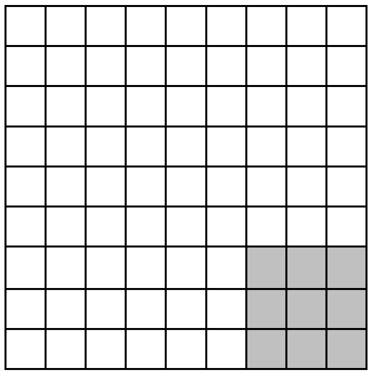
Input Image



Output Image

Convolution: basis of image filtering

- Spacial filter ⇨ “window going through the image”



Example of Convolution

Convolution: basis of image filtering

- Low-pass filtering

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$\frac{1}{16} \times$$

1	2	1
2	4	2
1	2	1

- High-pass filtering

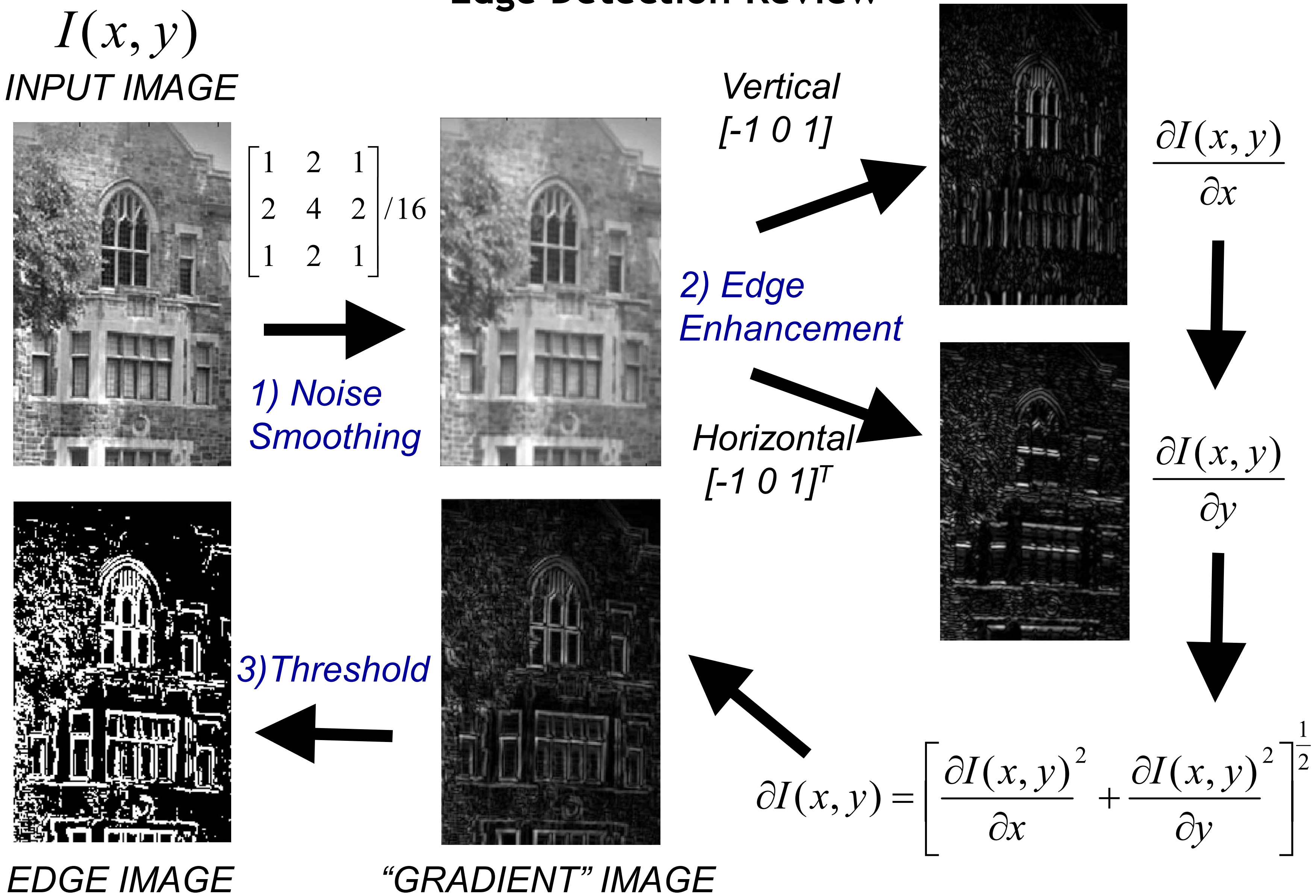
-1	0	1
-1	0	1
-1	0	1

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-2	0	2
-1	0	1

-1	-2	-1
0	0	0
1	2	1

Edge Detection Review



What are Interest Points?

- A point in an image which has a well-defined position and can be robustly detected.
- This means that an interest point can be a corner
  - Where two edges come together, i.e., intersection of two edges.
  - Where the image gradient has significant components in the *x* and *y* direction
- We will establish corners from the gradient rather than the edge images.

Moravec corner detector

- Defines a corner to be a point with low self similarity.
- For every pixel, it checks how similar a patch centred on the pixel is to nearby, largely overlapping patches.
- The similarity is measured by taking the sum of squared differences (SSD) between the two patches.

$$f(x,y) = \sum_{x_i=-\delta}^{\delta} \sum_{y_i=-\delta}^{\delta} [I(x_i, y_i) - I(x_i + \Delta x, y_i + \Delta y)]^2$$

Moravec corner detector (cont'd)

$$f(x,y)=\sum_{x_w=-\delta}^{\delta}\sum_{y_w=-\delta}^{\delta}[I(x_w,y_w)-I(x_w+\Delta x,y_w+\Delta y)]^2$$

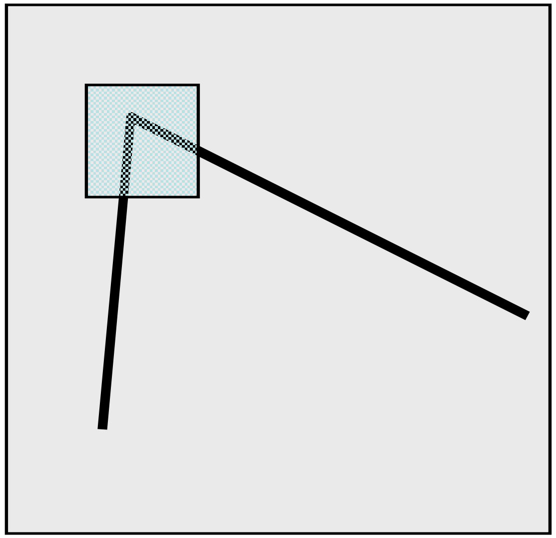
$$=\sum_{(x_w,y_w)\in W}[I(x_w,y_w)-I(x_w+\Delta x,y_w+\Delta y)]^2$$

– Four different shift directions $f_i(x,y)$: $f_{\mathrm{Moravec}}=\sum_{i=1}^{4} f_i$

– A corner is detected, when $f_{\mathrm{Moravec}}>th$

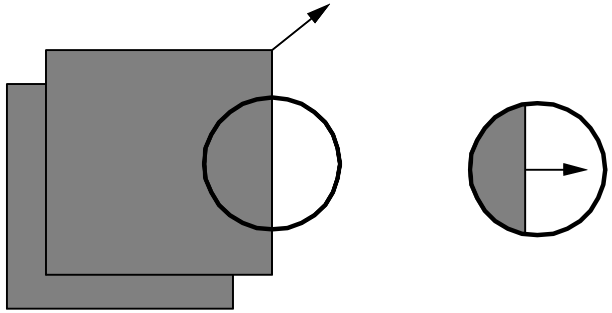
The Basic Idea

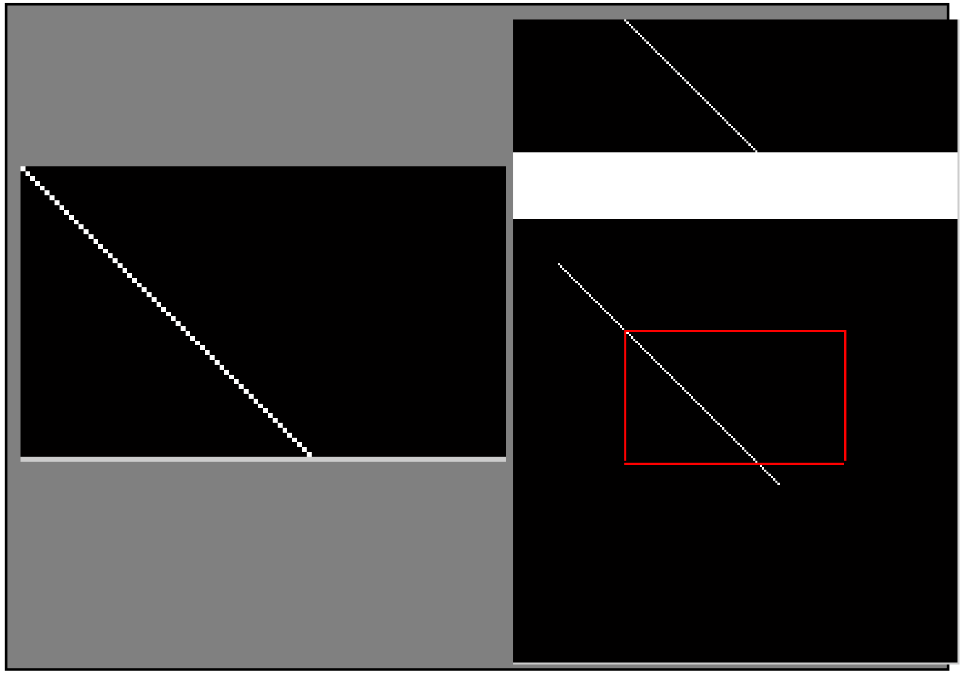
- We should easily recognize the point by looking through a small window
- Shifting a window in any direction should give a large change in intensity

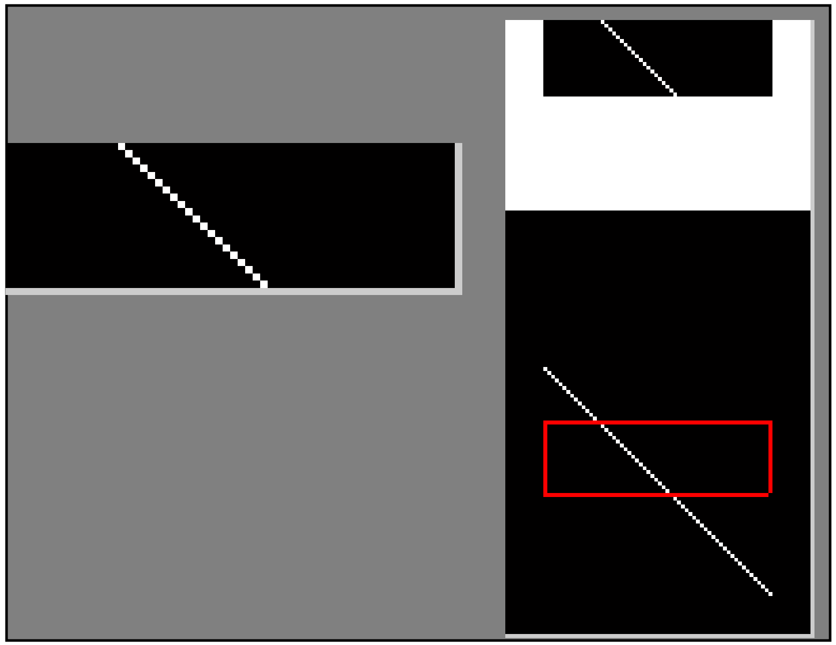


Aperture Problem

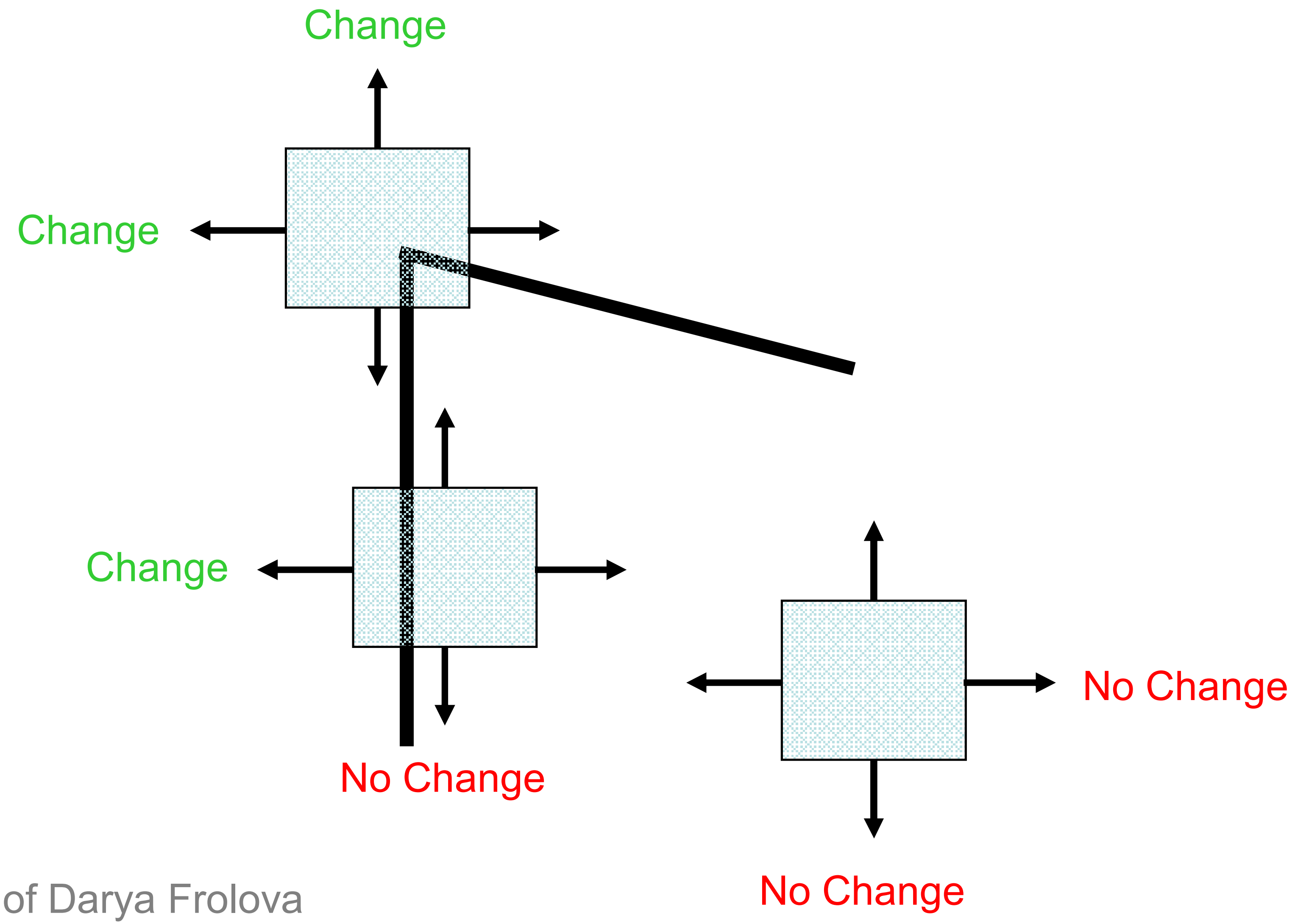
Consider the motion of an edge







Moravec Detector: Basic Idea



Courtesy of Darya Frolova

Harris corners (also based on 1st derivatives)

- Autocorrelation (*second moment*) matrix:
  - Avoids various shift directions
  - Approximate $I(x_w+\Delta x, y_w+\Delta y)$ by Taylor expansion:

$$I(x_w+\Delta x, y_w+\Delta y) \approx I(x_w, y_w) + \begin{pmatrix} I_x(x_w, y_w) & I_y(x_w, y_w) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

  - Rewrite $f(x,y)$:

$$f(x,y) = \sum_{(x_w,y_w)\in W} [\begin{pmatrix} I_x(x_w, y_w) & I_y(x_w, y_w) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}]^2$$

$$= \sum_{(x_w,y_w)\in W} \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \begin{pmatrix} I_x(x_w, y_w) \\ I_y(x_w, y_w) \end{pmatrix} \begin{pmatrix} I_x(x_w, y_w) & I_y(x_w, y_w) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

$$= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \mathbf{M} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$$

"second moment matrix **M**"

Harris Detector: Mathematics

Change of intensity for the shift [Δx, Δy]:

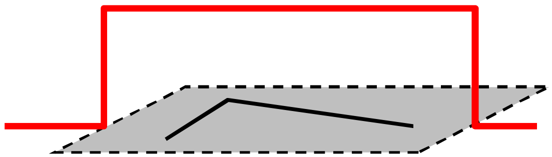
$$f(x,y) = \sum_{x,y} w(x,y)\left[I(x+\Delta x, y+\Delta y) - I(x,y)\right]^2$$

Window function

Shifted intensity

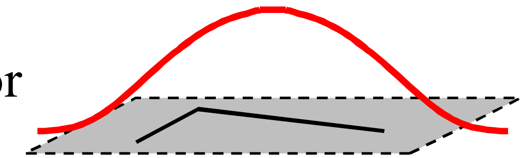
Intensity

Window function w(x,y) =



1 in window, 0 outside

or



Gaussian

Harris Detector: Mathematics

For small shifts [Δx, Δy] we have a bilinear approximation:

$$f(x,y) \cong \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \; M \; \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

(also known as squared gradient matrix or autocorrelation matrix)

Harris Detector: Mathematics

- Autocorrelation (second moment) matrix:

$$\mathbf{M} = \begin{pmatrix} \sum_{W} I_x^2 & \sum_{W} I_x I_y \\ \sum_{W} I_x I_y & \sum_{W} I_y^2 \end{pmatrix}$$

  - M can be used to derive a measure of "cornerness"
  - Independent of various displacements (Δx,Δy)
  - Corner: significant gradients in >1 directions ➔ rank **M** = 2
  - Edge: significant gradient in 1 direction ➔ rank **M** = 1
  - Homogeneous region ➔ rank **M** = 0

- Several variants of this corner detector:
  - KLT corners, Förstner corners

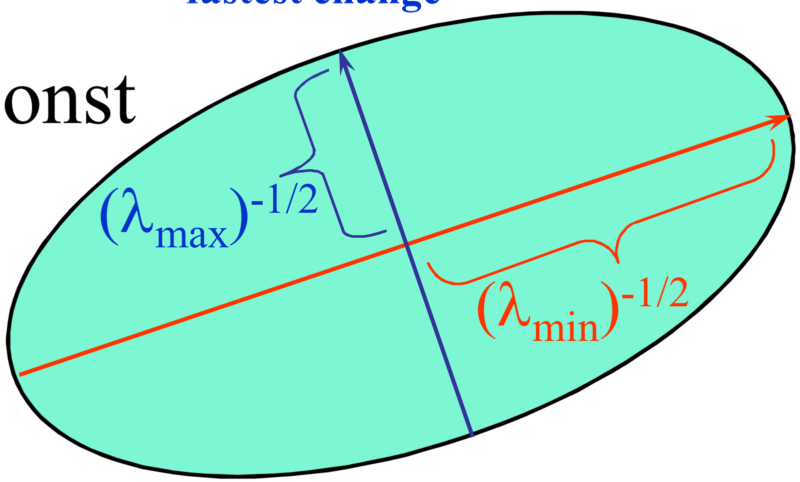
Harris Detector: Mathematics

Intensity change in shifting window: eigenvalue analysis

$$f(x,y) \cong [\Delta x \quad \Delta y] \; M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

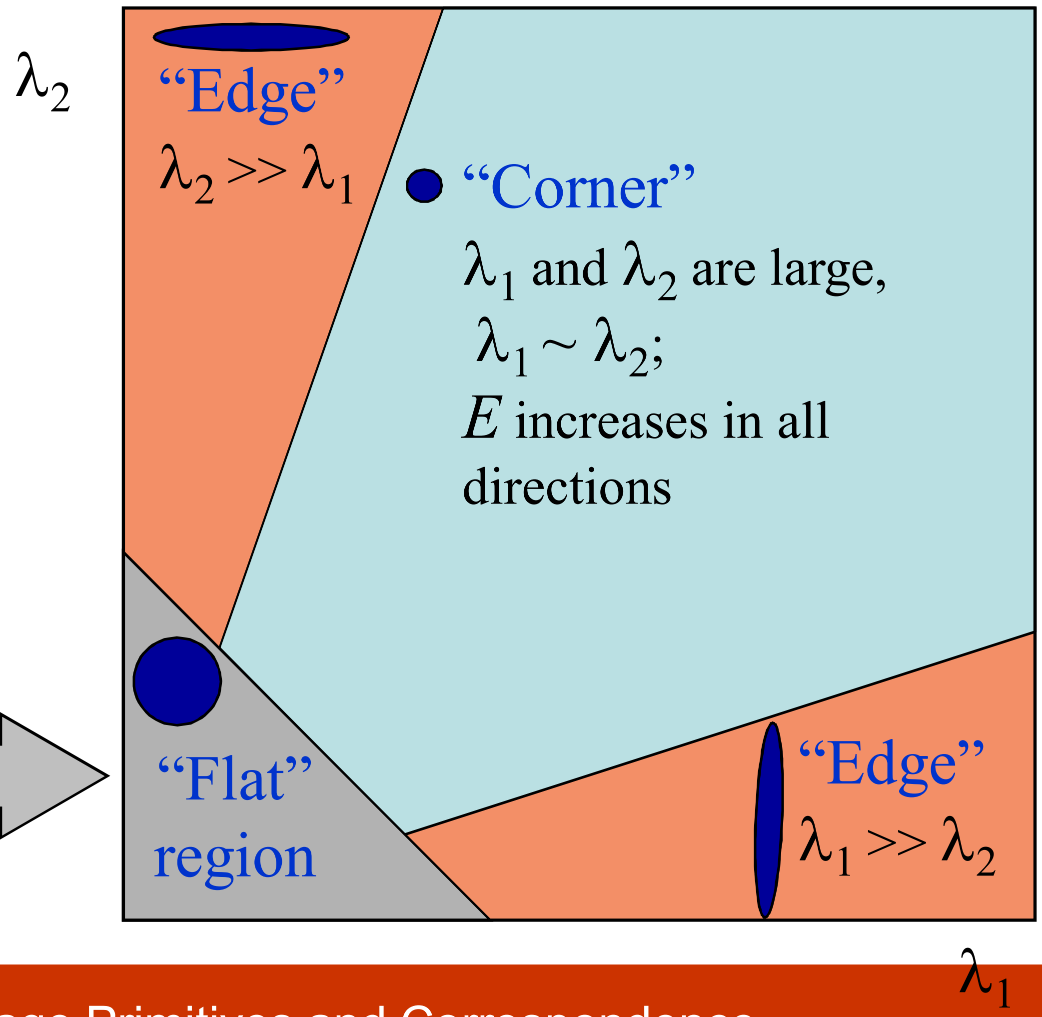
λ_1, λ_2 – eigenvalues of M

Ellipse $E(\Delta x, \Delta y) = \text{const}$



Harris Detector: Mathematics

Classification of image points using eigenvalues of M:



λ_1 and λ_2 are small;
E is almost constant in all directions

Harris Detector: Mathematics

- Harris and Stephens noted that exact computation of the eigen values is computationally expensive (since it requires a square root) and instead suggest the following function R (Measure of corner response):

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

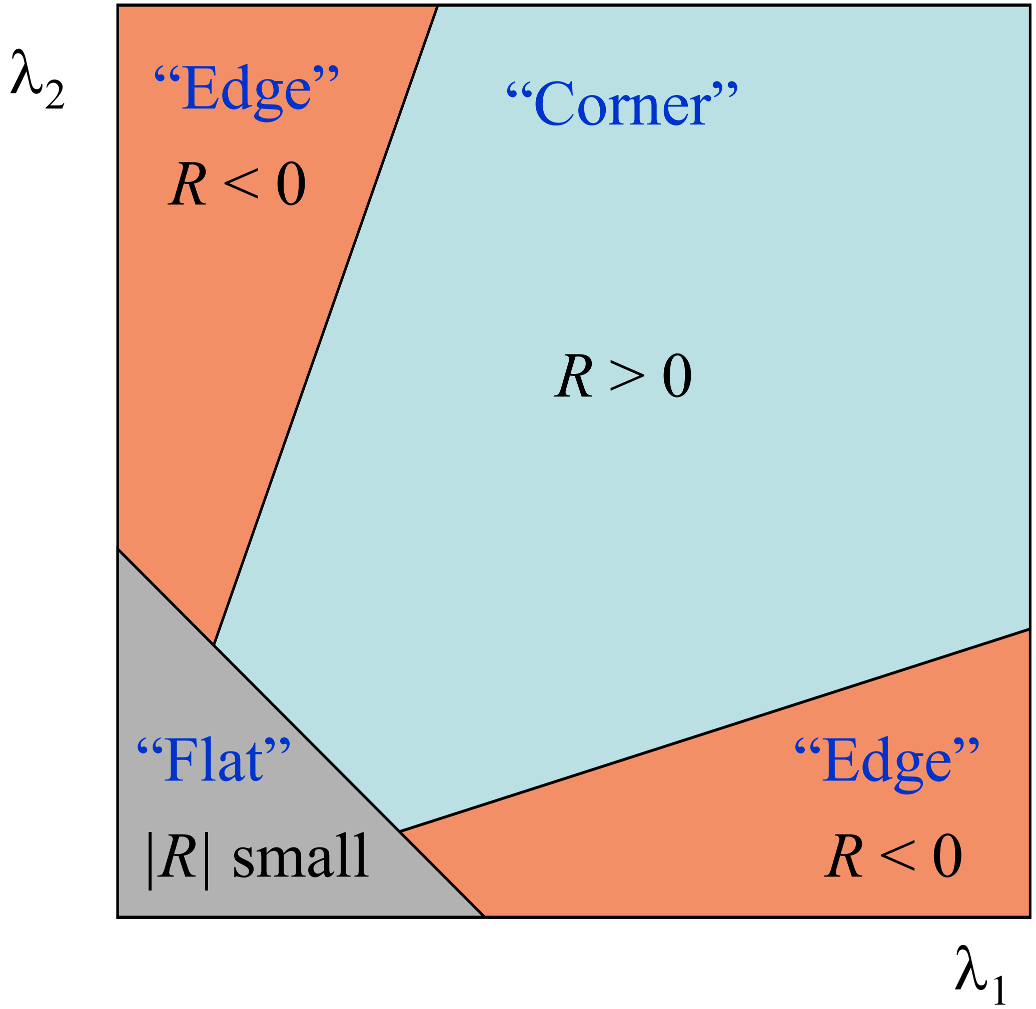
$$\det M = \lambda_1 \lambda_2$$

$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

(k – empirical constant, k = 0.04-0.06)

Harris Detector: Mathematics

- *R* depends only on eigenvalues of M
- *R* is large for a corner
- *R* is negative with large magnitude for an edge
- |*R*| is small for a flat region



Corner Detection Algorithm

1. Compute the image gradients

$$I_{x=}\frac{\partial I(x,y)}{\partial x}, I_{y=}\frac{\partial I(x,y)}{\partial y}$$

2. Define a neighborhood size as an area of interest around each pixel

10	11	12	12	10
11	11	10	10	10
13	13	50	55	55
13	15	53	55	58
18	19	53	60	61

3x3 neighborhood

Corner Detection Algorithm (cont'd)

3. ***For each image pixel (i,j)***, construct the following matrix from it and its neighborhood values

$$M_{(i,j)} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

local auto-correlation function of a signal

e.g.

$$I_x = \begin{matrix} 10 & 11 & 12 & 12 & 10 \\ 11 & 11 & 10 & 10 & 10 \\ 13 & 13 & 50 & 55 & 55 \\ 13 & 15 & 53 & 55 & 58 \\ 18 & 19 & 53 & 60 & 61 \end{matrix}$$

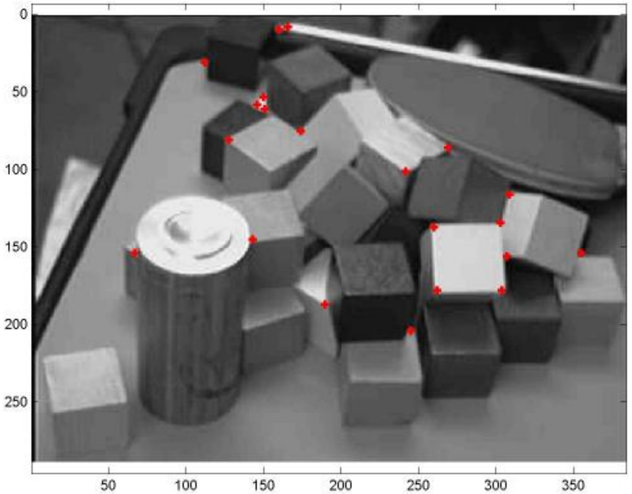
$$M_{(3,3)}[1,1] = 11^2 + 10^2 + 10^2 + 13^2 + 50^2 + 55^2 + 15^2 + 53^2 + 55^2$$

Corner Detection Algorithm (cont'd)

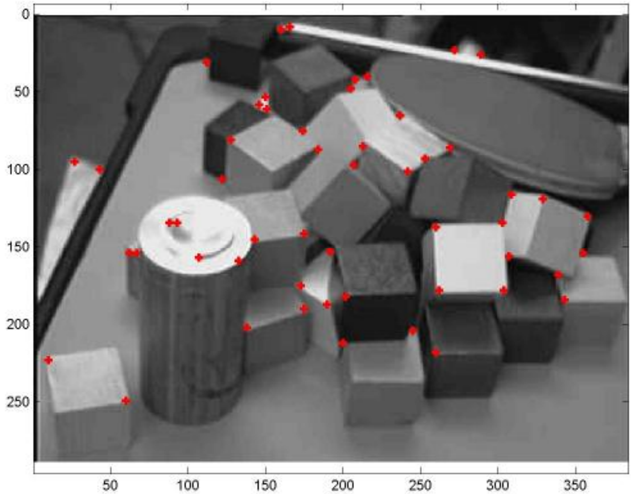
3. ***For each matrix*** $\boldsymbol{M_{(i,j)}}$, determine the 2 eigenvalues $\lambda_{(i.j)} = [\lambda_1, \lambda_2]$.
4. Construct R-image where $R(i,j)=\min(\lambda_{(i.j)})$.
5. Threshold R-image. Anything greater than threshold is a corner.

ISSUE: The corners obtained will be a function of the threshold !

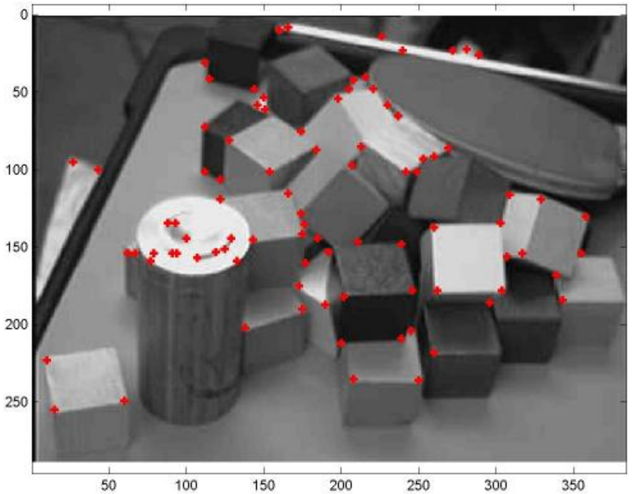
Corner Detection Sample Results



Threshold=25,000

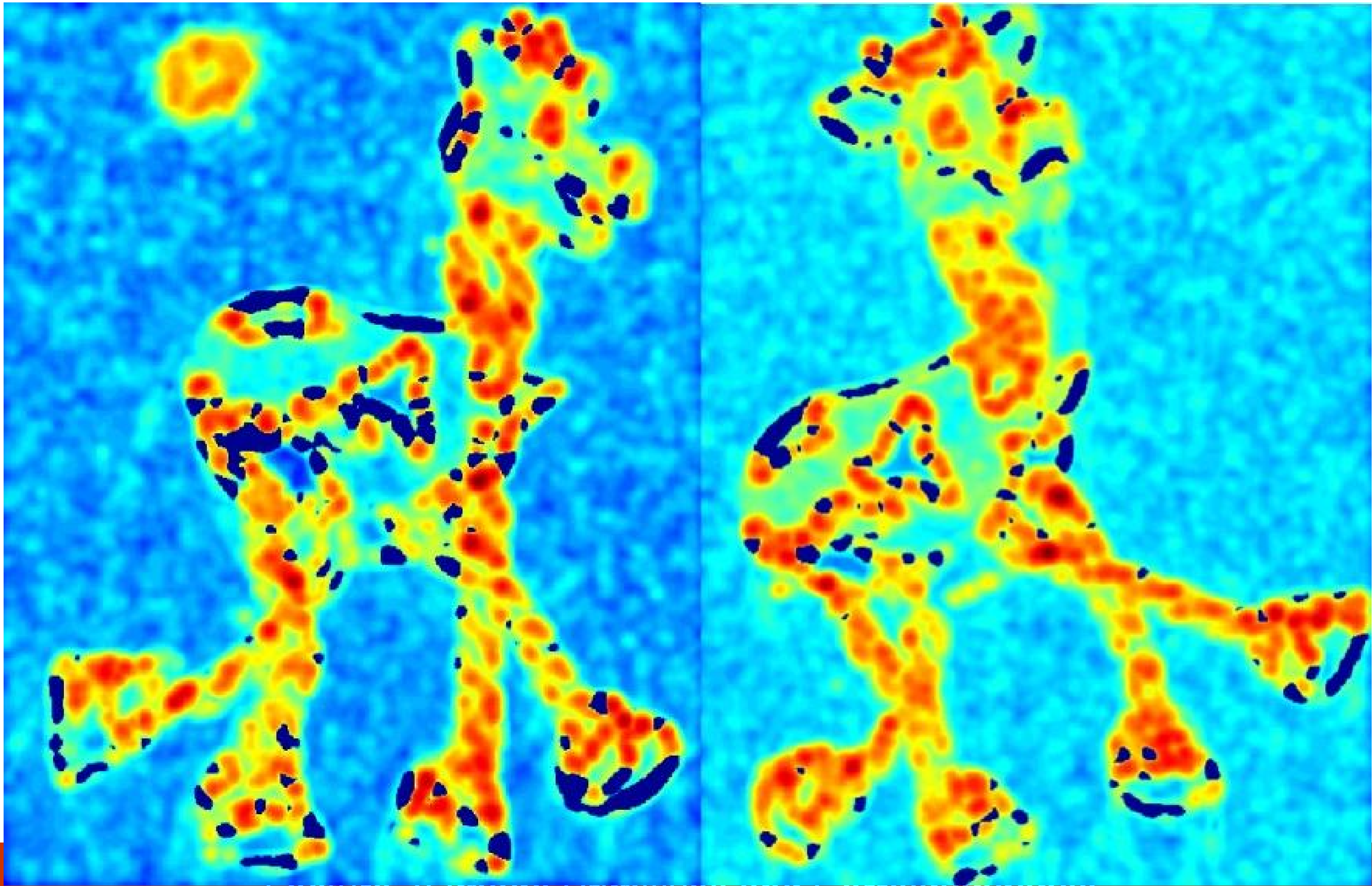


Threshold=10,000

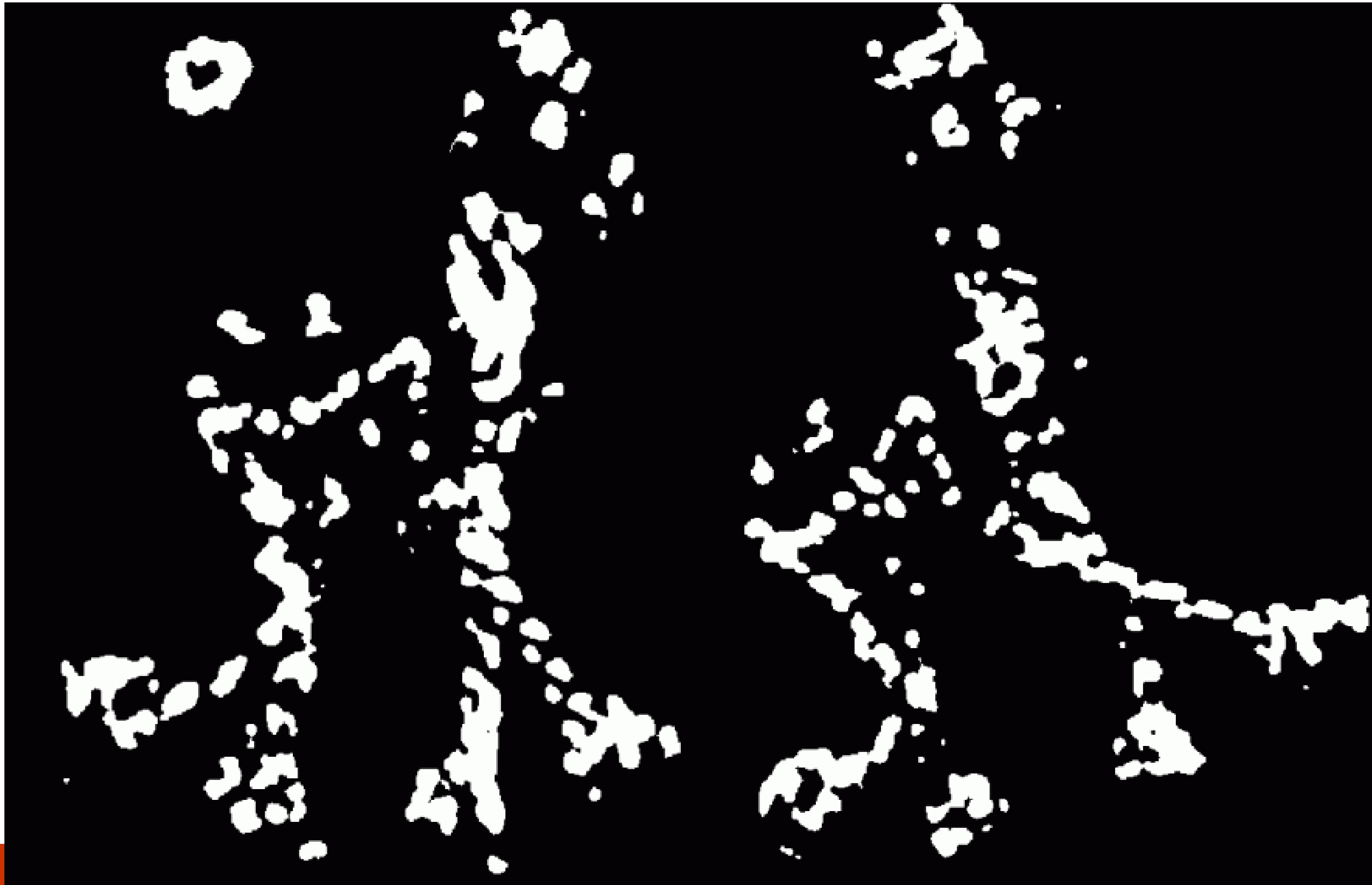


Threshold=5,000

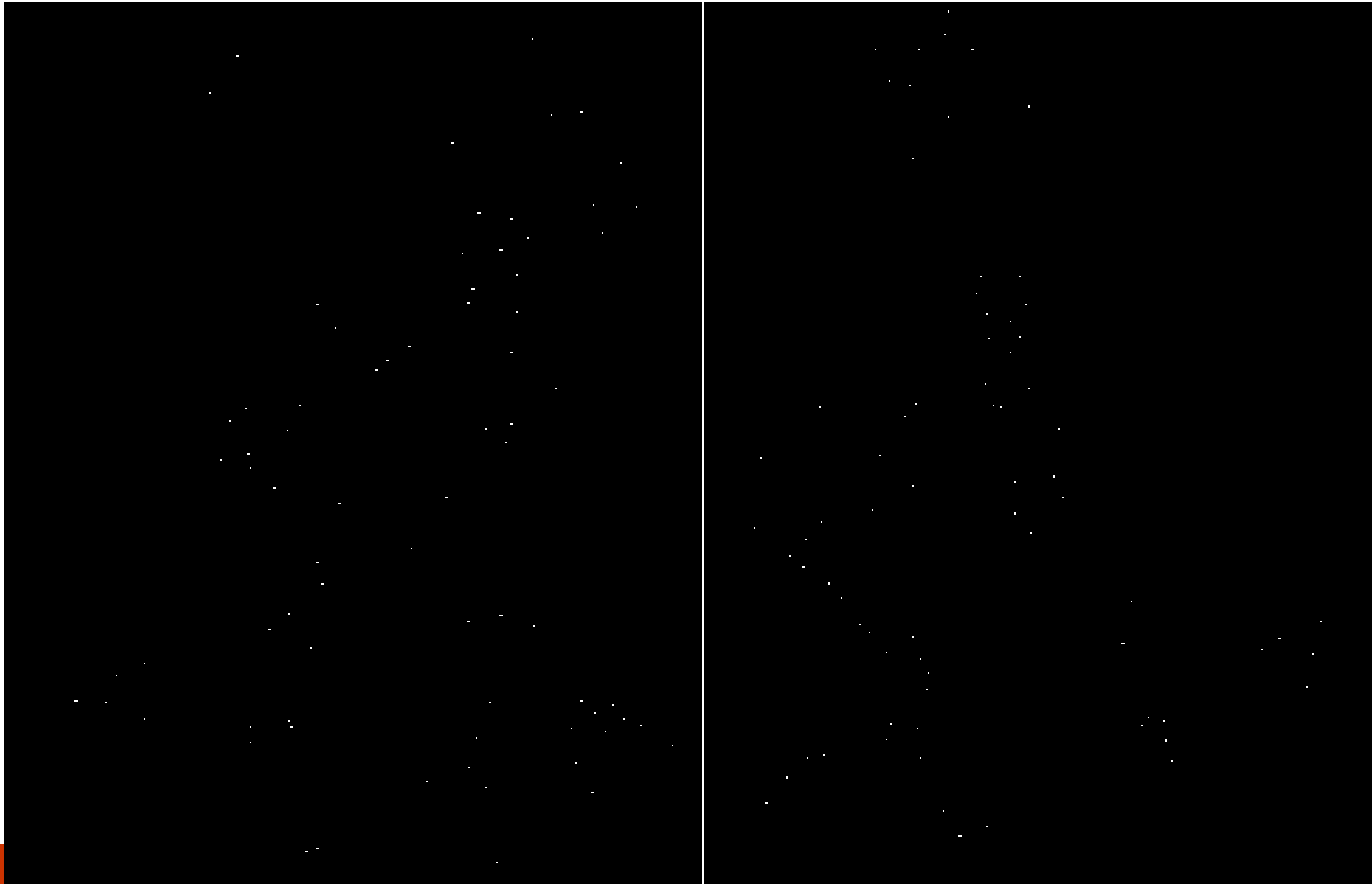
Compute corner response R



Find points with large corner response: R>threshold



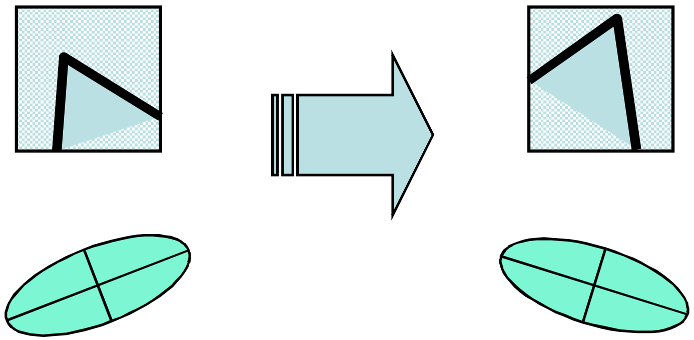
Take only the points of local maxima of R



Harris Detector: Workflow

Harris Detector: Some Properties

- Rotation invariance



Ellipse rotates but its shape (i.e. eigenvalues) remains the same

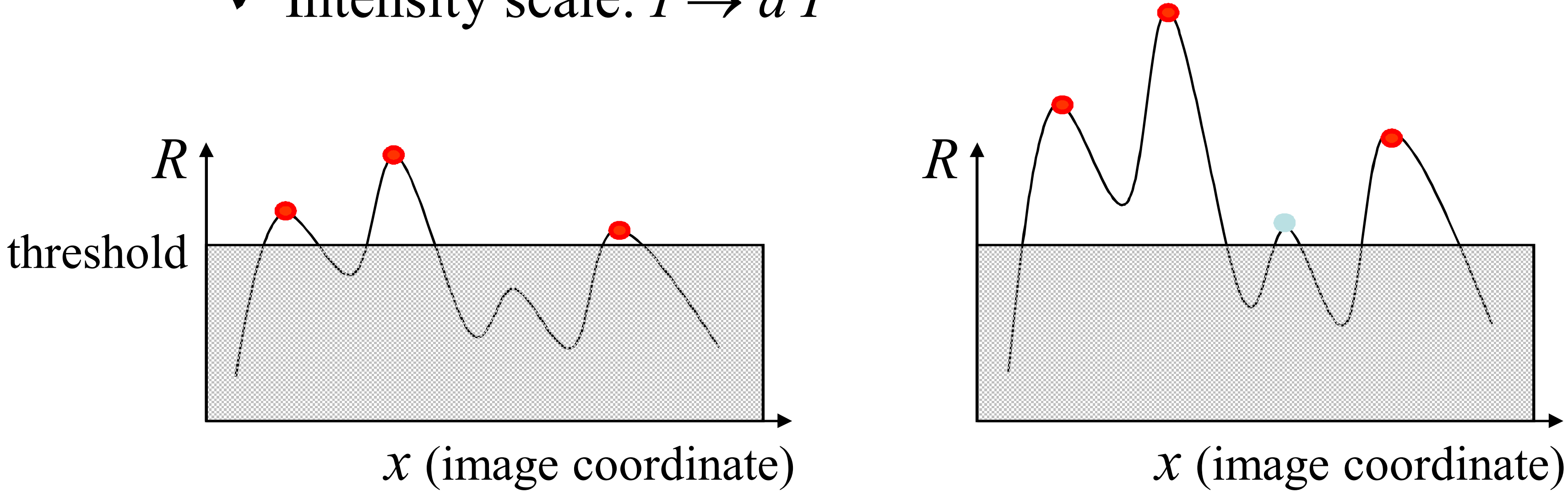
Corner response R is invariant to image rotation

Harris Detector: Some Properties

- Partial invariance to *affine intensity* change

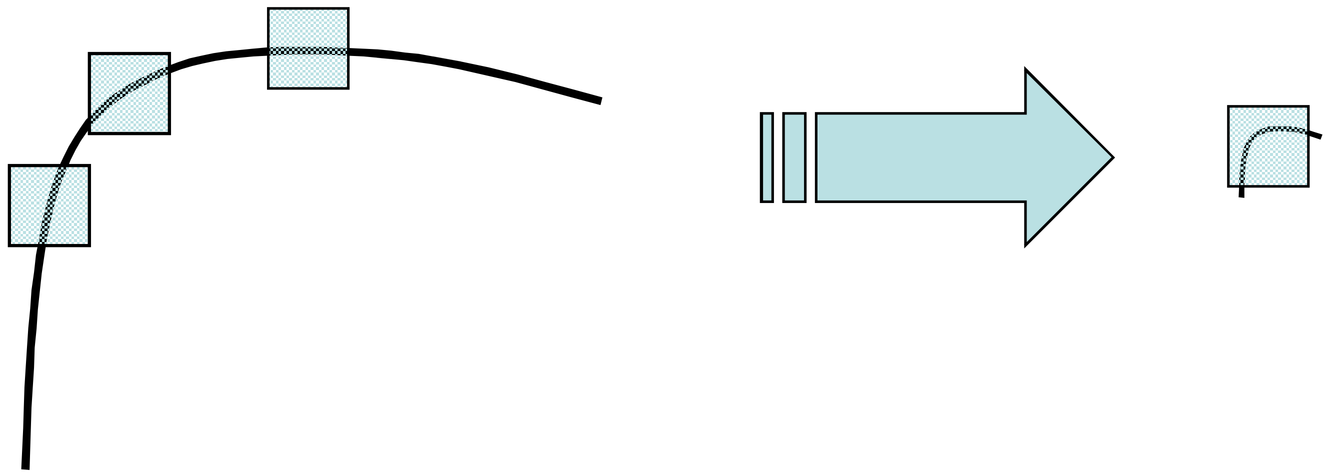
✓ Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$

✓ Intensity scale: $I \rightarrow a\,I$



Harris Detector: Some Properties

- But: non-invariant to *image scale*!



All points will be classified as edges

Corner !

Salient points based on 2nd derivatives

- Hessian: determinant

$$\det \mathbf{H} = \det \begin{pmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{pmatrix} = I_{xx} I_{yy} - I_{xy}^2$$

  - Local maxima of det **H** [Beaudet]
  - Zero crossings of det **H** [Dreschler+Nagel]
  - Invariant to rotation
  - Similar cornerness measure: local maxima of *K* [Kitchen+Rosenfeld]

$$K = \frac{I_{xx} I_y^2 - 2 I_{xy} I_x I_y + I_{yy} I_x^2}{I_x^2 + I_y^2}$$

Salient points based on 2nd derivatives

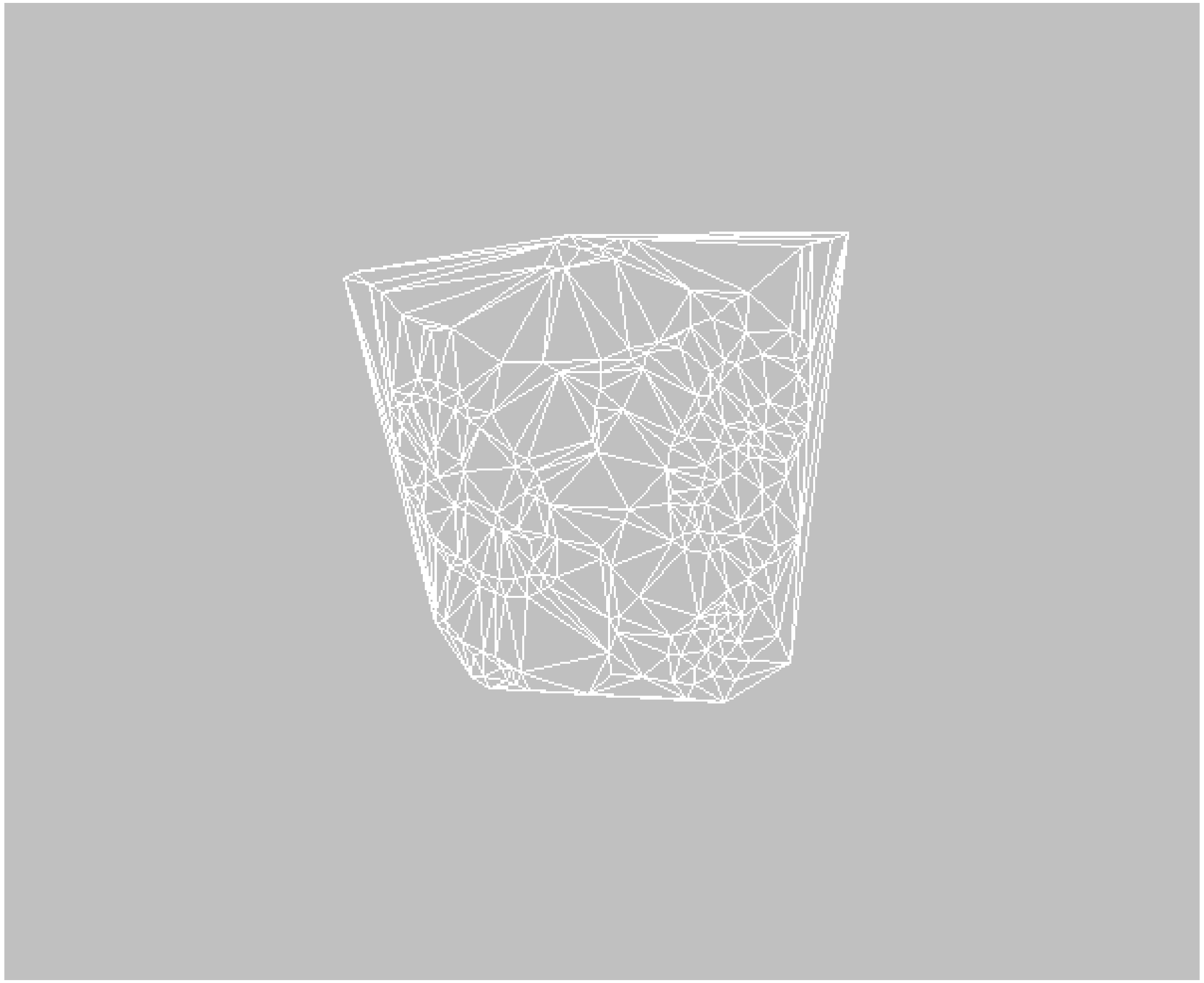
- Laplacian: trace of the **H** matrix

$$\text{trace } \mathbf{H} = \text{trace} \begin{pmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{pmatrix} = I_{xx} + I_{yy}$$

Any advantage w.r.t. Hessian?

Matching: remember our final aim…

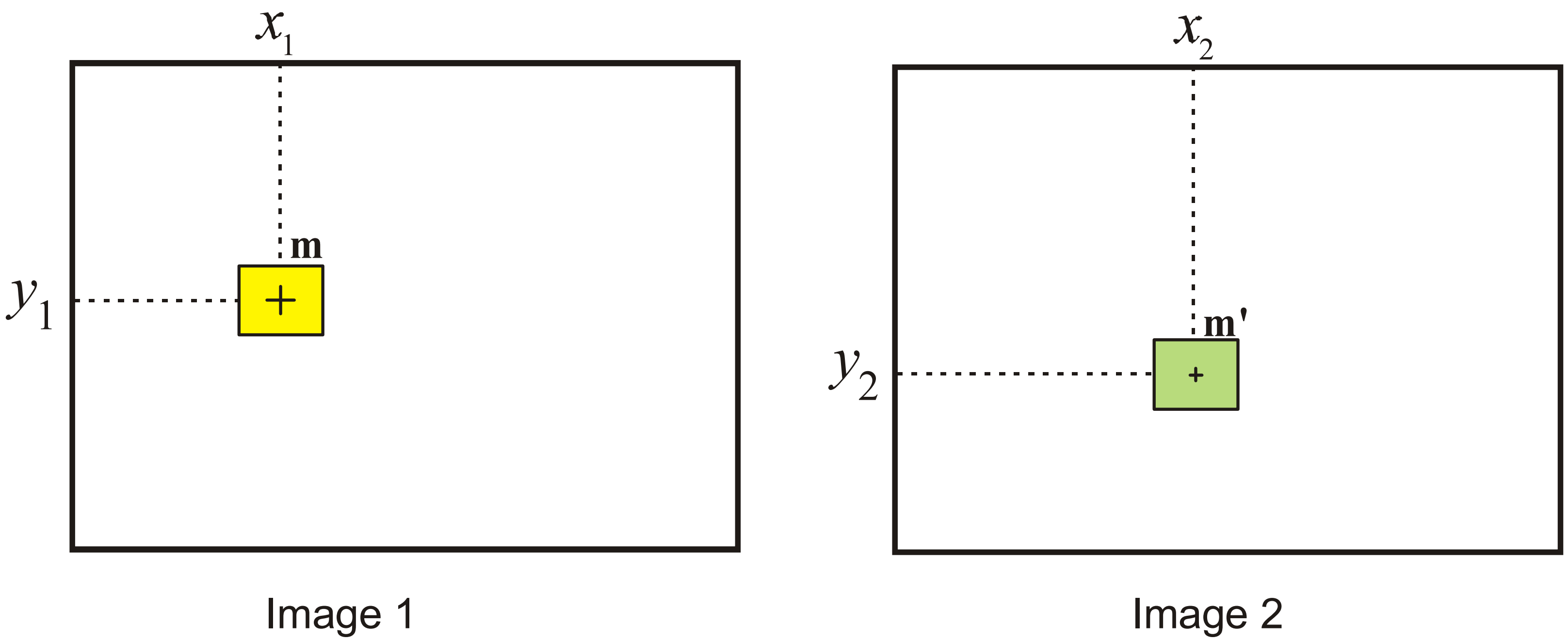
3D Reconstruction - Preview



Matching: Region based Similarity Metric

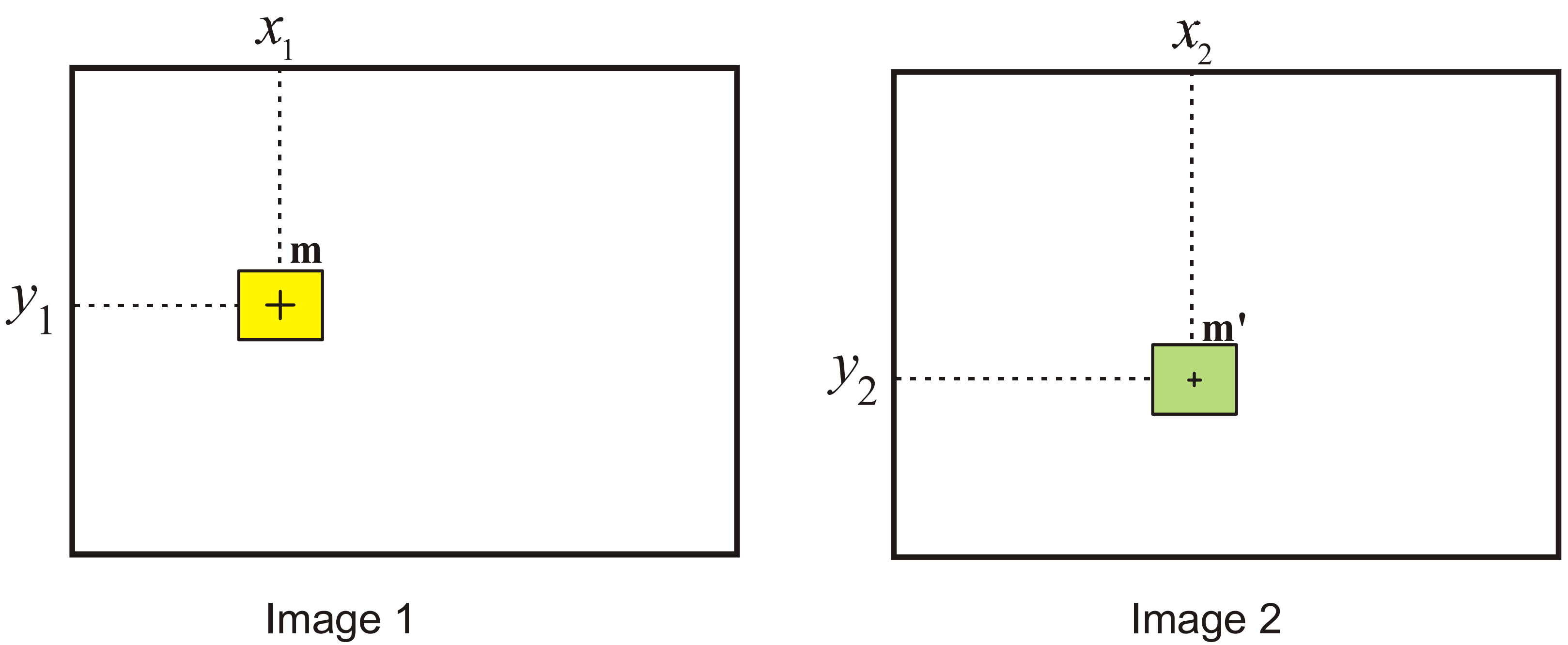
- Sum of squared differences
- Sum of absolute differences
- Cross-correlation
- Normalized cross-correlation

Sum of Squared Differences



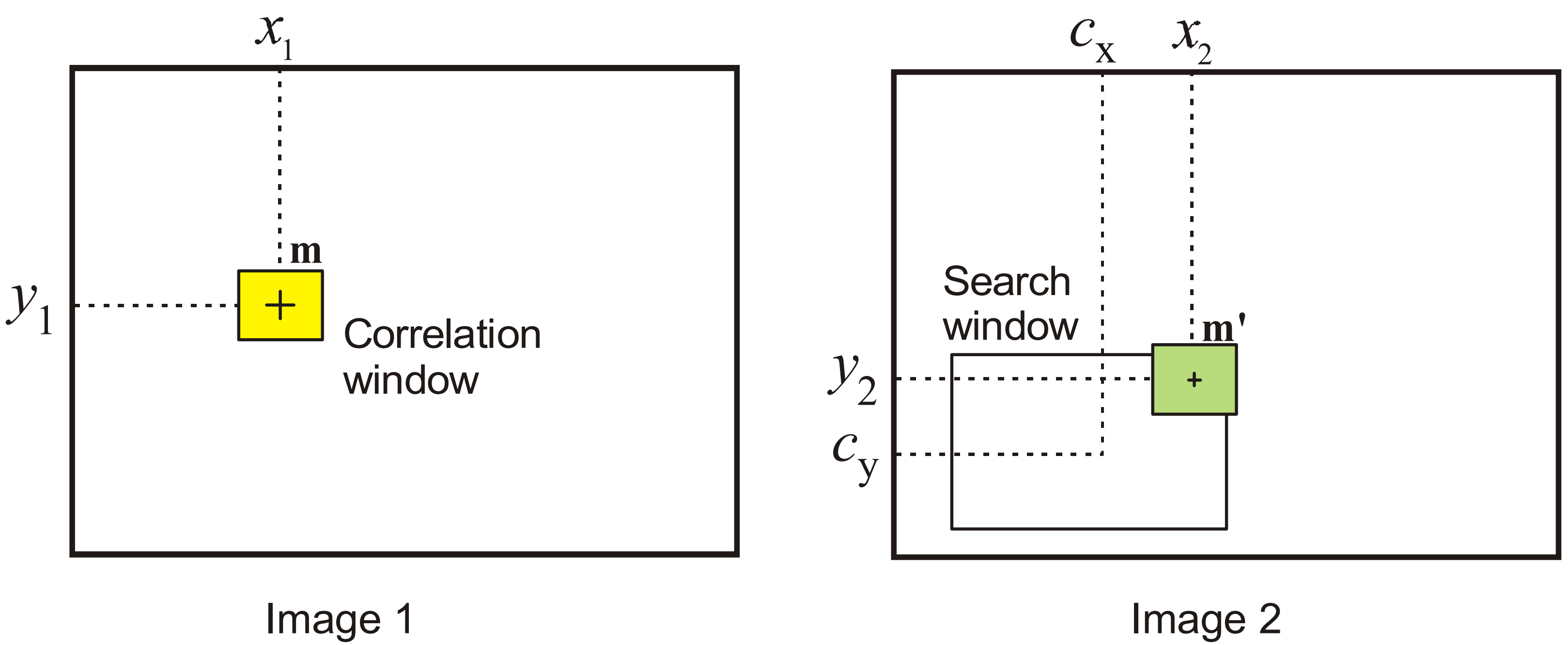
$$SSD(\mathbf{m},\mathbf{m}') = \sum_{i=-n/2}^{n/2} \sum_{j=-n/2}^{n/2} \left[I_1(x_1+i, y_1+j) - I_2(x_2+i, y_2+j) \right]^2$$

Sum of Absolute Differences



$$SAD(\mathbf{m},\mathbf{m}') = \sum_{i=-n/2}^{n/2} \sum_{j=-n/2}^{n/2} \left| I_1(x_1+i, y_1+j) - I_2(x_2+i, y_2+j) \right|$$

Normalized cross-correlation



$$corr(\mathbf{m},\mathbf{m}') = \frac{\sum_{i=-n/2}^{n/2} \sum_{j=-n/2}^{n/2} \left[I_1(x_1+i, y_1+j) - \overline{I_1(x_1,y_1)} \right] \cdot \left[I_2(x_2+i, y_2+j) - \overline{I_2(x_2,y_2)} \right]}{(n+1)^2 \sqrt{\sigma^2(I_1)\cdot\sigma^2(I_2)}}$$

Tracking in the image space

- Seek for image position p(x,y) which maximizes the similarity to template

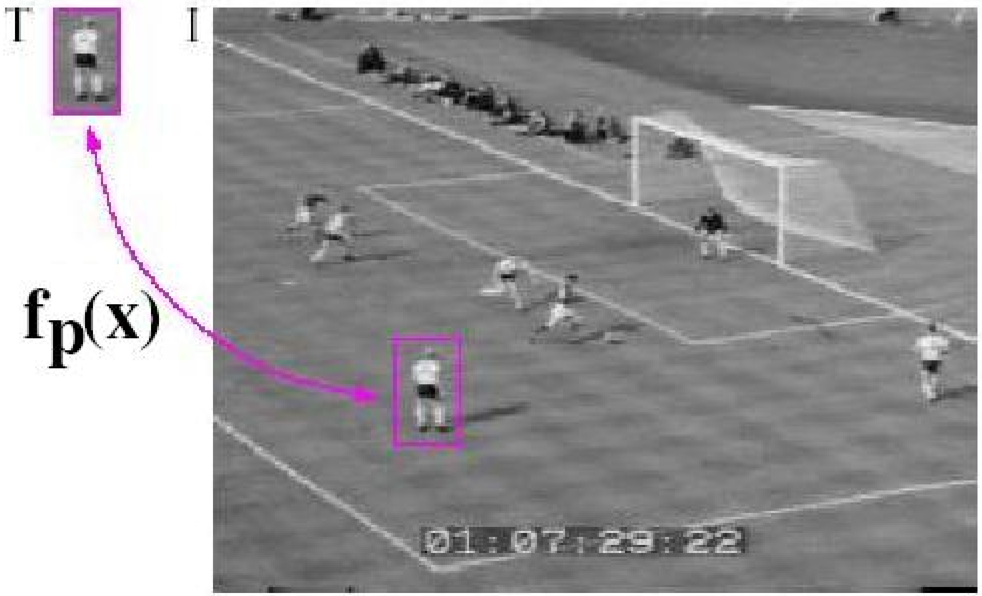


Image I and template T

SSD scores

Summary

- It is possible to detect points with well-defined position only if they are "corner-like".
- Eigen analysis helps in deciding…
- Several alternatives to detect interest points:
  - Moravec
  - Harris
  - Hessian/Laplacian
- We have seen some basic methods to solve the correspondence problem:
  - SSDs, SADs
  - Cross-correlation

Supporting References

- C. Harris and M.J. Stephens. A combined corner and edge detector. In Alvey Vision Conference, pages 147–152, 1988.
- C. Schmid, R. Mohr, and C. Bauckhage. Evaluation of interest point detectors. International Journal of Computer Vision, 37(2):151–172, June 2000.
- CVOnline

Next class: Planar transformations …

and then…

Outlier Rejection!