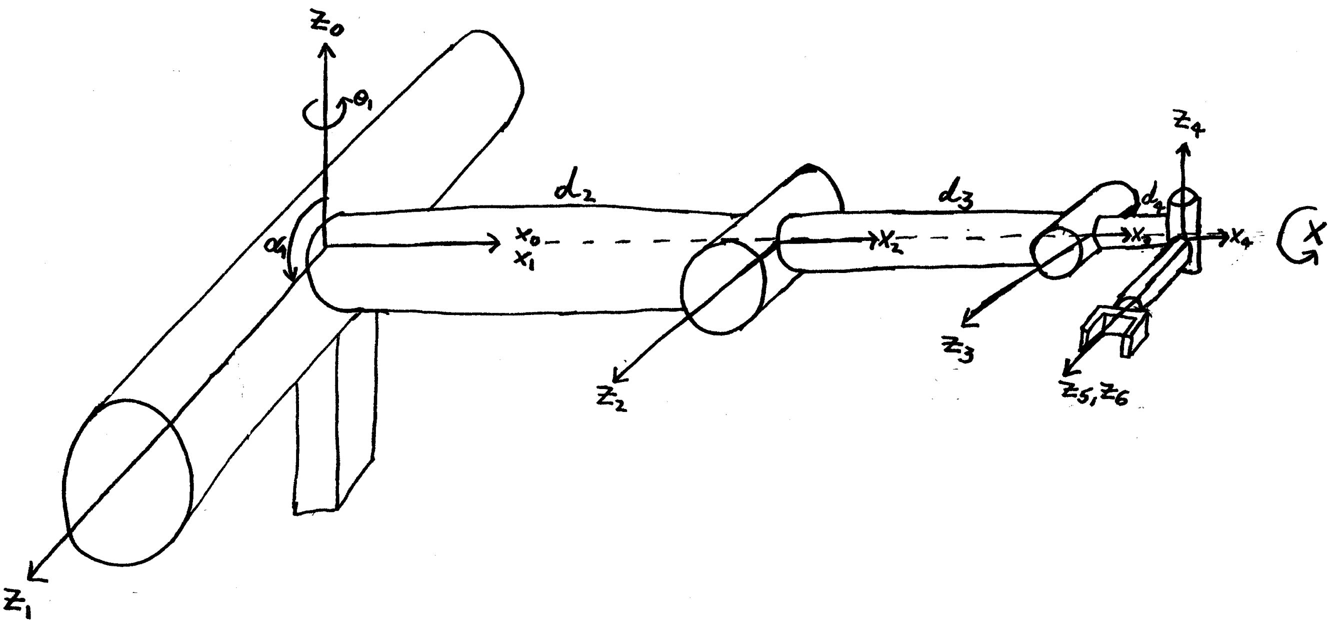
F21RO INTELLIGENT ROBOTICS TUTORIAL 2 "A" MATRICES

QUESTIONS

1. Explain what the *A* matrices in the Denavit-Hartenberg notation achieve.

2. Construct the general form of an *A* matrix for the 6 joint arm depicted in the Denavit-Hartenberg diagram below -



3. By substituting all the known paremeters for this arm into the general form, write down the 6 individual *A* matrices for the arm.

SOLUTIONS

1. Matrix A_i and its predecessors, A_{i-1}, A_{i-2}, etc. will perform the rotation for joint i and then correctly locate and orientate joint $i+1$ ready for its matrix, A_{i+1}, to carry out the appropriate rotation and displacement for the link which follows.

2. Each A matrix is the product of a rotation about z, representing the rotation (θ_i) of the current joint; a shift (d_i) along x, to the position of the next joint; and a rotation (α_i) about x, to align the new z axis with the rotation axis for the next joint -

$$A_n = \begin{bmatrix} Cos\theta_n & -Sin\theta_n & 0 & 0 \\ Sin\theta_n & Cos\theta_n & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 & d_n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & Cos\alpha_n & -Sin\alpha_n & 0 \\ 0 & Sin\alpha_n & Cos\alpha_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A_n = \begin{bmatrix} Cos\theta_n & -Sin\theta_n Cos\alpha_n & Sin\theta_n Sin\alpha_n & d_n Cos\theta_n \\ Sin\theta_n & Cos\theta_n Cos\alpha_n & -Cos\theta_n Sin\alpha_n & d_n Sin\theta_n \\ 0 & Sin\alpha_n & Cos\alpha_n & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. The d_i are all known for a given articulated arm because they are simpy the distances between the joints.

The θ_i represent the rotations of each of the joints and so will become known for any particular articulation of the arm.

The only unknowns are the α_i which are the angles necessary to re-align the z axis from the current joint's axis of rotation to the next joint's axis of rotation.

We note that some joints in the Denavit-Harteberg diagram are coincident and so the distance between them is 0 which will help to simply the A matrices. So, for the 6 joints, we have –

Joint 1:	$\alpha_1 = 90°$,	$d_1 = 0$
Joint 2:	$\alpha_2 = 0°$,	$d_2 \neq 0$
Joint 3:	$\alpha_3 = 0°$,	$d_3 \neq 0$
Joint 4:	$\alpha_4 = -90°$,	$d_4 \neq 0$
Joint 5:	$\alpha_5 = 90°$,	$d_5 = 0$
Joint 6:	$\alpha_6 = 0°$,	$d_6 = 0$

Substituting these values into the general A_n matrix yields the following set of A matrices -

$$A_1 = \begin{bmatrix} Cos\theta_1 & 0 & Sin\theta_1 & 0 \\ Sin\theta_1 & 0 & -Cos\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} Cos\theta_2 & -Sin\theta_2 & 0 & d_2Cos\theta_2 \\ Sin\theta_2 & Cos\theta_2 & 0 & d_2Sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} Cos\theta_3 & -Sin\theta_3 & 0 & d_3Cos\theta_3 \\ Sin\theta_3 & Cos\theta_3 & 0 & d_3Sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} Cos\theta_4 & 0 & -Sin\theta_4 & d_4Cos\theta_4 \\ Sin\theta_4 & 0 & Cos\theta_4 & d_4Sin\theta_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} Cos\theta_5 & 0 & Sin\theta_5 & 0 \\ Sin\theta_5 & 0 & -Cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} Cos\theta_6 & -Sin\theta_6 & 0 & 0 \\ Sin\theta_6 & Cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$