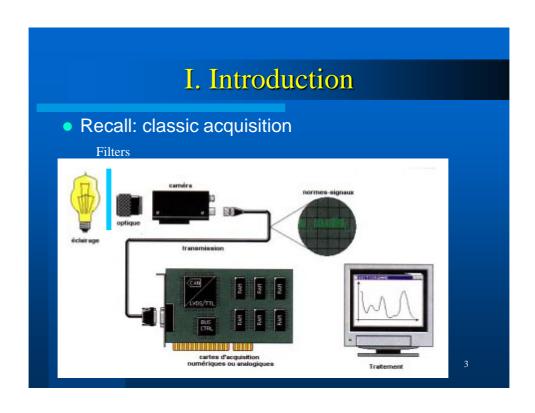


Plan du cours

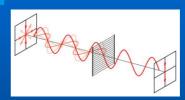
- Introduction
 several source of acquisition
 necessity to store high quantity of data
- II. Lossless methods
- III. Lossy methods
- IV JPEG norm
- **V MPEG Norm**





I. Introduction: rappels

- Polarizer Filter
 absorb a component
 => produce linearly
 polarized light
- Measure of 2 projections : degree of polarization





2 images of 1024x2014 12bits pixels Approx 3Mb





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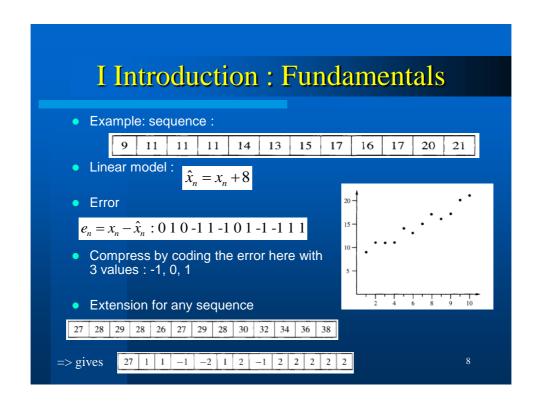
I Introduction: size of data

For instance, a still image of the resolution of 480x640 pixels with 24-bit intensity levels has the data size of 480x640x24 = 7.4 megabits!

Video images, 30 frames/second, of the resolution 768x1024 pixels with 24-bit levels has the data size of 566 megabits per second

Résolution	Format	Débit	sur cd
352x288	YUV	5 Mo/s	2 min
	MJPEG 6.6	768 Ko/s	14 min
	MPEG1	175 Ko/s	60 min
	DIVX 910 kbs	114 Ko/s	95 min
640x480	YUV	15 Mo/s	40 secondes
	MJPEG 6.6	2,3 Mo/s	4,5 min
	MPEG2	490 Ko/s	22 min
	DIVX 910 kbs	114 Ko/s	40 min

Introduction: Fundamentals Compression: Logic Physic Symmetric et asymmetric Adaptive or not Lossy or lossless Compression ratio: size of compressed image TC, size of original image TO T = TC / TO T small, small file Classic scheme: Extract the symbols, then model and code



I Introduction: Fundamentals

If n_1 and n_2 denote the number of information-carrying units in two data sets that represent the same information, the *relative data redundancy* R_D of the first data set, n_1 is defined as

$$R_D = 1 - \frac{1}{C_R}$$

where C_R , commonly called the *compression ratio*, is

$$C_R = \frac{n_1}{n_2} \,.$$

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I Introduction: Fundamentals

- If n1=n2, then CR=1 and RD=0.
 This indicates that the first data has no redundant data.
- If n1>>n2, then CR→∞ and RD=1.
 This implies significant compression and highly redundant data.
- If n2>>n1, then CR→0 and RD →∞.
 This implies data expansion, which is undesirable.

I Introduction: Definitions

- Lossless compression: the original image can be reconstructed perfectly as there is no information loss. Compression rates typically range from 2:1 to 50:1. Example: Run-length coding (JPEG), Huffman coding, LZW coding (TIFF, GIF), Bit-plane coding etc.
- Lossy compression: the original image and its reconstruction are not identical owing to some information loss.

Psychological properties of the eye may be taken into account.

Higher compression rates are possible than with lossless compression, typically up to 100:1. Example: DPCM, JPEG, MPEG etc.

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II Lossless compression

- Entropy: average self information (length) of a message $H = -\sum p_i \log_2(p_i)$
- Example 12323454567898910

$$P(1) = P(6) = P(7) = P(10) = \frac{1}{16}$$

 $P(2) = P(3) = P(4) = P(5) = P(8) = P(9) = \frac{2}{16}$. H = 3.25 bits

 Not possible to compress without data loss with a data flow inferior to the entropy of the source.

 Estimated entropy is reduced after coding with the sequence of residues

111-1111-11111 -111

P(1)=13/16 and P(-1)=3/16 so H=0.70 bits/symbol

• Reconstruction:

 $x_n = x_{n-1} + r_n$

Other sequence

12123333123333123312

P(1)=P(2)=1/4 and P(3)=1/2 => H=1.5 bits / symbol => 20 symbols => 30 bits to code

better here to code groups of symbols => H= 1 bit/symbol

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II Lossless compression

Coding based on statistics and propabilities

TABLE		ities of occurre the English al _l ititution.	
Letter	Probability	Letter	Probability
A	0.057305	N	0.056035
В	0.014876	О	0.058215
C	0.025775	P	0.021034
D	0.026811	Q	0.000973
E	0.112578	R	0.048819
F	0.022875	S	0.060289
G	0.009523	T	0.078085
Н	0.042915	U	0.018474
I	0.053475	v	0.009882
J	0.002031	W	0.007576
K	0.001016	X	0.002264
L	0.031403	Y	0.011702
M	0.015892	Z	0.001502

255/27/53/ 255/12/212/ 37/ 53/ 12/ 255/3/119

1) Run Length Encoding

- Work on byte
- Formats TIFF, BMP, PCX, fax ...
- Principle: reduce the size by encoding the redundancy:
 - AAAAAA coded 6A (redundancy/value)
 - but: ABCDEF => 1A1B1C1D1E1F
- Improvement: 3 bytes flag/counter/code
 - Rule: if flag = 255 then 3 bytes; if not flag then 1 byte
 - Ex: 28 times code ASCII 53
 - 13 times code 212
 - 1 times code 37
 - 1 times code 53
 - 1 times code 12
 - 4 times code 119

II Lossless compression 1) Run Length Encoding מסממממ מסממממם מסממממם IC ICØ4 nnnn IC 00000 مستومية ومستومه ومومومه **ICØ4** Gray level 200 FIGURE 8.3 FIGURE 8.3 Illustration of run-length coding: (a) original image. (b) Binary image with line 100 marked. (c) Line profile and binarization threshold. (d) Run-length code. 150 1023 Horizontal coordinate Line 100: $(1,63)\ (0,87)\ (1,37)\ (0,5)\ (1,4)\ (0,556)\ (1,62)\ (0,210)$

1) Run Length Encoding

 In the previous slide, the entire 1024×343 image was reduced to 12,166 runs. Thus, the compression ratio and the relative redundancy can be obtained by

$$C_R = \frac{n_1}{n_2} = \frac{1024 \cdot 343 \cdot 1}{12166 \cdot 11} = 2.63$$

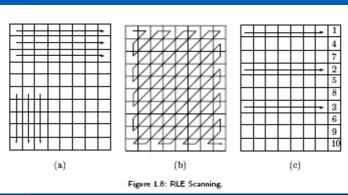
$$R_D = 1 - \frac{1}{C_R} = 1 - \frac{1}{2.63} = 0.62$$
.

• (Consider why 11 bits are used for each run.)

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1) Run Length Encoding

Scan strategy :



1) Run Length Encoding

- Conditional RLE : lossy
- Assuming an image with n grayscales, the method starts by assigning an n-bit code to each pixel depending on its near neighbors. It then concatenates the n-bit codes into a long string and calculates run lengths. The run lengths are assigned prefix codes (Huffman or other) that are written on the compressed stream
- Conditional probabilities P(X| A,B)
- Example with 4 bits codes

-			****	7770	7770	****	****	*****	****
A	В		W1	W2	W3	W4	W_5	W6	W7
2	15	value:	4	3	10	0	6	8	1
2	15	count:	21	6	5	4	2	2	1
3	0	value:	0	1	3	2	11	4	15
3	U	count:	443	114	75	64	56	19	12
3	1	value:	1	2	3	4	0	5	6
3	1	count:	1139	817	522	75	55	20	8
3	2	value:	2	3	1	4	5	6	0
3	2	count:	7902	4636	426	264	64	18	6
3	3	value:	3	2	4	5	1	6	7
3	3	count:	33927	2869	2511	138	93	51	18
3	4	value:	4	3	5	2	6	7	1
3	4	count:	2859	2442	240	231	53	31	13



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II Lossless compression

1b) Move To front

- Maintain the alphabet A of symbols as a list where frequently occurring symbols are located near the front
- Symbols encoded as the number of symbols that precede in the list
- The input stream abcddcbamnopponm is encoded as *C* = (0, 1, 2, 3, 0, 1, 2, 3, 4, 5, 6, 7, 0, 1, 2, 3)
- Without the move-to-front step it is encoded as C' = (0, 1, 2, 3, 3, 2, 1, 0, 4, 5, 6, 7, 7, 6, 5, 4).
- Both C and C' contain codes in the same range [0, 7], but the elements of C are smaller on the average, since the input starts with a concentration of abcd and continues with a concentration of mnop. (The average value of C is 2.5, while that of C' is 3.5.)
- Can be mixed with a variable length code

1b) Move To front

Example

a abcdmnop 0	a abcdmnop 0	a abedmnop 0	a abcdmnop 0
b abcdmnop 1	b abcdmnop 1	b abcdmnop 1	b abcdmnop 1
c bacdmnop 2	c abedmnop 2	c bacdmnop 2	c abcdmnop 2
d cbadmnop 3	d abcdmnop 3	d cbadmnop 3	d abcdmnop 3
d debamnop 0	d abcdmnop 3	m debamnop 4	m abcdmnop 4
c debamnop 1	c abcdmnop 2	n mdcbanop 5	n abcdmnop 5
b cdbamnop 2	b abcdmnop 1	o nmdcbaop 6	o abcdmnop 6
a bedamnop 3	a abcdmnop 0	p onmdebap 7	p abcdmnop 7
m abcdmnop 4	m abcdmnop 4	a ponmdeba 7	a abcdmnop 0
n mabednop 5	n abedmnop 5	b aponmdcb 7	b abcdmnop 1
o nmabedop 6	o abedmnop 6	e baponmde 7	c abcdmnop 2
p onmabedp 7	p abedmnop 7	d cbaponmd 7	d abcdmnop 3
p ponmabed 0	p abedmnop 7	m debaponm 7	m abcdmnop 4
o ponmabed 1	o abedmnop 6	n mdcbapon 7	n abcdmnop 5
n opnmabed 2	n abedmnop 5	o nmdcbapo 7	o abcdmnop 6
m nopmabed 3	m abcdmnop 4	p onmdebap 7	p abcdmnop 7
mnopabed		ponmdcba	
(a)	(b)	(c)	(d)
-	abla 1 14: Encoding With a	and Without Mayo to Event	

Table 1.14: Encoding With and Without Move-to-Front.

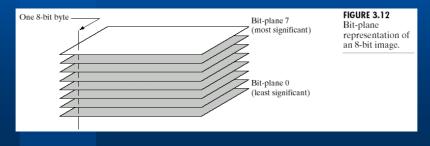
1c) Recursive Range Reduction

- Eliminating some most significant zero bits of integers
- Integer list is sorted
- Work well with data decreasing fast , but not too fast

Data	RR code	Rice code
	0101 = 6	
1011101	011101	$000001\ 1101$
1001011	1001011	$00001\ 1011$
0110001	0 110001	$0001\ 0001$
0101100	101100	$001\ 1100$
0001110	001110	$1\ 1110$
0001101	1101	$1\ 1101$
0001100	1100	$1\ 1100$
0001001	1001	$1\ 1001$
0000010	00 10	$1\ 0010$
0000001	01	1 0001
70	53	64 bits

2) Bit plane coding

- Another effective technique for reducing an image's inter-pixel redundancies is to process the image's bit planes individually. The technique, called bit-plane coding, is based on the concept of decomposing a multilevel image into a series of binary images and compressing each binary image by run-length coding
- The figure below illustrates bit-planes of an 8-bit image. Each bit generates a binary image of the same size as the original image.



II Lossless compression

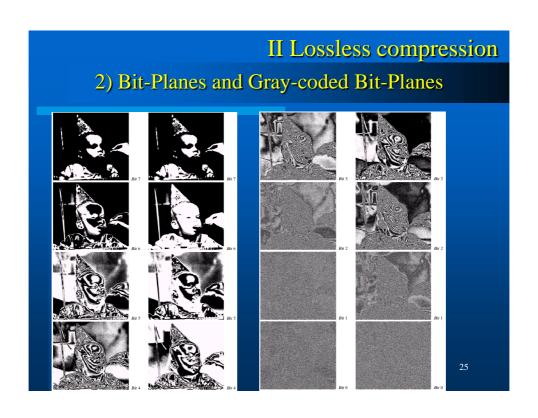
2) Bit plane coding

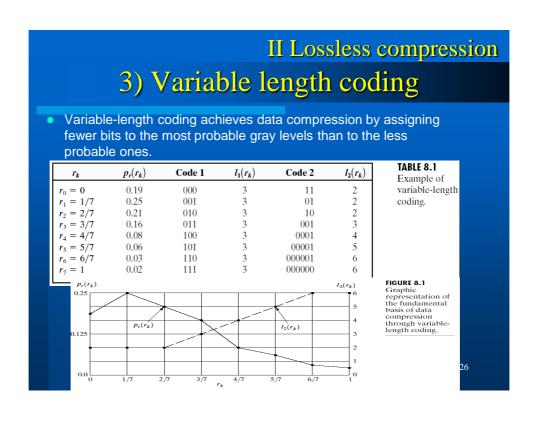
- The inherent disadvantage of bit-plane coding is that small changes in gray level can have a great effect on the resulting bit planes. For example, when a pixel value 127 (0111 11112) is adjacent to a pixel of value 128 (1000 00002), every bit plane will have a 0 to 1 (or 1 to 0) transition.
- This effect of small gray-level variations has a negative impact on the compression ratio.
- To cope with this problem, pixel values are converted to a Gray code that is obtained by

$$g_i = a_i \oplus a_{i+1}$$
 $0 \le i \le m-2$
 $g_{m-1} = a_{m-1}$,

where the gray levels of an *m*-bit gray-scale image can be represented in the form of the base 2 polynomial.

$$a_{m-1}2^{m-1} + a_{m-2}2^{m-2} + \dots + a_12^1 + a_02^0$$
.





3) Variable length coding

- Prefix property:
 once a certain bit pattern has been assigned as the
 code of a symbol, no other codes should start with that
 pattern
- Example:
- Code string : a1a3a2a1a3a3a4a2a1a1a2a2a1a1a3a1a1a2a3a1
- code 1 : ambiguous, code 2 no

Symbol	Prob.	Code1	Code2			
a_1	.49	1	1			
a_2	.25	01	01			
a_3	.25	010	000			
a_4	.01	001	001			
Table 2.2: Variable-Size Codes.						

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II Lossless compression

3) Variable length coding

Unary code: the positive integer n is defined as n = 1 ones followed by a single 0 or, alternatively, as n = 1 zeros followed by a single one

General unary codes : start/step/stop

n	Code	Alt. Code
1	0	1
2	10	01
3	110	001
4	1110	0001
5	11110	00001

n	$\begin{array}{c} a = \\ 3 + n \cdot 2 \end{array}$	nth codeword	Number of codewords	Range of integers
0	3	0xxx	$2^3 = 8$	0-7
1	5	10xxxxx	$2^5 = 32$	8-39
2	7	110xxxxxxxx	$2^7 = 128$	40-167
3	9	111xxxxxxxxxx	$2^9 = 512$	168 - 679
		Total	680	

Table 2.4: The General Unary Code (3,2,9)

3) Shannon-Fano coding

- Symbols sorted in decreasing order of their probabilities
- Division in two subsets representing aproximately half of the probabilities
- Rq: no code is the starting of an other one

	${\bf Prob.}$	Steps			Final			
1.	0.25	1	1					:11
2.	0.20	1	0					:10
3.	0.15	0		1	1			:011
4.	0.15	0		1	0			:010
5.	0.10	0		0		1		:001
6.	0.10	0		0		0	1	:0001
7.	0.05	0		0		0	0	:0000
4. 5. 6.	0.15 0.10 0.10	0 0 0	_	0	-	0	-	:010 :001 :0001

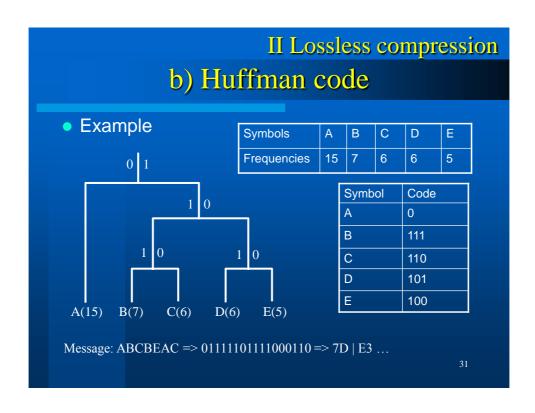
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II Lossless compression

3) Huffman coding

- Bit level coding
- Uses the statistics of the information
- Each symbol of the source is represented by a binary word of length inversely proportional to it's frequency
- Mechanism:
 - 1) build the occurrence table (search of probabilities p_i)
 - 2) reduction by grouping the 2 smallest frequencies; affect "0" to the smallest sort by decreasing order
 - Iterate to the root

ABLE 3.28	Huffman coding of 16-bit CD-quality audio.					
File Name	Original File Size (bytes)	Entropy (bits)	Estimated Compressed File Size (bytes)	Compression Ratio		
Mozart	939,862	12.8	725,420	1.30		
Cohn	402,442	13.8	349,300	1.15		
Mir	884,020	13.7	759,540	1.16		



II Lossless compression b) Huffman code Exercise 1: Obtain the Huffman code for the characters from A to G with the following frequencies. 29 A: 64 B: C: 32 12 D: 9 E: F: 66 23 G:

b) Huffman code

Exercise 2: Obtain the Huffman code for the image of 8 gray levels with the following occurrence probabilities, and also calculate the average bits/pixel, compression ratio C_R , and the relative data redundancy R_D .

Gray levels	Probabilities
0	0.21
1	0.27
2	0.24
3	0.11
4	0.08
5	0.05
6	0.03
7	0.01

II Lossless compression

c) Adaptative code

- Adapt to the incoming data stream
- Build the same dictionary onto the coder and the decoder => provide transmission. coder

```
Initialise_model();
Do {
C=getc(input);
Encode(c,output);
Update_model( c );
}
While ( (c=decode(input)) !=EOF) {
    put(c, output);
    update_model( c);
}
While (c!=EOF)
```

 Methods: Frequency coding, Adaptive Huffman and Lempel Ziv Welch (LZW)

II Lossless compression Frequency coding

- Work on byte
- 2 parts coding: header/body (variable size)
- Header: define size of the code (ex with 3 bits 8 possibilities)
- The last symbol fix the end of the message
- Compression rate <= 2
- Not used yet

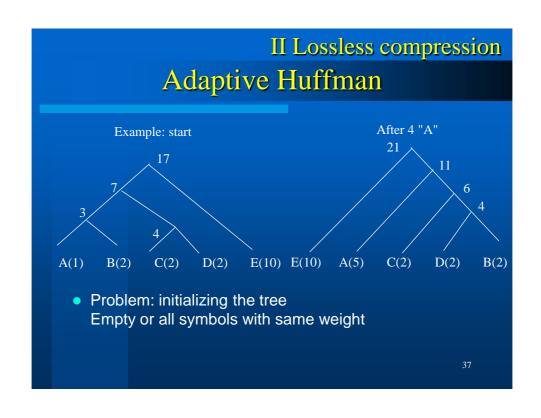
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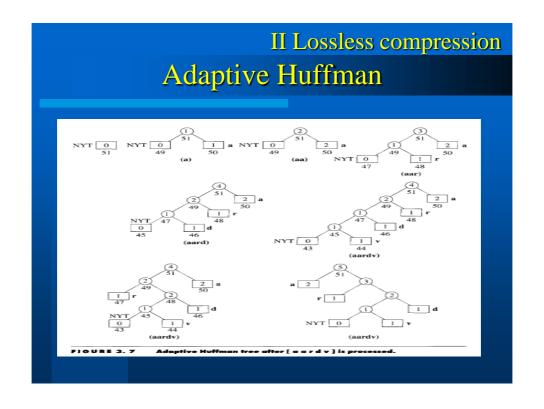
II Lossless compression

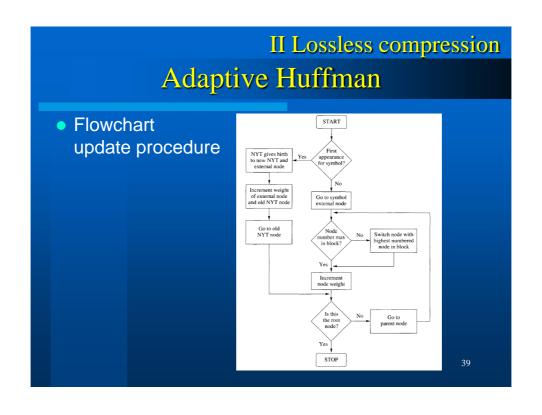
Adaptative Huffman

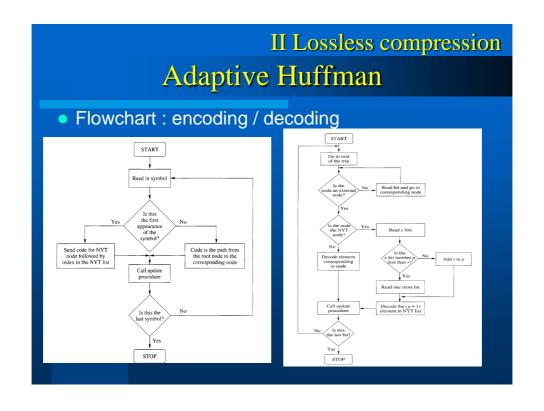
- Work on byte
- Property of "fraternity": for a binary tree with leaf having an increasing weight.
- When a symbol come:
 if non respect of the property
 then exchange 2 terms
 without changing the tree
 (actual symbol and symbol the most
 far symbol with same weight)

Parent
Weight=weight1+weight2
weight1 weight2









II Lossless compression Arithmetic coding

- 1. Start by defining the "current interval" as [0, 1).
- 2. Repeat the following two steps for each symbol s in the input stream:
 - 2.1. Divide the current interval into subintervals whose sizes are proportional to the symbols' probabilities.
 - 2.2. Select the subinterval for s and define it as the new current interval.
- 3. When the entire input stream has been processed in this way, the output should be any number that uniquely identifies the current interval (i.e., any number inside the current interval).

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II Lossless compression Arithmetic coding • Example Consider a three-letter alphabet $\mathcal{A} = \{a_1, a_2, a_3\}$ with $P(a_1) = 0.7$, $P(a_2) = 0.1$, and $P(a_3) = 0.2$. Using the mapping of Equation (4.1), $F_X(1) = 0.7$, $F_X(2) = 0.8$, and $F_X(3) = 1$. This partitions the unit interval as shown in Figure 4.1. $0.0 \quad a_1 \quad a_2 \quad 0.5460$ $a_1 \quad a_2 \quad 0.5460$ $a_2 \quad 0.5572 \quad a_3 \quad 0.5600$ FIGURE 4. 1 Restricting the interval containing the tag for the input sequence [a_1, a_2, a_3, ...].

Arithmetic coding

Example : SWISS MISS

Char	Freq	Prob.	Range	$\operatorname{CumFreq}$
		Total (CumFreq=	10
\mathbf{S}	5	5/10 = 0.5	[0.5, 1.0)	5
W	1	1/10 = 0.1	[0.4, 0.5)	4
I	2	2/10 = 0.2	[0.2, 0.4)	2
\mathbf{M}	1	1/10 = 0.1	[0.1, 0.2)	1
ш	1	1/10 = 0.1	[0.0, 0.1)	0

Update of Low and High bounds:
 NewHigh:=OldLow+Range*HighRange(X);
 NewLow:=OldLow+Range*LowRange(X);
 Range=OldHigh-OldLow and LowRange(X),
 HighRange(X) indicate the low and high limits of the range of symbol X

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II Lossless compression

Arithmetic coding

• Example : SWISS MISS

Char.		The calculation of low and high
S	L	$0.0 + (1.0 - 0.0) \times 0.5 = 0.5$
	H	$0.0 + (1.0 - 0.0) \times 1.0 = 1.0$
W	L	$0.5 + (1.0 - 0.5) \times 0.4 = 0.70$
	H	$0.5 + (1.0 - 0.5) \times 0.5 = 0.75$
I	L	$0.7 + (0.75 - 0.70) \times 0.2 = 0.71$
	Η	$0.7 + (0.75 - 0.70) \times 0.4 = 0.72$
S	L	$0.71 + (0.72 - 0.71) \times 0.5 = 0.715$
	Η	$0.71 + (0.72 - 0.71) \times 1.0 = 0.72$
S	L	$0.715 + (0.72 - 0.715) \times 0.5 = 0.7175$
	Η	$0.715 + (0.72 - 0.715) \times 1.0 = 0.72$
ш	L	$0.7175 + (0.72 - 0.7175) \times 0.0 = 0.7175$
	Η	$0.7175 + (0.72 - 0.7175) \times 0.1 = 0.71775$
M	L	$0.7175 + (0.71775 - 0.7175) \times 0.1 = 0.717525$
	Η	$0.7175 + (0.71775 - 0.7175) \times 0.2 = 0.717550$
I	L	$0.717525 + (0.71755 - 0.717525) \times 0.2 = 0.717530$
	Η	$0.717525 + (0.71755 - 0.717525) \times 0.4 = 0.717535$
S	L	$0.717530 + (0.717535 - 0.717530) \times 0.5 = 0.7175325$
	Η	$0.717530 + (0.717535 - 0.717530) \times 1.0 = 0.717535$
S	L	$0.7175325 + (0.717535 - 0.7175325) \times 0.5 = 0.71753375$
	Η	$0.7175325 + (0.717535 - 0.7175325) \times 1.0 = 0.717535$
		Table 2.48: The Process of Arithmetic Encoding.

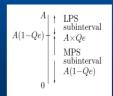
Char.	Code-low		Range
S	0.71753375 - 0.5	5 = 0.21753373	5/0.5 = 0.4350675
W	0.4350675 - 0.4	= 0.0350675	0.1 = 0.350675
I	0.350675 - 0.2	= 0.150675	/0.2 = 0.753375
\mathbf{S}	0.753375 - 0.5	= 0.253375	/0.5 = 0.50675
\mathbf{S}	0.50675 - 0.5	= 0.00675	/0.5 = 0.0135
ш	0.0135 - 0	= 0.0135	/0.1 = 0.135
\mathbf{M}	0.135 - 0.1	= 0.035	/0.1 = 0.35
I	0.35 - 0.2	= 0.15	/0.2 = 0.75
\mathbf{S}	0.75 - 0.5	= 0.25	0.5 = 0.5
\mathbf{S}	0.5 - 0.5	=0	/0.5 = 0

Table 2.50: The Process of Arithmetic Decoding.

QM coder

- JPEG
- main idea: classify each input symbol (which is a single bit) as either the more probable symbol (MPS) or the less probable symbol (LPS).
- Rules:

$$\begin{split} & \text{After MPS:} \quad C \text{ is unchanged,} \quad A \leftarrow A(1-Qe), \\ & \text{After LPS:} \quad C \leftarrow C + A(1-Qe), \quad A \leftarrow A \times Qe. \end{split}$$



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String compression

- compression methods based on strings of symbols can be more efficient than methods that compress individual symbols.
- Ex: huffman code of 2 symbols

String	Probability	Code
$\overline{a_1a_1}$	$0.8 \times 0.8 = 0.64$	0
a_1a_2	$0.8 \times 0.2 = 0.16$	11
a_2a_1	$0.2 \times 0.8 = 0.16$	100
a_2a_2	$0.2 \times 0.2 = 0.04$	101

String	Probability	Code
$a_{1}a_{1}a_{1}$	$0.8 \times 0.8 \times 0.8 = 0.512$	0
$a_{1}a_{1}a_{2}$	$0.8 \times 0.8 \times 0.2 = 0.128$	100
$a_{1}a_{2}a_{1}$	$0.8 \times 0.2 \times 0.8 = 0.128$	101
$a_{1}a_{2}a_{2}$	$0.8 \times 0.2 \times 0.2 = 0.032$	11100
$a_{2}a_{1}a_{1}$	$0.2 \times 0.8 \times 0.8 = 0.128$	110
$a_{2}a_{1}a_{2}$	$0.2 \times 0.8 \times 0.2 = 0.032$	11101
$a_{2}a_{2}a_{1}$	$0.2 \times 0.2 \times 0.8 = 0.032$	11110
$a_{2}a_{2}a_{2}$	$0.2 \times 0.2 \times 0.2 = 0.008$	11111

Dictionary methods LZ77; sliding window basis of pkzip, gzip; Window cutted in two parts: left is the search buffer and right the look ahead

 $\leftarrow \ \mathrm{coded} \ \mathrm{text...} \\ \mathbf{sir}_{\sqcup} \mathbf{sid}_{\sqcup} \mathbf{eastman}_{\sqcup} \mathbf{easily}_{\sqcup} \mathbf{t} \\ | \mathbf{eases}_{\sqcup} \mathbf{sea}_{\sqcup} \mathbf{sick}_{\sqcup} \mathbf{seals}_{||}... \leftarrow \ \mathrm{text} \ \mathrm{to} \ \mathrm{be} \ \mathrm{read}$

e coded (16,3,e)

buffer

sir⊔sid⊔eastman⊔	\Rightarrow	(0,0,``s")
s ir⊔sid⊔eastman⊔e	\Rightarrow	(0,0,"i")
$si r_{\sqcup}sid_{\sqcup}eastman_{\sqcup}ea$	\Rightarrow	(0,0,"r")
sir ⊔sid⊔eastman⊔eas	\Rightarrow	(0,0,"∟")
sir⊔sid⊔eastman⊔easi	\Rightarrow	(4,2,``d")

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II Lossless compression Lempel-Ziv Welch

- LZ78 with dictionnary, basis of compress, gif; improved in 1984 by Welch
- Work on byte
- Dictionnaire évolutif initialised with ASCII codes
- Learning step
- Example:

Symbole lu	Code numérie	que / binaire	Dictionnaire associé		
A	rien		Adresse	Séquence	
I	65	01000001	256	AI	
D	73	01001001	257	ID	
E	68	01000100	258	DE	
blanc	69	01000101	259	E blanc	
T	32	00100000	260	blanc T	
0	84	01010100	261	ТО	
1	79	01001111	262	OI	
blanc	73	01001001	263	1 blanc	
L	32	01000000	264	blanc L	
E	76	01001100	265	LE	
blanc	SP	11111111	266	E blanc C	
C	259	100000011	267	CI	
1	67	001000011	268	IE	
E	73	001001001	269	EL	
L	69	001000101	270	L blanc	
blanc	76	001001100	271	blanc T blanc	
T	rien		272	blanc A	
blanc	260	100000100	273	AID	
Α	32	000100000	274	DER	
1	rien		275	RA	
D	256	100000000			
E	rien				
R	258	100000010			
A	82	001010010			
	65	001000001			

II Lossless compression Lempel-Ziv Welch

A unique feature of the LZW coding is that the coding dictionary or code book is created while the data are being encoded. In addition, it is remarkable that an LZW decoder builds an identical decompression dictionary simultaneously the encoded data stream.

```
The LZW compression algorithm is summarized as follows:

w=NIL;

while (read a character k)

{

    if wk exists in the dictionary
        w=wk;
    else
        add wk to the dictionary
        output the code for w;
        w=k;
}
```

II Lossless compression

Lempel-Ziv Welch

The LZW decompression algorithm is as follows:

```
read a character k;
output k;
w=k;
while (read a character k)
/* k could be a character or a code */
{
    entry = dictionary entry for k;
    output entry;
    add w + entry[0] to dictionary;
    w = entry;
}
```

II Lossless compression Lempel-Ziv Welch

Exercise: complete the following dictionary of the LZW coding, and find the encoded message for "ABACABA".

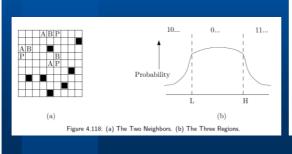
Index	
	А
	В
	С

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III Lossless compression

FELICS

- Fast, Efficient, Lossless Image Compression System
- code each pixel with a variable-size code based on the values of two of its previously seen neighbor pixels
- Code depends on P



D: 1	D 2	D' 1	
Pixel	Region	Pixel	
P	code	code	
L=15	0	0000	
16	0	0010	
17	0	010	
18	0	011	
19	0	100	
20	0	101	
21	0	110	
22	0	111	
23	0	0001	
H = 24	0	0011	
e 4.119: The	Codes for		
		32	

PPM

- Prediction with partial matching
- Use previous knowledge of the preceding data; order N context

Order 4	Order 3	Order 2	Order 1	Order 0
xyzz→x 2	xyz→z 2	xy→z 2	x → y 3	x 4
yzzx→y 1	yzz→x 2	→ x 1	$y \rightarrow z 2$	у З
zzxy→x 1	zzx→y 1	yz→z 2	\rightarrow x 1	z 4
$zxyx \rightarrow y = 1$	zxy→x 1	$zz\rightarrow x$ 2	$z \rightarrow z 2$	
xyxy→z 1	xyx→y 1	zx→y 1	\rightarrow x 2	
yxyz→z 1	yxy→z 1	$yx \rightarrow y = 1$		
		()		
		(a)		
Order 4	Order 3	Order 2	Order 1	Order 0
xyzz→x 2	xyz→z 2	xy→z 2	x—y 3	x 4
yzzx→y 1	yzz→x 2	$xy \rightarrow x = 1$	\rightarrow z 1	у З
\rightarrow z 1	zzx→y 1	yz→z 2	y → z 2	z 5
zzxy→x 1	→ z 1	$zz\rightarrow x$ 2	\rightarrow x 1	
zxyx→y 1	zxy→x 1	zx→y 1	$z \rightarrow z 2$	
xyxy→z 1	xyx→y 1	\rightarrow z 1	\rightarrow x 2	
yxyz→z 1	yxy→z 1	$yx \rightarrow y = 1$		
		(b)		

Table 2.72: (a) Contexts and Counts for "xyzzxyxyzzx". (b) Updated After Another z Is Input.

III Lossy compression

- In contrast to lossless coding methods, lossy coding is aimed at a greater data compression ratio by discarding fine details in an image.
- Many lossy encoding techniques can compress images by more than 100:1. If the compression ratio is at 10:1 to 50:1, a restored image can be even indistinguishable from its original.
- Psycho-visuals coding: the eye must not see the difference with the original image
- For example quantify the luminance
- Criteria:

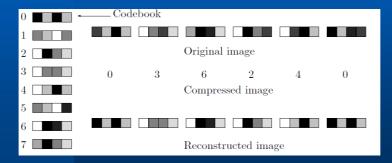
$$EQM = \frac{\sum_{i/j} (x_{ij} - \hat{x}_{ij})^2}{MN}$$

$$PSNR = 10 \log_{N} \frac{1}{MN} \sum_{i \neq j} (\hat{x}_{ij})^{2} dB$$

correct PSNR ≥ 30 dB

III Lossy compression 1) Vector Quantization

Intuitive methods : Subsampling **Vector Quantization**



III Lossy compression

1) Vector Quantization

- Choice of the codebook; LBG (1980) method
- Iterations
- Step 0: Select a threshold value e and set k=0 and $D^{(-1)}=\infty$. Start with an initial codebook with entries $C^{(k)}_i$ (where k is currently zero, but will be incremented in each iteration). Denote the image blocks by Bi (or training vectors : uses them to find the best codebook entries).
- Step 1: Pick up a codebook entry $C^{(k)}_i$. Find all the image blocks Bm that are closer to Ci than to any other Cj . This set (or partition) is denoted by $P^{(k)}_i$. Repeat for all values of i. It may happen that some partitions will be empty.
- of i. It may happen that some partitions will be empty. Step 2: Select an i and calculate the distortion $D^{(k)}$ between codebook entry C(k) and the set of training vectors (partition) $P^{(k)}$ i found for it in Step 1. Repeat for all i, then calculate the average $D^{(k)}$ of all the $D^{(k)}$ i. A distortion $D^{(k)}$ if or a certain i is calculated by computing the distances $d(C^{(k)}$, Bm) for all the blocks Bm in partition $P^{(k)}$, then computing the average distance. Alternatively, $D^{(k)}$ i can be set to the minimum of the distances $d(C^{(k)}$, Bm). Step 3: If $(D^{(k-1)} D^{(k)})/D^{(k)} \le e$, halt. The output of the algorithm is the last set of codebook entries $C^{(k)}$. This set can now be used to (lossy) compress the image with vector quantization. In the first iteration k is zero, so $D^{(k-1)} = D^{(-1)} = \infty > e$. This guarantees that the algorithm will not stop at the first iteration. Step 4: Increment k by 1 and calculate new codebook entries $C^{(k)}$: each equals the
- Step 4: Increment k by 1 and calculate new codebook entries $C^{(k)}i$; each equals the average of the image blocks (training vectors) in partition $P^{(k-1)}i$ that was computed in Step 1. (This is how the codebook entries are adapted to the particular image.) Go to Step 1.

1) Vector Quantization

Example : with 24 pixels

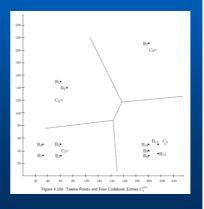
 $B_1=(32,32),\ B_2=(60,32),\ B_3=(32,50),\ B_4=(60,50),\ B_5=(60,150),\ B_6=(70,140),\ B_7=(200,210),\ B_8=(200,32),\ B_9=(200,40),\ B_{10}=(200,50),\ B_{11}=(215,50),\ and\ B_{12}=(215,35)$ (Figure 4.104). It is clear that the 12 points are concentrated in four regions. We select an initial codebook with the four entires $C_1^{(0)}=(70,40),\ C_2^{(0)}=(60,120),\ C_3^{(0)}=(210,200),\ and\ C_4^{(0)}=(225,50)$ (shown as x in the diagram). These entiries were selected more or less at random but we show later how the LBG algorithm selects them methodically, one by one. Because of the graphical nature of the data, it is easy to determine the four initial partitions. They are $P_1^{(0)}=(B_1,B_2,B_3,B_4),\ P_2^{(0)}=(B_5,B_6),\ P_2^{(0)}=(B_5,B_0),\ P_2^{(0)}=(B_5,B_0,B_0)$ and $P_2^{(0)}=(B_5,B_0,B_0,B_{11},B_{12})$. Table 4.105 shows how the average distortion $D^{(0)}$ is calculated for the first iteration (we use the Euclidean distance function). The result is

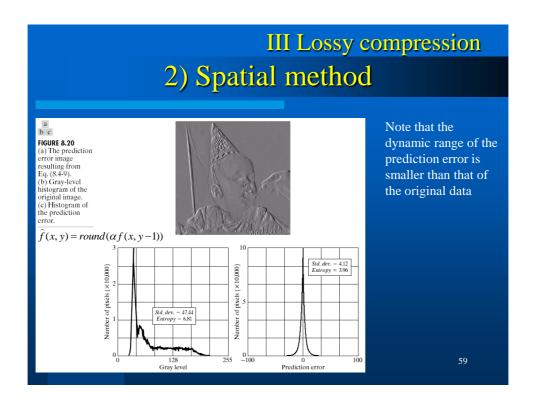
$$\begin{split} D^{(0)} = & (1508 + 164 + 1544 + 200 + 900 + 500 \\ & + 200 + 449 + 725 + 625 + 100 + 325)/12 \\ = & 603.33. \end{split}$$

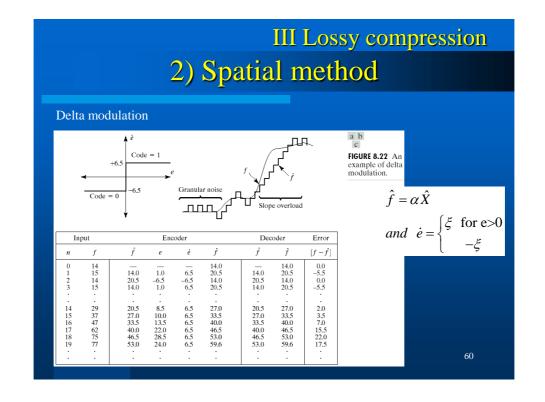
Step 3 indicates no convergence, since $D^{(-1)}=\infty$, so we increment k to 1 and calculate four new codebook entries $C_i^{(1)}$ (rounded to the nearest integer for simplicity)

$$\begin{split} C_1^{(1)} &= (B_1 + B_2 + B_3 + B_4)/4 = (46, 41), \\ C_2^{(1)} &= (B_5 + B_6)/2 = (65, 145), \\ C_3^{(1)} &= B_7 = (200, 210), \end{split} \tag{4.29}$$

 $C_4^{(1)} = (B_8 + B_9 + B_{10} + B_{11} + B_{12})/5 = (206, 41).$

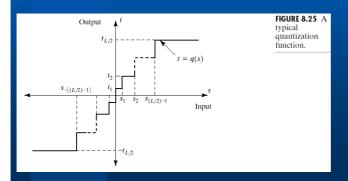






2) Spatial method

- Differential Pulse Coded Modulation DPCM: a staircase-like quantization function is employed (see next slide).
- In addition, the prediction is performed by a linear combination of m previous pixels (Eqs. (8.5-16) to (8.5-19))

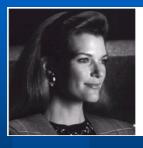


Slowly changing regions are finely quantized, while the rapidly changing areas are quantized more coarsely.

III Lossy compression

2) Spatial method

Prediction Errors in Four Cases



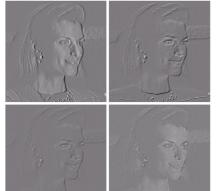
 $\hat{f}(x, y) = 0.97 f(x, y-1)$

FIGURE 8.23 A 512 × 512 8-bit monochrome image.

FIGURE 8.24 A

 $\hat{f}(x, y) = 0.75 f(x, y-1) + 0.75 f(x-1, y) - 0.5 f(x-1, y-1)$

Prediction error



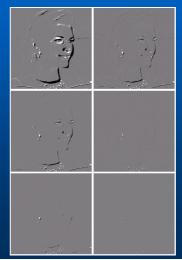
 $(0.97 f(x, y-1) \text{ if } \Delta h \leq \Delta v$ 0.97 f(x-1, y) otherwise

 $\hat{f}(x, y) = 0.5 f(x, y-1) + 0.5 f(x-1, y)$

2) Spatial method

Reconstruction errors of DPCM





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III Lossy compression

2) Spatial method

 The table below lists the RMS errors of the difference images, as well as for a number of other combinations of predictor and quantizers.

$$s_{i} = \begin{cases}
s_{i-1} (s - t_i) p(s) ds = 0 & i = 1, 2, ..., L/2 \\
0 & i = 0 \\
(t_i - t_{i+1})/2 & i = 1, 2, ..., L/2 - 1 \\
\infty & i = L/2
\end{cases}$$

	Lloyd-Max Quantizer			Lloyd-Max Quantizer Adaptive Quantizer			tizer
Predictor	2-level	4-level	8-level	2-level	4-level	8-level	
Eq. (8.5-16)	30.88	6.86	4.08	7.49	3.22	1.55	
Eq. (8.5-17)	14.59	6.94	4.09	7.53	2.49	1.12	
Eq. (8.5-18)	9.90	4.30	2.31	4.61	1.70	0.76	
Eq. (8.5-19)	38.18	9.25	3.36	11.46	2.56	1.14	
Compression	8.00:1	4.00:1	2.70:1	7.11:1	3.77:1	2.56:1	

TABLE 8.11 Lossy DPCM root-mean-square error summary.

2 a) Roberts Method

- Diminish the size of the code (classically 8 bits per pixel)
- 1st step: X1=X + random noise b
- 2nd step: reduction of the number of bits
- After construction: subtract a random noise with same dynamic as the signal
- Low rate 8/3 before Huffmann coding

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III Lossy compression

3) Block Coding

- Low modification of luminance between adjacent pixels
- Blocks of 4x4 or 8x8
- Parameters : mean

$$\overline{X} = \frac{1}{N} \sum_{i=1}^{N} X_i$$

mean shift

$$\varepsilon = \frac{1}{N} \sum_{i=1}^{N} |X_i - \overline{X}|$$

threshold

$$X_i \ge \overline{X} \Rightarrow p = 1$$
$$X_i < \overline{X} \Rightarrow p = 0$$

Example: compression

110 100 88 89
$$\overline{X} = 94$$
 1 1 0 0 121 77 80 83 $\varepsilon = 16$ 1 0 0 0 0 70 75 81 79 0 0 0 0 0

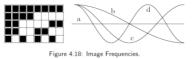
• Reconstruction from 2 states:

$$Y_1 = \overline{X} + \frac{n\varepsilon}{2n_1} \qquad Y_0 = \frac{1}{2n_1} = \frac{n\varepsilon}{2n_1}$$

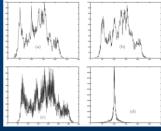
$$Y_0 = \bar{X} - \frac{n\varepsilon}{2n_0}$$

4) Method with transform

- Image transform; linear, orthogonal
- To decorrelate input datas (ex RGB colors)



 Linear transform : rotations , consider each pair of pixels as a point



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III Lossy compression

4) Method with transform

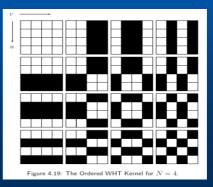
- Classical transforms :
- 1. The Walsh-Hadamard transform (WHT, Section 4.5.2) is fast and easy to compute (it requires only additions and subtractions), but its performance, in terms of energy compaction, is lower than that of the DCT
- 2. The Haar transform [Stollnitz et al. 96] is a simple, fast transform. It is the simplest wavelet transform.
- 3. The Karhunen-Loève transform (KLT) is the best one theoretically, in the sense of energy compaction (or, equivalently, pixel decorrelation). However, its coefficients are not fixed; they depend on the data to be compressed. Calculating these coefficients (the basis of the transform) is slow, as is the calculation of the transformed values themselves. Since the coefficients are data dependent, they have to be included in the compressed stream. For these reasons and because the DCT performs almost as well, the KLT is not generally used in practice.
- 4. The discrete cosine transform (DCT). This important transform is almost as efficient as the KLT in terms of energy compaction, but it uses a fixed basis, independent of the data. There are also fast methods for calculating the DCT. This method is used by JPEG and MPEG audio.

4) Method with transform

- Walsh-Hadamard transform
- Given an NxN block of pixels Pxy (where N must be a power of 2, N = 2n), its two-dimensional WHT and inverse WHT are defined by Equations

$$\begin{split} H(u,v) &= \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} p_{xy}g(x,y,u,v) \\ &= \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} p_{xy}(-1) \sum_{i=0}^{n-1} [b_i(x)p_i(u) + b_i(y)p_i(v)], \\ P_{xy} &= \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} H(u,v)h(x,y,u,v) \\ &= \frac{1}{N} \sum_{u=0}^{N-1} \sum_{v=0}^{N-1} H(u,v)(-1) \sum_{i=0}^{n-1} [b_i(x)p_i(u) + b_i(y)p_i(v)], \end{split}$$

$$\begin{split} p_0(u) &= b_{n-1}(u), \\ p_1(u) &= b_{n-1}(u) + b_{n-2}(u), \\ p_2(u) &= b_{n-2}(u) + b_{n-3}(u), \\ &\vdots \\ p_{n-1}(u) &= b_1(u) + b_0(u). \end{split}$$



III Lossy compression

4) Method with transform

• Haar transform; first wavelet

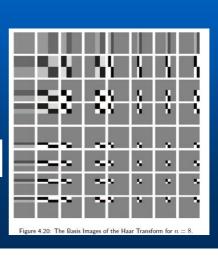
$$h_0(x) \stackrel{\text{def}}{=} h_{00}(x) = \frac{1}{\sqrt{N}}, \text{ for } 0 \le x \le 1,$$

$$h_k(x) \stackrel{\text{def}}{=} h_{pq}(x) = \frac{1}{\sqrt{N}} \begin{cases} 2^{p/2}, & \frac{q-1}{2^p} \le x < \frac{q-1/2}{2^p}, \\ -2^{p/2}, & \frac{q-1/2}{2^p} \le x < \frac{q}{2^p}, \\ 0, & \text{otherwise for } x \in [0, 1]. \end{cases}$$

Matrix example for N=2

$$\mathbf{A}_2 = \begin{pmatrix} h_0(0/2) & h_0(1/2) \\ h_1(0/2) & h_1(1/2) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Given an image block **P** of order $N \times N$ where N = 2n, its Haar transform is the matrix product $\mathbf{A}N\mathbf{P}\mathbf{A}N$



- Karhunen-Loève (KLT); PCA based approach
- Given an image, we break it up into k blocks of n pixels each, where n
 is typically 64 but can have other values, and k depends on the image
 size.

vectors

$$v^{(i)} = b^{(i)} - \overline{b}$$

$$w^{(i)} = Av^{(i)}$$

nxk matrix

$$V = (v^{(1)} \dots v^{(k)})$$
 and $W = (w^{(1)})$

 $w^{(k)}$

The *n* coefficient vectors $\mathbf{c}(j)$ of the Karhunen-Loève transform are given by

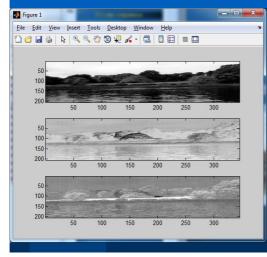
$$c^{(j)} = (w^{(1)}_{j} \dots w^{(k)}_{j}), j=1,2,...,n$$

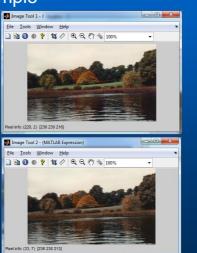
Matrix **A:** its rows are the basis vectors of the KLT, and the energies (variances) of the transformed vectors are the eigenvalues $\lambda 1$, $\lambda 2$, . . . , λn of $\mathbf{V} \cdot \mathbf{V}^T$.

$$\mathbf{W} \cdot \mathbf{W}^T = \mathbf{A} (\mathbf{V} \cdot \mathbf{V}^T) \mathbf{A}^T = \begin{pmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \lambda_n \end{pmatrix}$$

III Lossy compression

Karhunen-Loève (KLT), example





III Lossy compression

4) Method with transform

DCT

$$DCT(i, j) = \sqrt{\frac{2}{N}}C(i)C(j)\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}pixel(x, y) \cos\left(\frac{(2x+1)i\pi}{2N}\right)\cos\left(\frac{(2y+1)j\pi}{2N}\right)$$

$$C(i) \text{ ou } C(j) = \begin{cases} \frac{1}{\sqrt{2}} & \text{if } i = 0\\ 1 & \text{otherwise} \end{cases}$$

- Example of 1D data: p = (12, 10, 8, 10, 12, 10, 8, 11)
- DCT: 28.6375, 0.571202, 0.46194, 1.757, 3.18198, -1.72956, 0.191342, -0.308709
- Quantization 1: 28.6, 0.6, 0.5, 1.8, 3.2, –1.8, 0.2, –0.3; IDCT: 12.0254, 10.0233, 7.96054, 9.93097, 12.0164, 9.99321, 7.94354, 10.9989
- Quantization 2: 28, 0, 0, 2, 3, -2, 0, 0; IDCT: 11.236, 9.62443, 7.66286, 9.57302, 12.3471, 10.0146, 8.05304, 10.6842
 Max error 0.764 (or 6.4% of 12).

Transform

Quantization

Re-ordering of coefficients

Coding 73

III Lossy compression

4) Method with transform

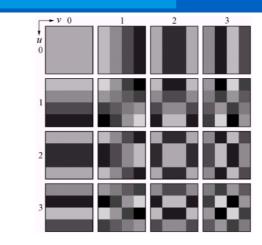
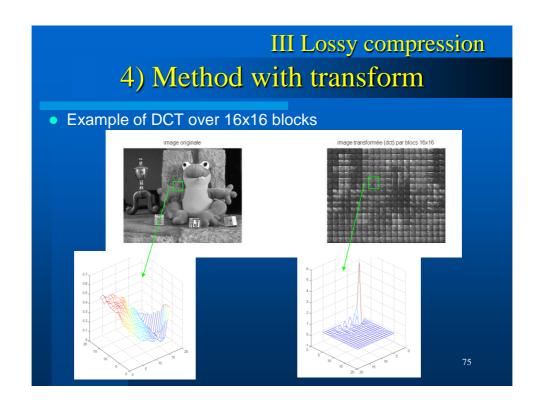
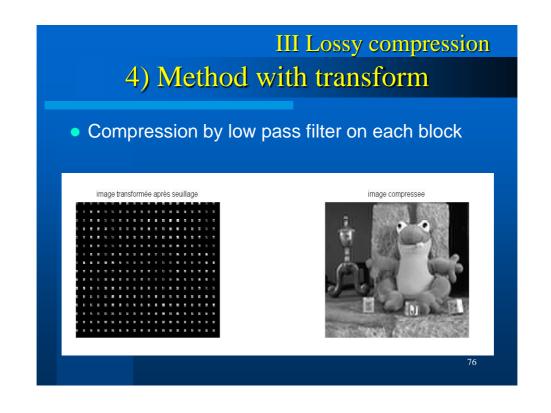
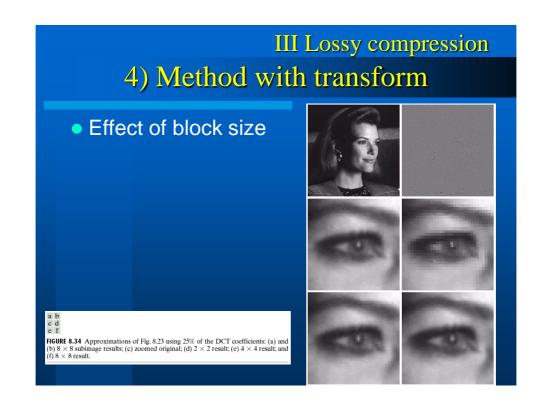


FIGURE 8.30 Discrete-cosine basis functions for N = 4. The origin of each block is at its top left.

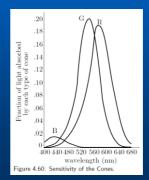


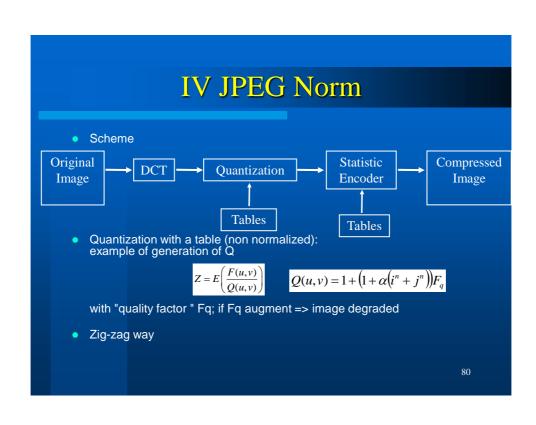


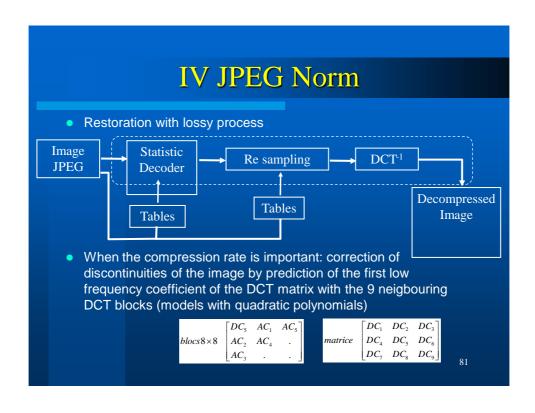


IV JPEG norm

- Joint Photographic Expert group; 1991
- Describe the scheme; freedom on implementation
- 2 classes of process: lossless or lossy
- 4 modes: sequential DCT, progressive DCT (successive approx.), lossless or hierarchic (approx. on several resolution)
- Samples of 8 ou 12 bits
- Color Images treated in Y,Cr,Cb (color downsampled)

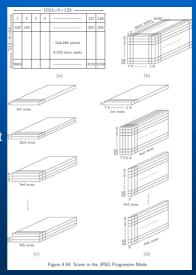






IV JPEG Norm

- Progressive mode
- higher-frequency DCT coefficients are written on the compressed stream in blocks called "scans."
- Each scan that is read and processed by the decoder results in a sharper image.
 The idea is to use the first few scans to quickly create a low-quality, blurred preview of the image, and then either input the remaining scans or stop the process and reject the image.
- Trade-off: the encoder has to save all the coefficients of all the data units in a memory buffer before they are sent in scans, and also go through all the steps for each scan, slowing down the progressive mode



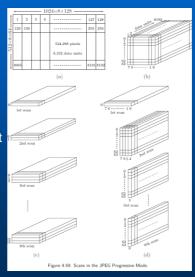
IV JPEG Norm

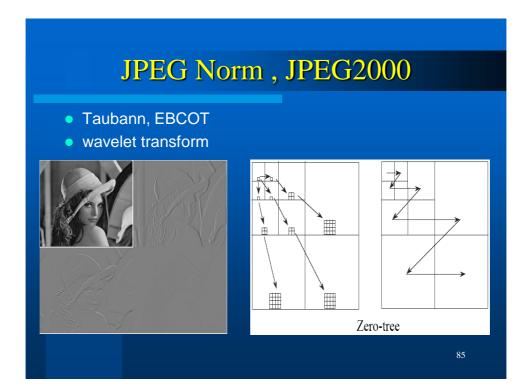
- Example
- Frequency encoding
- Colorshades
- Layers



IV JPEG Norm

- Progressive mode
- higher-frequency DCT coefficients are written on the compressed stream in blocks called "scans."
- Each scan that is read and processed by the decoder results in a sharper image.
 The idea is to use the first few scans to quickly create a low-quality, blurred preview of the image, and then either input the remaining scans or stop the process and reject the image.
- Trade-off: the encoder has to save all the coefficients of all the data units in a memory buffer before they are sent in scans, and also go through all the steps for each scan, slowing down the progressive mode





JPEG Norm, JPEG2000

- Wanted functionalities:
 - Gray and binary coding, lossless or lossy
 - Progressive mode by resolution/quality
 - ROI access
 - Error robustness, security
- Technical elements:
 - Tiles partitioning of the image and coding by bit planes (code blocks)

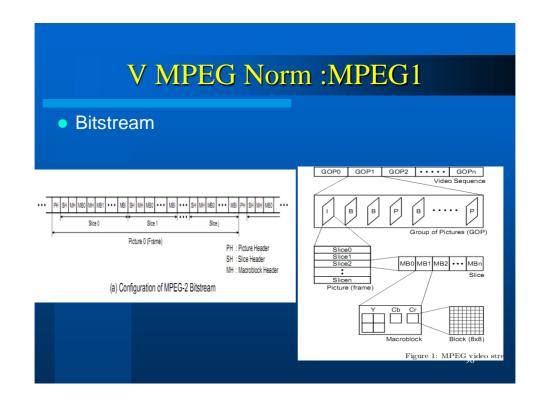
V MPEG Norm

- MPEG1 since 1992; principles of JPEG but exploit the temporal redundancies.
 Low resolution images; flow of 1,5 Mbits/s with sound => VHS quality on CDRom (VideoCD)
- MPEG2 since 1994: higher image quality (3 à 300 Mbits/s)
 Broadcasting and DVD
- MPEG4 objects coding
- AVI Files: container => use the right coder/decoder

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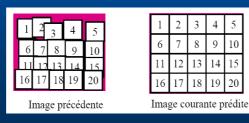
V MPEG Norm Color space: Y Cr Cb Sampling frequency: CCIR 601 norm; 13.75 MHz, "4", for Y and 6.75 MHz for Cr and Cb, "2" 4:2:2 4:1:1 4:2:0 4:4:4 Suppresion Indicators Indicators Suppresion Indicators Suppresion Indicators Suppresion Indicators Suppresion Indicators Indicators Suppresion Indicators Indicators Suppresion Indicators Indicators Suppresion Indicators Indicator

Source Intermediate Format: suppress data before compression: one frame over 2 and one point over 2 in each line => 320 pixels x288 at 25 images Hz (code 2:1:0); 31,5 MHz before compression Groups of image: Intra JPEG coded, Predicted (from image I or P), Bidirectional (movement vectors between images I or P neighboring) | District Norm of the prediction of the presentation | Transmission | Transmis



V MPEG Norm: MPEG1

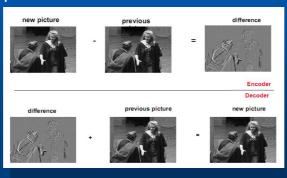
- Movement estimator: predict the displacements
- work on macroblocks: 6 blocks (4 of luminance 2 of chrominance)
- Work on a lookup zone
- Keeping of movement vectors (<> of spatial position of the mblocks between 2 images)

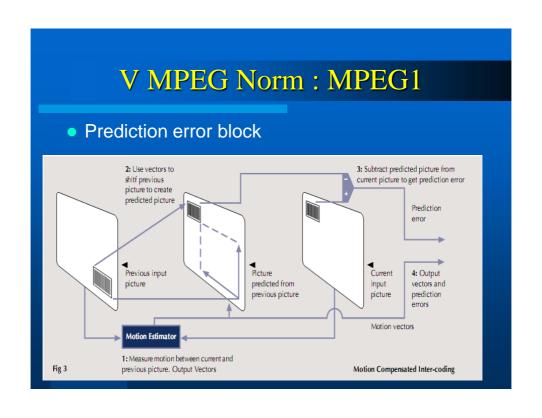


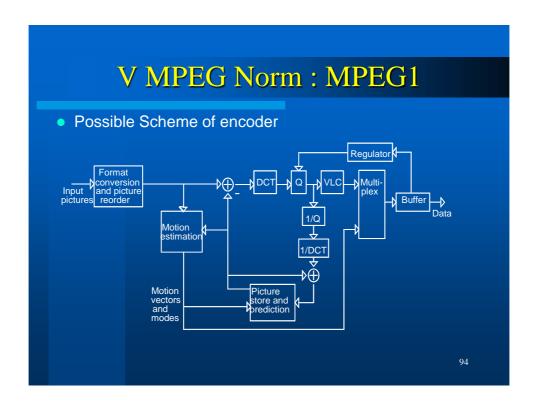
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V MPEG Norm: MPEG1

- Predicted Pictures
- The difference between the current picture and the previous one.







V MPEG Norm: MPEG2

- Support higher resolution
- Processing of interlaced (50 frames /s) and progressive scan => complexity of movement detection
- hierarchic Coding for transmitting at different quality levels
- Compatibility with MPEG1
- 6 profiles for broadcasting
- 4 levels corresponding to the maximum image resolution

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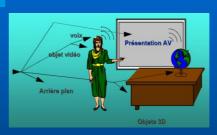
V MPEG Norm: MPEG2

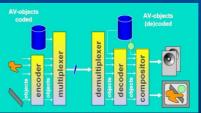
• Profiles:

Profil Niveau	Simple	Main	4:2:2	SNR	Spatial	High
High		4:2:0 1920x1152 80 Mbits/s I,P,B	4:2:0, 4:2:2 1920x1152 300 Mbits/s I,P,B			4:2:0, 4:2:2 1920x1152 15 Mbits/s I,P,B
High 1440		4:2:0 1440x1152 60 Mbits/s I,P			4:2:0, 4:2:2 1440x1152 60 Mbits/s I,P,B	4:2:0, 4:2:2 1440x1152 80 Mbits/s I,P,B
Main	4:2:0 720x576 15 Mbits/s I,P	4:2:0 720x576 15 Mbits/s I,P	4:2:0, 4:2:2 720x608 50 Mbits/s I,P,B	4:2:0, 4:2:2 720x576 15 Mbits/s I,P,B		4:2:0, 4:2:2 720x576 20 Mbits/s I,P,B
Low		4:2:0 352x288 4 Mbits/s I,P,B		4:2:0, 4:2:2 352x288 15 Mbits/s I,P,B		

V MPEG Norm: MPEG4

- Objects decomposition of the scene
- Natural Video Processing tools, 2D and 3D compression
- Synthesized objects texture statistic and dynamic coding





V MPEG Norm: MPEG4

Large applications : Real time communication Video surveillance

Multimedia mobile phone Storage and content based search

Streaming without uploading all the source

Simultaneous scene visualization (vision conference)

Data transmission (all types of data: video, audio,...)

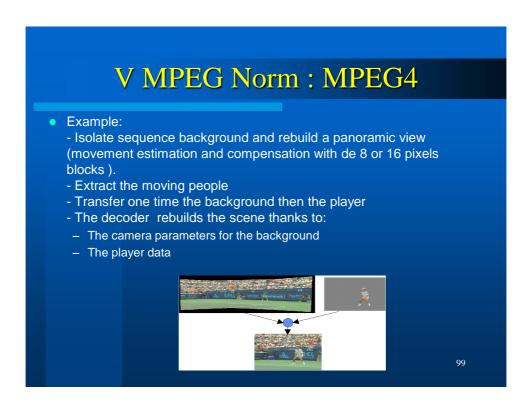
Post production (movies, tv)

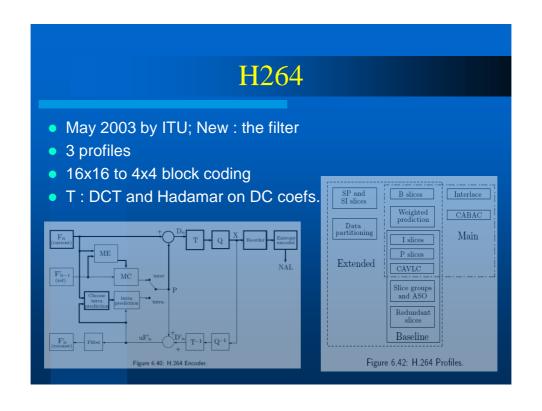
DVD

Face tracking

Hierarchy and audio objects in a scene

- Interactivity
- DivX/Xvid : based on the vidéo part of MPEG4





references

- Digital Image Processing : R.Gonzales, E.Woods
- Data compression, the complete reference, David Salomon, Springer
- D.S. Taubman et M.W. Marcellin, "JPEG2000 Image compression fundamentals, standards and practice", Kluwer Academic Publishers (2001).