

Eigenvalues and eigenvectors

NOTE

You can discuss the problems with each other, but your solutions must be independently written with your own words and formulations.

Problem 1

The first problem of this homework is the **problem 17** in the problems list. Here we give some ideas to help you finding a solution.

This problem is an example of what is called *Markov processes*. To solve the problem we first need to find the transition matrix or Markov matrix.

- For a given day, say Day n , we divide the citizens of *Computer Vision Village* into two groups: group 1 contains all citizens who read the paper on Day n , and group 2 contains all citizens who do not buy the paper on Day n . We also define a vector

$$v_n = \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \in \mathbb{R}^2,$$

where g_1 is the size of group 1 and g_2 the size of group 2.

- Construct a 2×2 matrix $M = (m_{ij})$, where the entry m_{ij} gives the probability that a citizen in group j one day will be in group i the next day. M is the transition matrix of the Markov process (check that the columns of M add to 1).
 - Show that we have $Mv_n = v_{n+1}$.
 - Now, you are ready to answer questions 1 and 2.
- To answer question 3, think about eigenvalues of M . What does 'sales figures become stable' means in terms of eigenvalues/eigenvectors?

Problem 2

This second problem is about another application of eigenvalues/eigenvectors. It deals with the Google PageRank algorithm.

- Read the paper *How Google ranks Web pages?* which is available from Edomo and write a summary of the ranking method.

2. Explain how the eigenvalue problem is solved. You don't need to rewrite what is in the paper, just put down what you understand.
3. Starting with a 5×5 Markov matrix, apply the algorithm described in Section 5 of the paper to find the eigenvector associated with eigenvalue 1.
4. **Additional question** (If you like and have time) Write this algorithm as a Matlab function.
5. Any other comment?