F20DL and F21DL: Part 2: Machine Learning Lecture 6.2: Linear Classifiers, ctd

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Course Feedback on Vision

- closes on Monday, 19th November
- please find 10 mins or so to give constructive feedback

Our progress

last time, we derived the Linear Regression by working with derivatives of the error function.

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Today:

- finishing regression
- starting Neural Nets

Customer transactions, converted to numeric

Trans.	Music on CD?	Music on MP3?	Board Games	On-line Games	Output
T1	0	1	0	1	1
T2	1	0	0	0	0
T3	1	0	0	1	1
T4	1	0	1	0	0
T5	0	1	0	0	0
T6	0	1	1	0	0
T7	0	0	0	1	1
T8	0	1	1	1	0
T9	1	1	0	0	0
T10	1	1	0	1	1

Running this data set in Weka

Functions ⇒ Linear Regression:
(Use "More Options" ⇒ "Output Predictions")
Gives output:

```
Linear Regression Model

Buys =

     -0.4     * gamesHard +
      0.72    * gamesOnline +
      0.16
```

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```

How do you interpret this, in "human" terms?

Accuracy?

```
=== Predictions on test data ===

    inst#     actual  predicted      error
        1      1          0.846     -0.154
        1      1          0.846     -0.154
        1      0          1          1
        1      0          0.222      0.222
        1      0          0.222      0.222
        1      1          0.846     -0.154
        1      1          0.846     -0.154
        1      0         -0.387     -0.387
        1      0          0.222      0.222
        1      0         -0.387     -0.387
```

Functions for Classification

In classification tasks, there are normally two values - 0 and 1, so linear function is not well suited.
For classification, one uses **squashed linear function** of the form

$$f(X_1, \ldots, X_n) = G(w_0 + w_1X_1 + \ldots + w_nX_n)$$

where G is **an activation function** from real numbers to $[0, 1]$.

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Example: A step function

$S(x) = 1 \text{ if } x \geq 0 \text{ and } S(x) = 0 \text{ if } x \leq 0$

- ... was used in Perceptron [Rosenblatt, 1958] - one of the first methods for learning.
- Disadvantage: not differentiable.

Sigmoid (logistic) activation function

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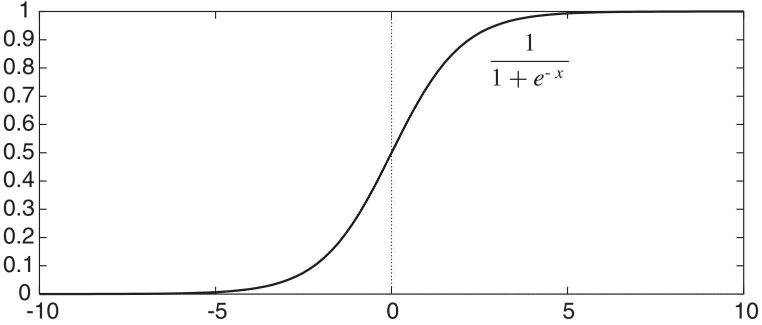

If the function is differentiable...

we can use gradient descent to update the weights:
For the sigmoid function σ, the derivative is
$\sigma'(x) = \sigma(x) \times (1 - \sigma(x))$
We use this to change line 15 in the algorithm *LinearLearner* to

$$w_i := w_i + \eta \times \delta \times pval^{\overline{\omega}}(e, Y) \times [1 - pval^{\overline{\omega}}(e, Y)] \times val(e, X_i).$$

where $pval^{\overline{\omega}}(e, Y) = \sigma(\sum_i w_i \times val(e, X_i))$, and δ as before.
– The resulting algorithm will be called "Logistic Regression"

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If your maths is good, try computing the derivative
$((val(e, Y) - pval(e, Y))^2)' =$
$2 \times \delta \times pval(e, Y) \times [1 - pval(e, Y)] \times val(e, X_i)$,
where $pval(e, Y) = \sigma(w_i \times val(e, X_i))$, for each weight w_i.
(very similar to our last lecture exercise)

Logistic regression

Algorithm LogisticLearner(X,Y,E,η)
Inputs:
    X: set of input features, $X = \{X_1, \ldots, X_n\}$
    Y: target feature
    E: set of examples from which to learn
    η: - learning rate
Output: parameters $w_0, \ldots, w_n$.
    Local $w_0, \ldots, w_n$ - real numbers
    $pval(e, Y) = \sigma(w_0 + w_1 \times val(e, X_1) + \ldots + w_n \times val(e, X_n))$
initialise $w_0, \ldots, w_n$ randomly
repeat
    for each example e in E **do**
        $\delta := val(e, Y) - pval(e, Y)$
        for each $i \in [0, n]$ **do**
            $w_i := w_i + \eta \times \delta \times pval(e, Y) \times [1 - pval(e, Y)] \times val(e, X_i)$
until termination
return $w_0, \ldots, w_n$

Example

Our old "Reading mail example" (from lecture on Decision trees) can be classified correctly by the following function:

$$Reads = \sigma(-8 + 7Short + 3New + 3Known),$$

where σ is the sigmoid function. A function similar to this can be found with about 3,000 iterations of gradient descent with a learning rate $\eta = 0{,}05$.

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where σ is the sigmoid function. A function similar to this can be found with about 3,000 iterations of gradient descent with a learning rate $\eta = 0{,}05$.
According to this function, *Reads* is true if and only if *Short* is true and either *New* or *Known* is true.

Running our Lecture1 exercise in Weka

```
Logistic Regression with ridge parameter of 1.0E-8
Coefficients...
                                   Class
Variable                            buys
====================================
musicCD                           0.3475
musicMP3                          6.6939
gamesHard                       -52.6445
gamesOnline                      51.9035
Intercept                       -38.6139
```

Running our Lecture1 exercise in Weka

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Logistic Regression with ridge parameter of 1.0E-8
Coefficients...
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musicCD                            0.3475
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=== Predictions on test data ===

 inst#     actual  predicted error prediction
     1     1:buys     1:buys       1
     1     1:buys     1:buys       1
     1     1:buys     1:buys       1
     1     1:buys     1:buys       1
     1  2:cancels  2:cancels       1
     1  2:cancels     1:buys   +   1
     1  2:cancels  2:cancels       1
     1  2:cancels  2:cancels       1
     1  2:cancels  2:cancels       1
     1  2:cancels  2:cancels       1
```

Accuracy?

```
Correctly Classified Instances           9

=== Confusion Matrix ===

 a b   <-- classified as
 4 0 | a = buys
 1 5 | b = cancels
```

Bias in linear classifiers and decision trees

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w_0	w_1	$\cdots$	w_k

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-15	10	$\cdots$	10

  This concept forms a large decision tree.

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  This concept forms a large decision tree.
- Consider representing a conditional: "If X_7 then X_2 else X_3":
  - Simple in a decision tree.
  - Complicated (possible?) for a linear separator

Conclusions

- We have learned about linear classifiers
- Next time, we will discuss their limitations and ways to overcome the limitations
- As always – check related Chapters in the Course textbook: §4.6, pp.124-129, §11.4, 459-469

Homework/Lab: Test 4

- Take the small emotion recognition data set (fer10.arff) again, available on Vision (attached to this lecture slides)
- It is not directly suitable for Linear Regression (Weka will not even allow to run it). Why?

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- Convert your data set to numeric values, like this:

```
@attribute 'musicCD' numeric
@attribute 'musicMP3' numeric
@attribute 'gamesHard' numeric
@attribute 'gamesOnline' numeric
@attribute 'Buys' numeric
```

- Run, in Weka: Linear regression with the settings: weka.classifiers.functions.LinearRegression -S 2 -R 1.0E-8 -num-decimal-places 4

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- Take your small emotion recognition data set again
- Convert your data set to numeric values and NOMINAL class, like this:

```
@attribute 'musicCD' numeric
@attribute 'musicMP3' numeric
@attribute 'gamesHard' numeric
@attribute 'gamesOnline' numeric
@attribute 'Buys' {Buys, Cancels}
```

- Run, in Weka: Logistic Regression with the settings: weka.classifiers.functions.Logistic -R 1.0E-8 -M -1 -num-decimal-places 4

Homework/Lab: Test 4

- We use the same test set as for Decision trees
- Compare performance of these two linear classifiers with your results for Decision trees, on the same test set.
- Be ready to answer test questions about your conclusions
- For both Linear regression and Logistic regression, note the weights w_0, w_1, w_2, w_3, w_4 computed by Weka.
- For both cases, write the *pval* formula in the general form (as we did in the lecture) and compare to what Weka gives.