Path Control

We now address the problem of how to get the TCP of a robotic manipulator to follow a path in space with a prescribed velocity. This is the sort of control we need for paint-spraying operations for instance.

We shall consider the off-line computation of paths, so avoiding the problem of real-time processing for now.

One procedure we could use is to **Store and Playback** -

a) Choose a total time for the motion and divide it up into equal intervals $\mathbf{\Delta t}$.
b) Determine the hand position, **Th,** for each interval.
c) Use $\mathbf{Th^{-1}}$ to find the joint angles for each **Th** position.
d) Store all the joint angles at each step along the path (i.e. at each time interval, $\mathbf{\Delta t}$).
e) Play back the stored joint angles to achieve the motion.

The problem with this approach is that the resulting TCP motion will be jerky because it doesn't ensure a smooth transition from one set of joint angles to the next.

Also, consider a straight-line path between two points in 3D space, starting from rest and ending at rest and with no orientation change along the way.

The velocities along the three Cartesian axes must be constant for straight-line motion.

Consider just the x-component of the velocity for now. We can treat the y and z components in a similar fashion.

Let the velocity along x be $1ms^{-1}$ and the time interval, $\mathbf{\Delta t}$, be 5ms.

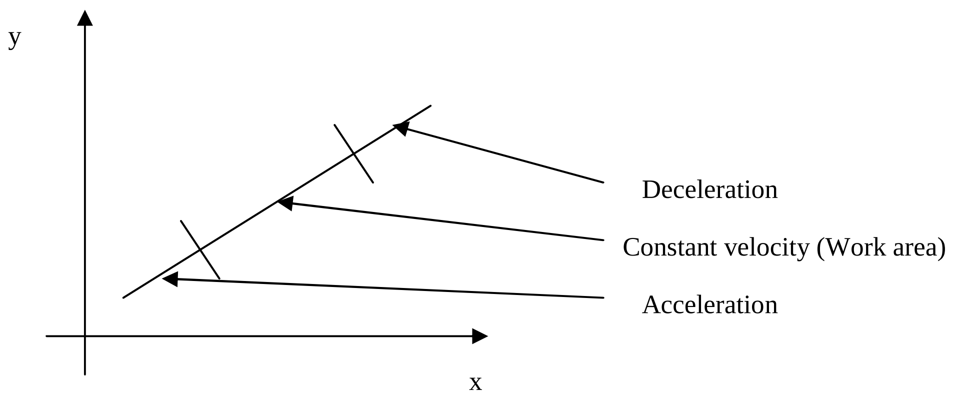
The position values for x will need to be -

Time (ms)	x (mm)
5	5
10	10
15	15

The first 5ms will need an acceleration in x of $400ms^{-2}$ This is just not realistic!

We need to plan our path much more carefully than this.

There are really 3 parts to a controlled motion along a path–

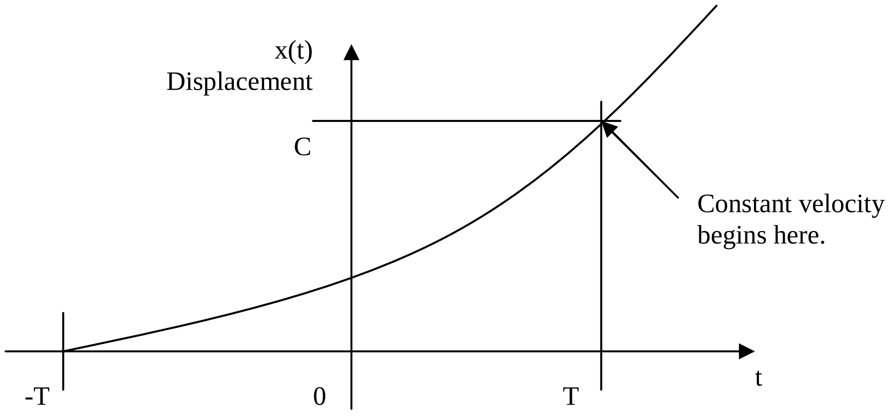


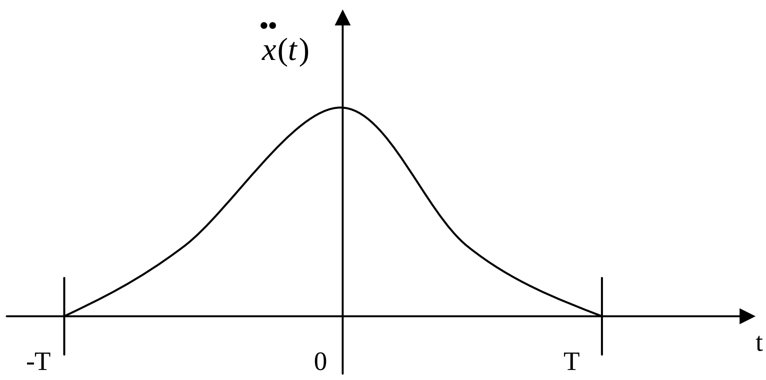
Path Control Polynomials

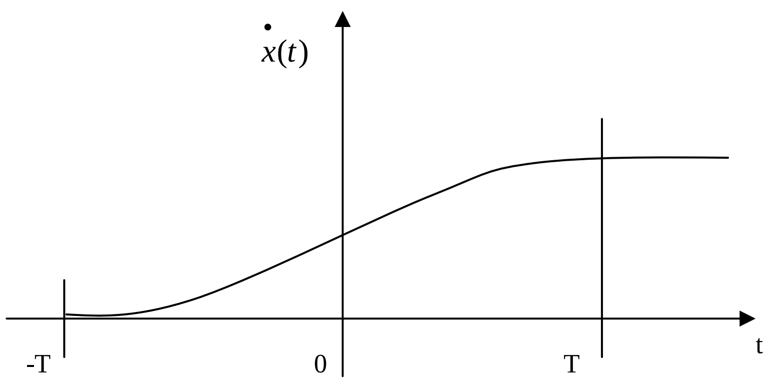
Let *x(t)* be the *x* co-ordinate of the robot hand at the time *t*. *x(t)* tells us what the *x* co-ordinate of the TCP has to be at successive times, *t* , in order to arrive at the start of our constant velocity at the right time and in the right place.

Suppose we wish to accelerate from rest until at a time, *T*, we have moved a distance *C* along *x* and arrived at a constant velocity of *C/T* along *x*.

We make *t=0* the mid point of our acceleration -







We have 6 parameters in our problem. For each end of the motion we specify -

1. Position – $\mathbf{x(t)}$
2. Velocity – $\dot{\mathbf{x}}\mathbf{(t)}$
3. Acceleration – $\ddot{\mathbf{x}}\mathbf{(t)}$

So *x(t)* will need up to 6 free variables to describe it. This means it can be described by a polynomial of degree 5.

Our problem here is symmetric though so we only need a degree 4 polynomial -

$$\mathbf{x(t) = a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0}$$

$$\dot{\mathbf{x}}\mathbf{(t) = 4a_4 t^3 + 3a_3 t^2 + 2a_2 t + a_1}$$

$$\ddot{\mathbf{x}}\mathbf{(t) = 12a_4 t^2 + 6a_3 t + 2a_2}$$

The boundary conditions state the following -

$$\mathbf{x(-T) = 0} \qquad \dot{\mathbf{x}}\mathbf{(-T) = 0} \qquad \ddot{\mathbf{x}}\mathbf{(-T) = 0}$$
$$\mathbf{x(T) = C} \qquad \dot{\mathbf{x}}\mathbf{(T) = {}^{C}\!/_{T}} \qquad \ddot{\mathbf{x}}\mathbf{(T) = 0}$$

Consider our equation for the acceleration at times **–T** and **+T**

$$\mathbf{t = -T}$$
$$\mathbf{0 = 12a_4 T^2 - 6a_3 T + 2a_2}$$
$$\mathbf{t = +T}$$
$$\mathbf{0 = 12a_4 T^2 + 6a_3 T + 2a_2}$$
$$\therefore \mathbf{a_3} = 0$$

Similarly the equation for velocity gives -

$$\mathbf{a_1 = \frac{C}{2T}}$$

Setting **t=-T** in these same equations gives -

$$\mathbf{a}_2 = \mathbf{\frac{3C}{8T^2}}$$

and

$$\mathbf{a_4 = \frac{-C}{16T^4}} \qquad \mathbf{a}_0 = \frac{3\mathbf{C}}{16}$$

follow.