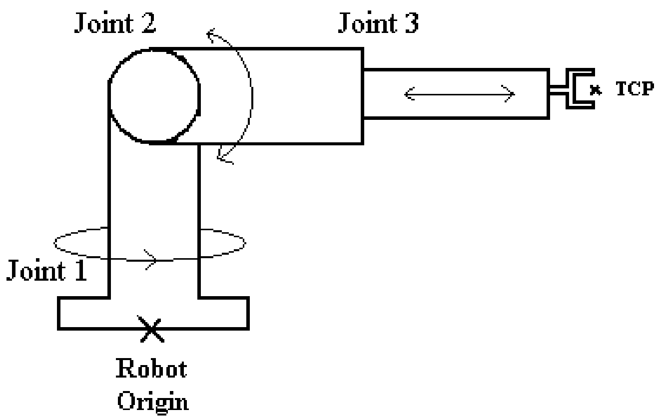
PAST PAPER QUESTION FROM REVISION LECTURE

Q1 Consider the following three-joint manipulator geometry –



The waist and shoulder joints are revolute and the radial joint is prismatic. Joint 2 is located a distance d_1 above the robot origin, i.e. the TCP is at a height of d_1 in the above diagram. The TCP is located a distance d_2 from Joint 2 when the radial joint is fully retracted, i.e. when the arm extension is 0.

(a) Derive the inverse kinematic equations for the three joints of this manipulator. [Hint: A geometric solution can be used here] (9)

(b) Hence derive the inverse Jacobian for the manipulator. (11)

SOLUTION

(a) Derive the inverse kinematic equations for the three joints of this manipulator. [Hint: A geometric solution can be used here]

Let the joint angles be θ_1 for the waist rotation, θ_2 for the shoulder rotatation and r for the radial extension

Viewing the arm from above we obtain the waist rotation –

$$\tan(\theta_1) = \frac{y}{x}$$

$$\boxed{\therefore \theta_1 = ATAN2(y, x)}$$

3 MARKS

Viewing the side elevation we can determine the shoulder rotation –

$$\tan(\theta_2) = \frac{z - d_1}{\sqrt{x^2 + y^2}}$$

$$\boxed{\therefore \theta_2 = ATAN2\left(z - d_{1'} \sqrt{x^2 + y^2}\right)}$$

3 MARKS

Also from the side elevation we can determine the radial extension –

$$r + d_2 = \sqrt{(z - d_1)^2 + x^2 + y^2}$$

$$\boxed{\therefore r = \sqrt{(z - d_1)^2 + x^2 + y^2} - d_2}$$

3 MARKS

b). Hence derive the inverse Jacobian for the manipulator.

We differentiate each of the inverse kinematics equations with respect to time –

$$\tan(\theta_1) = \frac{y}{x}$$

$$\therefore \sec^2(\theta_1)\frac{\partial \theta_1}{\partial t} = \frac{-y}{x^2}\frac{\partial x}{\partial t} + \frac{1}{x}\frac{\partial y}{\partial t}$$

Noting that $\sec^2(\theta_1) = \frac{(r + d_2)^2}{x^2}$ because sec(x) = 1/cos(x)

$$\therefore \dot{\theta}_1 = \frac{-y}{x^2\left((r+d_2)^2 / x^2\right)}\dot{x} + \frac{1}{x\left((r+d_2)^2 / x^2\right)}\dot{y}$$

$$\boxed{\therefore \dot{\theta}_1 = \frac{-y}{(r+d_2)^2}\dot{x} + \frac{x}{(r+d_2)^2}\dot{y}}$$

3 MARKS

$$\tan(\theta_2) = \frac{z - d_1}{\sqrt{x^2 + y^2}}$$

$$\therefore \sec^2(\theta_2)\frac{\partial \theta_2}{\partial t} = (z - d_1)\frac{\partial\left(1 \big/ \sqrt{x^2+y^2}\right)}{\partial t} + \frac{1}{\sqrt{x^2+y^2}}\frac{\partial z}{\partial t}$$

$$\therefore \dot{\theta}_2 \sec^2(\theta_2) = \frac{1}{\sqrt{x^2+y^2}}\dot{z} + (z - d_1)\left[\frac{-x\dot{x} - y\dot{y}}{\left(x^2+y^2\right)^{3/2}}\right]$$

Noting that $\sec(\theta_2) = 1 \big/ \cos(\theta_2) = 1 \Big/ \frac{\sqrt{x^2+y^2}}{r+d_2} = \frac{r+d_2}{\sqrt{x^2+y^2}}$ $\therefore \sec^2(\theta_2) = \frac{(r+d_2)^2}{x^2+y^2}$

$$\boxed{\therefore \dot{\theta}_2 = \frac{\sqrt{x^2+y^2}}{(r+d_2)^2}\dot{z} + \frac{z-d_1}{(r+d_2)^2\sqrt{x^2+y^2}}\left[-x\dot{x} - y\dot{y}\right]}$$

3 MARKS

$$(r + d_2)^2 = (z - d_1)^2 + x^2 + y^2$$

$$(2r + 2d_2)\frac{\partial r}{\partial t} = (2z - 2d_1)\frac{\partial z}{\partial t} + 2x\frac{\partial x}{\partial t} + 2y\frac{\partial y}{\partial t}$$

$$\boxed{\therefore \dot{r} = \frac{z-d_1}{r+d_2}\dot{z} + \frac{x}{r+d_2}\dot{x} + \frac{y}{r+d_2}\dot{y}}$$

3 MARKS

So the inverse Jacobian is therefore –

$$\boxed{\begin{pmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{r} \end{pmatrix} = \begin{pmatrix} \frac{-y}{(r+d_2)^2} & \frac{x}{(r+d_2)^2} & 0 \\ \frac{-(z-d_1)x}{(r+d_2)^2\sqrt{x^2+y^2}} & \frac{-(z-d_1)y}{(r+d_2)^2\sqrt{x^2+y^2}} & \frac{\sqrt{x^2+y^2}}{(r+d_z)^2} \\ \frac{x}{r+d_2} & \frac{y}{r+d_2} & \frac{z-d_1}{r+d_2} \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}}$$

2 MARKS