Education and Culture

Erasmus Mundus

Vibot

Universitat de Girona

Visual Perception

Lecture 2

Camera Calibration

UNIVERSITÉ DE BOURGOGNE

UdG

HERIOT WATT UNIVERSITY

Contents

2. Camera Calibration

2.1 Calibration introduction

2.2 The pinhole model

2.3 The method of Hall

2.4 The method of Faugeras-Toscani – Modelling

2.5 The method of Faugeras-Toscani – Calibration

2.6 The method of Faugeras-Toscani with distortion

2.7 Experimental comparison of methods

Contents

2. Camera Calibration

2.1 Calibration introduction

2.2 The pinhole model

2.3 The method of Hall

2.4 The method of Faugeras-Toscani – Modelling

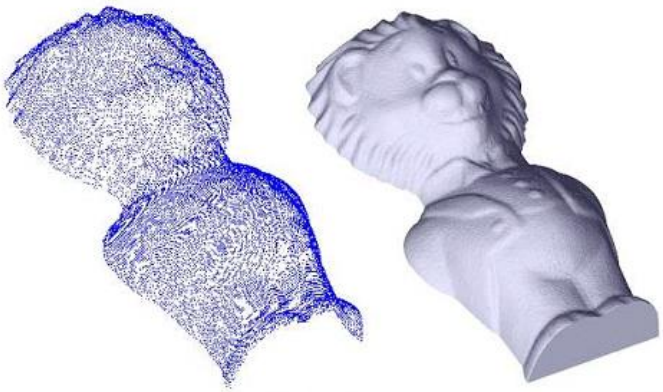
2.5 The method of Faugeras-Toscani – Calibration

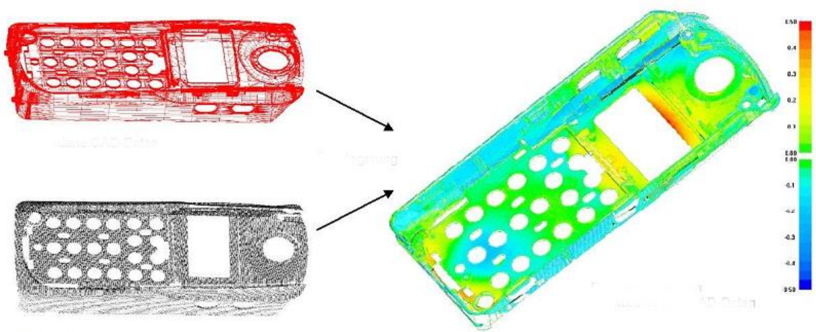
2.6 The method of Faugeras-Toscani with distortion

2.7 Experimental comparison of methods

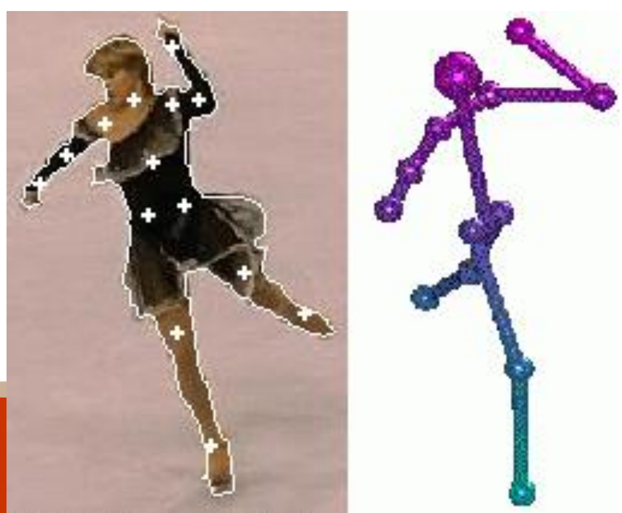
2.1 Calibration Introduction

- Some applications of this capability include
  - Dense reconstruction
  - Visual inspection





  - Object localization
  - Camera localization



2.1 Calibration Introduction – Perspective Imaging

"The Scholar of Athens," Raphael, 1518

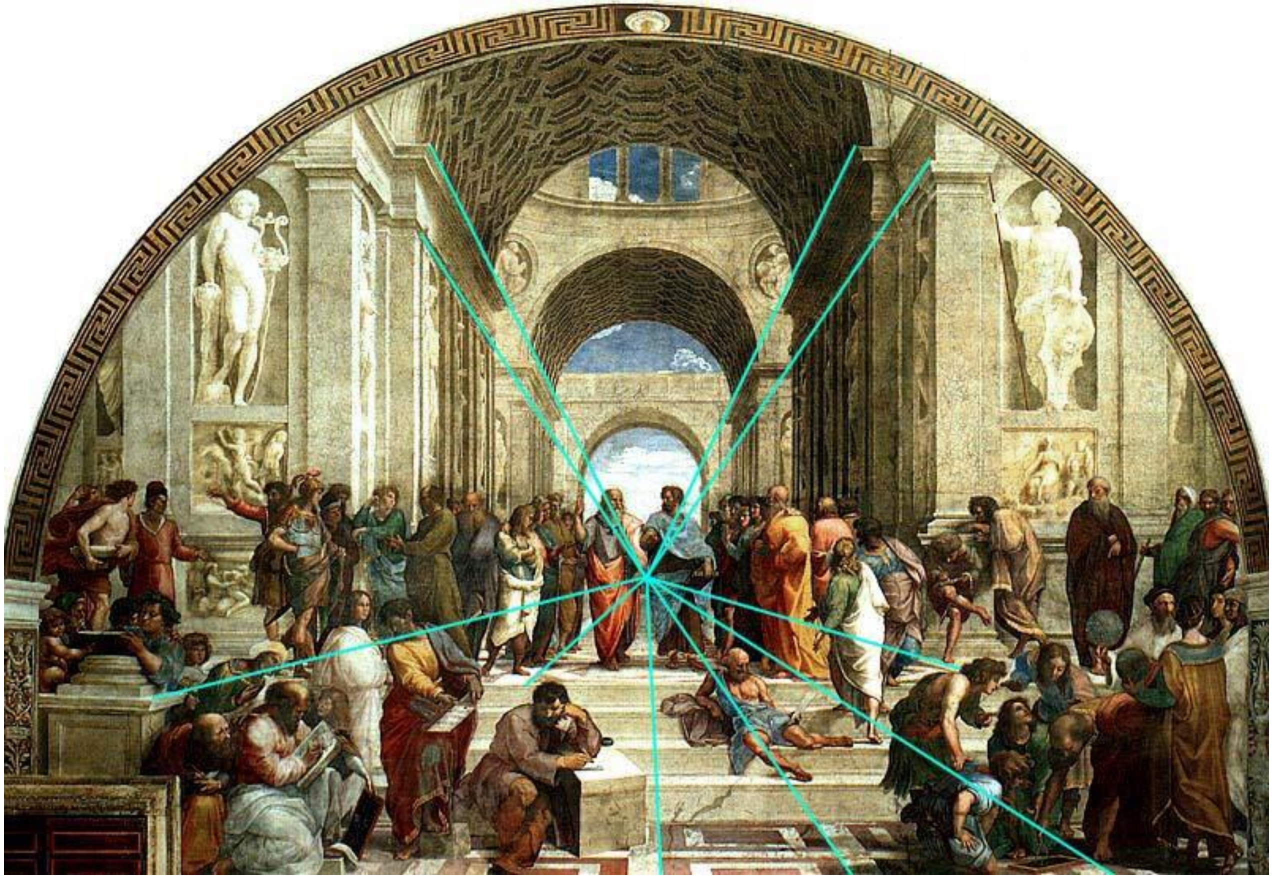
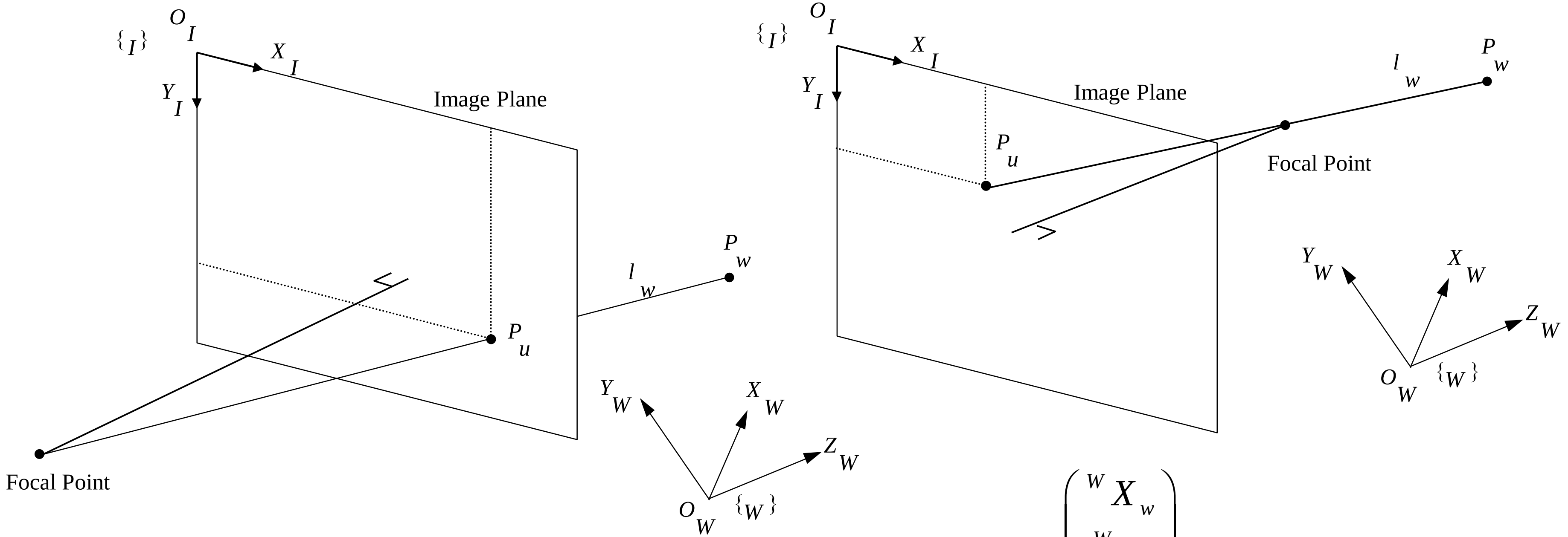


Image courtesy of C. Taylor

2.1 Calibration Introduction



$$^{W}P_{w} = \begin{pmatrix} ^{W}X_{w} \\ ^{W}Y_{w} \\ ^{W}Z_{w} \\ 1 \end{pmatrix}$$

?

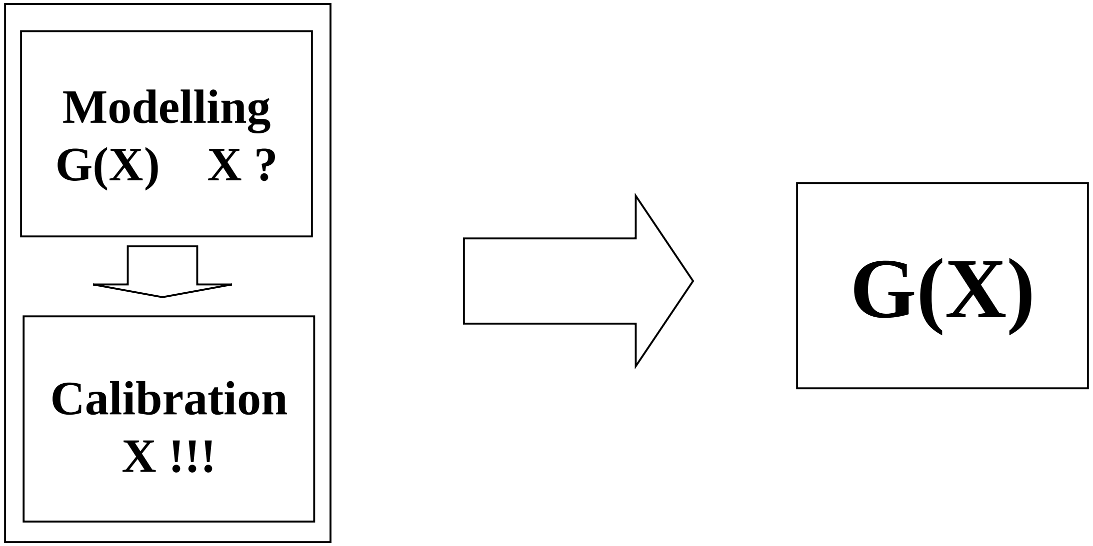
$$^{I}P_{u} = \begin{pmatrix} ^{I}X_{u} \\ ^{I}Y_{u} \\ 1 \end{pmatrix}$$

In pixels

In metrics

$$^{W}l_{w} = \begin{pmatrix} ^{W}X_{w} \\ ^{W}Y_{w} \\ ^{W}Z_{w} \\ 0 \end{pmatrix}$$

2.1 Calibration Introduction



Modelling:

- Determine the equation that approximates the camera behaviour.
- Define the set of unknowns in the equation (camera parameters).
- The camera model is an approximation of the physics & optics of the camera.

Calibration:

- Get the numeric value of every camera parameter.

Contents

2. Camera Calibration

2.1 Calibration introduction

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2.3 The method of Hall

2.4 The method of Faugeras-Toscani – Modelling

2.5 The method of Faugeras-Toscani – Calibration

2.6 The method of Faugeras-Toscani with distortion

2.7 Experimental comparison of methods

Contents

2. Camera Calibration

2.1 Calibration introduction

2.2 The pinhole model

2.3 The method of Hall

2.4 The method of Faugeras-Toscani – Modelling

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2.6 The method of Faugeras-Toscani with distortion

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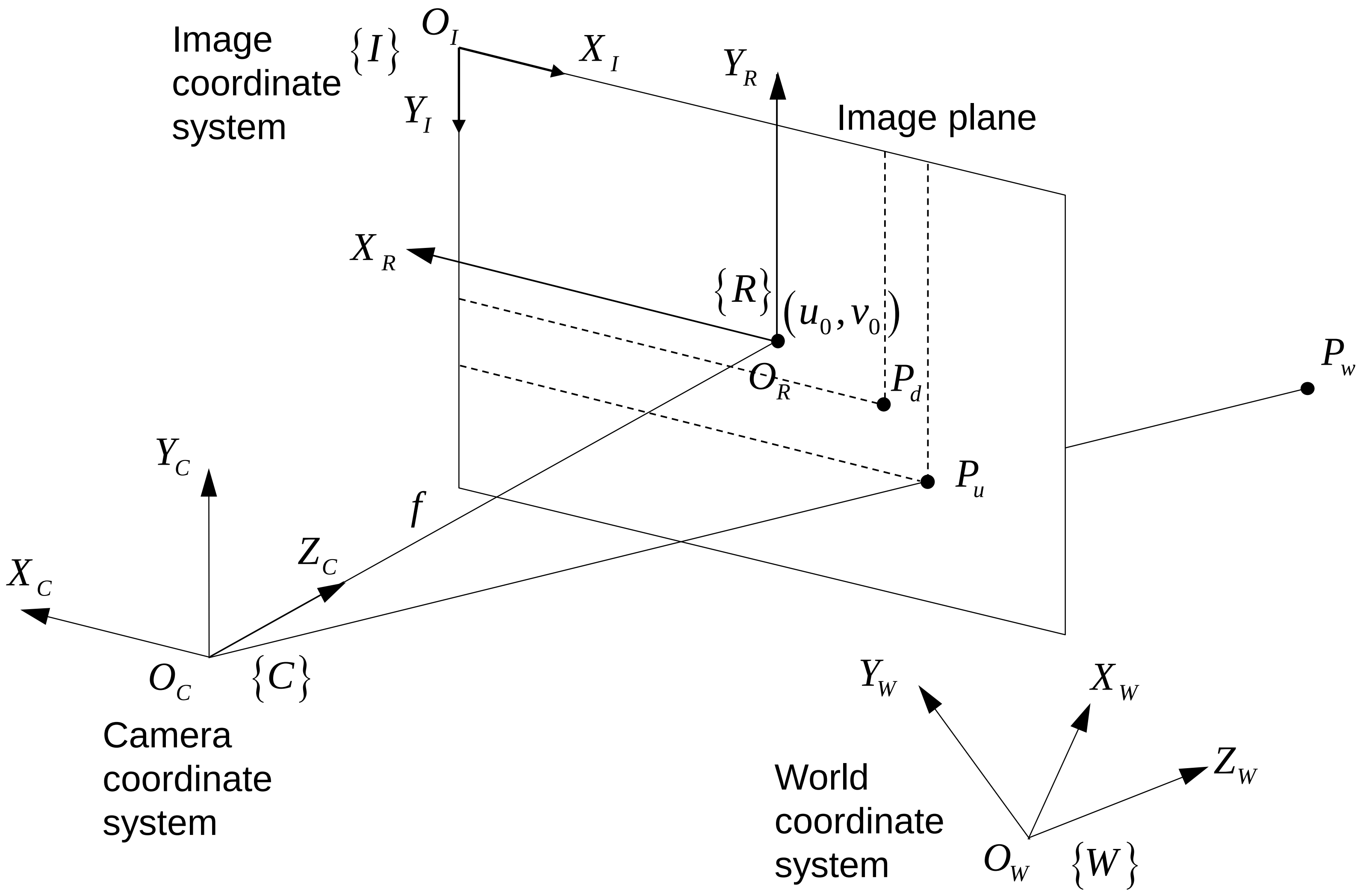
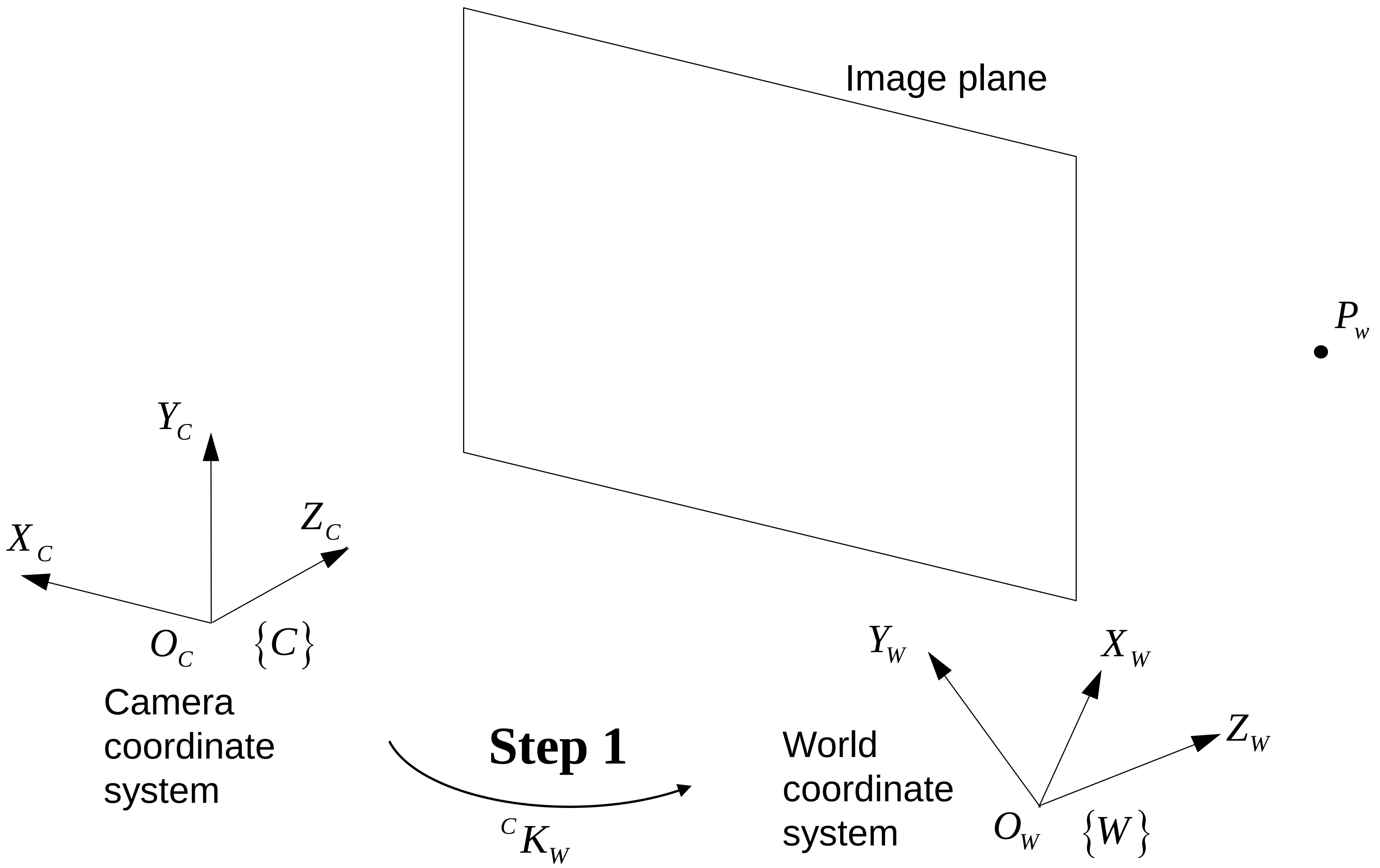
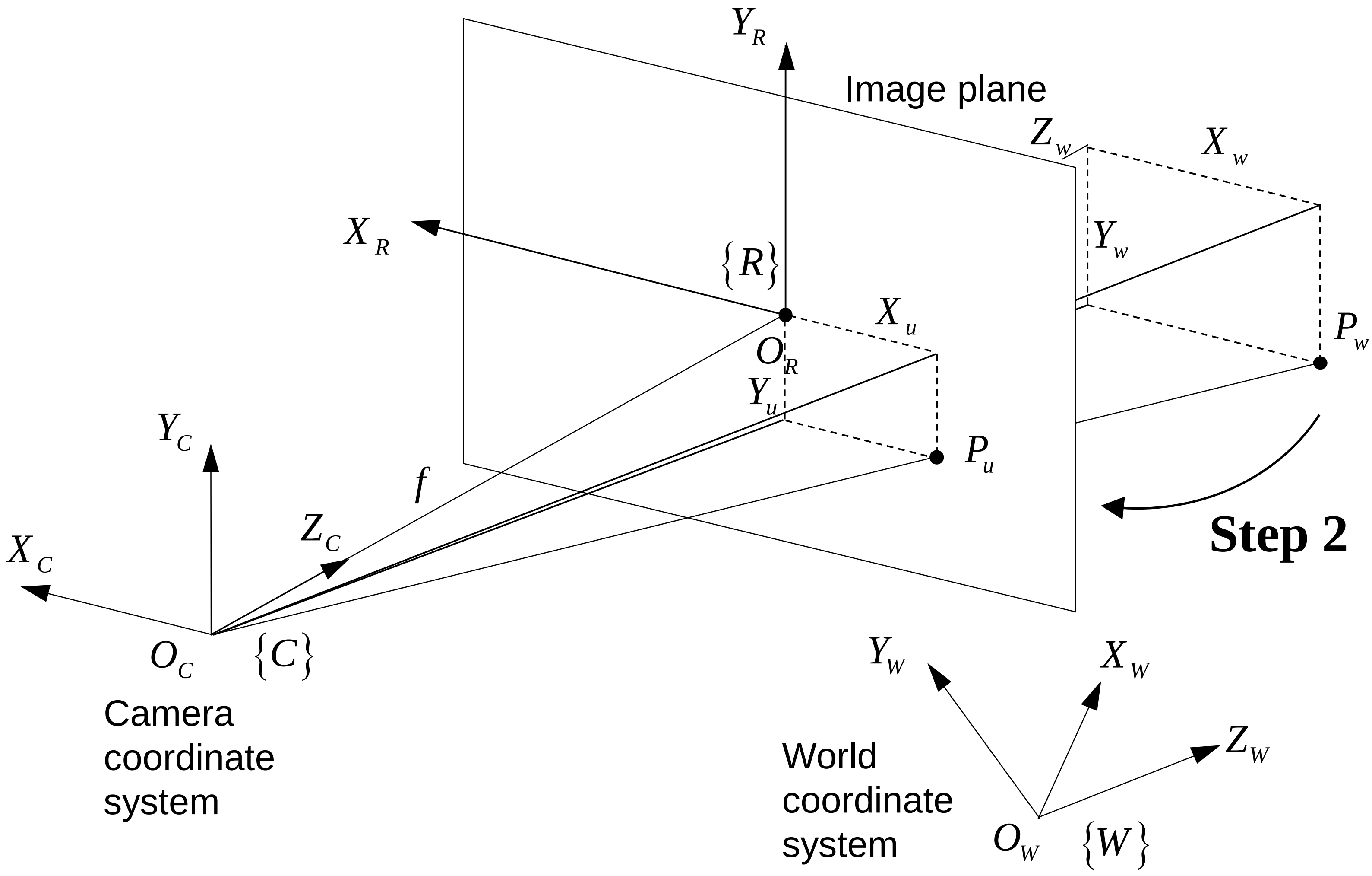
2.2 Pinhole Model



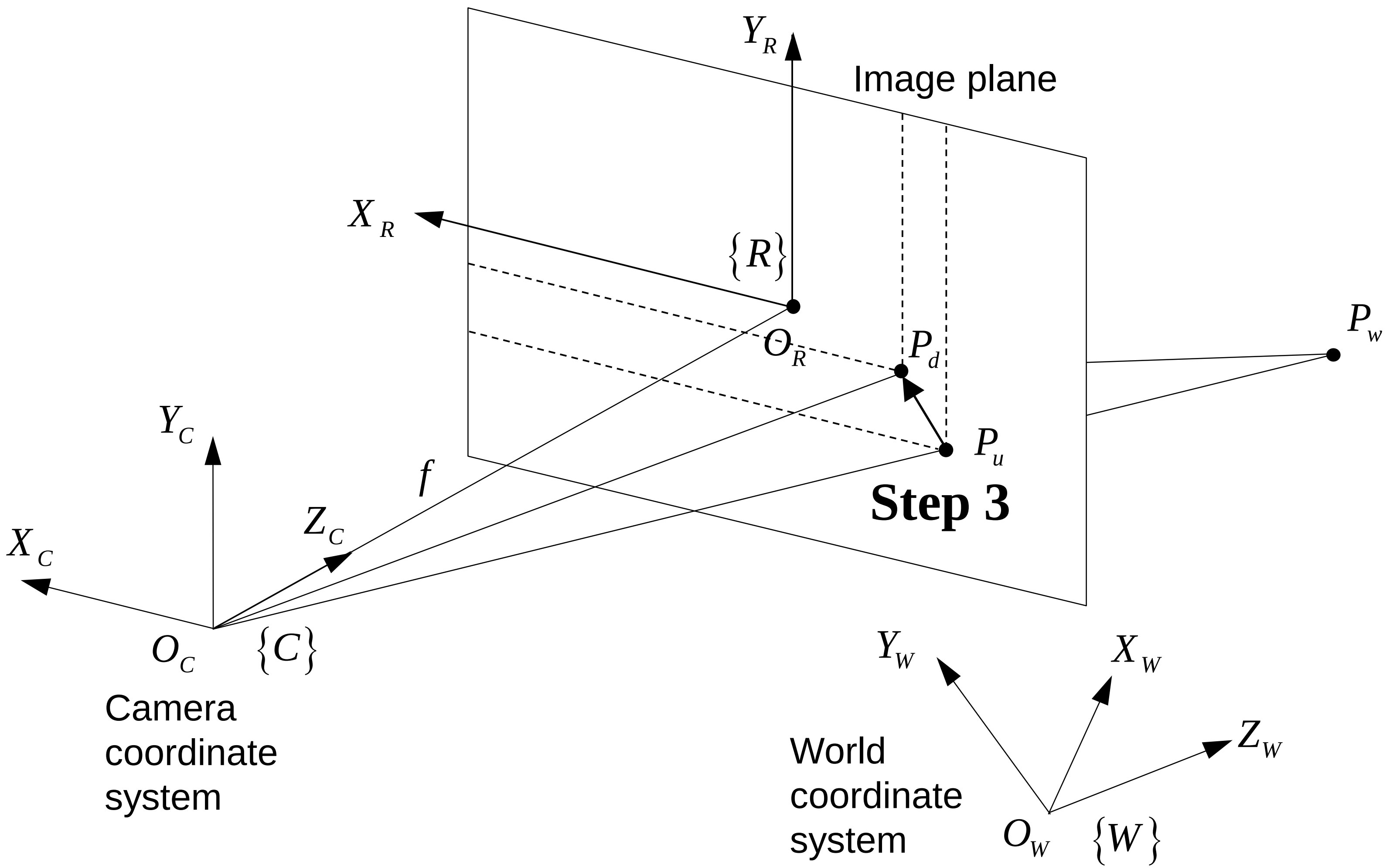
2.2 Pinhole Model (Step 1: World to Camera)



2.2 Pinhole Model (Step 2: Projection)

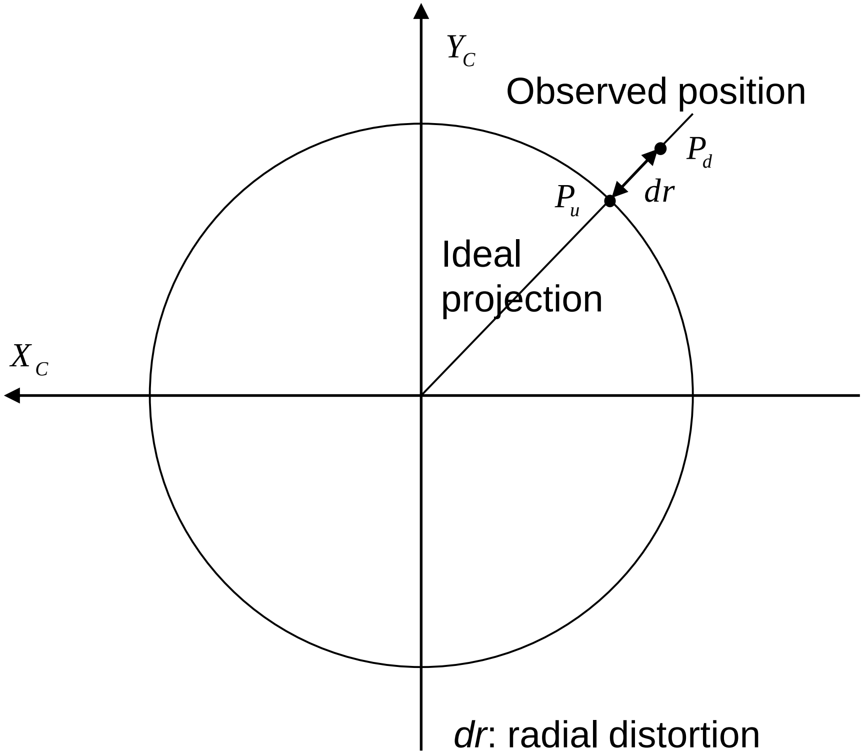


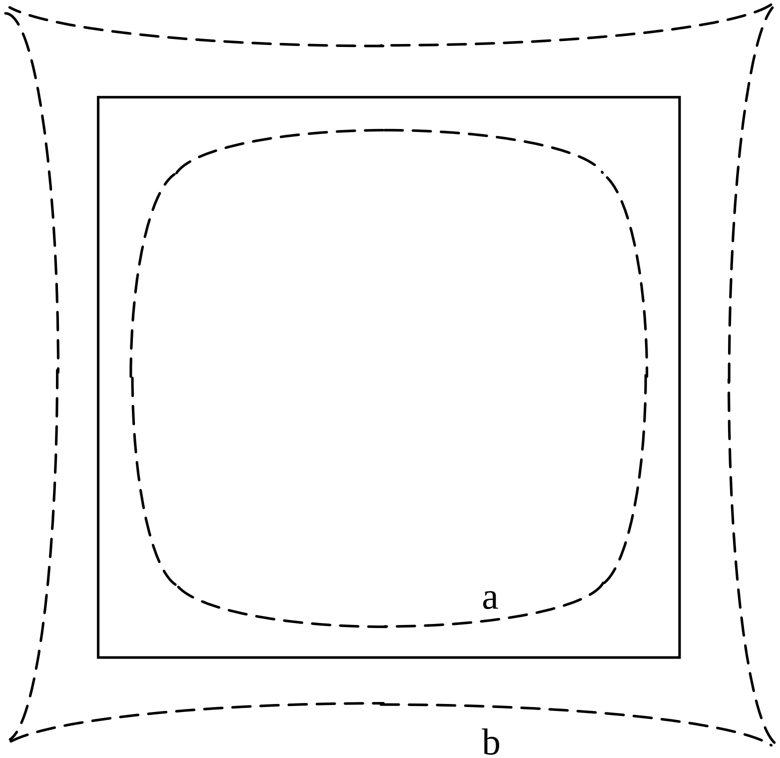
2.2 Pinhole Model (Step 3: Lens Distortion)



2.2 Pinhole Model (Step 3: Lens Distortion)

Radial Distortion





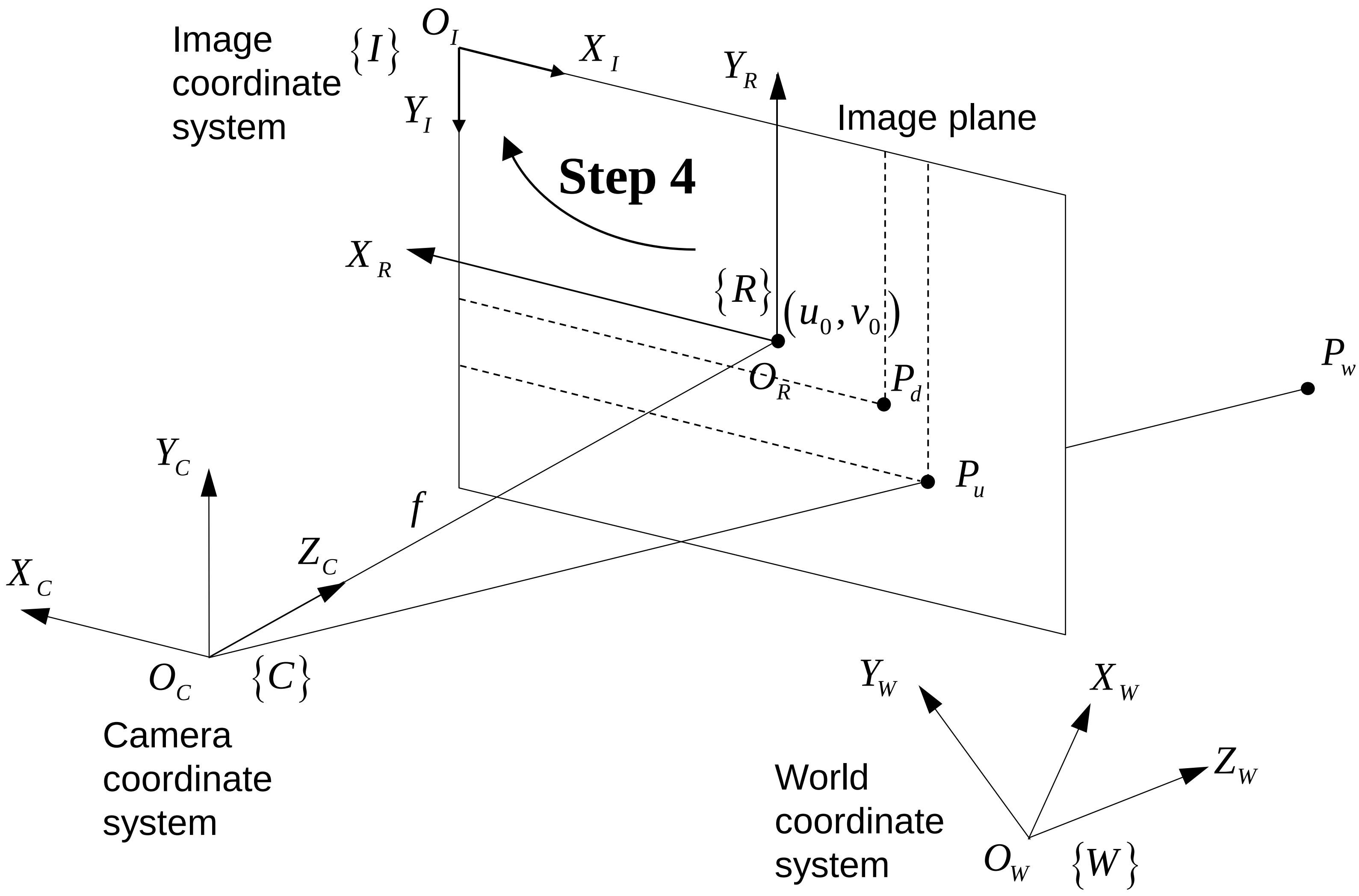
Radial distortion effect (a: negative, b: positive)

2.2 Pinhole Model (Step 3: Lens Distortion)

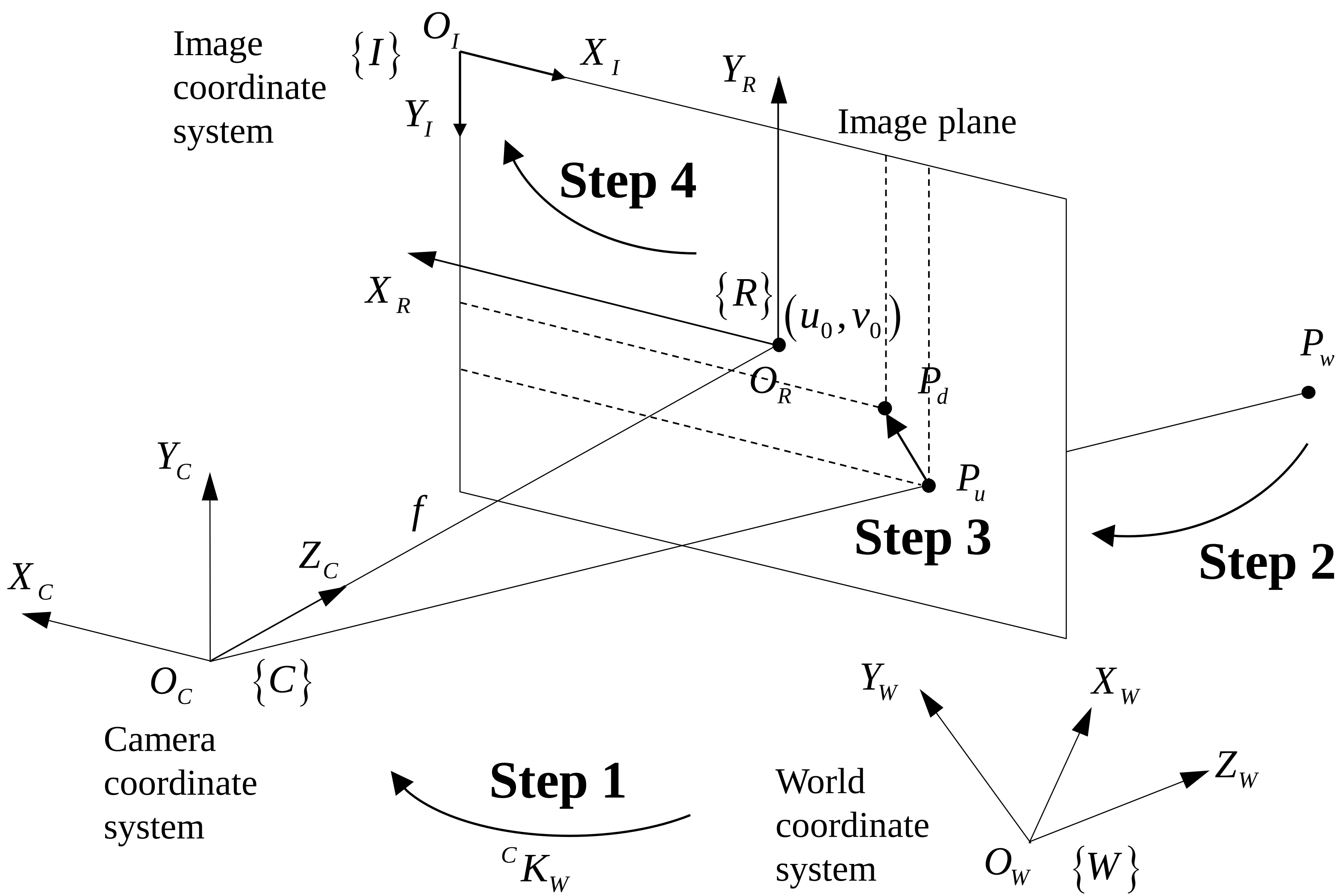
Radial and Tangential Distortion

Image without distortion

2.2 Pinhole Model (Step 4: Camera to Image)



2.2 Pinhole Model



2.2 Calibration Methods (I)

- Method of Hall
  - Lineal method
  - Transformation matrix
- Method of Faugeras-Toscani
  - Lineal method
  - Obtaining camera parameters
- Method of Faugeras-Toscani with distortion
  - Iterative method
  - Radial distortion
- Method of Tsai
  - Iterative method
  - Radial distortion
  - Focal distance estimation
- Method of Weng
  - Iterative method
  - Radial and tangential distortion
- … and many more

Contents

2. Camera Calibration

- 2.1 Calibration introduction
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- 2.3 The method of Hall
- 2.4 The method of Faugeras-Toscani – Modelling
- 2.5 The method of Faugeras-Toscani – Calibration
- 2.6 The method of Faugeras-Toscani with distortion
- 2.7 Experimental comparison of methods

Contents

2. Camera Calibration

- 2.1 Calibration introduction
- 2.2 The pinhole model
- 2.3 The method of Hall
- 2.4 The method of Faugeras-Toscani – Modelling
- 2.5 The method of Faugeras-Toscani – Calibration
- 2.6 The method of Faugeras-Toscani with distortion
- 2.7 Experimental comparison of methods

2.3 The Method of Hall

- Method of Hall
  - Lineal method
  - Transformation matrix
- Method of Faugeras-Toscani
  - Lineal method
  - Obtaining camera parameters
- Method of Faugeras-Toscani with distortion
  - Iterative method
  - Radial distortion
- Method of Tsai
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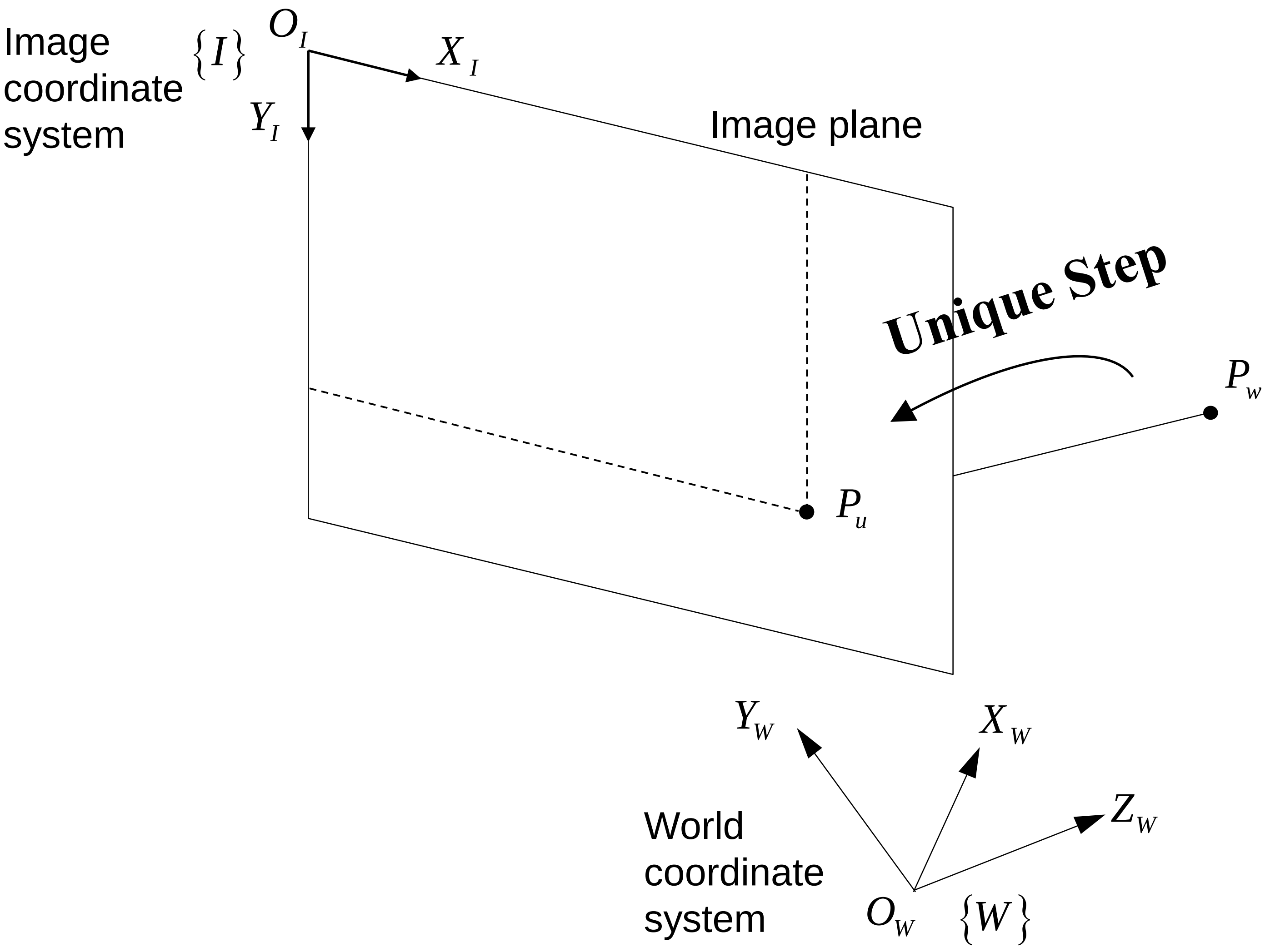
  - Radial distortion
  - Focal distance estimation
- Method of Weng
  - Iterative method
  - Radial and tangential distortion
- … and many more

2.3 The Method of Hall - Modelling



2.3 The Method of Hall - Modelling

Assume light is captured on the image plane by a linear projection

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

The matrix is defined up to a scale factor → Multiple Solutions
A component is fixed to the unity → Unique Solution

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & 1 \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

2.3 The Method of Hall - Calibration

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & 1 \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

$$ {}^{I}X_u = \frac{A_{11}\,{}^{W}X_w + A_{12}\,{}^{W}Y_w + A_{13}\,{}^{W}Z_w + A_{14}}{A_{31}\,{}^{W}X_w + A_{32}\,{}^{W}Y_w + A_{33}\,{}^{W}Z_w + 1}$$

$$ {}^{I}Y_u = \frac{A_{21}\,{}^{W}X_w + A_{22}\,{}^{W}Y_w + A_{23}\,{}^{W}Z_w + A_{24}}{A_{31}\,{}^{W}X_w + A_{32}\,{}^{W}Y_w + A_{33}\,{}^{W}Z_w + 1}$$

$$A_{11}\,{}^{W}X_w - A_{31}\,{}^{I}X_u\,{}^{W}X_w + A_{12}\,{}^{W}Y_w - A_{32}\,{}^{I}X_u\,{}^{W}Y_w + A_{13}\,{}^{W}Z_w - A_{33}\,{}^{I}X_u\,{}^{W}Z_w + A_{14} = {}^{I}X_u$$

$$A_{21}\,{}^{W}X_w - A_{31}\,{}^{I}Y_u\,{}^{W}X_w + A_{22}\,{}^{W}Y_w - A_{32}\,{}^{I}Y_u\,{}^{W}Y_w + A_{23}\,{}^{W}Z_w - A_{33}\,{}^{I}Y_u\,{}^{W}Z_w + A_{24} = {}^{I}Y_u$$

2.3 The Method of Hall - Calibration

$$A_{11}\,{}^{W}X_{w} - A_{31}\,{}^{I}X_{u}\,{}^{W}X_{w} + A_{12}\,{}^{W}Y_{w} - A_{32}\,{}^{I}X_{u}\,{}^{W}Y_{w} + A_{13}\,{}^{W}Z_{w} - A_{33}\,{}^{I}X_{u}\,{}^{W}Z_{w} + A_{14} = {}^{I}X_{u}$$

$$A_{21}\,{}^{W}X_{w} - A_{31}\,{}^{I}Y_{u}\,{}^{W}X_{w} + A_{22}\,{}^{W}Y_{w} - A_{32}\,{}^{I}Y_{u}\,{}^{W}Y_{w} + A_{23}\,{}^{W}Z_{w} - A_{33}\,{}^{I}Y_{u}\,{}^{W}Z_{w} + A_{24} = {}^{I}Y_{u}$$

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{21} & A_{22} & A_{23} & A_{24} & A_{31} & A_{32} & A_{33} \end{pmatrix}^{\mathrm{T}}$$

Obtaining 11 unknowns and each 2D point gives two equations
So, at least 6 points are needed. More points leads to a more accurate solution.

$$QA = B$$

$$Q_{2i-1} = \begin{pmatrix} {}^{W}X_{wi} & {}^{W}Y_{wi} & {}^{W}Z_{wi} & 1 & 0 & 0 & 0 & 0 & -{}^{I}X_{ui}\,{}^{W}X_{wi} & -{}^{I}X_{ui}\,{}^{W}Y_{wi} & -{}^{I}X_{ui}\,{}^{W}Z_{wi} \end{pmatrix}$$

$$Q_{2i} = \begin{pmatrix} 0 & 0 & 0 & 0 & {}^{W}X_{wi} & {}^{W}Y_{wi} & {}^{W}Z_{wi} & 1 & -{}^{I}Y_{ui}\,{}^{W}X_{wi} & -{}^{I}Y_{ui}\,{}^{W}Y_{wi} & -{}^{I}Y_{ui}\,{}^{W}Z_{wi} \end{pmatrix}$$

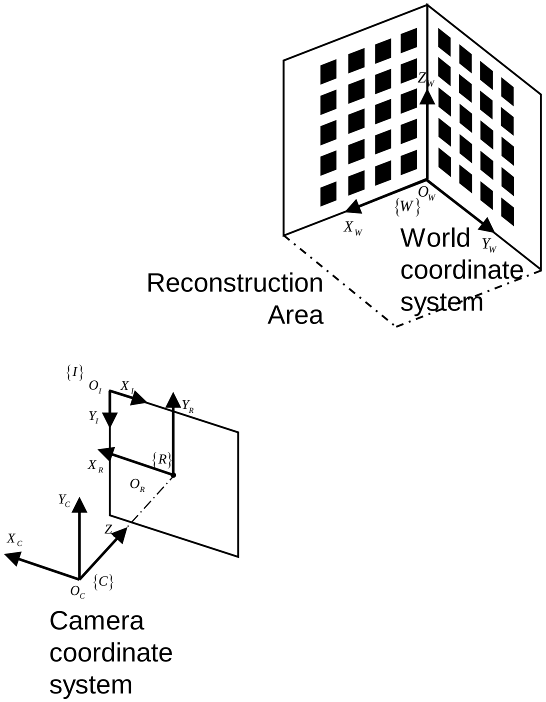
$$B_{2i-1} = \left({}^{I}X_{ui} \right)$$

$$B_{2i} = \left({}^{I}Y_{ui} \right)$$

Pseudoinverse leads to a unique solution:

$$A = Q^{-1}B \quad \Rightarrow \quad \boxed{A = \left(Q^{t}Q\right)^{-1} Q^{t} B}$$

2.3 The Method of Hall - Calibration



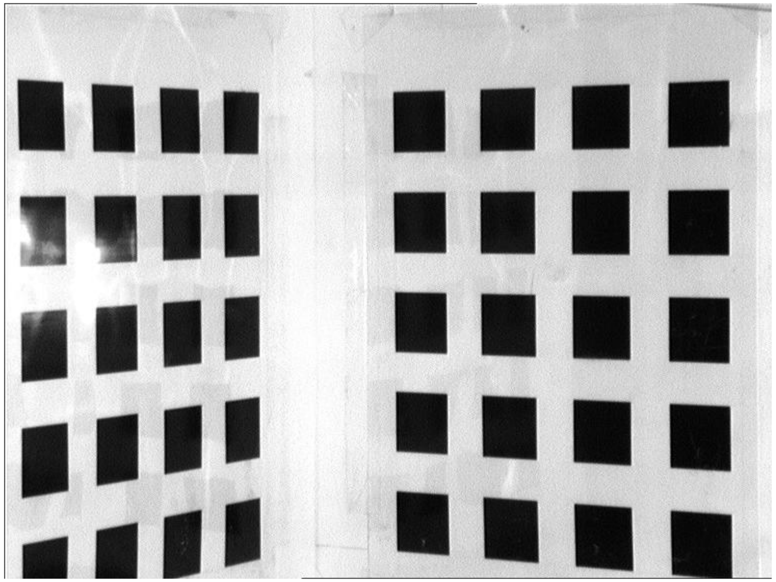


Image of the calibrating pattern

$$Q_{2i-1} = \left({}^{W}X_{wi} \quad {}^{W}Y_{wi} \quad {}^{W}Z_{wi} \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad -{}^{I}X_{ui}\,{}^{W}X_{wi} \quad -{}^{I}X_{ui}\,{}^{W}Y_{wi} \quad -{}^{I}X_{ui}\,{}^{W}Z_{wi} \right)$$

$$Q_{2i} = \left(0 \quad 0 \quad 0 \quad 0 \quad {}^{W}X_{wi} \quad {}^{W}Y_{wi} \quad {}^{W}Z_{wi} \quad 1 \quad -{}^{I}Y_{ui}\,{}^{W}X_{wi} \quad -{}^{I}Y_{ui}\,{}^{W}Y_{wi} \quad -{}^{I}Y_{ui}\,{}^{W}Z_{wi} \right)$$

$$B_{2i-1} = \left({}^{I}X_{ui} \right)$$

$$B_{2i} = \left({}^{I}Y_{ui} \right)$$

$$A = \left(Q^{t} Q \right)^{-1} Q^{t} B$$