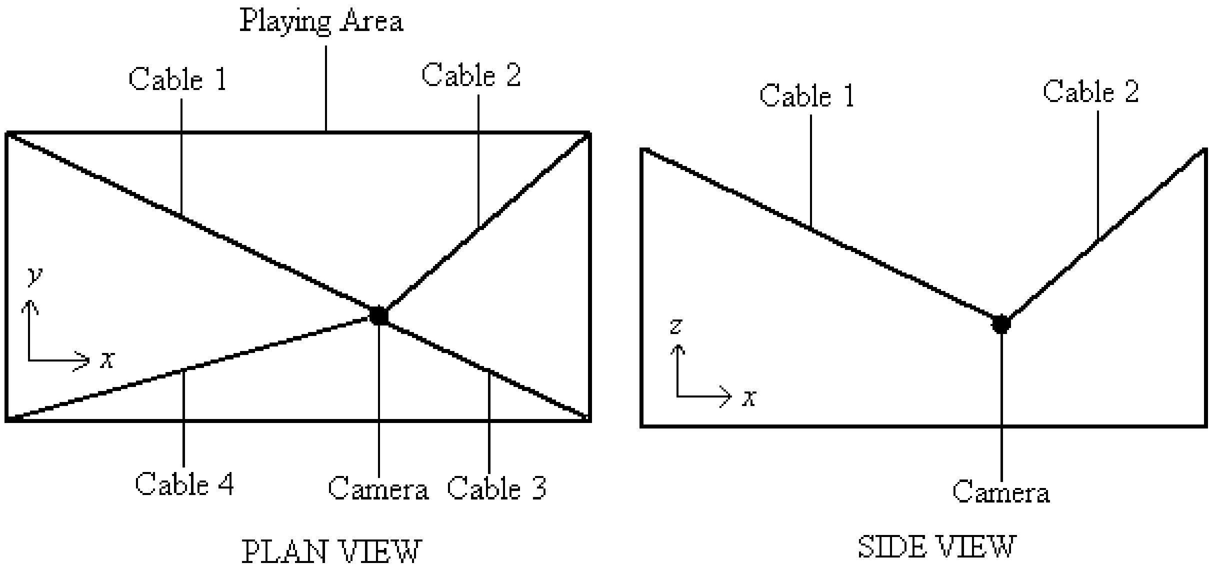
F21RO INTELLIGENT ROBOTICS TUTORIAL 4 INVERSE KINEMATICS

Consider the following diagrams depicting the plan and side elevations of an overhead camera system mounted above a rectangular sports playing area, such as a football or rugby pitch -



The camera is moved around above the players' heads by means of four actuators each of which "reels in" or "plays out" one of the cables. By this means the camera can be located at any (x,y,z) position above the pitch. Let the dimensions of the pitch be X and Y and the height of the actuators (maximum elevation of the camera) be Z.

QUESTION 1.

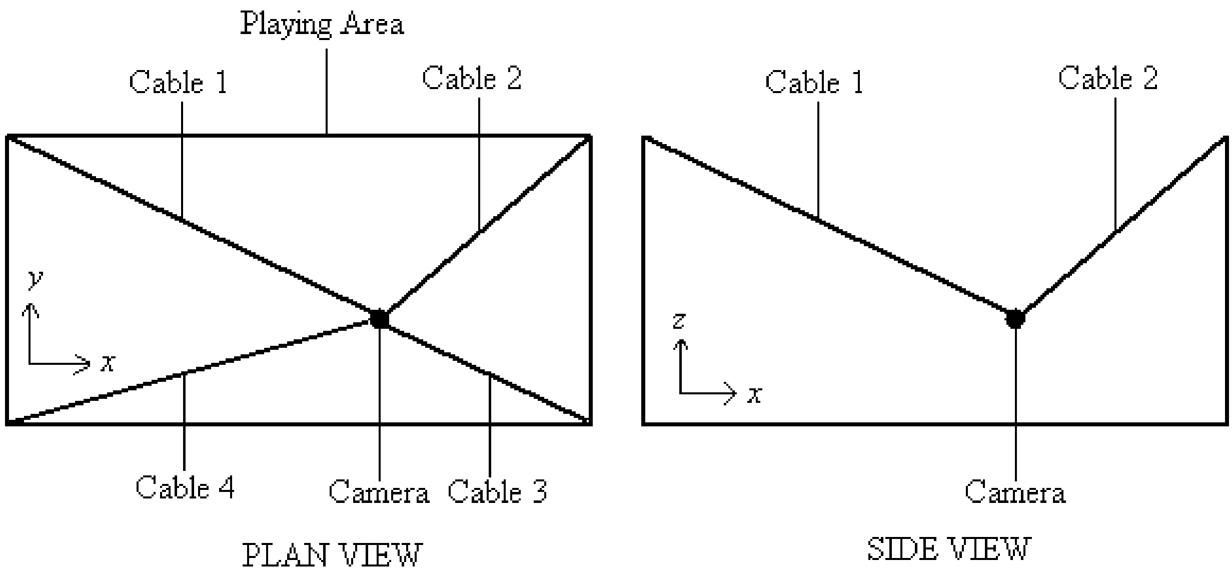
Derive the inverse kinematic equations for the four actuators which will enable the operator to drive the camera to any (x,y,z) position. I.e. derive equations for the cable lengths necessary to achieve a given (x,y,z) position for the camera.

QUESTION 2.

Suppose that the video image captured by the camera is fed into a ball-tracking system which is able to continuously derive the (x,y) position of the ball automatically. Suppose also that the instantaneous velocity of the ball can be continuously determined by the tracking system.

Given the x and y components of the instantaneous velocity of the ball, derive the inverse kinetic equations which will enable the four actuators to operate at the correct speed for the camera to follow the ball whilst maintaining a constant height, H, above the playing surface.

SOLUTION 1.



From the Plan view (i.e. ignoring the camera height for now) and assigning d_1, d_2, d_3 and d_4 to the 4 cable lengths as projected onto the *xy* plane respectively we have –

$$d_1 = \sqrt{[x^2 + (Y-y)^2]}$$

$$d_2 = \sqrt{[(X-x)^2 + (Y-y)^2]}$$

$$d_3 = \sqrt{[(X-x)^2 + y^2]}$$

$$d_4 = \sqrt{[x^2 + y^2]}$$

As these are not the actual cable lengths required but their projections onto the *xy* plane we now need to consider the Side view to introduce the height element. The actual cable lengths required are the hypoteneuse's formed from each d_i and $(Z-z)$ for a given height z.

Let the final cable lengths be given by D_1, D_2, D_3 and D_4. The inverse kinematic equations will thefore be –

$$D_1 = \sqrt{[d_1^2 + (Z-z)^2]} = \sqrt{[x^2 + (Y-y)^2 + (Z-z)^2]}$$

$$D_2 = \sqrt{[d_2^2 + (Z-z)^2]} = \sqrt{[(X-x)^2 + (Y-y)^2 + (Z-z)^2]}$$

$$D_3 = \sqrt{[d_3^2 + (Z-z)^2]} = \sqrt{[(X-x)^2 + y^2 + (Z-z)^2]}$$

$$D_4 = \sqrt{[d_4^2 + (Z-z)^2]} = \sqrt{[x^2 + y^2 + (Z-z)^2]}$$

SOLUTION 2.

We start with the inverse kinematic equations from part (a), setting $z = H$ –

$$D_1 = = \sqrt{[x^2 + (Y\text{-}y)^2 + (Z\text{-}H)^2]}$$
$$D_2 = \sqrt{[(X\text{-}x)^2 + (Y\text{-}y)^2 + (Z\text{-}H)^2]}$$
$$D_3 = \sqrt{[(X\text{-}x)^2 + y^2 + (Z\text{-}H)^2]}$$
$$D_4 = \sqrt{[x^2 + y^2 + (Z\text{-}H)^2]}$$

Squaring them yields –

$$D_1^2 = x^2 + (Y\text{-}y)^2 + (Z\text{-}H)^2$$

$$D_2^2 = (X\text{-}x)^2 + (Y\text{-}y)^2 + (Z\text{-}H)^2$$

$$D_3^2 = (X\text{-}x)^2 + y^2 + (Z\text{-}H)^2$$

$$D_4^2 = x^2 + y^2 + (Z\text{-}H)^2$$

We note that each of the D_i and x and y are functions of time and can be used to generate the kinetic equations if differentiated with respect to time –

$$d/dt(D_1(t)^2) = d/dt(x(t)^2 + (Y\text{-}y(t))^2 + (Z\text{-}H)^2)$$

$$d/dt(D_2(t)^2) = d/dt((X\text{-}x(t))^2 + (Y\text{-}y(t))^2 + (Z\text{-}H)^2)$$

$$d/dt(D_3(t)^2) = d/dt((X\text{-}x(t))^2 + y(t)^2 + (Z\text{-}H)^2)$$

$$d/dt(D_4(t)^2) = d/dt(x(t)^2 + y(t)^2 + (Z\text{-}H)^2)$$

Performing the differentiations and dropping the *(t)* again for convenience we get –

$$2D_1.dD_1/dt = 2x.dx/dt - 2(Y\text{-}y).dy/dt$$

$$2D_2.dD_2/dt = -2(X\text{-}x).dx/dt - 2(Y\text{-}y).dy/dt$$

$$2D_3.dD_3/dt = -2(X\text{-}x).dx/dt + 2y.dy/dt$$

$$2D_4.dD_4/dt = 2x.dx/dt + 2y.dy/dt$$

Rearranging provides the final kinetic equations which identify the 4 instantaneous cable velocities necessary to achieve instantaneous velocities of dx/dt and dy/dt for the camera at any constant height (including H) –

$$dD_1/dt = x/D_1.dx/dt - (Y\text{-}y)/D_1.dy/dt$$

$$dD_2/dt = -(X\text{-}x)/D_2.dx/dt - (Y\text{-}y)/D_2.dy/dt$$

$$dD_3/dt = -(X\text{-}x)/D_3.dx/dt + y/D_3.dy/dt$$

$$dD_4/dt = x/D_4.dx/dt + y/D_4.dy/dt$$