Intensity based Image Registration

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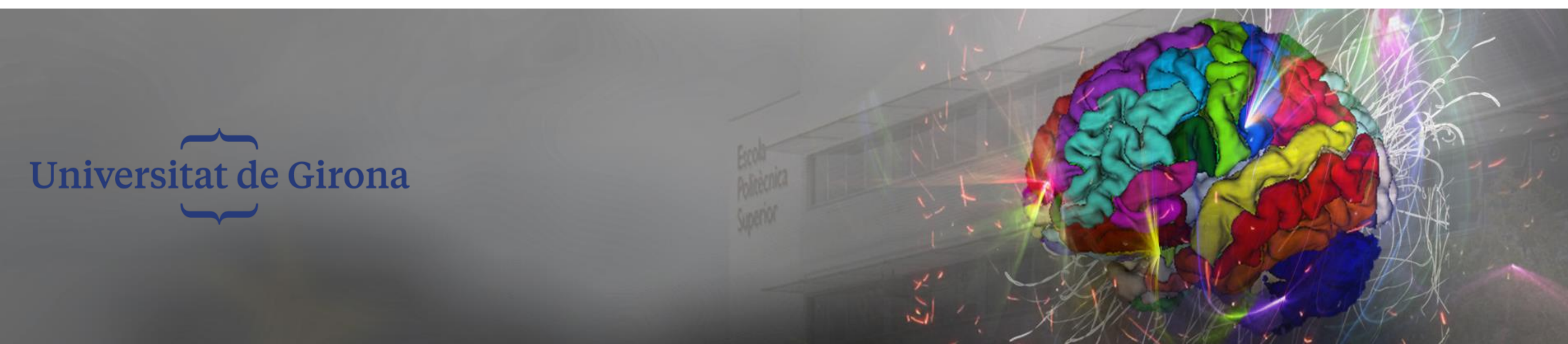


Image Registration . Overview

- Find the transformation or mapping function f

$$T(x', y') = R(f(x, y))$$

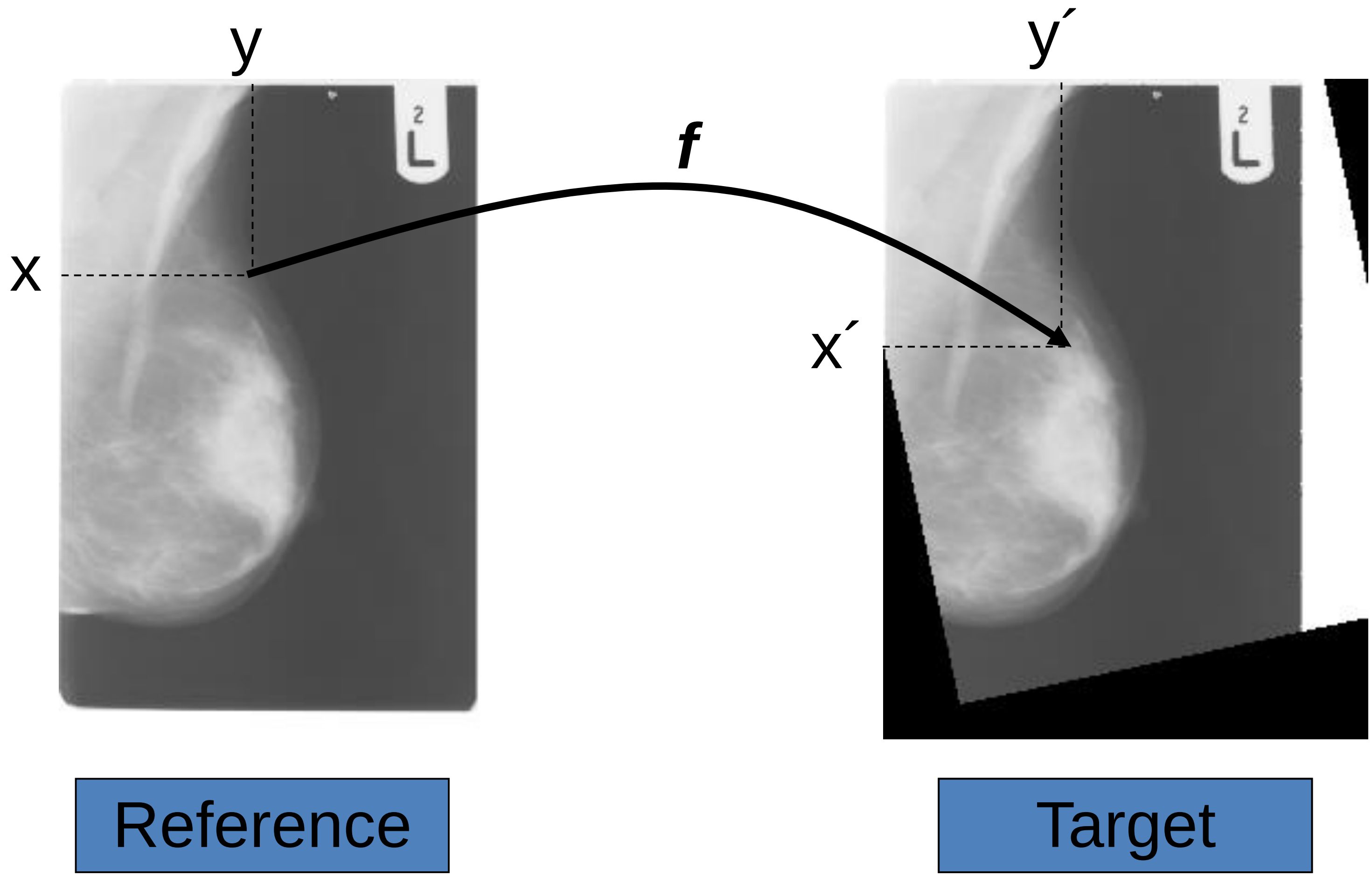
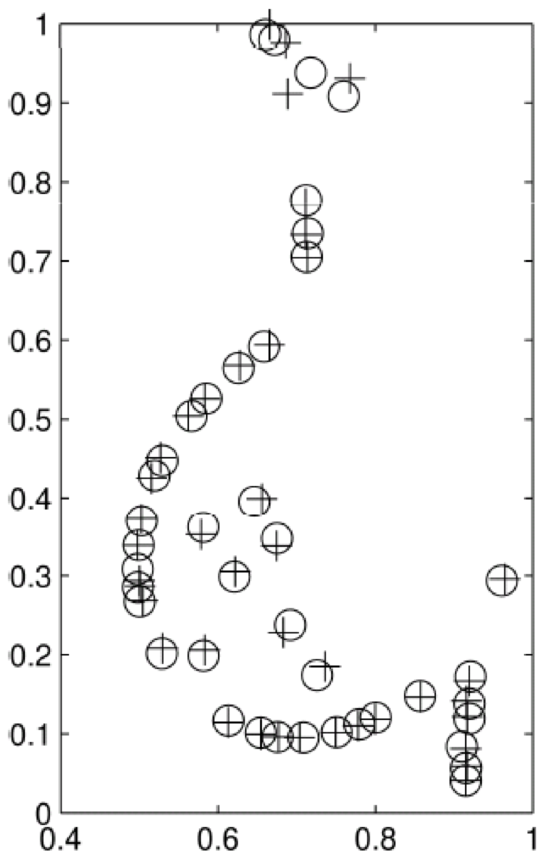


Image Registration. Components

- Classification:
  - Intensity based vs feature based (aka dense vs sparse)



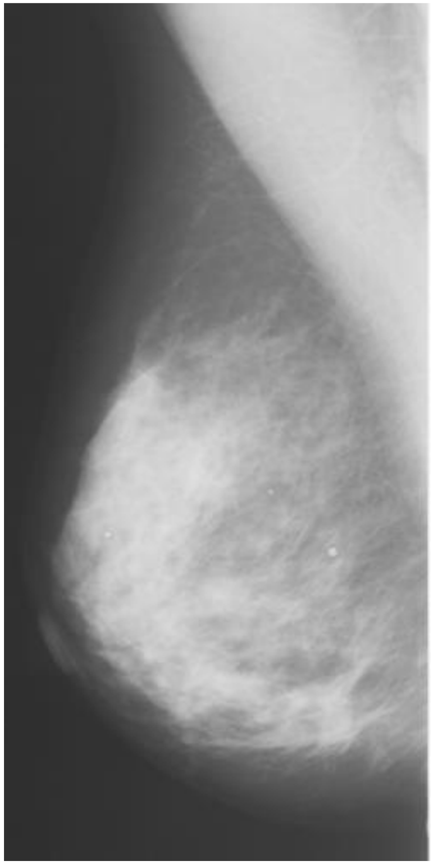
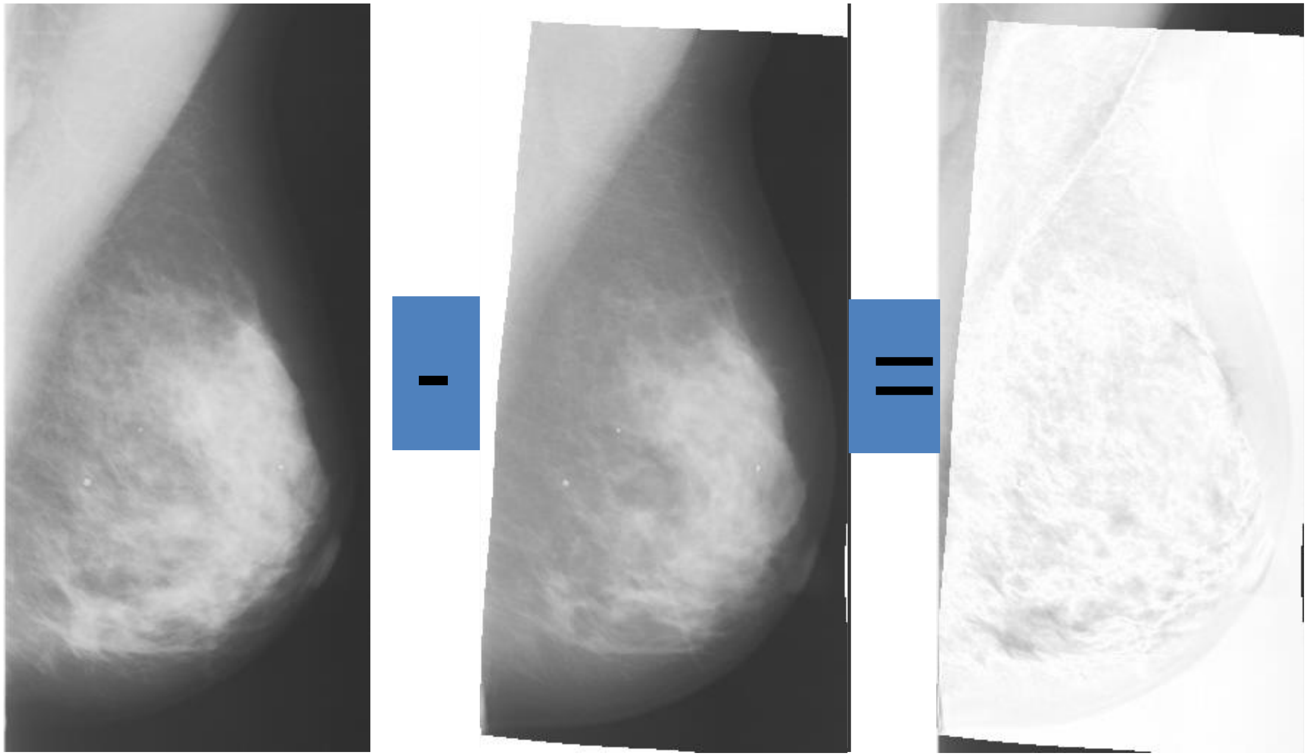


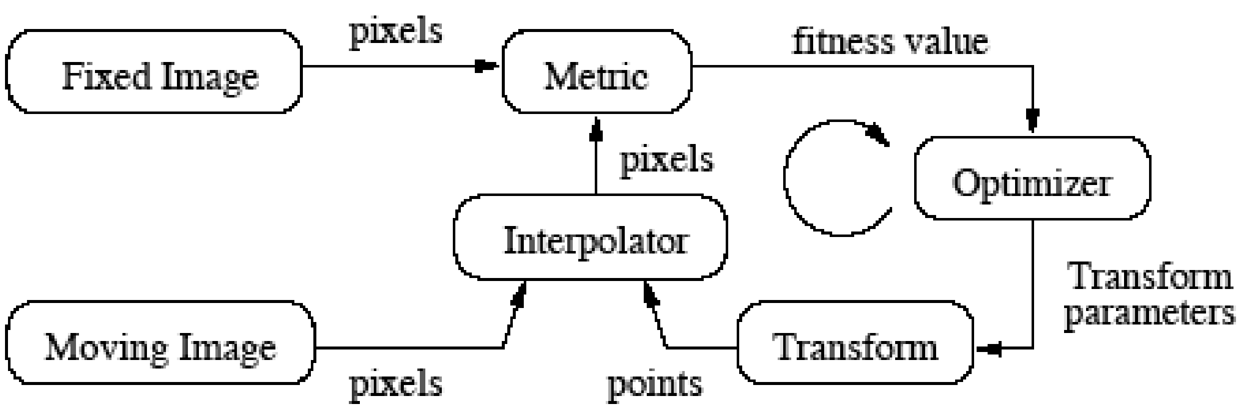
Image Registration. Classification

- Classification:
  - Rigid vs non-rigid (aka deformable, local)



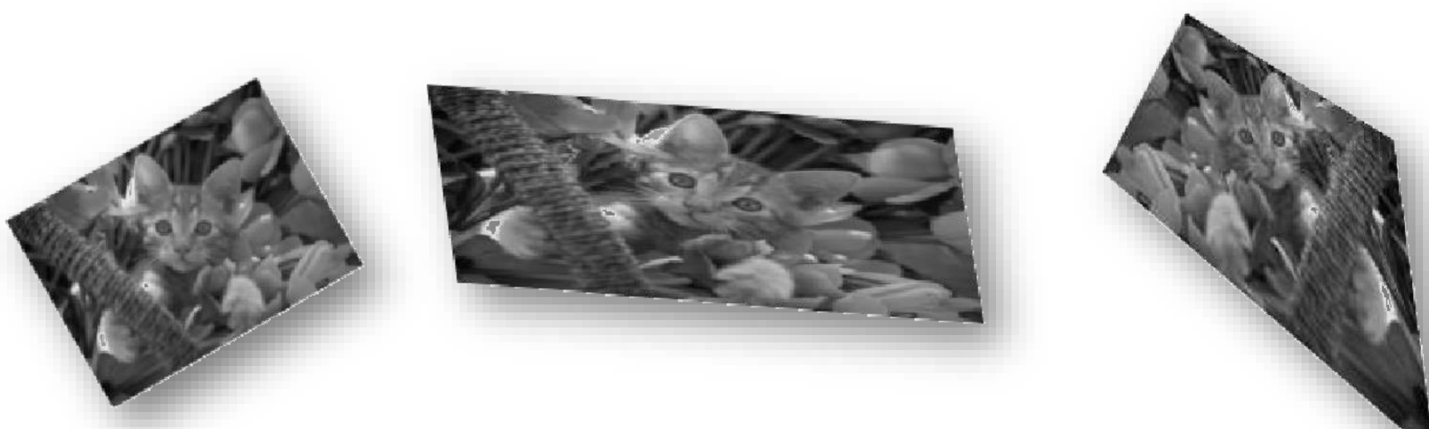
Components

- **Transform:** allowed movement of the moving image, depends on some parameters.
- **Metric:** Measure of similarity between fixed and moving.
- **Interpolator:** Determines the pixel intensity given the position and neighbour pixels.
- **Optimizer:** finds the best metric value with respect to the transformation parameters.



Transform

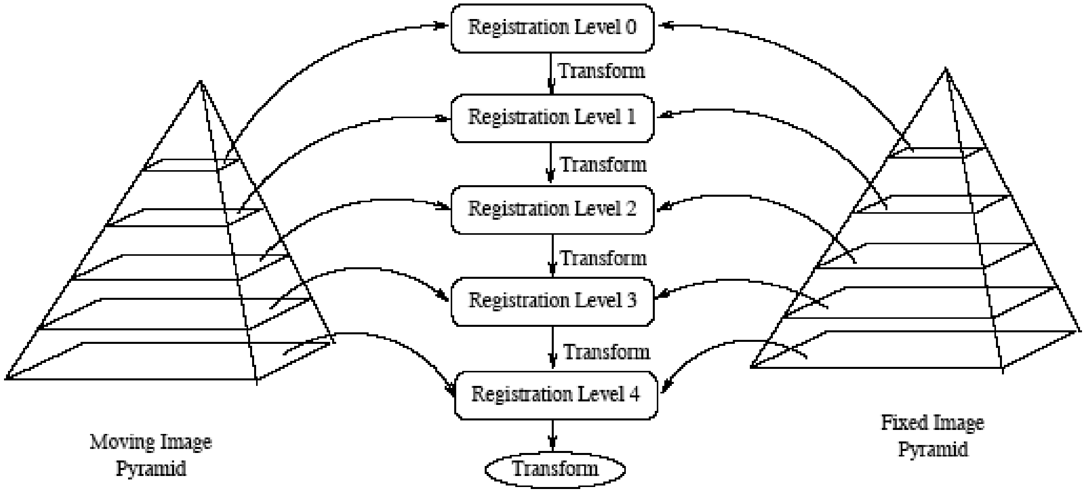
- **Many different classifications**
  - Global vs Local
  - Parametric vs Non-parametric
- Global: all pixels suffer the “same” transformation.
  - Rigid: Rotation and translation
  - Similarity: Rigid + Scaling
  - Affine: Similarity + Shearing and tearing (keeps parallelism)
  - Perspective



[1] *Zitová et al., Image registration methods: A survey, IVC, 2003*

Transform

- **Local:** pixels can move independently at a local level.
  - Thin Plate Splines. Based on point correspondence and spline interpolation
  - B-Splines (Free-Form deformations). Deform grids of points based on B-spline interpolation.
  - Demons, Finite Element Modelling, Poly Rigid, Local Affine, …
- **Variations**
  - Global + Local
  - Multi-resolution



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Global Transform

- **Rigid**

$$f_x(x,y) = x\cos\phi + y\sin\phi + t_x$$
$$f_y(x,y) = -x\sin\phi + y\cos\phi + t_y$$

- **Affine**

$$f_x(x,y) = a_x x + a_y y + t_x$$
$$f_y(x,y) = b_x x + b_y y + t_y$$

- **Projective**

$$f_x(x,y) = \frac{a_x x + a_y y + t_x}{c_x x + c_y y + 1}$$
$$f_y(x,y) = \frac{b_x x + b_y y + t_y}{c_x x + c_y y + 1}$$

Local (non-rigid) Transformations

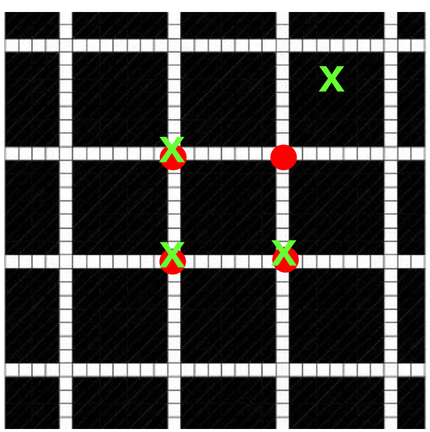
- **Thin Plate Splines**

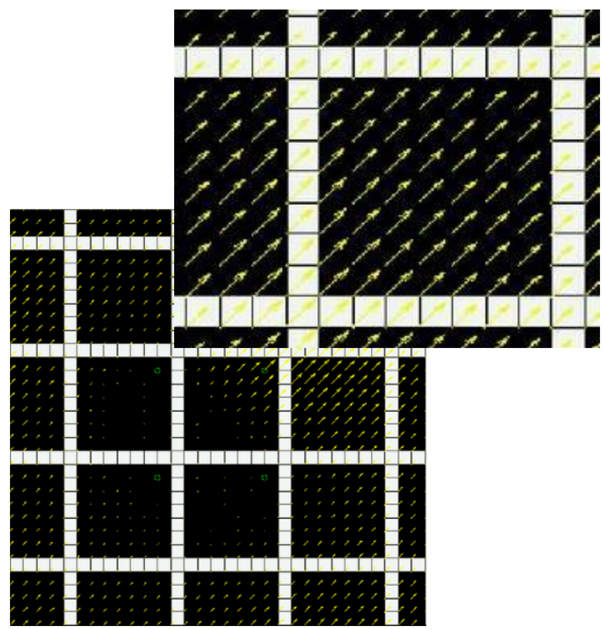
$$\iint_{R^2}\left(\left(\frac{\partial^2 f}{\partial x^2}\right)^2+2\left(\frac{\partial^2 f}{\partial x\partial y}\right)^2+\left(\frac{\partial^2 f}{\partial y^2}\right)^2\right)dx\,dy$$

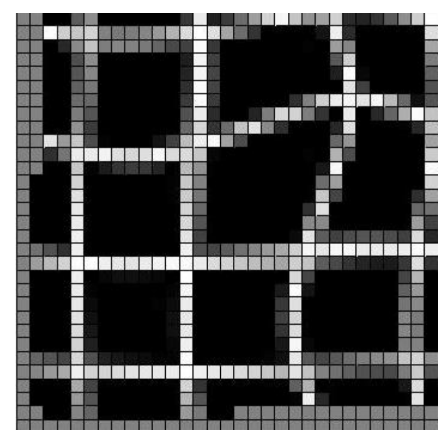
$$f_x(x,y)=a_x x+a_y y+t_x+\sum_{i=1}^{N} w_i U(|\vec{p}_i-(x,y)|)$$

$$f_y(x,y)=b_x x+b_y y+t_y+\sum_{i=1}^{N} w_i U(|\vec{p}_i-(x,y)|)$$

$$U(r)=r^2\log r^2$$





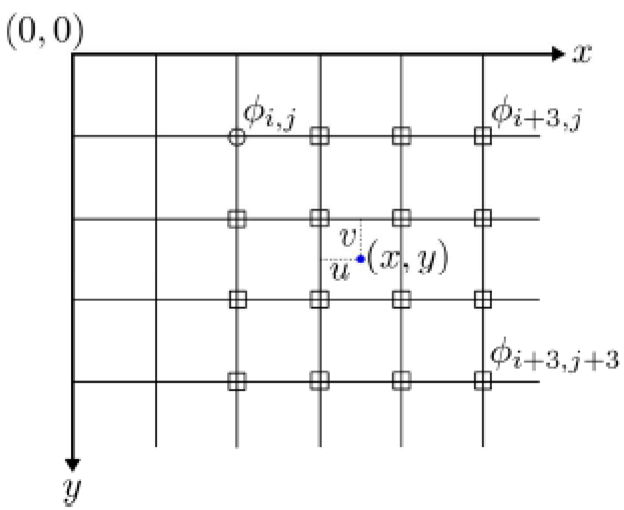


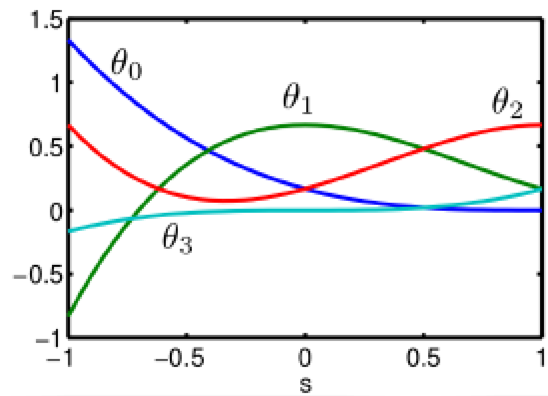
Local (non-rigid) Transformations

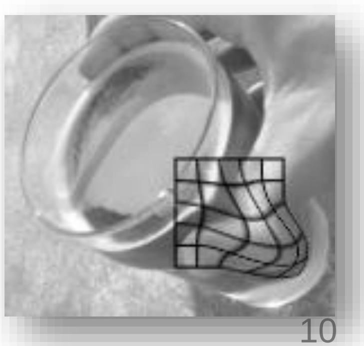
- **B-Splines grid (Free-form deformations)**

$$\mathbf{T}_{\mathrm{local}}(x, y, z) = \sum_{l=0}^{3}\sum_{m=0}^{3}\sum_{n=0}^{3} B_l(u)B_m(v)B_n(w)\phi_{i+l,\, j+m,\, k+n}$$

$$\begin{aligned}
B_0(u) &= (1-u)^3/6 \\
B_1(u) &= (3u^3 - 6u^2 + 4)/6 \\
B_2(u) &= (-3u^3 + 3u^2 + 3u + 1)/6 \\
B_3(u) &= u^3/6.
\end{aligned}$$

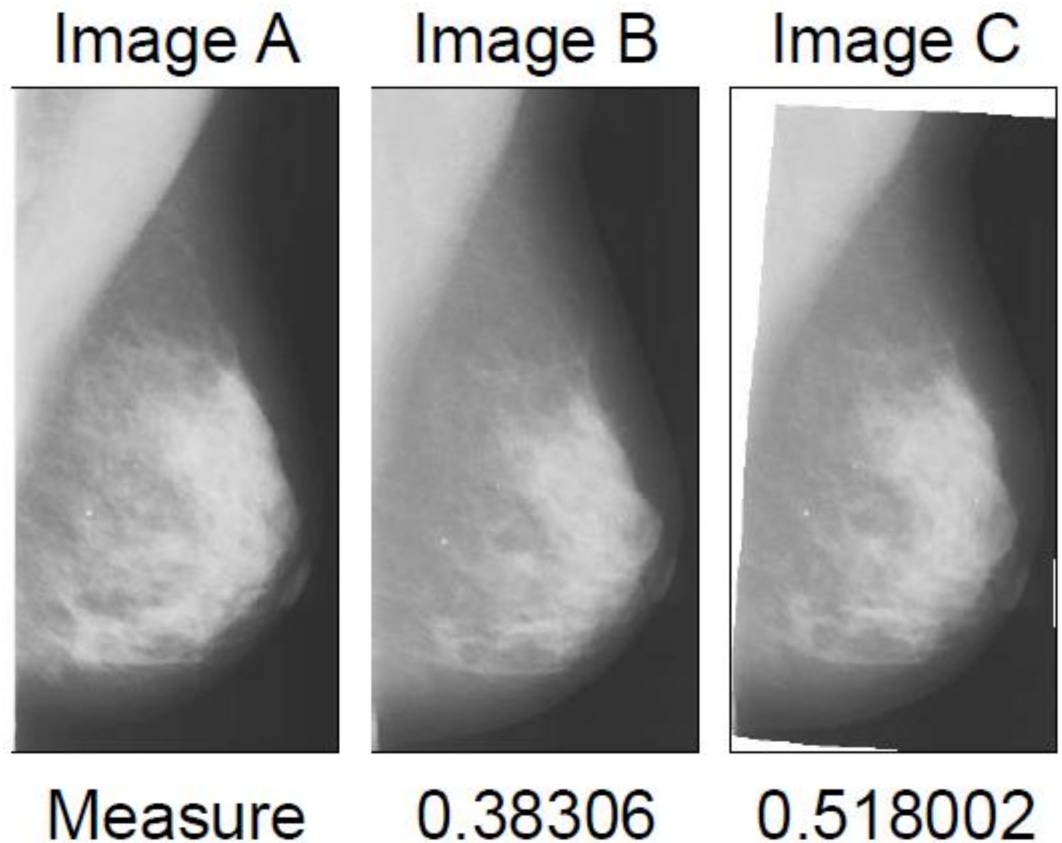






Similarity Metrics

How similar two images are?



Similarity Metrics

- Measure the similarity between images
- Definition of a metric between image A and B (C).

$$
\begin{aligned}
&R1) \quad d(\mathbf{A},\mathbf{B}) > 0 \\
&R2) \quad d(\mathbf{A},\mathbf{B}) = 0 \ \textit{iff} \ \mathbf{A} = \mathbf{B} \\
&R3) \quad d(\mathbf{A},\mathbf{B}) = d(\mathbf{B},\mathbf{A}) \\
&R4) \quad d(\mathbf{A},\mathbf{B}) \leq d(\mathbf{A},\mathbf{C}) + d(\mathbf{C},\mathbf{B})
\end{aligned}
$$

Similarity Metrics

- Simple Intensity difference (Root mean squared)

$$RMS(A,B) = \frac{1}{N}\sqrt{\sum_{x=0}^{X}\sum_{y=0}^{Y}(A(x,y) - B(x,y))^2}$$

- Normalised cross correlation
  - Correlation between pixels in both images.
  - High values denote high similarity
  - Intensity, gradient, etc.

$$R(A,B) = \frac{\sum_{x=0}^{X}\sum_{y=0}^{Y}(A(x,y) - \overline{I_A})(B(x,y) - \overline{I_B})}{\sqrt{\sum_{x=0}^{X}\sum_{y=0}^{Y}(A(x,y) - \overline{I_A})^2 \sum_{x=0}^{X}\sum_{y=0}^{Y}(B(x,y) - \overline{I_B})^2}}$$

Similarity Metrics

- Entropy related measures
  - Mutual information
  - Entropy of the difference image
    - Low values, low entropy, hence similar images?

$$S_{AB} = \sum_{x=0}^{X} \sum_{y=0}^{Y} A(x,y) - sB(x,y)$$

$$H(S_{AB}) = -\sum_{i=0}^{I} p(i) \log(p(i))$$

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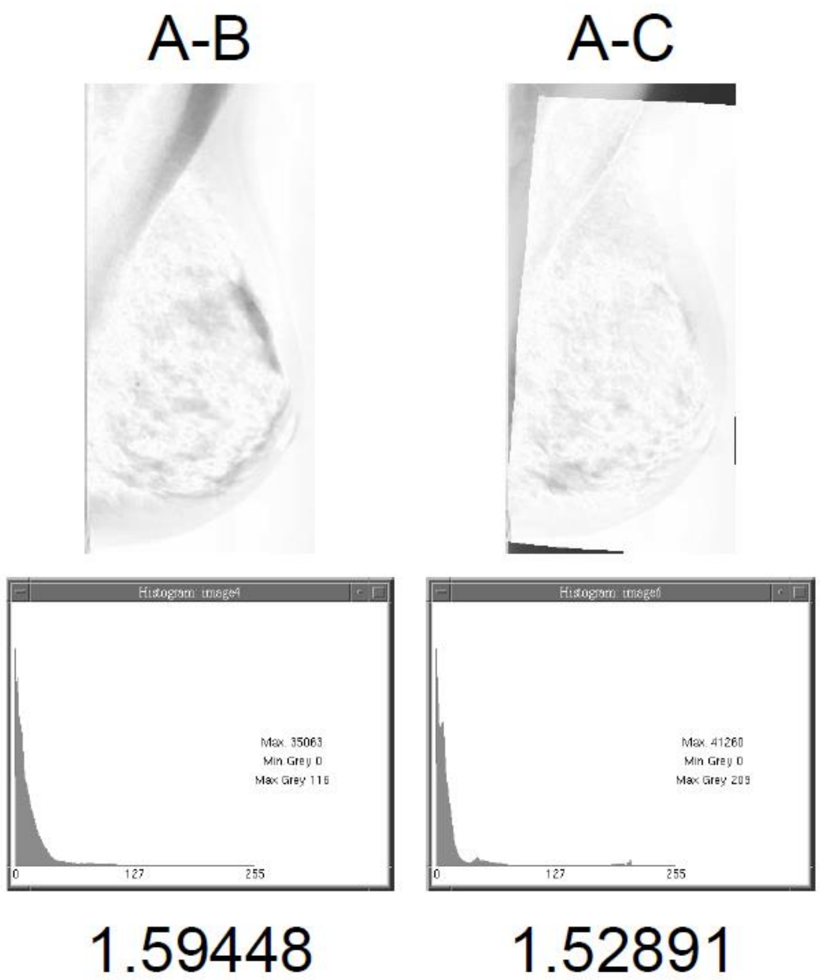
Similarity Metrics

- Entropy of the difference image



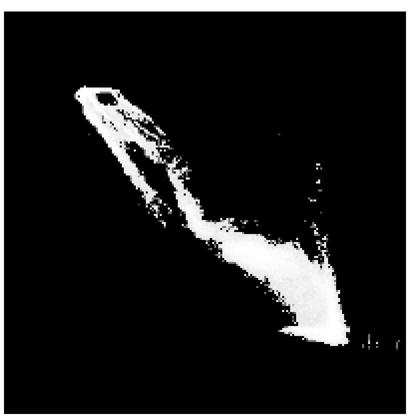
Similarity Metrics

- Mutual Information

$$C(A,B) = \sum_{i=0}^{I}\sum_{j=0}^{J} p_{AB}(i,j)\log\frac{p_{AB}(i,j)}{p_A(i)p_B(j)}$$

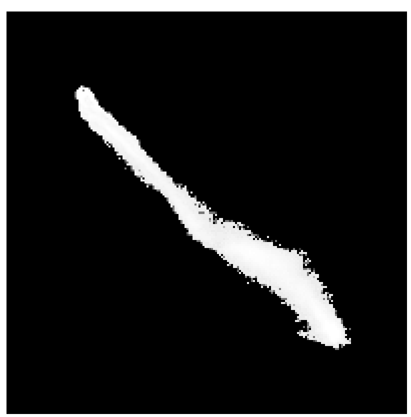
$$= \quad H(A) + H(B) - H(A,B)$$

Joint histogram



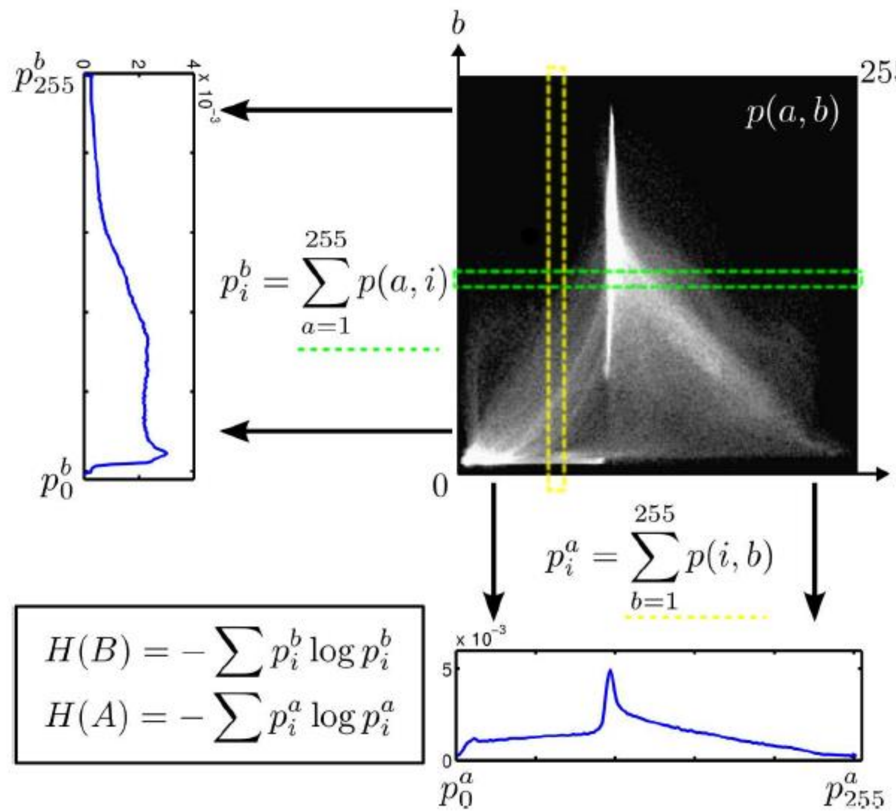
Before

0.38306



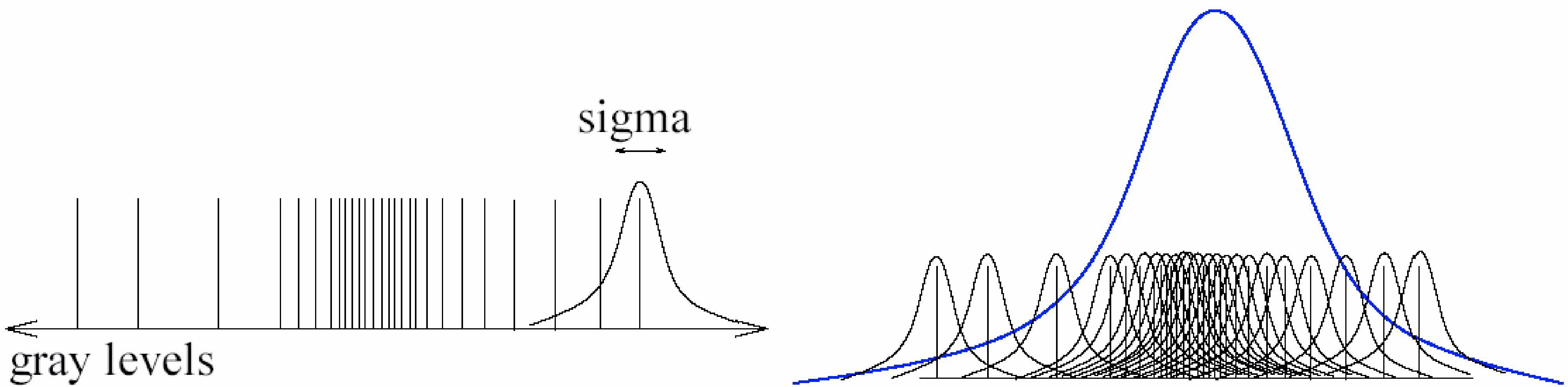
After

0.518002



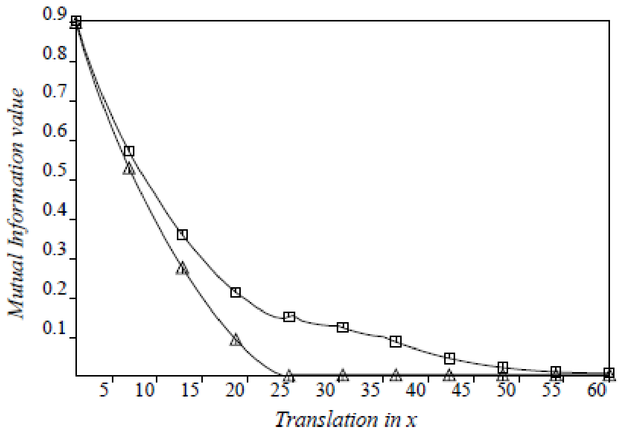
Similarity Metrics

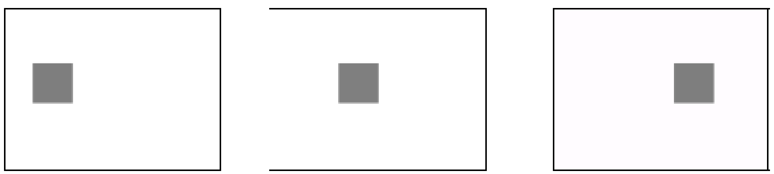
- How to estimate the histogram?
- Has to be a continous distribution!
- Parzen Window estimation



Similarity Metrics

- Variations: Non-overlapping MI.

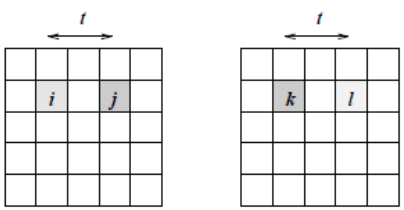




MI	0.012189	0.012189
Novel	0.183060	0.037711

- Incorporate spatial information into MI
- Pixel pairs instead of single pixels (GLCM)

Image A Image B



A-B	A-C
0.335626	0.441571

R. Marti, R. Zwiggelaar and C. Rubin *A Novel Similarity Measure to Evaluate Image Correspondence* ICPR. IEEE Computer Society, pp. 171-174, 2000, Barcelona (Spain)

Similarity Metrics

- Mathematically express the human perception of similarity
- Does not always work!

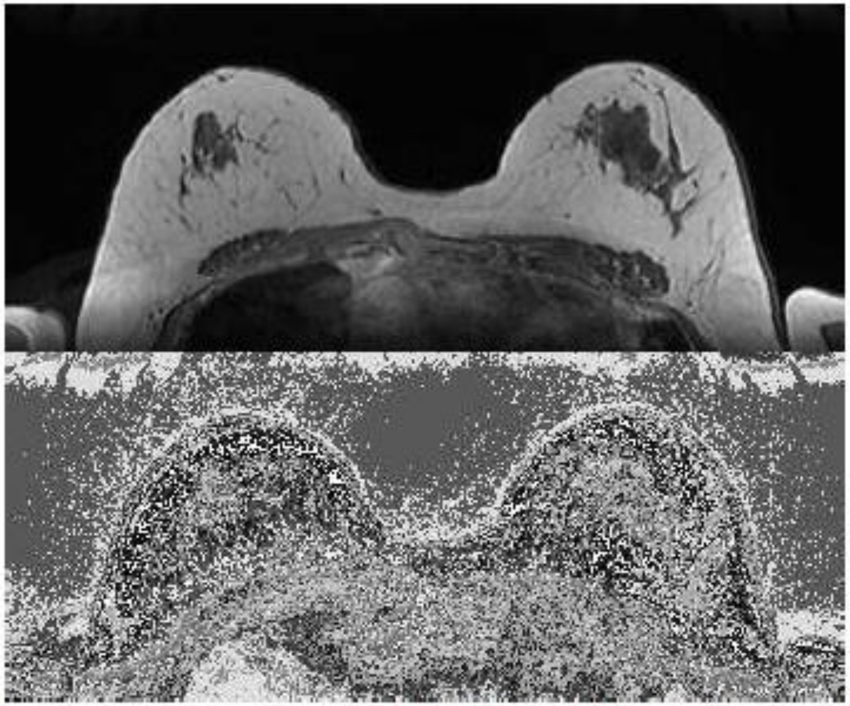


Figure 1. Example of image intensity swapping for an axial slice though an MRI breast volume. Pixel intensity values in the original image **a** are binned and arbitrarily rearranged to give image **b**. The mutual information (MI) between these two images is maximal (here, for 64 bins, MI=6) and the images may be regarded as being identical.

Similarity Metrics

- Mathematically express the human perception of similarity
- Does not always work!

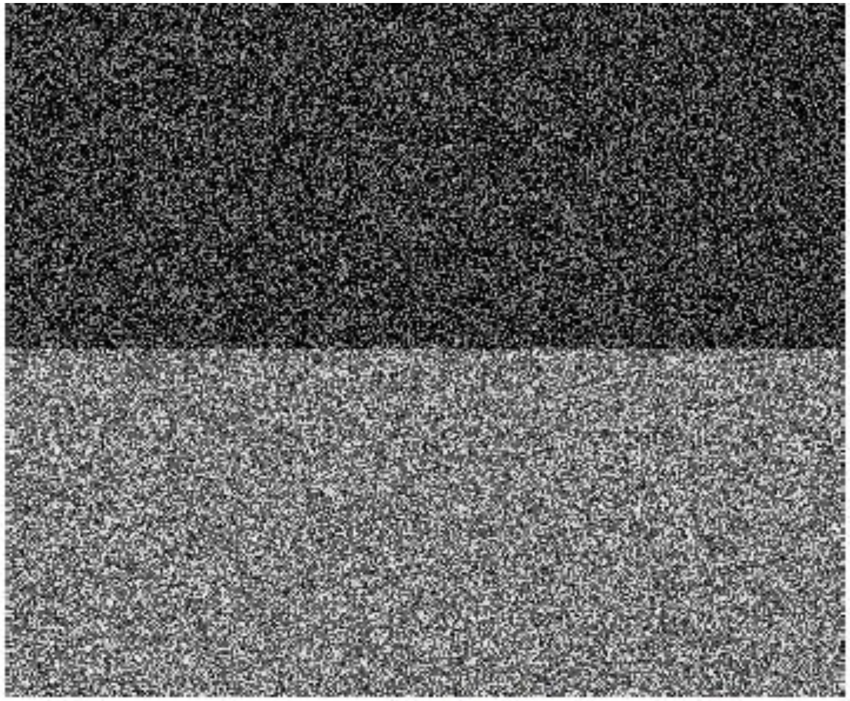
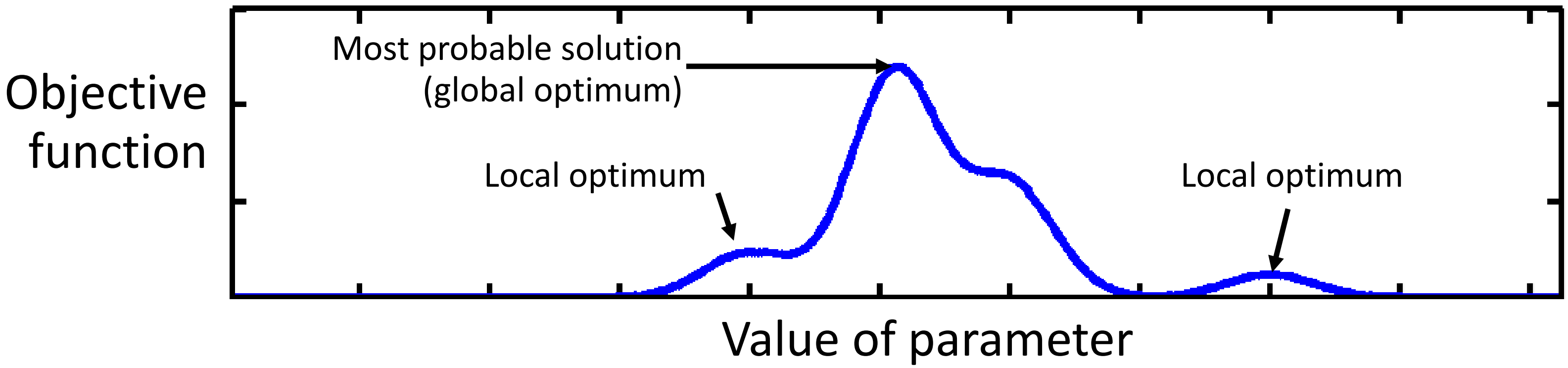


Figure 2. Example of intensity position swapping for an axial slice though an MRI breast volume (1**a**). The pixel positions are arbitrarily rearranged in space. These images have the same marginal entropy as Figure 1**a**. Figure 2**b** has had the intensity values swapped as in Figure 1**b**. Again, the mutual information (MI) between these two images is maximal (here, for 64 bins, MI=6) and the 'images' may be regarded as being identical

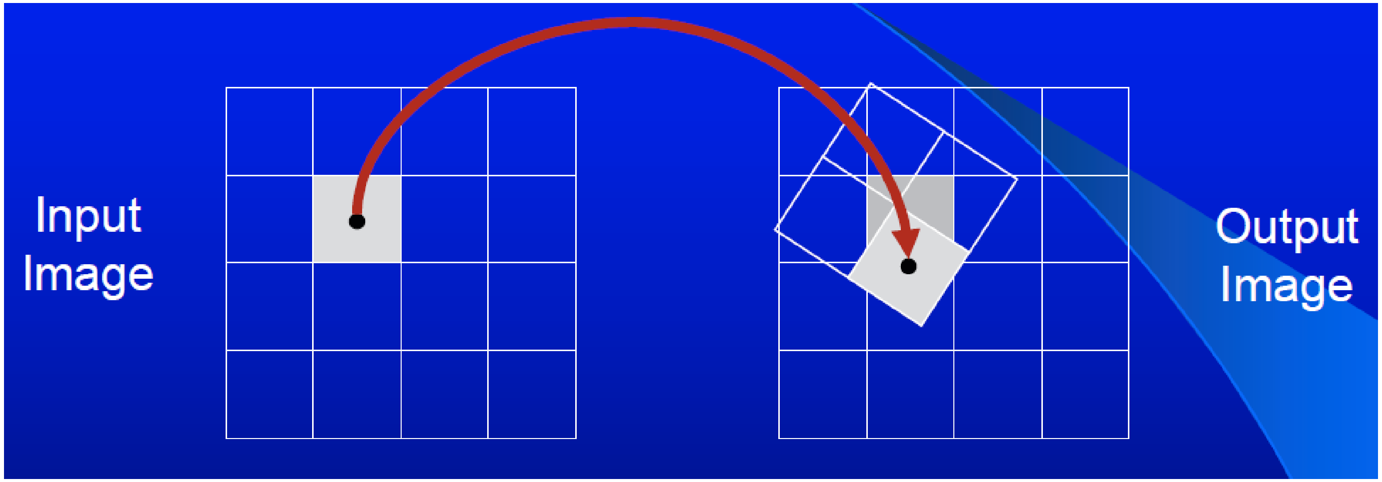
Optimizer

- Finds the transformation parameters that maximises a similarity measure
- Gradient descent algorithms
- Usually need to specify the gradient of the metric with respect to the transformation parameters



Mapping

- Forward mapping
  - Input image pixel is mapped onto the output image
  - Output pixels with more than one hit: overlap
  - Value must be accumulated from overlapping pixels
  - Output pixels with no hits: hole



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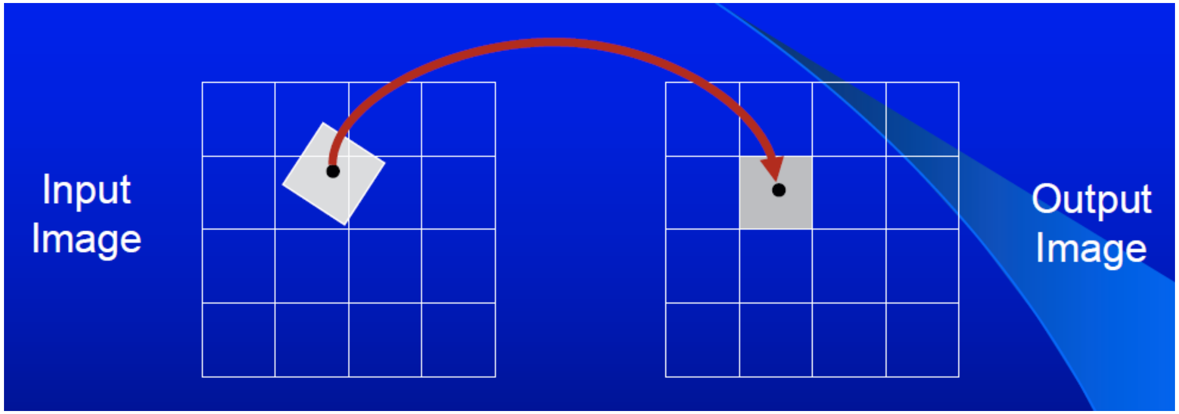
Mapping

- Inverse mapping
  - Output pixels are mapped back onto the input image
  - Output pixel value must be interpolated from a neighborhood in the input image
  - Scheme avoids any holes and overlaps in the output image because all pixels are scanned sequentially



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Interpolator

- Transforming the image

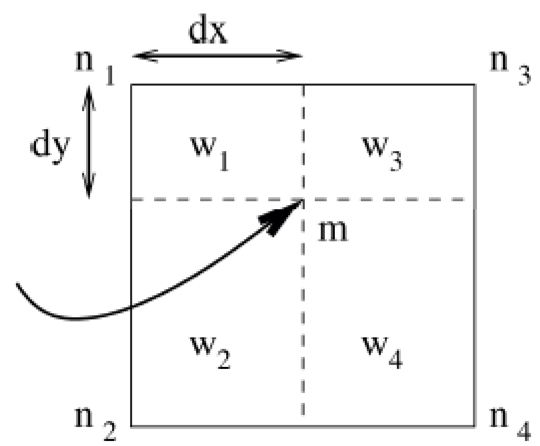
```
for y=1..n_y  % loop over rows
   for x=1..n_x % loop over columns
       x' = f_x(x,y) % transform
       y' = f_y(x,y)
       if 1≤x'≤ n_x, & 1≤y'≤n_y, then  % voxel in range
           newB(x,y) = B(x',y')%assign re-sampled value
       end % voxel in range
   end
end
```

  - What if x',y' are not integers?
  - Inverse transform

Interpolator

- Transformed coordinates are usually non-integers, $m = (x', y')$
- Assign the intensity of
  - Nearest Neighbour

$$B(m) = B(\min_{\forall i}(d_E[m, n_i]))$$



  - Linear interpolation

$$B(m) = \sum_i w_i B(n_i)$$

$$w_1 = (1 - dx)(1 - dy) \quad w_2 = (1 - dx)dy \quad w_3 = dx(1 - dy) \quad w_4 = dxdy$$

  - Cubic Interpolation

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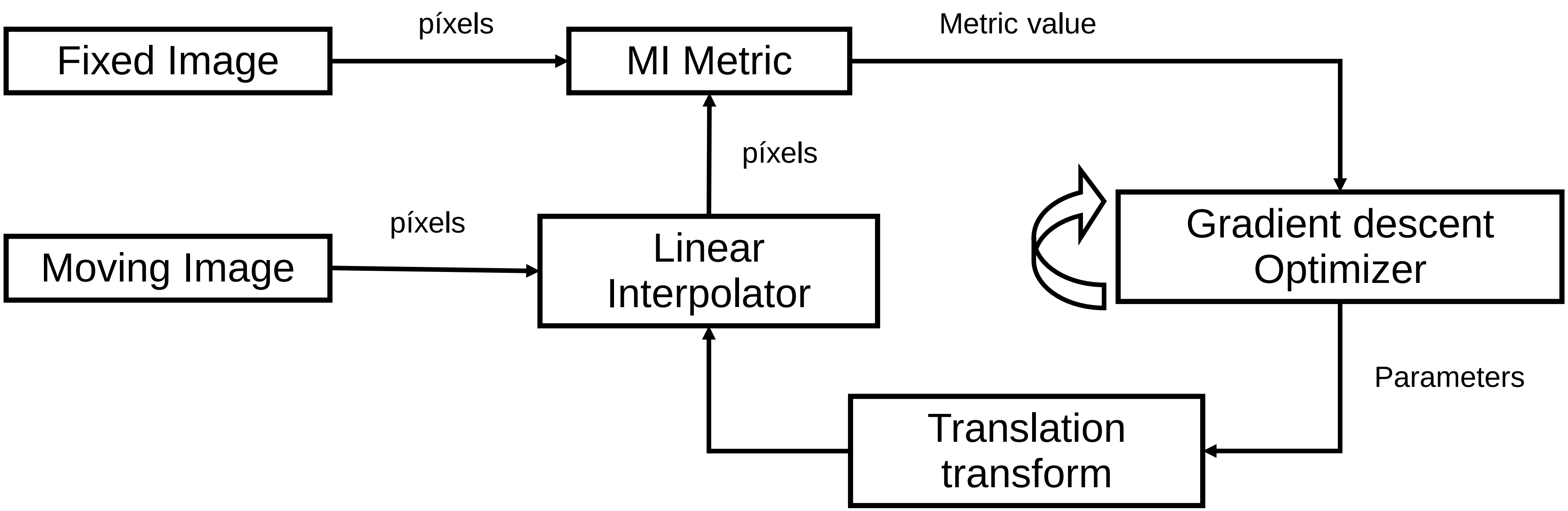
Hello world Registration example



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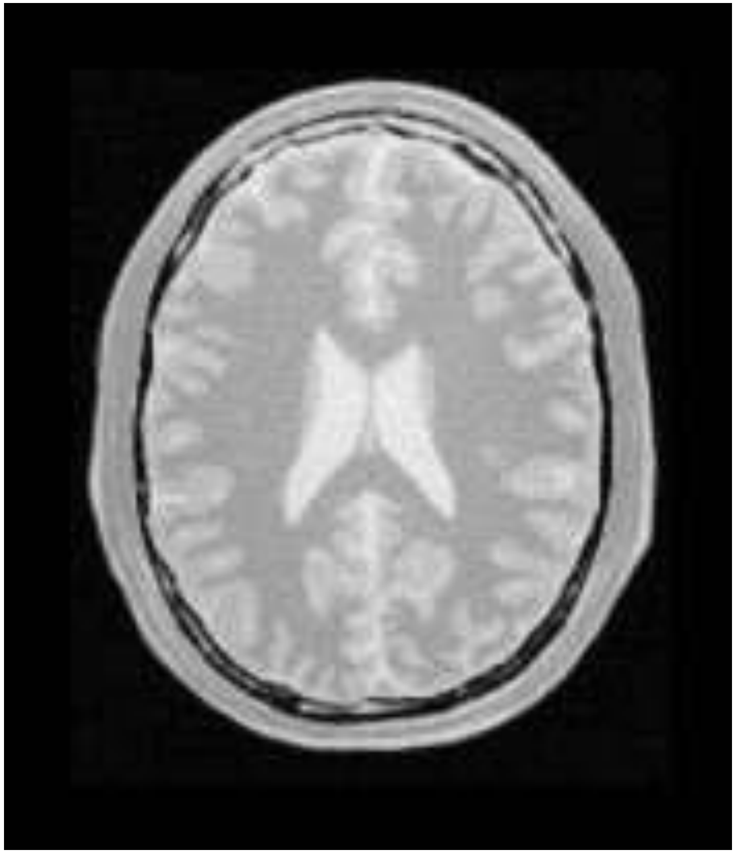
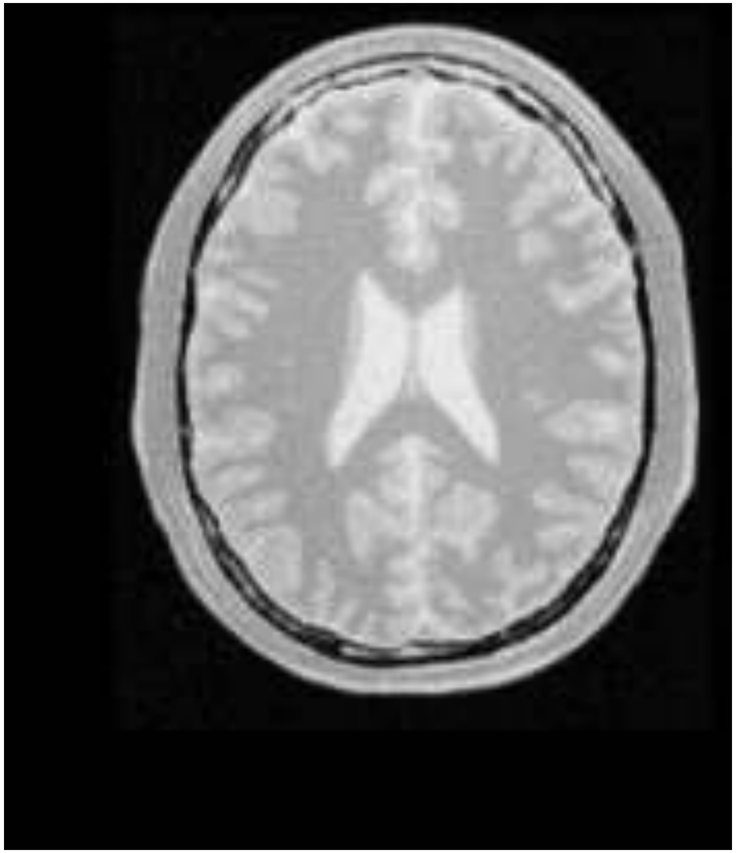
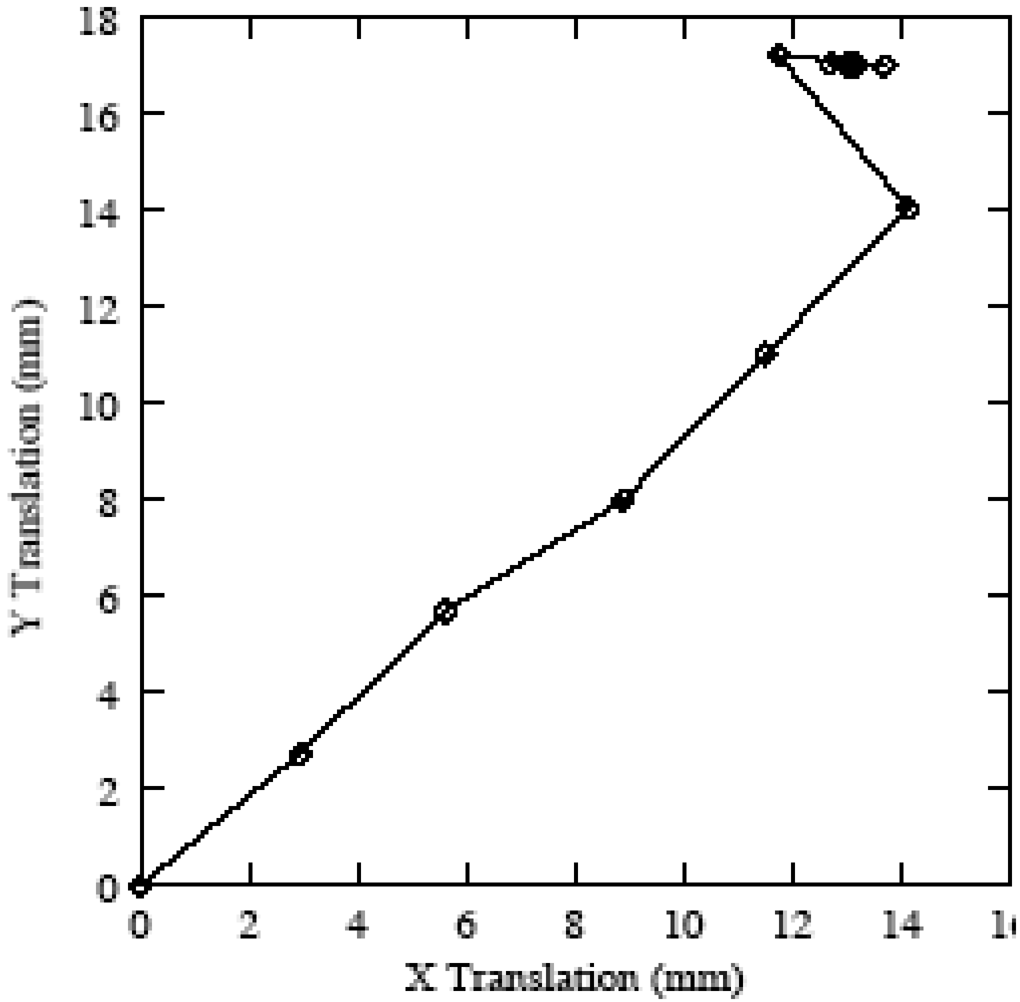
Hello world input





X & Y translation vs. time



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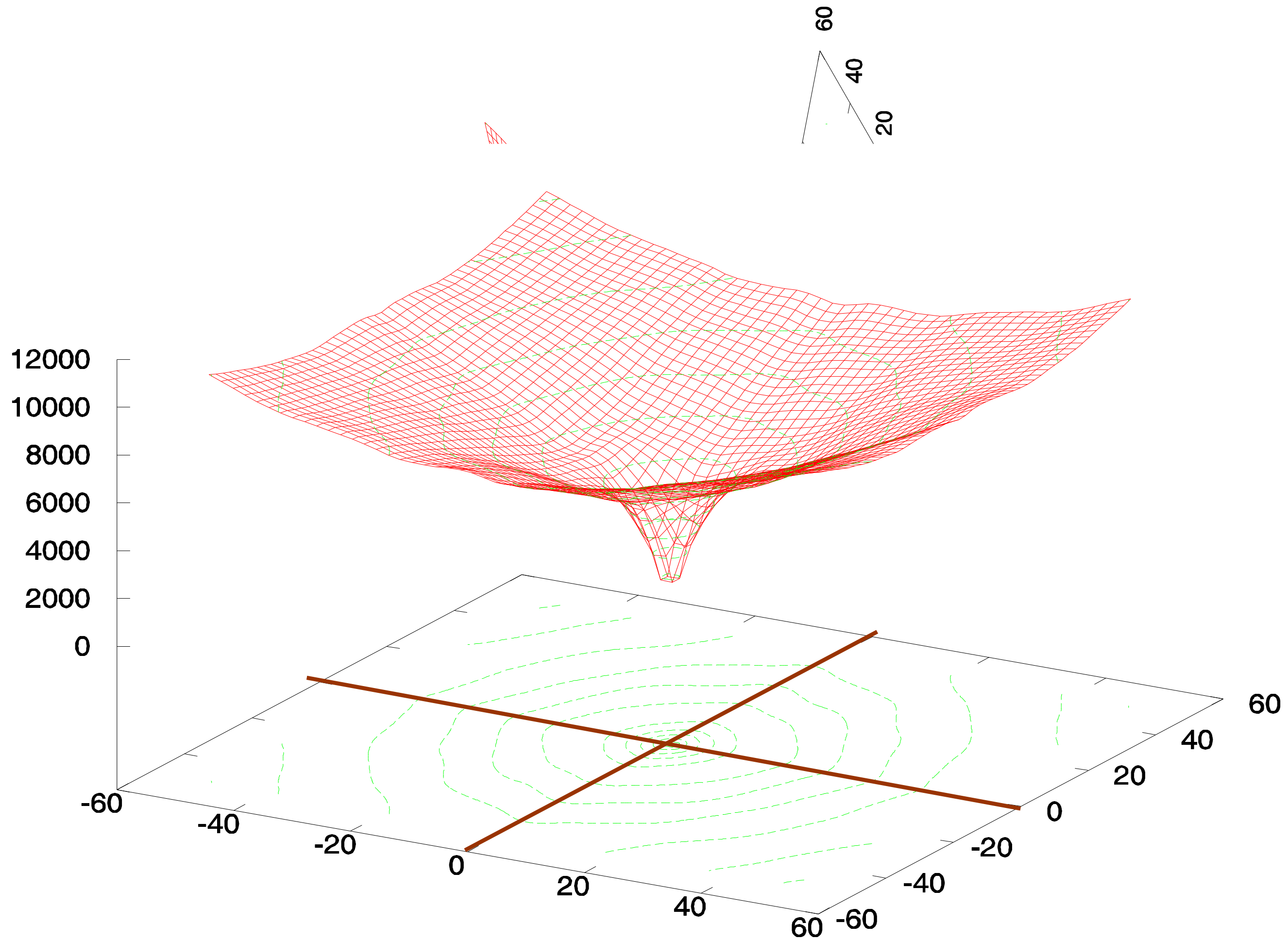
X & Y translation vs. time



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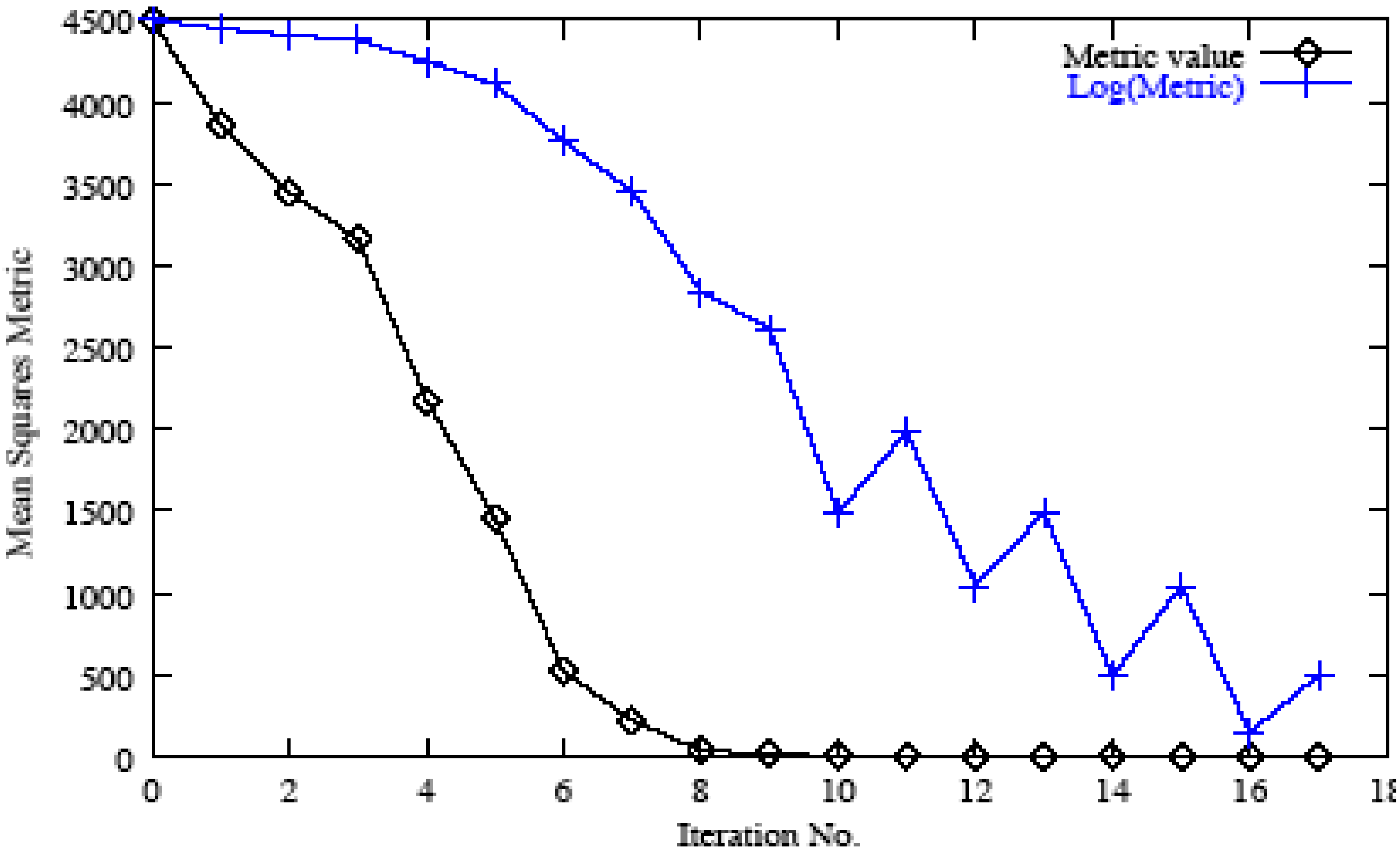
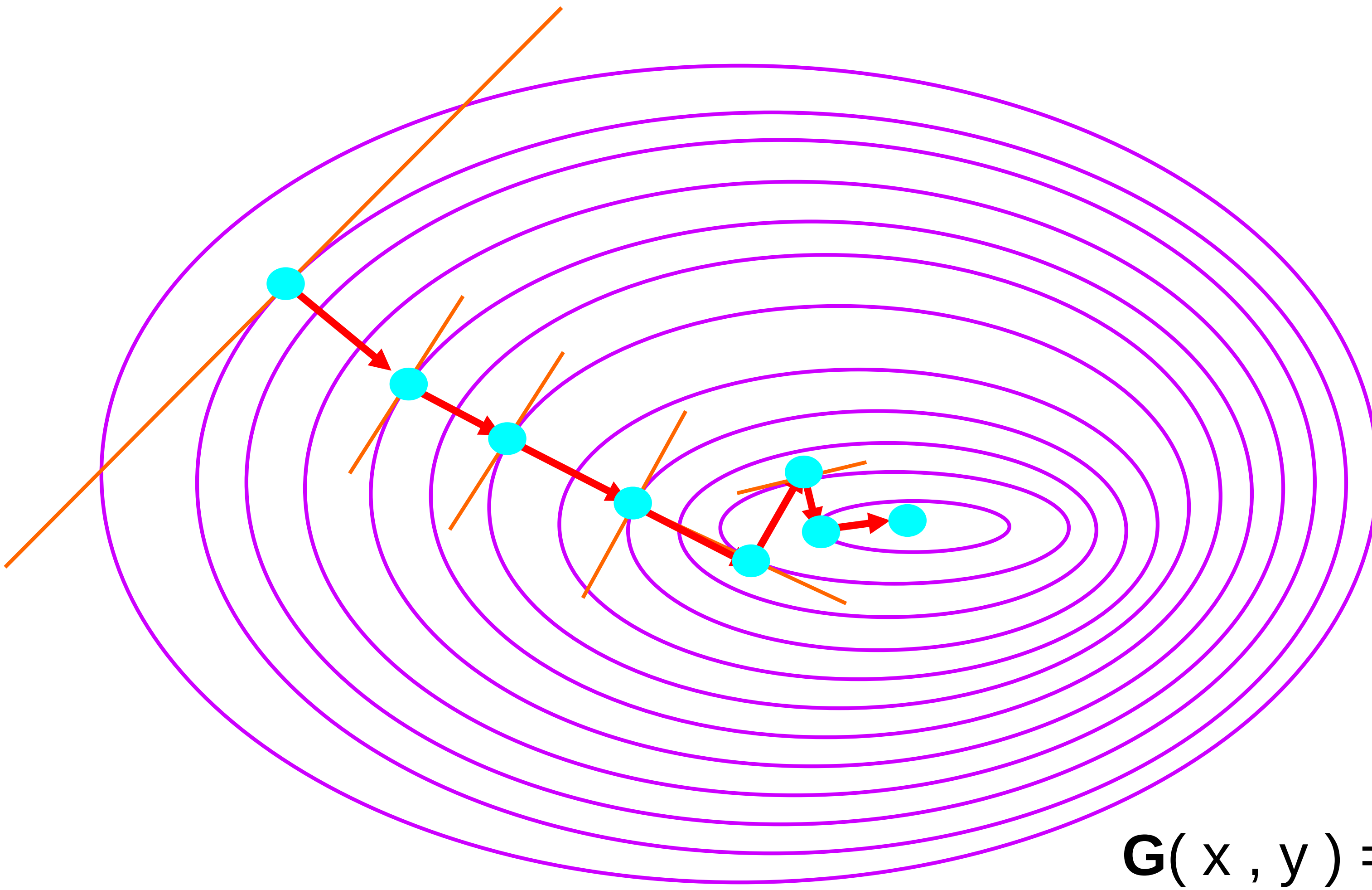
Metric vs. time



Gradient Descent

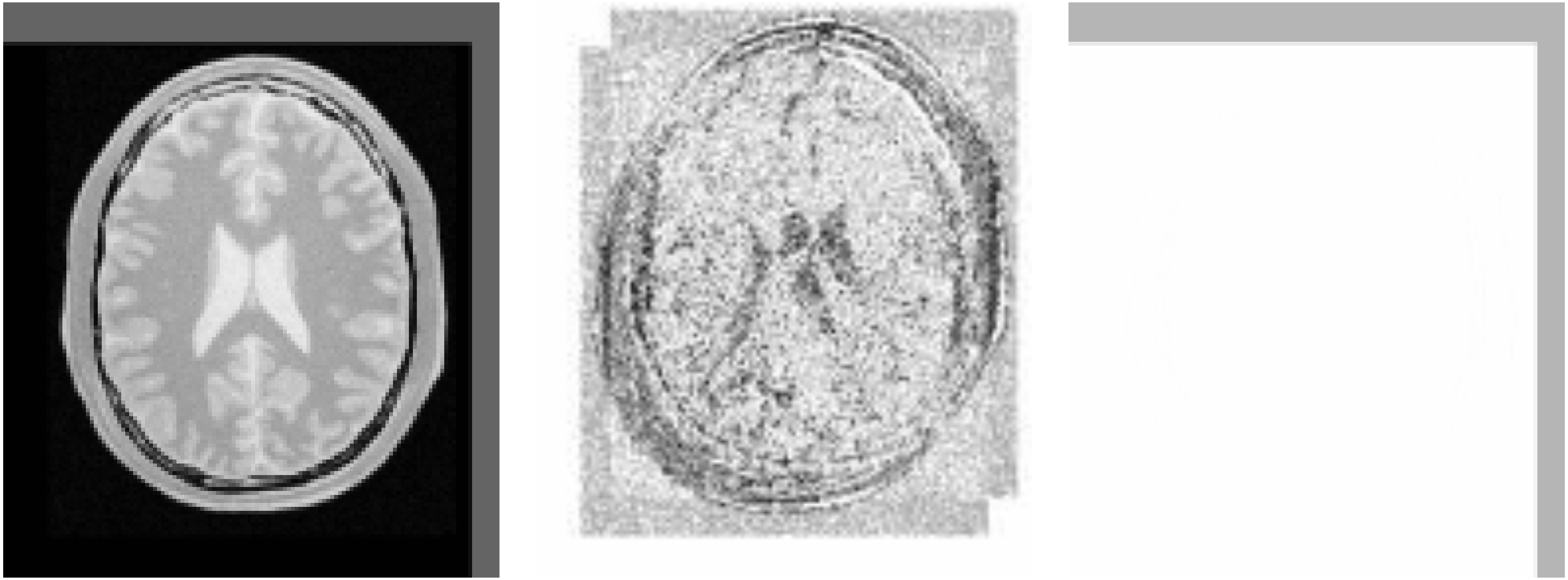
Gradient Descent Optimizer



$f(x, y)$

$\mathbf{G}(x, y) = \nabla f(x, y)$

Image comparison



Evaluating your registration

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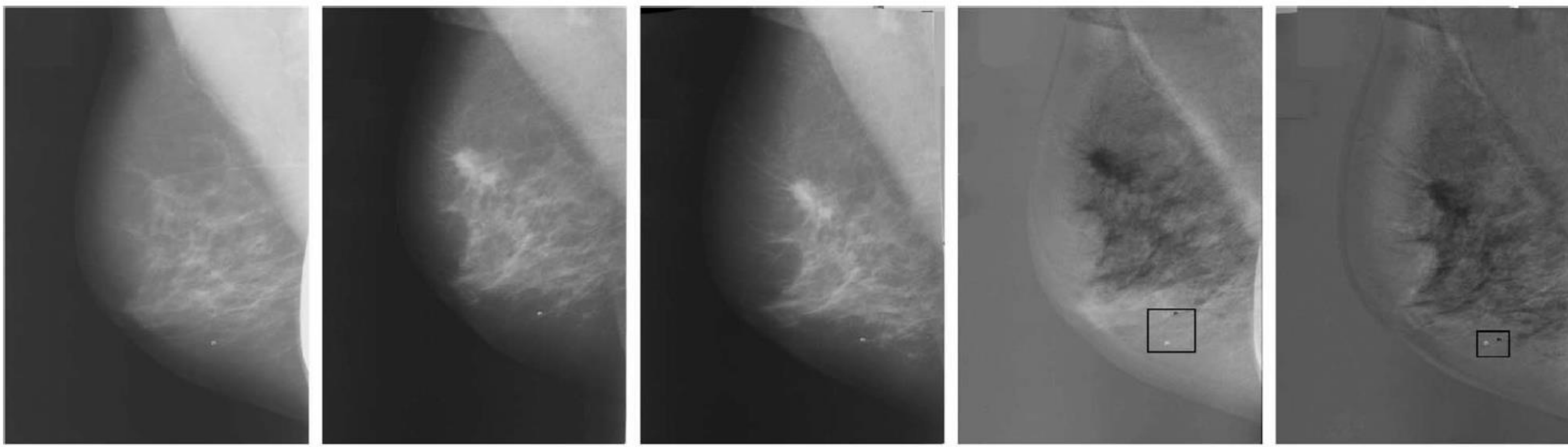
How to evaluate?

You have developed your image registration algorithm... now what?



Fixed | Moving | Registered Moving | Diff before | Diff after

Validation of algorithms

- Technical validation
  - Accuracy, Robustness, speed, reliability
- Clinical Validation
  - Is it useful for the doctors?
  - Does it help to improve diagnosis/treatment?
- Comercial System
  - FDA Approval

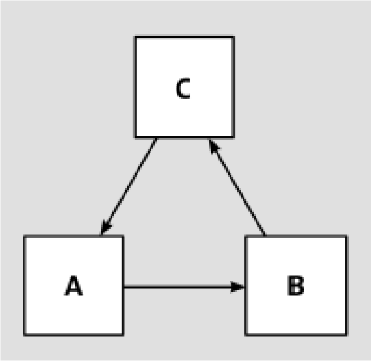
Evaluation criteria

- Robustness
  - Variability for starting conditions of the registration
  - Noise, allowed misregistration, landmark error, etc.
- Consistency
- Visual assessment
  - Subtraction images
  - Checkerboard, overlay
- Quantitative analysis
  - Landmark error
  - Simulated deformations
  - Similarity metrics.
- Ground Truth!?

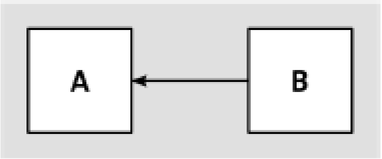
Consistency

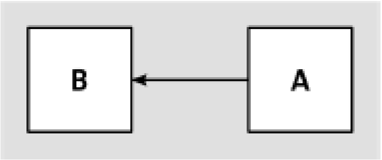
- Apply various registration in order to evaluate the consistency.

$$T_{AB}(T_{BC}(T_{CA}(x))) = I(x) = x$$



- Forward /inverse consistency
  - Bijectivity/diffeomorphic registration





- Does not measure accuracy!!

Visual Assessment

- Qualitative assessment. Observer opinion.
- Intra – inter observer variability!
- Subtraction images
- Overlay
  - Checkboard
  - Contours
  - Colors

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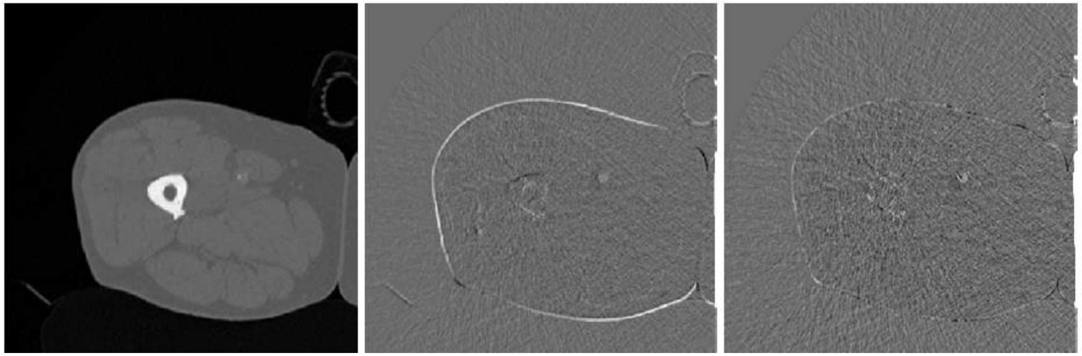
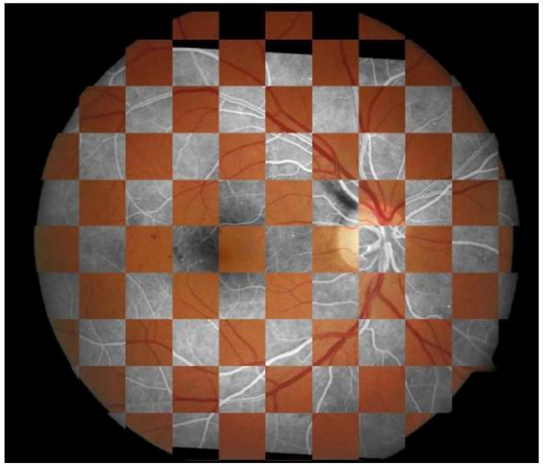
Visual Assessment

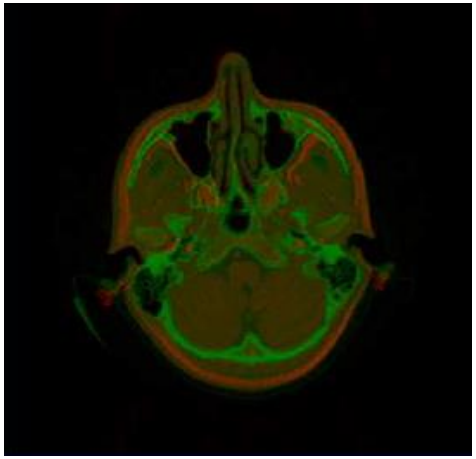
- Subtraction images
  - Absolute?



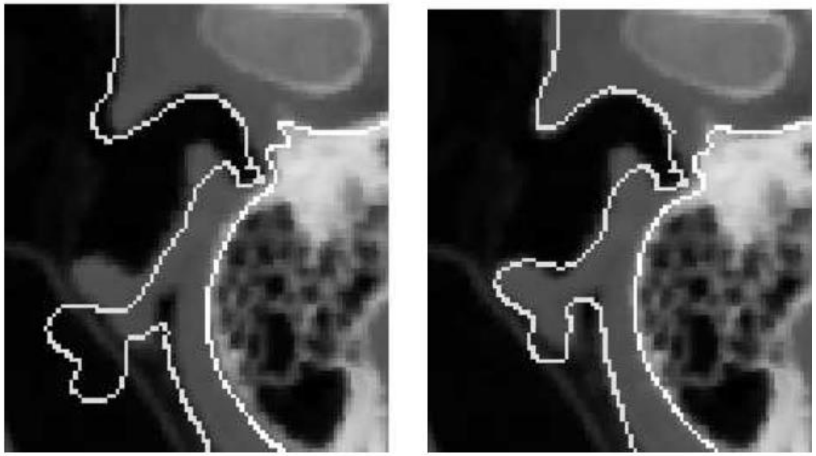
- Overlay



Checker board



Color (R & G)



Contour

Visual Assessment

- Example
  - Observer study

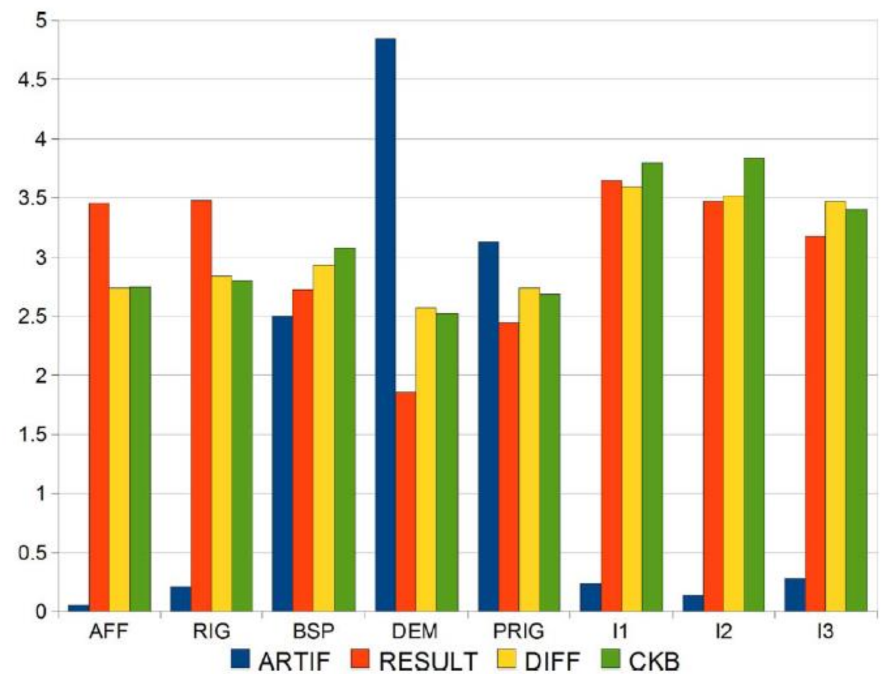
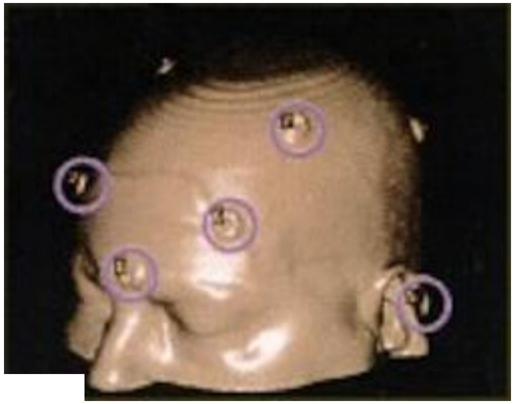


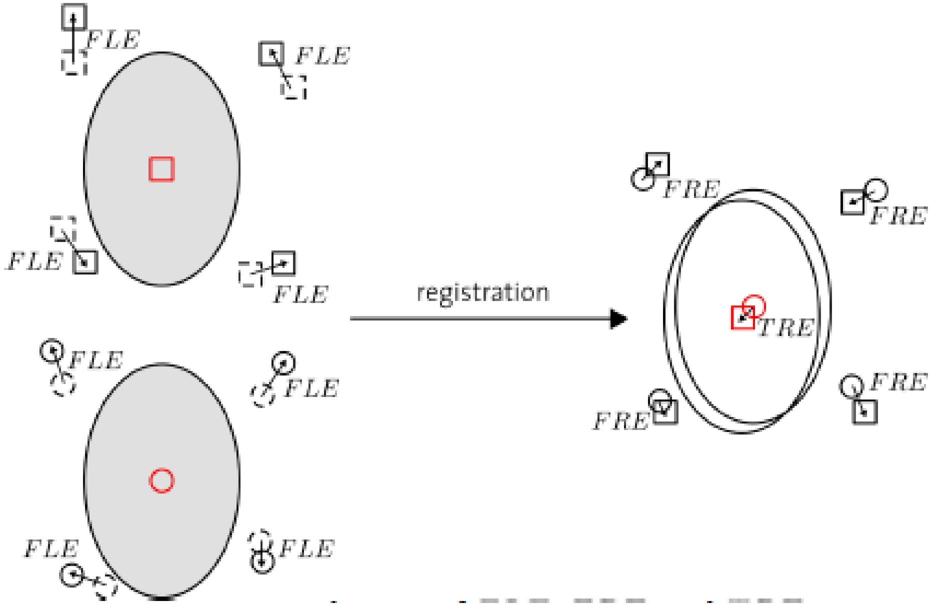
Fig. 4. Summary of the observer study. Bars show observer perception, in all cases higher means better except for the number of artifacts (ARTIF) were lower means better. AFF = affine, RIG = rigid, BSP = B-Splines FFD, DEM = Demons, PRIG = polyrigid, I1 = MR BSP, I2 = AFF + BSP, I3 = MR AFF + MR BSP.

Y.Díez, A.Oliver, X.Lladó, J.Freixenet, J.Martí, J.C.Vilanova, and R.Martí. Revisiting intensity-based image registration appplied to mammography. IEEE Trans. on Information Technology in BioMedicine, 15(5), pp 716-725, 2011.

Errors

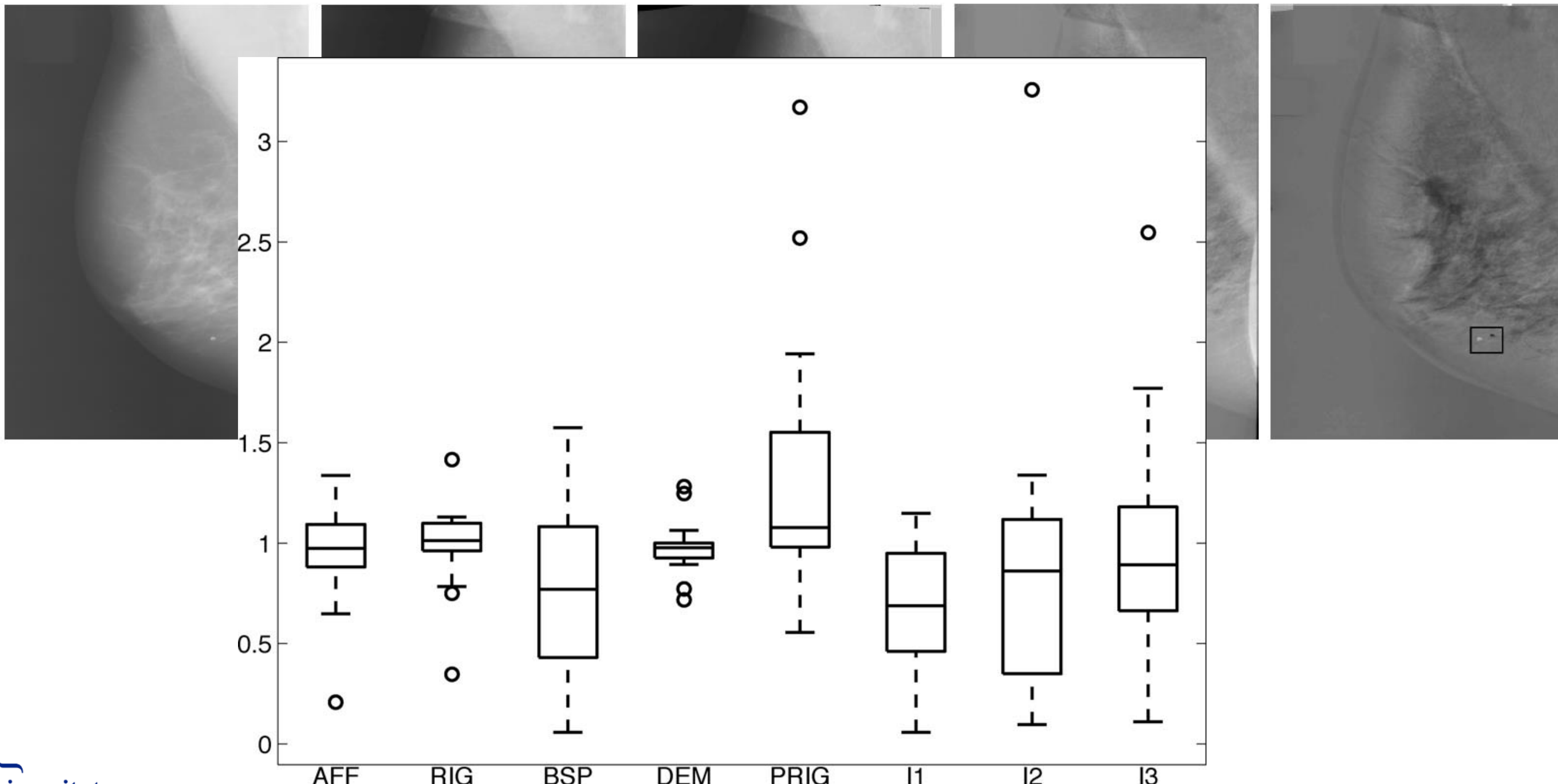
- Fiducial Localization Error (FLE)
- Fiducial Registration Error (FRE)
- Target Registration Error (TRE)





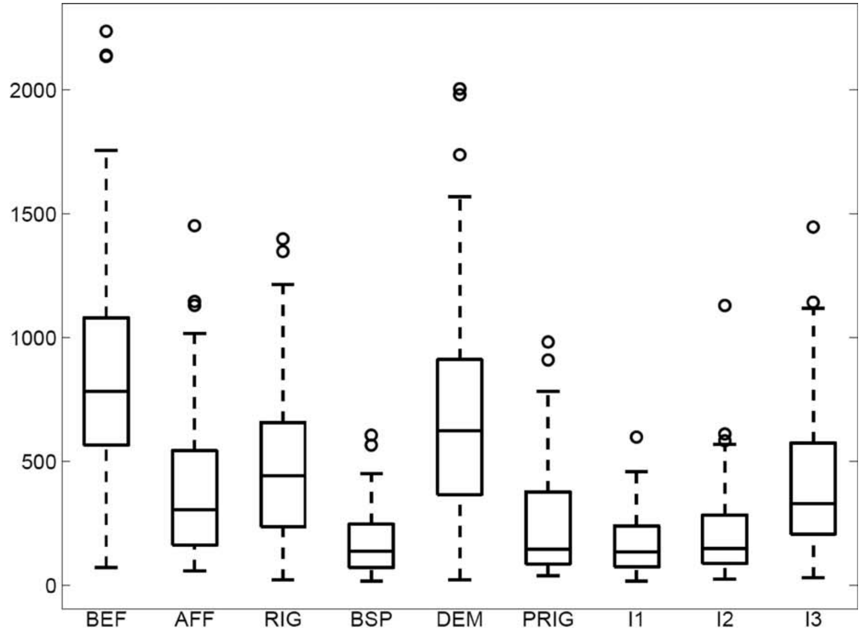
Errors. example

- TRE: Computing error after registration between microcalcifications.

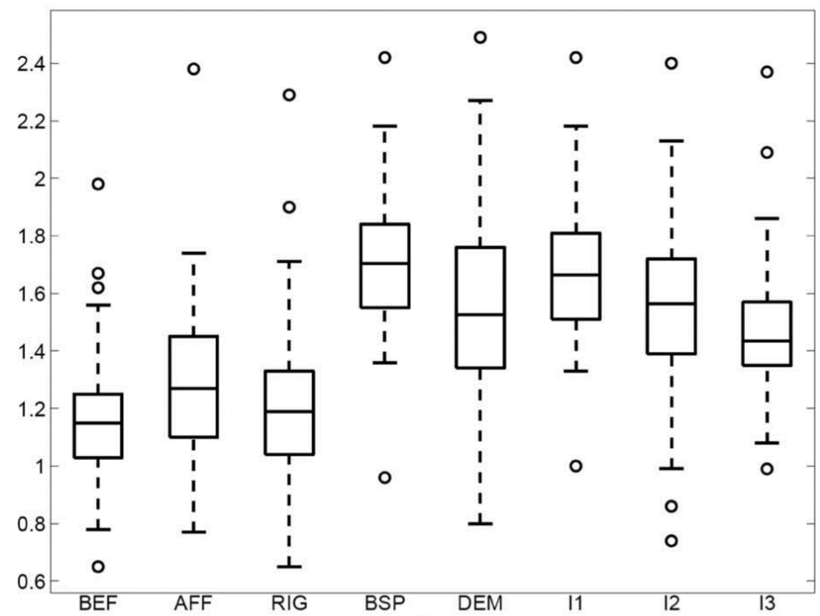


Similarity metrics

- Metric comparison
  - A higher metric DOES NOT always mean better registration



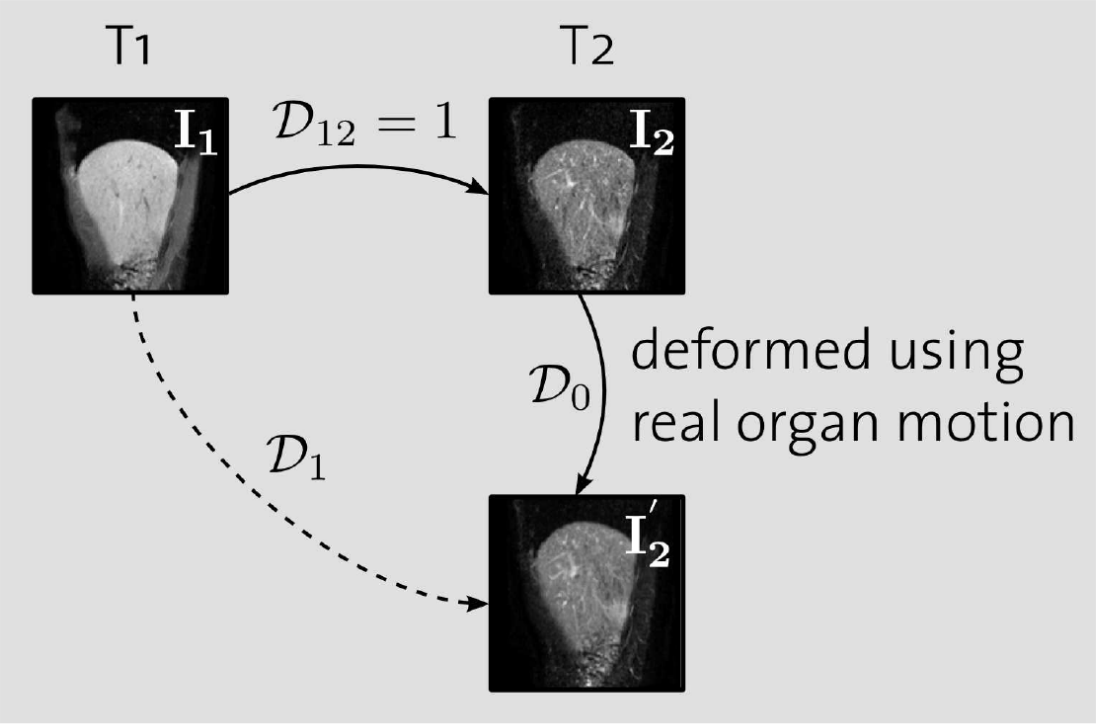
SSD



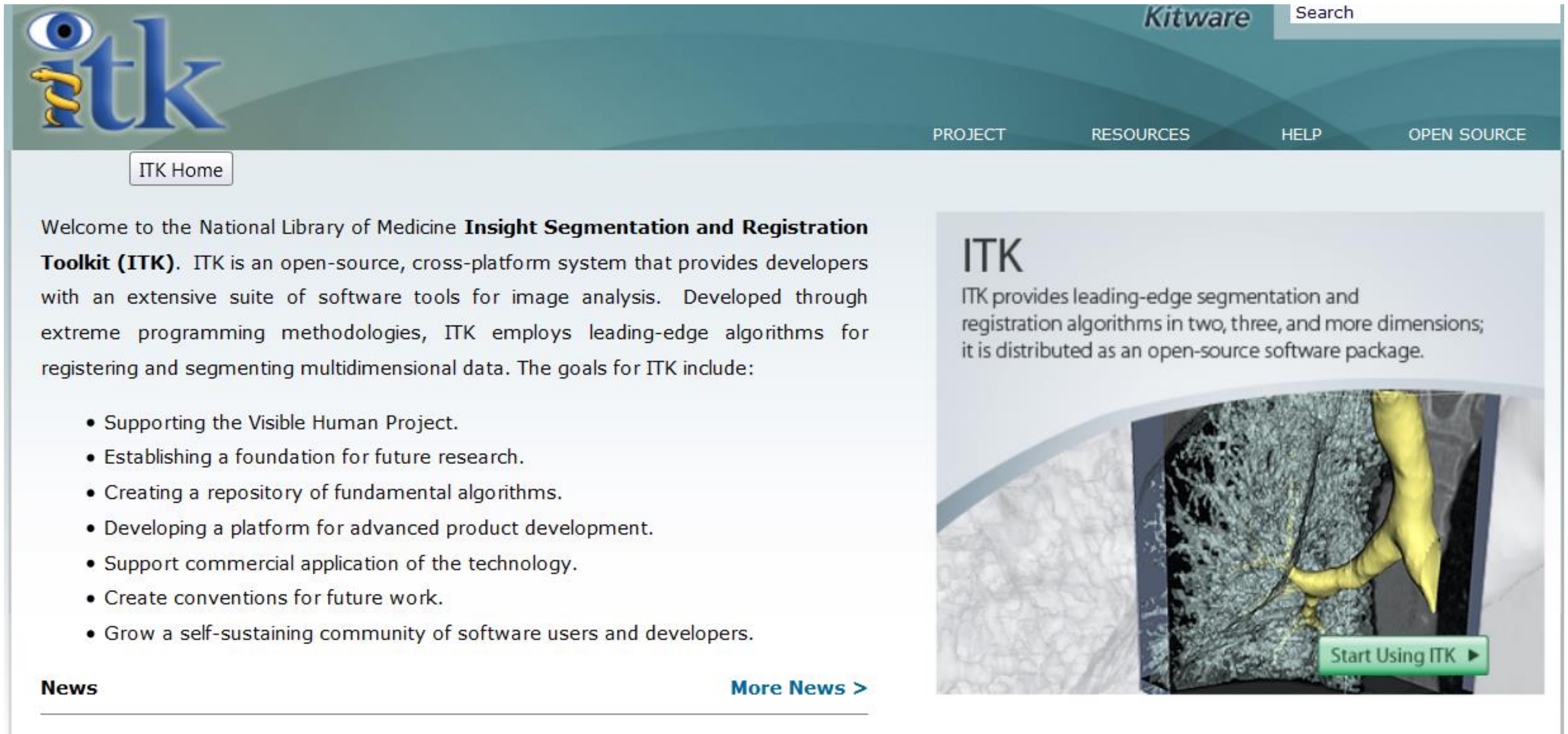
MI

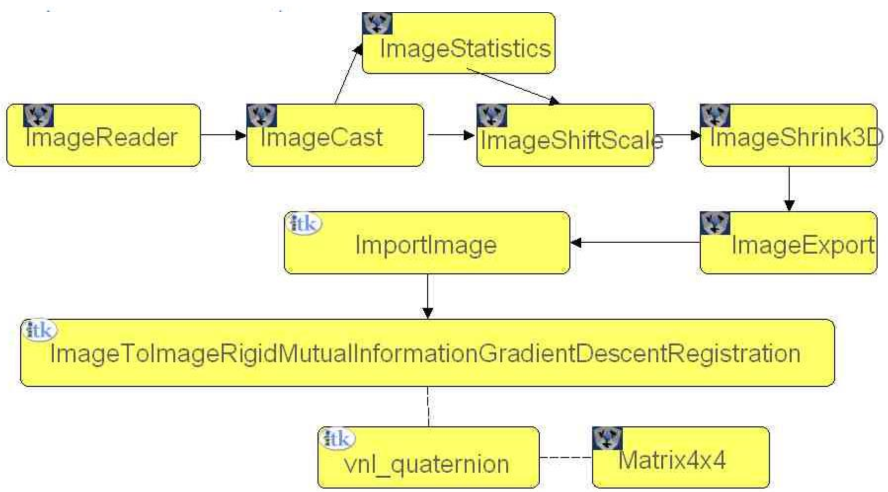
Synthetic / Simulated models

- Difficulty of obtaining the ground truth
- Synthetic/Simulated examples
- Simulate deformation and recover it with IR.



ITK & Image registration





Matlab & Image registration

The imregconfig function makes it easy to pick the correct optimizer and metric configuration to use with imregister. These two images have different intensity distributions, which suggests a multimodal configuration.

```
[optimizer,metric] = imregconfig('multimodal');
```

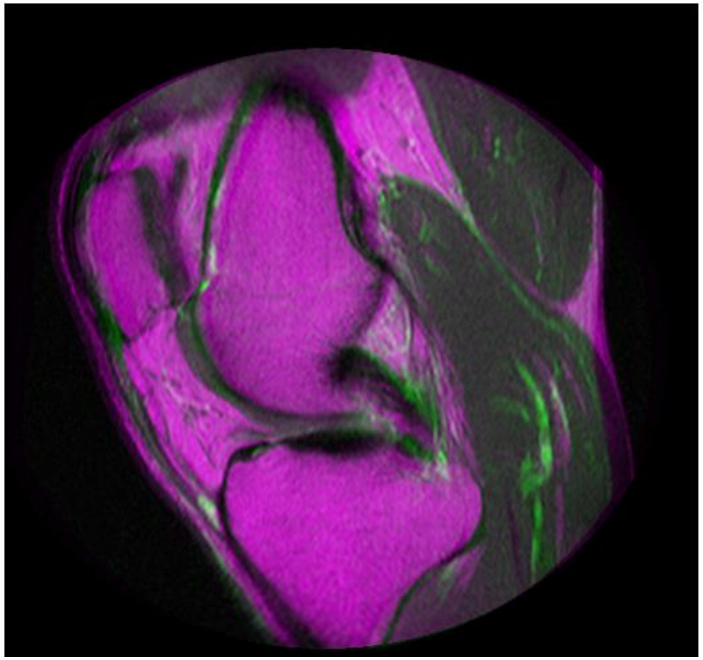
The distortion between the two images includes scaling, rotation, and (possibly) shear. Use an affine transformation to register the images.

It's very, very rare that imregister will align images perfectly with the default settings. Nevertheless, using them is a useful way to decide which properties to tune first.

```
movingRegisteredDefault = imregister(moving, fixed, 'affine', optimizer, metric);

f3 = figure;
imshowpair(movingRegisteredDefault, fixed);
figure(f3);
title('A: Default registration');
```

A: Default registration



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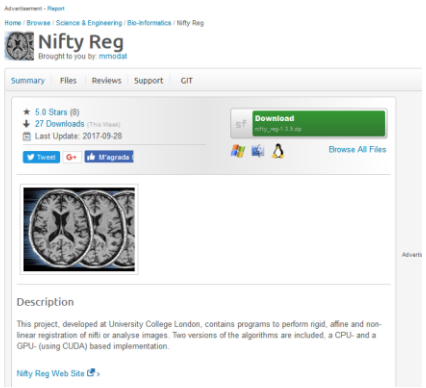
Software & Libraries

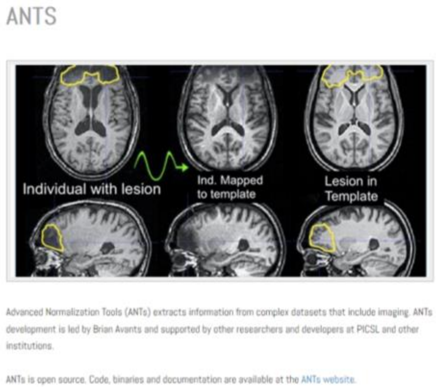
- Niftyreg

https://sourceforge.net/projects/niftyreg/



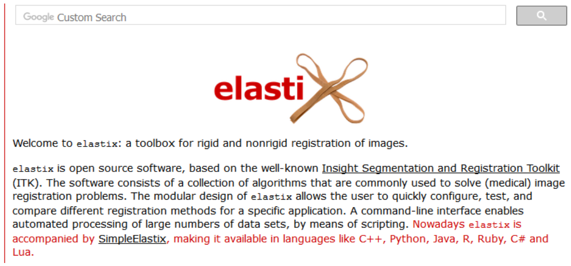
- ANTS

http://picsl.upenn.edu/software/ants/



- Elastix

http://elastix.isi.uu.nl/



http://www0.cs.ucl.ac.uk/opensource_mia_ws_2012/links.html

ITK

- Getting Started
  - http://www.itk.org/CourseWare/Training/GettingStarted-I.pdf
  - http://www.itk.org/CourseWare/Training/GettingStarted-III.pdf
  - Example code: ImageRegistration1.cxx
- Software Guide http://www.itk.org/ItkSoftwareGuide.pdf

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To know more…

- *Insight into Images: Principles and Practice for Segmentation, Registration and Image Analysis, Terry S. Yoo (Editor)*
- *Handbook of Medical Imaging: Processing and Analysis, Isaac Bankman (Editor)*
- Fundamentals of Medical Imaging, P. Suetens, Cambridge University Press 2002
- A. Melbourne, G. Ridgway, D. J. Hawkes. Image Similarity Metrics in Image Registration. CMIC, UCL London.
- Medical Image Analysis, A. Dhawan, Wiley 2003
- ITK Software Guide (www.itk.org)