The History and Mathematics of Quantitative Trading

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Abstract

This paper will seek to define what Quantitative Trading is, it's history, and the mathematics used behind it. Quantitative analysis uses a mathematical analysis of the measurable figures of a company, such as the value of assets or projected sales. This information is then used to trade stocks on a high frequency basis. Quantitative analysts aim to represent a given reality in terms of a numerical value.

1 Introduction

Quantitative analysis (QA) is employed for several reasons, including measurement, performance evaluation or valuation of a financial instrument, and predicting real-world events, such as changes in a country's gross domestic product (GDP). A typical problem for a mathematically oriented quantitative trader would be to develop a model for pricing, hedging, and risk-managing a complex derivative product. These quantitative analysts tend to rely more on numerical analysis than statistics and econometrics. In the financial services industry, QA is used to analyze investment opportunities, such as when to purchase or sell stocks and derivatives. Quantitative analysis ranges from the examination of simple statistical data to complex calculations such as option pricing.

2 History of Quantitative Analytics

2.1 Theory of Speculation

Louis Jean-Baptiste Alphonse Bachelier (1870-1946) was a French mathematician. In 1900 he published his doctoral thesis "Theory of Speculation". In this paper Bachlelier introduces the concept of what is now known as Stochastic Analysis. This calculus was developed to cope with questions arising in probability theory in which processes are modelled by motion along paths which typically are not differentiable. Stochastic calculus is the area of mathematics that deals with processes containing a stochastic component and thus allows the modeling of random systems (e.g. stock prices).

2.2 Portfolio Selection

In 1952, Harry Markowitz, an American economist, published his doctoral thesis "Portfolio Selection" and it was one of the first efforts by economics journals to formally adapt mathematics to concepts of finance. His paper formalized the notion of mean return and co-variances for common stocks and thus led him to quantify the concept of 'diversification'.

2.3 Stochastic Calculus in Finance

In 1956, another American Economist, Paul Samuelson introduced stochastic calculus into the study of finance. The basic hypothesis under which once operates is that the price of an asset is a random variable. More precisely, the hypothesis is that the price of an asset X is represented by two components:
1. A deterministic component which represents some structure in the price movement
2. A stochastic component, which represents the noise in price movement.

2.4 Black–Scholes Model

In 1973, a pair of economists, Fischer Black and Myron Scholes developed the Black–Scholes model. The Black-Scholes Merton (BSM) model is a differential equation used to solve for options prices .The formula, is perhaps the world's most well-known options pricing model. It was introduced in their 1973 paper, "The Pricing of Options and Corporate Liabilities," published in the Journal of Political Economy.
Here is an example of their "Call Option" Formula:

$$C = S_t N(d_1) - Ke^{-rt} N(d_2)$$

where:

$$d_1 = \frac{ln\frac{S_t}{K} + (r + \frac{\sigma_v^2}{2})t}{\sigma_s \sqrt{t}}$$

and

$$d_2 = d_1 - \sigma_s \sqrt{t}$$

where:
C= Call option price
S = Current stock price
K= Strike price
r= Risk-free interest rate
t= Time to maturity
N = A normal distribution

3 Jim Simons and Renaissance Technologies

Jim Simons, is known as the "Quant King," having started one of the most successful quant funds in the world—Renaissance Technologies, "Rentech". Before Rentech, Simons spent time at the National Security Agency and taught at the Massachusetts Institute of Technology and Harvard.

3.1 RenTech

In 1978, the mathematician started the hedge fund Monemetrics, which was the predecessor to the highly successful Renaissance Technologies. Simons didn't think to apply mathematics to his hedge fund at first; however, over time, he realized he could use mathematical and statistical models to interpret data. By 1988, Simons decided to solely use quantitative analysis to decide which trades to enter. Simons only sought experts in mathematics, data analysis, and many other scientific-related fields to work with him at the fund. The Quant King filled the fund with programmers, mathematicians, physicists, and cryptographers. The company thrived due to the complex mathematical formulas these scientists developed.

4 Mathematics Behind Quantitative Analysis

4.1 Finite Difference Method

The Finite Difference is used to solve partial differential equations and is focused on approximating them. FDM describes how an option price evolves over time by a set of difference equations. The discrete difference equations may then be solved iteratively to calculate a price for the option. The mathematics is based on Taylor series approximation. First, we are assuming the function whos derivatives are to be approximated are properly behaved. So by Taylor's theorem:

$$f(x_0+h) = f(x_0) + \frac{f'(x_0}{1!}h + \frac{f''(x_0}{2!}h^2 + ... + \frac{f^n(x_0)}{n!}h^n + R_n(x)$$

where $R_n(x)$ is a remainder term. So to find an approximation for the first derivative:

$$f(x_0+h) = f(x_0) + f'(x_0)h + R_1(x)$$

Setting $x_0 = a$ we have:

$$f(a+h) = f(a) + f'(a)h + R_1(x)$$

Dividing across by h gives:

$$\frac{f(a+h)}{h} = \frac{f(a)}{h} + f'(a) + \frac{R_1(x)}{h}$$

Solving for $f'(a)$:

$$f'(a) = \frac{f(a+h) - f(a)}{h} - \frac{R_1(x)}{h}$$

Assuming that $R_1(x)$ is small enough:

$$f'(a) \approx \frac{f(a+h) - f(a)}{h}.$$

4.2 Monte Carlo Method

The Monte Carlo Method is also used to solve partial differential equations, but Monte Carlo simulation is also common in risk management. Los Alamos scientists aiming to take advantage of the first "super" computer MANIAC, invented a statistical sampling-based technique to solve problems related to stochastic neutron diffusion in atomic bomb project and for estimating eigenvalues of the Schrödinger equation, which came to be called the Monte Carlo. The method is used for more than three or four state variables. In this instance, finite-difference methods face several difficulties and is not practical. What's the difference between the Monte Carlo method and the Monte Carlo simulation? Monte Carlo method is a technique that can be used to solve a mathematical or statistical problem, and a Monte Carlo simulation uses repeated sampling to obtain the statistical properties of some phenomenon. An example to demonstrate:

Simulation: Drawing one pseudo-random uniform variable from the interval [0,1] can be used to simulate the tossing of a coin: If the value is less than or equal to 0.50 designate the outcome as heads, but if the value is greater than 0.50 designate the outcome as tails. This is a simulation, but not a Monte Carlo simulation.

Monte Carlo method: Pouring out a box of coins on a table, and then computing the ratio of coins that land heads versus tails is a Monte Carlo method of determining the behavior of repeated coin tosses, but it is not a simulation.

Monte Carlo simulation: Drawing a large number of pseudo-random uniform variables from the interval [0,1] at one time, or once at many different times, and assigning values less than or equal to 0.50 as heads and greater than 0.50 as tails, is a Monte Carlo simulation of the behavior of repeatedly tossing a coin. Monte Carlo is used in corporate finance to model components of project cash flow, which are impacted by uncertainty.

4.3 Ordinary least squares

Ordinary least squares (OLS) is a type of linear least squares method for estimating the unknown parameters in a linear regression model. There are a few assumptions that we must take before doing the OLS: a) Regression assumes that variables have normal distributions.

b) Standard multiple regression can only accurately estimate the relationship between dependent and independent variables if the relationships are linear in nature.

c) Homoscedasticity means that the variance of errors is the same across all levels of the

independant variables.
The most common application of this method, which is sometimes referred to as "linear" or "ordinary", aims to create a straight line that minimizes the sum of the squares of the errors that are generated by the results of the associated equations
Suppose the data we have, consist of n observations $\{y_i, x_i\}_{i=1}^{n}$. Each observation i includes a scalar response y_i and a column vector x_i of values of p predictors x_{ij} for $j = 1, ..., p$. In a linear regression model, the response variable y_i, is a linear function of the regressors:

$$y_i =_1 x_{i1} +_2 x_{i2} + ... +_p x_{ip} + \varepsilon_i$$

which, in vector form would be:

$$y_i = x_i^T + \varepsilon_i$$

4.4 Spline Interpolation

Spline interpolation is used to interpolate values from spot and forward interest rates curves. Given $(x_0, y_0), (x_1, y_1), ...(x_n, y_n)$ find the value of 'y' at the value of 'x' that is not given.

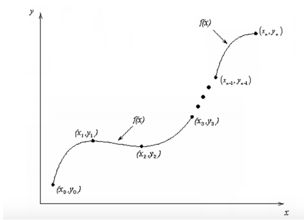

Figure 1: Example of Spline Interpolation

5 Conclusions

This paper aimed to give a context at the brief history and mathematics behind quantitative analysis in regards to its application to financial markets. By introducing stochastic calculus, the understanding of finance markets was greatly improved and models were able to be developed. The Black-Scholes Model, finite difference methods, monte carlo method, OLS, and spline interpolation all allow for increased awareness when it comes to options pricing. Quantitative analysis is most commonly used to analyze investment opportunities, such as when to purchase or sell stocks and derivatives, like options.

References

Haugh, Martin. A Brief Introduction to Stochastic Calculus. A Brief Introduction to Stochastic Calculus, www.columbia.edu/ mh2078/FoundationsFE/IntroStochCalc.pdf.

Kenton, Will. “How Quantitative Analysis (QA) Works.” Investopedia, Investopedia, 29 Jan. 2020, www.investopedia.com/terms/q/quantitativeanalysis.asp.

Kenton, Will. “How the Black Scholes Price Model Works.” Investopedia, Investopedia, 6 Feb. 2020, www.investopedia.com/terms/b/blackscholes.asp.

Kenton, Will. “How the Least Squares Method Works.” Investopedia, Investopedia, 29 Jan. 2020, www.investopedia.com/terms/l/least-squares-method.asp.

“Mathematics Institute.” Stochastic Analysis, warwick.ac.uk/fac/sci/maths/research/interests/stochastica

“Real Options with Monte Carlo Simulation.” Real Options with Monte Carlo Simulation, marcoagd.usuarios.rdc.puc-rio.br/monte-carlo.html.

Sawilowsky, Shlomo S. “You Think You’Ve Got Trivials?” Journal of Modern Applied Statistical Methods, vol. 2, no. 1, 2003, pp. 218–225., doi:10.22237/jmasm/1051748460.