

# Cosmology

## Pre-class reading 3

Distribution functions: In thermal equilibrium, relativistic particles follow the Fermi-Dirac or Bose-Einstein distributions if they are fermions or bosons. Non-relativistic particles follow the Maxwell-Boltzmann distribution regardless if they are fermions or bosons.

From the second law of thermodynamics, it follows that entropy density (proportional to statistical weight times  $T$  to the third power) times scale factor to the third power is a constant. As the Universe expands and cools, particles change from being relativistic (changing their statistical weights), contributing to the entropy density to being non-relativistic thereby transferring their contribution to other relativistic particles.

Decoupling of stable particles from the relativistic fluid: Initially, stable particles are coupled to the relativistic plasma through particle interactions. Following the expansion of the Universe, which decreases the specific entropy and results in cooling, the stable particles “freeze-out”/ decouple from the relativistic plasma. If the cross section for interactions is low, the freeze-out of the stable particles happens in the relativistic regime, in which case, the freeze-out abundances will be comparable to that of the photons. If the cross section is higher, the freeze-out takes place in the non-relativistic regime, in which case their freeze-out abundances are exponentially suppressed compared to the photons.

Proton and neutron abundances are set by the following weak interactions:

$p + e \rightarrow n + \text{electron neutrino}$

$n + e^+ \rightarrow p + \text{electron anti neutrino}$

Formation of light nuclei takes place through a chain of 2 body interactions starting with  $p$  and  $n$  when the Universe has cooled to a few giga Kelvin. These interactions produce  $D$ ,  $3H$ ,  $3He$ ,  $4He$ , and  $7Li$ .

Observations vs. model for light elements:

$4He$ : Observations suggest  $\eta = 1.2-8 \times 10^{-10}$

$D$ : Observations suggest  $D/H \sim 2.8 \times 10^{-5}$  consistent with  $\eta$  obtained from WMAP (CMB)

$3He$ : Observations suggest  $D+3He/H < 1 \times 10^{-4}$  which gives  $\eta > 3 \times 10^{-10}$

${}^7\text{Li}$ : Observations suggest  ${}^7\text{Li}/\text{H} \sim 1.1 \times 10^{-10}$  inconsistent by a factor of 4 for  ${}^7\text{Li}/\text{H}$  of  $\sim 5.2 \times 10^{-10}$  obtained from WMAP

1a. Initially, neutrinos are in thermal equilibrium with the photon fluid through weak interactions with electrons and positrons. At a temperature of  $T_f \sim 1$  MeV, the interaction rate for these reactions drops below the expansion rate of the Universe and the neutrinos decouple from the photons. However, as they remain relativistic, the statistical weights don't change and the temperature of the neutrinos remains the same as the photons. Later at  $T \sim 0.51$  MeV, the electron pair annihilation rate drops below the expansion rate and the electrons freeze out from the photon fluid. The entropy released in this process (as electrons become non-relativistic) is then transferred to photons which increases its temperatures but not to the decoupled neutrinos who conserve their entropy separately. There is thus expected to be a neutrino background whose temperature is lower than that of the photon background.

1b. If we assume a constant annihilation cross-section, the equilibrium abundances of relativistic particles as a function of temperature is constant. For non-relativistic particles, the equilibrium abundances are exponentially suppressed as a function of  $1/T$

1c. Primordial nucleosynthesis begins at temperatures of a few giga Kelvin. However at this time, the number densities of p and n are already too low to form heavy elements through direct many body interactions. Nucleosynthesis proceeds through a chain of two body interactions producing D,  ${}^3\text{H}$ ,  ${}^3\text{He}$ ,  ${}^4\text{He}$ , and  ${}^7\text{Li}$ . However, since there are no stable elements with atomic weight 5 or 8, heavier elements beyond  ${}^7\text{Li}$  are not produced through this chain.

1d. I'm not sure how the CMB temperature comes into play for setting the n/p ratio as the CMB is from the epoch of recombination when the photons decouple from the ion plasma. But, assuming a higher CMB temperature implies a higher temperature where neutrinos freeze out of the relativistic plasma causing n and p freeze out, we will have a **lower** n/p ratio. Since almost all primordial n gets locked up in primordial  ${}^4\text{He}$ , we would have a reduced abundance of  ${}^4\text{He}$ . **Having lower abundances of  ${}^4\text{He}$  in stellar cores would mean longer stellar lifetimes.**

1e. I don't think the baryon to photon ratio should affect the *freeze-out* n/p ratio as that depends only on the Q-factor between neutron and proton masses and the temperature at which neutrino freeze-out occurs. A higher baryon to photon ratio implies a lower  $|\ln \eta|$  (as  $\eta \sim 10^{-9}$  to  $10^{-10}$ ), which implies a higher T at

which primordial nucleosynthesis begins. At this time  $n/p$  would be slightly higher than at lower temperatures because not many  $n$  would have decayed to  $p$ , which would lead to more  $4\text{He}$  formed.

2. I have been using the MBW book, but it seems to be unnecessarily complicated so I have been supplementing this with random online lecture notes