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Cosmology

Pre-class reading 8

1) The Press-Schechter formalism relies on the spherical collapse model's prediction of a critical overdensity, δ_c~1.686 which serves as a threshold for gravitational collapse. In this model, a homogeneous spherical perturbation in an expanding universe eventually detaches from the Hubble flow, reaches a maximum radius (turnaround), and collapses under its own gravity. After virialization, the region settles into a stable configuration where kinetic and potential energies satisfy the virial theorem. The critical density is derived analytically under the assumption of spherical symmetry, enabling a direct mapping between the linear density field and nonlinear halo formation. This universality allows the Press-Schechter approach to statistically predict halo abundance by counting regions in the initial density field that exceed δ_c when linearly extrapolated to the collapse epoch.

2) The spherical collapse model underestimates the abundance of high-mass halos and over predicts low-mass ones compared to cosmological simulations. The ellipsoidal collapse model incorporates the effects of tidal shear, arising from anisotropic gravitational forces exerted by the surrounding matter distribution. Unlike the spherical case, which depends only on density, the ellipsoidal model has a mass-dependent critical density parameter, which is modulated by the local shear field.
The tidal forces alter the collapse timescale, causing halos in overdose environments to virtualize earlier. Additionally, the ellipsoidal model predicts earlier formation times for low-mass halos in underdone regions, aligning with observed assembly bias trends.

3) The viral radius demarcates the boundary within which a halo is gravitationally bound and approximately virialized. Inside r_vir, DM follows a density profile characterized by a steep rise towards the center and a gradual decline at larger radii. A commonly adopted boundary is where the enclosed mass and mean density within the boundary is ~200 times the critical density. Outside the viral radius, matter remains part of the cosmic web, undergoing infall and remaining unvirialized.
The viral radius is significant because it defines a halo edge for observational mass estimates, enabling us to apply the viral theorem. This also standardizes comparisons between theoretical predictions and simulations, which define quantities relative to this radius.