

COL352 Problem Sheet 1

January 9, 2025

Problem 1. Create a DFA that represents the language $\{\text{binary representation of } n \in \mathbb{N} \mid n \bmod 8 \text{ is either 4 or 1}\}$

Problem 2. Create a regular expression over the alphabet $\{0, 1\}$ that represents the language mentioned in Problem 1.

Problem 3. Construct a DFA which accept the language $L = \{w \mid w \in \{a, b\}^* \text{ and } Na(w) \bmod 3 = Nb(w) \bmod 3\}$, where $Na(w)$ and $Nb(w)$ return the number of occurrences of a and b in w respectively.

Problem 4. Construct a DFA that recognizes the following language over the alphabet $\{0, 1\}$.

$$\{x \mid 01 \text{ and } 10 \text{ have equal number of occurrences as substrings in } x\}$$

Problem 5. Let $C_n = \{x \mid x \text{ is a binary number that is a multiple of } n\}$. Show that for each $n \geq 1$, the language C_n is regular. Provide a general construction for C_i for $i \geq 0$.

Problem 6. You already know from the lectures that regular languages are closed under complementation. Given DFAs D_1 and D_2 that recognize languages L_1 (over Σ_1) and L_2 (over Σ_2) respectively, construct an automaton D recognizing the following languages, if you believe that the class of regular languages is closed under the following operations. Provide a counterexample otherwise.

1. **Difference:** $L_1 \setminus L_2 := \{x \mid x \in L_1 \text{ and } x \notin L_2\}$
2. **Star:** $L_1^* := \{w_1 w_2 \dots w_n \mid w_i \in L_1 \text{ for } n \geq 0, \text{ and every } 1 \leq i \leq n\}$

Problem 7. Design an efficient algorithm that takes input the description of a DFA D and determines if the resulting language $L(D)$ is

1. Empty
2. Infinite
3. Σ^*

Problem 8. Given two DFAs, D_1 and D_2 , design an efficient algorithm to determine if $L(D_1) = L(D_2)$.

Problem 9. $L_1 = (0|1)^*0(0|1)^*1(0|1)$, $L_2 = (0|1)^*01(0|1)^*$. Show that the two languages are equal.

Problem 10. Let L_1 be a regular language and L_2 be any language (not necessarily regular) over the same alphabet Σ . Prove that the language $L = \{x \in \Sigma^* \mid x \cdot y \in L_1 \text{ for some } y \in L_2\}$ is regular by defining a DFA for L starting from a DFA for L_1 and the language L_2 .

Problem 11. $\phi : \Sigma^* \rightarrow \Gamma^*$ is called a homomorphism over strings if for all $x, y \in \Sigma^*$, $\phi(xy) = \phi(x)\phi(y)$. Show that if L is a regular language, then $\phi(L) := \{y \in \Gamma^* \mid y = \phi(x), x \in L\}$ where ϕ is a homomorphism as defined above is also regular.

Problem 12. Prove that the class of regular languages is closed under inverse homomorphisms. That is, prove that if $L \subseteq \Gamma^*$ is a regular language and $\phi : \Sigma^* \rightarrow \Gamma^*$ is a string homomorphism, then $\phi^{-1}(L) = \{x \in \Sigma^* \mid \phi(x) \in L\}$ is regular.

Problem 13. Prove that the class of **non-regular** languages is **not** closed under the Union operation.
Hint: use the closure properties mentioned in the lectures.

Problem 14. Let L be an arbitrary regular language. Prove that the following languages are regular.

1. $\{x \mid x \cdot \text{reverse}(x) \in L\}$
2. $\{x \mid x \cdot \text{reverse}(x) \cdot x \in L\}$.
3. $\{x \mid xxx \in L\}$

Problem 15. Let L be a regular language with DFA D . We define $\text{Pre}(L) = \{x \mid x \text{ is a prefix of some } y \in L\}$. Show that $\text{Pre}(L)$ is regular by constructing a DFA.

Problem 16. Let A be any language. Define $\text{DROP-OUT}(A)$ to be the language containing all strings that can be obtained by removing one symbol from a string in A . Thus, $\text{DROP-OUT}(A) = \{xz \mid xyz \in A \text{ where } x, z \in \Sigma^*, y \in \Sigma\}$. Show that the class of regular languages is closed under the DROP-OUT operation.