Final Project

ENGR 441

Due Date: December 12th, 2025 9:30am

Instructions

In this project, you are expected to perform state estimation for a spacecraft in orbit around the Earth. You may work with one partner to complete the assignment.

Please provide a single PDF document with requested explanations, plots, and derivations, and include your Python code at the end for verification. Upload your work to Gradescope.

Overview

You are working at NASA GSFC on the navigation team for an Earth orbiting science satellite launched into orbit on a Falcon-9. SpaceX designed its launch to deliver the payload into low Earth orbit with reference coordinates of

$$\boldsymbol{X}_{\text{œ}} = \begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 7000 & \text{km} \\ 0.2 & \text{-} \\ 45 & \text{deg} \\ 0 & \text{deg} \\ 270 & \text{deg} \\ 78.75 & \text{deg} \end{bmatrix}$$

however it's the navigation's team responsibility to verify this orbit and/or determine any errors in its delivery. To accomplish this, the navigation team has access to range and range-rate measurements from the spacecraft provided by the Deep Space Network (DSN). The latitude and longitude (ϕ, λ) of the three DSN ground stations are provided below:

- DSN # 0: Goldstone, USA: (35.297°, -116.914°)
- DSN # 1: Madrid, Spain: (40.4311°, -4.248°)
- DSN # 2: Canberra, Australia: (-35.4023°, 148.9813°)

These ground stations are positioned on a spherical Earth of radius 6378.137 km, with a rotation rate of $\omega_{\mathcal{E}/\mathcal{N}} = 7.292115 \times 10^{-5}$ rad/s and a Local Sidereal Time of $\gamma_0 = 0°$.

`Project-Measurements-Easy.npy` contains the range ρ and range-rate $\dot{\rho}$ measurements alongside additional information. Specifically, each row in the datafile is formatted as:

$$[t, i, \rho, \dot{\rho}]$$

where t corresponds with the time the measurement was received and i corresponds to the ground station index (as labeled above).

As best as you are aware, the spacecraft's motion is governed by the following differential equation:

$$\ddot{\boldsymbol{r}} = -\frac{\boldsymbol{\mu}}{r^3} \cdot \boldsymbol{r} \tag{1}$$

where $\boldsymbol{r}$ is the position of the spacecraft in the inertial frame. Similarly, the measurements provided can be computed using the following equations:

$$\boldsymbol{\rho} = \boldsymbol{r} - \boldsymbol{R}_{\text{site,i}} \tag{2}$$

$$\rho = \|\boldsymbol{r} - \boldsymbol{R}_{\text{site,i}}\| \tag{3}$$

$$\dot{\rho} = \frac{\boldsymbol{\rho} \cdot (\dot{\boldsymbol{r}} - \dot{\boldsymbol{R}}_{\text{site,i}})}{\rho} \tag{4}$$

$$\dot{\boldsymbol{R}}_{\text{site,i}} = \boldsymbol{\omega}_{\mathcal{E}/\mathcal{N}} \times \boldsymbol{R}_{\text{site,i}} \tag{5}$$

where $\boldsymbol{R}_{\text{site,i}}$ is the location of the DSN station also in the inertial frame. Note that the measurements provided by the DSN have some intrinsic noise which can be modeled as Gaussian white noise. Explicitly, the noise in the range is characterized by a variance of 1 m^2, and by $1 \text{ cm}^2/\text{s}^2$ variance in the range-rate.

Using your knowledge of the spacecraft's dynamics and the measurements provided by the DSN, generate a report which culminates in an estimate of the spacecraft's state over time. To help facilitate your progress, please generate your report in the following order, answering the following intermediate questions:

Part 1. Problem Setup

a) Express the non-linear system in continuous time state-space form, clearly defining the vectors $\boldsymbol{f}(X(t))$ and $\boldsymbol{h}(X(t))$.

b) Define the linearized dynamics and measurement matrices $A(t)$ and $C(t)$.

c) Show how these matrices are converted to their discrete time forms F_k and H_k. Recall F_k is the state transition matrix $\Phi(t_j, t_i)$ which requires integration.

d) Define your noise matrices Q_k and R_k, and discuss their relationship to the aforementioned system of equations.

e) Plot the measurements as a function of time.

Part 2. Plan Filter Implementation

a) Provide pseudocode from which you will base your extended Kalman filter implementation. Highlight the major steps in your algorithm and any noteworthy modifications or subtle details required for this problem that you want the graders to be aware of. Be comprehensive, as this is what the grading team will primarily reference if the results / plots don't look quite right.

Part 3. Pure Prediction

a) Specify your choice of $\boldsymbol{x}_0$, P_0, Q_0, and R_0. Explain your reasoning. (For $\boldsymbol{x}_0$, express in cartesian coordinates.)

b) Implement just the prediction step of the extended Kalman Filter (i.e. do not perform measurement updates)

c) Plot the covariance of the state estimate as a function of time, centered about a mean of zero — i.e. $\pm 3\sigma$ bounds taken from $P_{\hat{\boldsymbol{x}},k}^-(t)$. Choose appropriate y-limits for your plot to maximize readability. Explain what you see and if it makes sense.

Part 4. Measurement Updates

a) Implement the measurement update step of the extended Kalman Filter.

b) Plot the pre- and post-measurement update $\pm 3\sigma$ bounds on the same plot. Explain any differences.

c) Plot the difference between the pre- and post-measurement update state estimates, $\boldsymbol{\mu}_{\hat{\boldsymbol{x}},k}^{+}(t_k) - \boldsymbol{\mu}_{\hat{\boldsymbol{x}},k}^{-}(t_k)$, inside of the pre-measurement covariance $\pm 3\sigma$ bounds, $P_{\hat{\boldsymbol{x}},k}^{+}(t_k)$.

Part 5. Filter Solutions

a) With your EKF implemented, run the simulation through to completion, and compute the post-fit measurement residuals as a function of time via:

$$\delta \boldsymbol{Y}(t_k) = \boldsymbol{Y}(t_k) - h(\boldsymbol{\mu}_{\hat{\boldsymbol{x}},k}^{+}(t_k))$$

b) Do your measurement residuals give you confidence that you estimated the state correctly? Why or why not?

c) Plot the estimated state $\boldsymbol{\mu}_{\hat{\boldsymbol{x}},k}^{+}(t)$ and its associated $\pm 3\sigma$ bounds taken from $P_{\hat{\boldsymbol{x}},k}^{+}(t)$ as a function of time.

d) Report your best state estimate at the final time-step and its associated uncertainty.

Part 6. (Optional) Debugging Efforts

Use this section to outline any of your debugging efforts for if things aren't going your way. This is a good place to earn some partial credit. This should be a "research" log of what experiments you performed and why. A list of guiding questions if your stuck include:

1. Consider how process noise matrix is used in the filter. What happens if there are large gaps between measurements?

2. Consider if the values used in your measurement noise matrix are appropriate. Should these values only reflect the uncertainty in the sensor?

3. If your filter is diverging, does the divergence start at the beginning or mid-way through? What possible reasons exist for either outcome?

4. If you had to define a single scalar metric to evaluate your filter's quality, what would it be, and can you use this to help you determine optimal tuning values?

5. Does plotting your best estimate in a different reference frame or element description help?

Part 7. (Bonus) Challenge Orbit

a) Perform a second analysis with the more difficult dataset `Project-Measurements-Hard.npy`. Use

$$\begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 7000 & \text{km} \\ 0.6 & - \\ 45 & \text{deg} \\ 180 & \text{deg} \\ 0 & \text{deg} \\ 45 & \text{deg} \end{bmatrix}$$

as your initial guess. Your analysis can amount to your debugging process, and points will be awarded based on how thoughtful your experimentation is and the quality of your solution.