ENAE 441 - 0101
Final Project: Extended Kalman Filter

Due on December 16th, 2025 at 12:30 PM

Dr. Martin, 09:30 AM

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Problem 1: Problem Setup

a. Express the non-linear system in continuous time state-space form, clearly defining the vectors $\boldsymbol{f}\left(X\left(t\right)\right)$ and $\boldsymbol{h}\left(X\left(t\right)\right)$

b. Define the linearized dynamics and measurement matrices $A\left(t\right)$ and $C\left(t\right)$.

c. Show how these matrices are converted to their discrete time forms F_k and H_k. Recall F_k is the state transition matrix $\boldsymbol{\Phi}\left(t_j, t_i\right)$ which requires integration.

d. Define your noise matrices Q_k and R_k, and discuss their relationship to the aforementioned system of equations.

e. Plot the measurements as a function of time.

Solution

Part A

Answer

Part B

Answer

Part C

Answer

Part D

Answer

Part E

Answer

Problem 2: Plan Filter Implementation

Provide pseudocode from which you will base your extended Kalman filter implementation. Highlight the major steps in your algorithm and any noteworthy modifications or subtle details required for this problem that you want the grades to be aware of. Be comprehensive, as this is what the grading team will primarily reference if the results/plots don't quite look right.

Solution

Answer

Problem 3: Pure Prediction

a.

b.

c.

Solution

Code

See the Python code for this assignment.

Problem 4: Measurement Updates

a.

b.

c.

Solution

Code

See the Python code for this assignment.

Problem 5: Filter Solutions

a.

b.

c.

d.

Solution

Code

See the Python code for this assignment.

Problem 6: Debugging Efforts *(Optional)*

Use this section to outline any of your debugging efforts for if things aren't going your way. This is a good place to earn some partial credit. This should be a "research" log of what experiments you performed and why. A list of guiding questions if your stuck include:

1. Consider how process noise matrix is used in the filter. What happens if there are large gaps between measurements?

2. Consider if the values used in your measurement noise matrix are appropriate. Should these values only reflect the uncertainty in the sensor?

3. If your filter is diverging, does the divergence start at the beginning or mid-way through? What possible reasons exist for either outcome?

4. If you had to define a single scalar metric to evaluate your filter's quality, what would it be, and can you use this to help you determine optimal tuning values?

5. Does plotting your best estimate in a different reference frame or element description help?

Solution

Answer

Problem 7: Challenge Orbit *(Bonus)*

Perform a second analysis with the more difficult dataset `Project-Measurements-Hard.npy`. Use

$$\begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 7000\,\text{km} \\ 0.6 \\ 45° \\ 180° \\ 0° \\ 45° \end{bmatrix}$$

as your initial guess. Your analysis can amount to your debugging process, and points will be awarded based on how thoughtful your experimentation is and the quality of your solution.

Solution

Answer

Code

See the Python code for this assignment.

Code

```python
import numpy as np
import matplotlib.pyplot as plt
import os


def main():
    print("Hello World!")

    #######
    # p02 #
    #######

    x = np.linspace(0, 10, 100)
    y = np.sin(x)

    with open("./outputs/text/s02.txt", "w", encoding="utf-8") as f:
        f.write((f"x:\n{x}\n\ny:\n{y}"))

    plt.figure()
    plt.plot(x, y)

    save_path = os.path.join(os.path.dirname(__file__), "./outputs/figures/s02.png")
    plt.savefig(save_path)
    plt.show()


if __name__ == "__main__":
    main()
```