ENAE 441 - 0101

HW02: Reference Frames and Ground Tracks

Due on October 11th. 2025 at 11:59 PM

Dr. Martin, 09:30 AM

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October 8, 2025

Problem 1: Reference Frame Conversions

Program the transformations between the following reference frames:

1. Perifocal → ECI
2. ECI → ECEF
3. ECEF → Topocentric

Solution

Part A

Answer

Part B

Answer

Code

See the Python code for this assignment.

Problem 2: Orbit in Different Reference Frames

Given the following orbital elements of a satellite:

$$\boldsymbol{X}(t_0)_\infty = \begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 7 \times 10^3\,\mathrm{km} \\ 0.05 \\ 45^\circ \\ 30^\circ \\ 60^\circ \\ 0^\circ \end{bmatrix}$$

the gravitational parameter of the Earth $\mu = 3.986 \times 10^5\,\frac{\mathrm{km}^3}{\mathrm{s}^2}$, and its rotation rate $\omega_{\mathcal{E}/\mathcal{N}} = 7.2911 \times 10^{-5}\,\frac{\mathrm{rad}}{\mathrm{s}}$,

1. Plot the trajectory for 24 hours in the following reference frames:
   (a) ECI
   (b) Perifocal
   (c) ECEF

   Make sure to label the axes and title each plot.
2. Compare the characteristics of the orbit in the different reference frames. Briefly discuss the advantages and limitations of using each frame to represent the satellite's orbit.

Solution

Part A

Answer

Part B

Answer

Code

See the Python code for this assignment.

Problem 3: Ground Tracks of Different Orbits

For each of the following four spacecraft / orbital element sets:

$$\boldsymbol{X}_1 = \begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 6.789 \times 10^3\,\text{km} \\ 0.007 \\ 51.6^\circ \\ 0^\circ \\ 215^\circ \\ 0^\circ \end{bmatrix};$$

$$\boldsymbol{X}_2 = \begin{bmatrix} 26.56 \times 10^3\,\text{km} \\ 0.02 \\ 55^\circ \\ 0^\circ \\ 215^\circ \\ 0^\circ \end{bmatrix}; \quad \boldsymbol{X}_3 = \begin{bmatrix} 26.6 \times 10^3\,\text{km} \\ 0.74 \\ 63.4^\circ \\ 270^\circ \\ 80^\circ \\ 0^\circ \end{bmatrix}; \quad \boldsymbol{X}_4 = \begin{bmatrix} 42.164 \times 10^3\,\text{km} \\ 0.02 \\ 0^\circ \\ 0^\circ \\ 35^\circ \\ 0^\circ \end{bmatrix}$$

1. Propagate the orbit for three periods and plot its ground track.
2. Using the ground tracks, identify where the spacecraft is closest to the Earth? A general geographic region is sufficient. Explain.
3. Identify a potential use case for the specific orbit you've just plotted. Why might this particular ground track be advantageous?

Solution

Part A

Answer

Part B

Answer

Part C

Answer

Code

See the Python code for this assignment.

Problem 4: Measurements from the Deep Space Network Stations

Place an observer at the Goldstone Deep Space Network (DSN) location's latitude and longitude $(\phi, \lambda) = (35.2967°, -116.9141°)$. Propagate each spacecraft's trajectory from Problem 2 for one orbit:

1. Compute the azimuth and elevation of each spacecraft with respect to the observer. Plot these angular measurements in a polar plot.
2. Compute the range to the spacecraft, and plot the range as a function of time. Be sure to mask any measurements generated when the spacecraft's elevation falls below $10°$.
3. Interpret the above plots. Which spacecraft are visible to the station at some point along their orbit? Explain.

Solution

Part A

Answer

Part B

Answer

Part C

Answer

Code

See the Python code for this assignment.