ENAE 441 - 0101

HW02: Reference Frames and Ground Tracks

Due on October 11th. 2025 at 11:59 PM

Dr. Martin, 09:30 AM

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October 16, 2025

Problem 1: Reference Frame Conversions

Program the transformations between the following reference frames:

(a) Perifocal → ECI

(b) ECI → ECEF

(c) ECEF → Topocentric

Solution

Part A

```
[ 843.39611889 6163.06502372 2351.13004745   -6.19057628   -1.00614988    4.85812138]
```

Part B

```
[ 847.88944677 6162.44845736 2351.130047     -6.19130795   -1.00163612    4.858121  ]
```

Part C

```
[-2033.40663814  5315.73998516 -9817.57349847]
```

Code

See the Python code for this assignment.

Problem 2: Orbit in Different Reference Frames

Given the following orbital elements of a satellite:

$$\boldsymbol{X}\left(t_0\right)_\infty = \begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 7\times 10^3\,\text{km} \\ 0.05 \\ 45^\circ \\ 30^\circ \\ 60^\circ \\ 0^\circ \end{bmatrix}$$

the gravitational parameter of the Earth $\mu = 3.986 \times 10^5\ \frac{\text{km}^3}{\text{s}^2}$, and its rotation rate $\omega_{\mathcal{E}/\mathcal{N}} = 7.2911 \times 10^{-5}\ \frac{\text{rad}}{\text{s}}$,

(a) Plot the trajectory for 24 hours in the following reference frames:

  (1) ECI

  (2) Perifocal

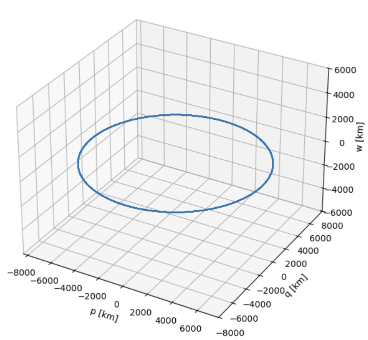
  (3) ECEF

  Make sure to label the axes and title each plot.

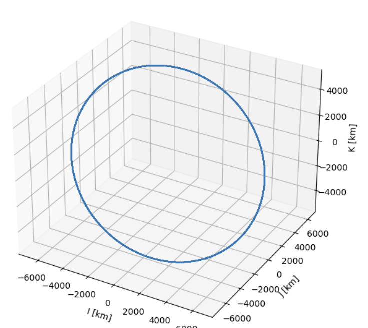
(b) Compare the characteristics of the orbit in the different reference frames. Briefly discuss the advantages and limitations of using each frame to represent the satellite's orbit.

Solution

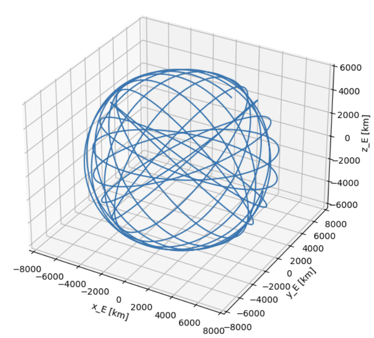
Part A



(a) Trajectory in Perifocal Frame



(b) Trajectory in ECI Frame



(c) Trajectory in ECEF Frame

Part B

```
Perifocal: orbit is a fixed ellipse in the spacecraft orbital plane
    Great for two-body analytics and interpreting anomalies (θ), but has no Earth context.
ECI: ellipse is fixed in inertial space (no Earth rotation)
    Good for multi-body perturbations and inter-frame transforms; longitude/latitude are not obvious.
ECEF: Earth-fixed axes rotate with the planet; the trajectory appears to sweep over the rotating Earth
```

Problem 2 continued on next page...

```
    This is the natural frame for **ground tracks**, access windows, and station visibility,
    ↪  but motion
    mixes orbital dynamics with Earth rotation.
```

Code

See the Python code for this assignment.

Problem 3: Ground Tracks of Different Orbits

For each of the following four spacecraft / orbital element sets:

$$\boldsymbol{X}_1 = \begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 6.789 \times 10^3 \,\text{km} \\ 0.007 \\ 51.6^\circ \\ 0^\circ \\ 215^\circ \\ 0^\circ \end{bmatrix};$$

$$\boldsymbol{X}_2 = \begin{bmatrix} 26.56 \times 10^3 \,\text{km} \\ 0.02 \\ 55^\circ \\ 0^\circ \\ 215^\circ \\ 0^\circ \end{bmatrix}; \quad \boldsymbol{X}_3 = \begin{bmatrix} 26.6 \times 10^3 \,\text{km} \\ 0.74 \\ 63.4^\circ \\ 270^\circ \\ 80^\circ \\ 0^\circ \end{bmatrix}; \quad \boldsymbol{X}_4 = \begin{bmatrix} 42.164 \times 10^3 \,\text{km} \\ 0.00 \\ 0^\circ \\ 0^\circ \\ 35^\circ \\ 0^\circ \end{bmatrix}$$

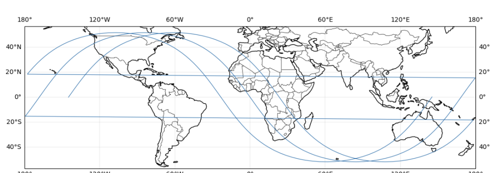
(a) Propagate the orbit for three periods and plot its ground track.

(b) Using the ground tracks, identify where the spacecraft is closest to the Earth? A general geographic region is sufficient. Explain.

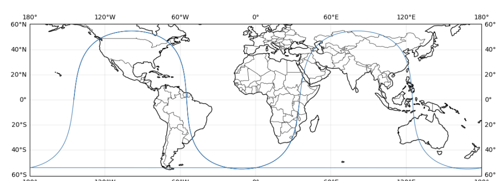
(c) Identify a potential use case for the specific orbit you've just plotted. Why might this particular ground track be advantageous?

Solution

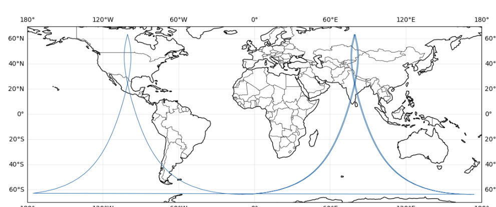
Part A



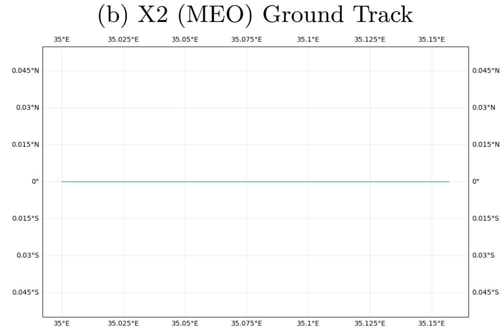
(a) X1 (LEO) Ground Track



(b) X2 (MEO) Ground Track



(c) X3 (Molniya) Ground Track



(d) X4 (GEO) Ground Track

Part B

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Closest approach to Earth occurs at perigee. Approximate subsatellite points (at t=0) are:
X1 (LEO): perigee SSP ≈ lat +0.0°, lon -145.0°
X2 (MEO): perigee SSP ≈ lat +0.0°, lon -145.0°
X3 (Molniya): perigee SSP ≈ lat -63.4°, lon -10.0°
X4 (GEO): perigee SSP ≈ lat +0.0°, lon +35.0°
```

Part C

```
X1 (LEO): human spaceflight / Earth observation; frequent revisits, moderate global coverage
X2 (MEO): GNSS constellation-style navigation; near 12-hour period gives repeating ground
↪ tracks
X3 (Molniya): long dwell over high northern latitudes—ideal for comms and ISR in high-lat
↪ regions
X4 (GEO): continuous coverage over a fixed longitude—telecom, weather, and broadcast
```

Code

See the Python code for this assignment.

Problem 4: Measurements from the Deep Space Network Stations

Place an observer at the Goldstone Deep Space Network (DSN) location's latitude and longitude $(\phi, \lambda) = (35.2967°, -116.9141°)$. Propagate each spacecraft's trajectory from Problem 2 for one orbit:

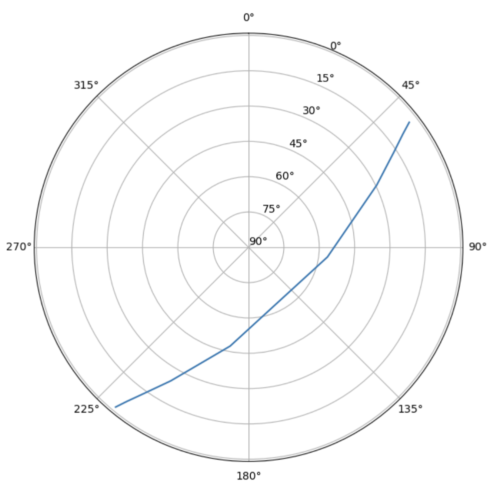
(a) Compute the azimuth and elevation of each spacecraft with respect to the observer. Plot these angular measurements in a polar plot.

(b) Compute the range to the spacecraft, and plot the range as a function of time. Be sure to mask any measurements generated when the spacecraft's elevation falls below 10°.

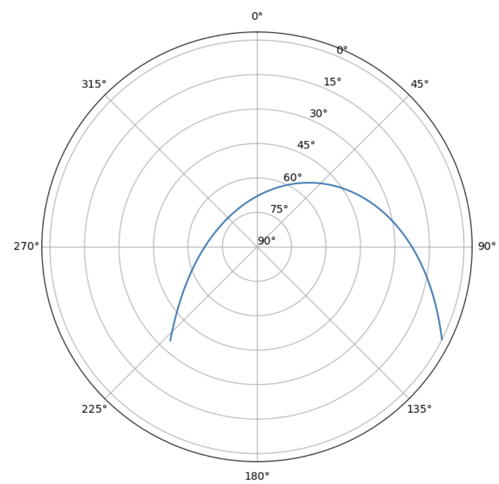
(c) Interpret the above plots. Which spacecraft are visible to the station at some point along their orbit? Explain.

Solution

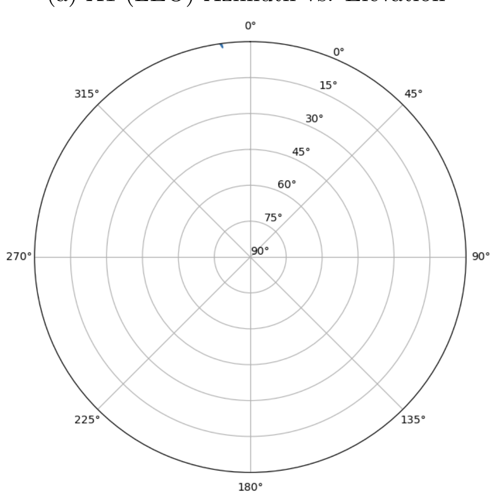
Part A



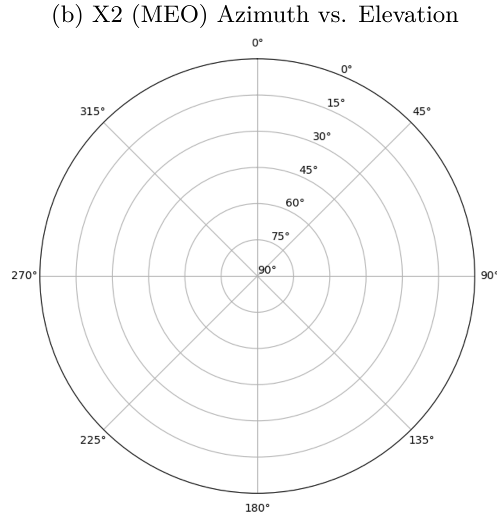
(a) X1 (LEO) Azimuth vs. Elevation



(b) X2 (MEO) Azimuth vs. Elevation



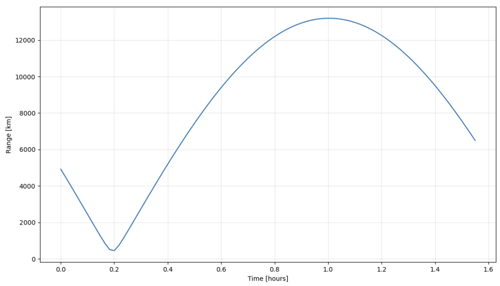
(c) X3 (Molniya) Azimuth vs. Elevation



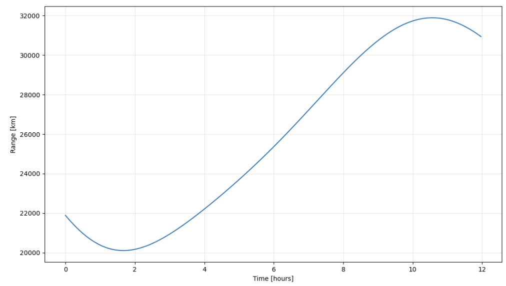
(d) X4 (GEO) Azimuth vs. Elevation

Problem 4 continued on next page...

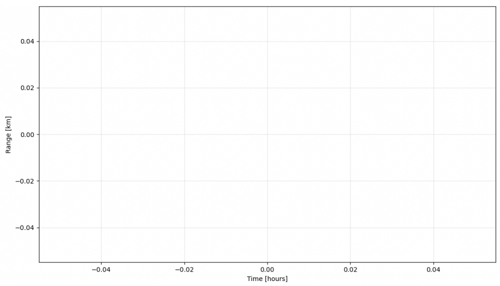
Part B



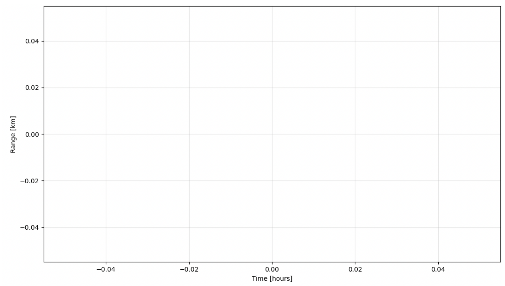
(a) X1 (LEO) Range vs. Time



(b) X2 (MEO) Range vs. Time



(c) X3 (Molniya) Range vs. Time



(d) X4 (GEO) Range vs. Time

Part C

```
X1 (LEO): VISIBLE during one orbit from Goldstone.
X2 (MEO): VISIBLE during one orbit from Goldstone.
X3 (Molniya): NOT VISIBLE during one orbit from Goldstone.
X4 (GEO): NOT VISIBLE during one orbit from Goldstone.
```

Code

See the Python code for this assignment.