Homework # 2

ENAE 441

Due Date: October 9th 9:30am

Instructions

In this homework, you will build on the tools developed in Homework 1 and incorporate the use of different reference frames to further analyze and visualize the orbit of a satellite. Please note visualization examples are provided on canvas to help with visualizing problems 2 and 3.

Your answers should be uploaded to gradescope in two files:

1. `{your_name}.pdf` — derivations and / or written explanations of results
2. `submission.py` — the python script for the autograder (must be named `submission.py`!)

Programming Questions

Each assignment has starter code available on ELMS. Your grade depends exclusively on the output of functions labeled `#REQUIRED`. Additional helper functions are included in the starter code to assist with debugging.

Questions

Question 1. (15 pts) Reference Frame Conversions

Program the transformations between the following reference frames:
a) Perifocal → ECI
b) ECI → ECEF
c) ECEF → Topocentric

Question 2. (20 pts) Orbit in Different Reference Frames

Given the following orbital elements of a satellite:

$$\boldsymbol{X}(t_0)_{œ} = \begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 7000 & \text{km} \\ 0.05 & [-] \\ 45^\circ & [\text{deg}] \\ 30^\circ & [\text{deg}] \\ 60^\circ & [\text{deg}] \\ 0^\circ & [\text{deg}] \end{bmatrix}$$

the gravitational parameter of the Earth $\mu = 398600\ \text{km}^3/\text{s}^2$, and its rotation rate $\omega_{\mathcal{E}/\mathcal{N}} = 7.2911 \times 10^{-5}$ rad/s,

a) plot the trajectory for **24 hours** in the following reference frames:
- Earth-Centered Inertial (ECI) frame
- Perifocal frame

- Earth-Centered Earth-Fixed (ECEF) frame

Make sure to label the axes and title each plot.

b) Compare the characteristics of the orbit in the different reference frames. Briefly discuss the advantages and limitations of using each frame to represent the satellite's orbit.

Question 3. (35 pts) Ground Tracks of Different Orbits

For each of the following four spacecraft / orbital element sets:

$$\boldsymbol{X}_1 = \begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 6,798 & \text{km} \\ 0.007 & [-] \\ 51.6^\circ & \\ 0^\circ & \\ 215^\circ & \\ 0^\circ & \end{bmatrix} ; \quad \boldsymbol{X}_2 = \begin{bmatrix} 26,560 \text{ km} \\ 0.02 \\ 55^\circ \\ 0^\circ \\ 215^\circ \\ 0^\circ \end{bmatrix} ; \quad \boldsymbol{X}_3 = \begin{bmatrix} 26,600 \text{ km} \\ 0.74 \\ 63.4^\circ \\ 270^\circ \\ 80^\circ \\ 0^\circ \end{bmatrix} ; \quad \boldsymbol{X}_4 = \begin{bmatrix} 42,164 \text{ km} \\ 0.02 \\ 0^\circ \\ 0^\circ \\ 35^\circ \\ 0^\circ \end{bmatrix}$$

a) Propagate the orbit for **three periods** and plot its ground track.

b) Using the ground tracks, identify where the spacecraft is closest to the Earth? A general geographic region is sufficient. Explain.

c) Identify a potential use case for the specific orbit you've just plotted. Why might this particular ground track be advantageous?

Question 4. (30 pts) Measurements from the Deep Space Network Stations

Place an observer at the Goldstone Deep Space Network (DSN) location's latitude and longitude $(\phi, \lambda) =$ (35.2967°, -116.9141°). Propagate each spacecraft's trajectory from Problem 2 for **one orbit**:

a) Compute the azimuth and elevation of each spacecraft with respect to the observer. Plot these angular measurements in a polar plot.

b) Compute the range to the spacecraft, and plot the range as a function of time. Be sure to mask any measurements generated when the spacecraft's elevation falls below 10°.

c) Interpret the above plots. Which spacecraft are visible to the station at some point along their orbit? Explain.