Homework # 1

ENAE 441

Due Date: September 25th 9:30am

Instructions

Please answer the following questions. Your answers should be uploaded to gradescope in two files:

1. `{your_name}.pdf` — derivations and / or written explanations of results
2. `submission.py` — the python script for the autograder (must be named `submission.py`!)

Programming Questions

Each assignment has starter code available on ELMS. Your grade depends exclusively on the output of functions labeled `#REQUIRED`. Additional helper functions are included in the starter code to assist with debugging.

Development Environment

If you want to ensure that your development environment matches that of the autograder (e.g. python version and packages match), then consider installing the same development environment in Visual Studio Code. This can be accomplished following the instructions on Canvas: `Files/VS Code Setup/install.pdf`

Questions

Question 1. (15 pts) The Eccentricity vector

a) Using the definition of the eccentricity vector $\boldsymbol{e}$ to prove that the total energy of the system, $\epsilon = \frac{\boldsymbol{v}\cdot\boldsymbol{v}}{2} - \frac{\mu}{r}$ is equal to

$$\epsilon = -\frac{\mu}{2a}$$

b) Using the expression $\epsilon = \frac{\boldsymbol{v}\cdot\boldsymbol{v}}{2} - \frac{\mu}{r}$, prove that $\frac{d\epsilon}{dt} = 0$

Question 2. (15 pts) Prove that

$$\ddot{\boldsymbol{r}} \times \boldsymbol{h} = \mu \frac{d}{dt}\left(\frac{\boldsymbol{r}}{r}\right)$$

Hint: Use polar coordinates for $\boldsymbol{r}$ and $\dot{\boldsymbol{r}}$.

Question 3. (40 pts) Orbital Element Calculations

Implement the following calculations in Python:

(a) Takes state vector $\boldsymbol{X}$ and converts to orbital elements, **œ**
(b) Takes vector of the orbital elements **œ** and converts to $\boldsymbol{X}$
(c) Takes ake a set of orbital elements, and calculates the orbital period.

To validate your conversions, you should be able to convert between these two state vectors:

$$^{\mathcal{N}}\boldsymbol{X}(t_0)_{\text{cart}} = \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} 594.193479 & \text{km} \\ -5881.90168 & \text{km} \\ -4579.29092 & \text{km} \\ 5.97301650 & \text{km/s} \\ 2.50988687 & \text{km/s} \\ -2.44880269 & \text{km/s} \end{bmatrix} \rightarrow \boldsymbol{X}(t_0)_{\text{œ}} = \begin{bmatrix} a \\ e \\ i \\ \omega \\ \Omega \\ \theta \end{bmatrix} = \begin{bmatrix} 6798.1366 & \text{km} \\ 0.1 & [-] \\ \pi/4 & [\text{rad}] \\ \pi/3 & [\text{rad}] \\ \pi/4 & [\text{rad}] \\ \pi & [\text{rad}] \end{bmatrix}$$

Assume that $\mu = 398600.4418$ km^3/s^2.

Question 4. (30 pts) Propagate and Visualize the Orbit.

Using SciPy's `solve_ivp` function that takes the cartesian state vector from Question 3, $\boldsymbol{X}(t_0)$, and integrate the orbit for one period using the following differential equation:

$$^{\mathcal{N}}\dot{\boldsymbol{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ -\mu x/r^3 \\ -\mu y/r^3 \\ -\mu z/r^3 \end{bmatrix}; \quad \boldsymbol{X} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

(a) Plot the resulting orbit in 2D and 3D. Be sure that all axes have the same limits.
(b) Calculate the orbital elements at each timestep, and plot them as a function of time.
(c) In a few sentences, explain what you see and if it makes sense.