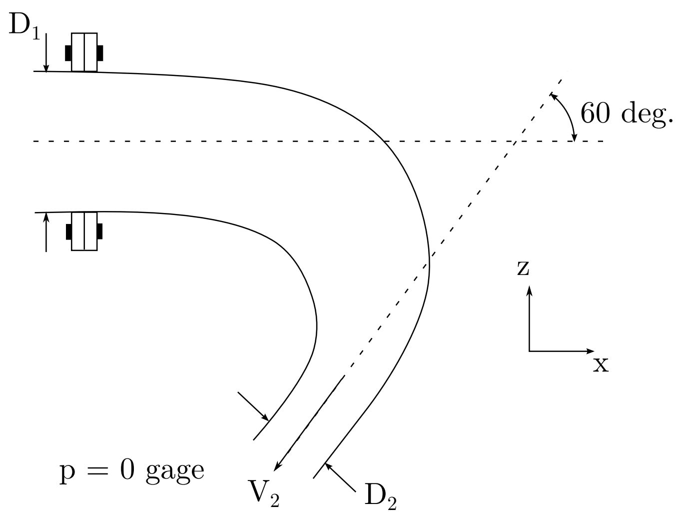
ENAE311H Homework 4

Due: Thursday, November 6th; online submission

1. The pipe bend below discharges water into the atmosphere. Determine the magnitude and direction of the external force components at the flange required to hold the bend in place. The bend lies in the horizontal plane. Assume the interior volume of the bend is $0.25\,\mathrm{m}^3$, D_1=60 cm, D_2=30 cm and V_2=10 m/s. The density of water is $1000\,\mathrm{kg/m}^3$.



2. Consider a flow field where the velocity components are as follows:

$$u = \frac{1}{y},\ v = -\frac{1}{x}. \tag{1}$$

Derive the equations of the streamlines of the flow field, and sketch them (also showing the direction of the flow). Is this flow field rotational or irrotational?

3. Consider a flow field where the velocity components are as follows:

$$u = \frac{x}{x^2+y^2},\ v = \frac{y}{x^2+y^2}. \tag{2}$$

Derive the equations of the streamlines of the flow field, and make a sketch of them (also showing the direction of the flow). Now rewrite the velocity components in cylindrical coordinates (u_r, u_θ) and calculate the vorticity. Note that the vorticity in a two-dimensional cylindrical coordinate system is given by:

$$\xi = \frac{1}{r}\frac{\partial(ru_\theta)}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta} \tag{3}$$

where

$$\begin{aligned} x &= r\cos\theta && (4)\\ y &= r\sin\theta && (5)\\ u_r &= u\cos\theta + v\sin\theta && (6)\\ u_\theta &= -u\sin\theta + v\cos\theta. && (7) \end{aligned}$$

Find the velocity potential in cylindrical coordinates and sketch equipotential lines on the streamline plot. What is the relationship between the streamlines and the equipotential lines?

4. a) In class we saw that for the flow along a streamline, pressure changes are related to velocity changes through the Euler's equation:

$$dp = -\rho V dV. \tag{8}$$

If we assumed ρ to be constant, we could integrate this directly to obtain the incompressible Bernoulli's equation. Let us now drop our assumption that ρ is constant, but assume that the flow along the streamline is *isentropic*. Using this assumption and starting from Euler's equation, derive the compressible Bernoulli's equation:

$$\frac{1}{2}V^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = const. \tag{9}$$

b) A Learjet is flying at close to 10 km altitude and is equipped with a combined Pitot/static probe. The measured Pitot and static pressure are 50.0 kPa and 26.4 kPa, respectively. A total temperature probe is also used to measure the temperature at the Pitot-probe stagnation point, and reads 267.7 K. Using the compressible Bernoulli's equation, what is the airspeed of the aircraft? What would the error be if you used the incompressible equation (i.e., assumed that the free-stream density were equal to the density at the stagnation point)? Assume γ=1.4.