University of Maryland at College Park

Dept. of Aerospace Engineering

ENAE 432: Aerospace Control Systems

Problem Set #7

Issued: 29 Mar. 2025
Due By: 4 Apr 2025

Question 1:

A dynamic system has the transfer function

$$G(s) = \frac{7}{(s/5+1)^3}$$

Note that this system is stable. To get it to accurately track commanded behaviors $y_d(t)$, we propose to use the proportional control law $u(t) = Ke(t)$ with $K > 0$.

a.) Do a Nyquist analysis for this system, showing that the closed-loop dynamics will be stable for small values of K, unstable for large values of K. Show a representative plot for each case, count the number of -1 encirclements, and apply the stability criterion. Use this analysis, together with the Bode diagrams of the system, to determine the maximal value of K for which the closed-loop system will be stable.

b.) Determine the value of K for which $L(s)$ will have a phase margin of $55°$. Determine the closed-loop poles which result from this choice of K. Estimate from the closed-loop pole locations the expected settling time and percent overshoot of the response for $y(t)$ when $y_d(t)$ is a unit step. Use Matlab to obtain an exact step response of $T(s)$ and quantify the relevant features. Comment on the exact response vs. your initial estimates, and discuss possible reasons for any discrepancies.

c.) Use Matlab to compute the input $u(t)$ generated by the controller during the step response in b.) Determine the maximal and steady-state values of $|u(t)|$ in this case.

d.) Use the properties of the sensitivity transfer function $S(s)$ to predict the steady-state tracking error which will be seen for the step response of the feedback loop in b.) If instead the feedback loop were asked to track $y_d(t) = A\sin(\omega t)$, for what range of $\omega > 0$ (if any) can you ensure that $|e_{ss}(t)| < A/2$?

Question 2:

The dynamics of a physical system are given by

$$\begin{aligned} \dot{x}_1 &= x_1 - x_2 + u \\ \dot{x}_2 &= -u \\ y &= 2x_1 \end{aligned}$$

This system is unstable (why?). We will attempt to stabilize it using the proportional control law $u(t) = Ke(t)$.

a.) Perform a Nyquist analysis to determine the range of K for which the closed-loop system will be stable. Show the Nyquist diagram and corresponding Bode plots you use to draw your conclusions.

b.) Choose a value for K so that the magnitude crossover frequency is 2 rad/sec. What is resulting the phase margin? Show how you calculated these results from the Bode diagrams.

c.) Generate a closed-loop step response using the value of K in b). What is the percentage overshoot and settling time?

d.) Suppose you double the gain from that found in b). How will the margins and crossovers change? Based on these changes, would you expect the settling time to increase or decrease? What about the overshoot? Explain your reasoning. Generate the new closed-loop step responses and comment on the actual changes. (To save space and facilitate comparison, you can overlay this on top of the step response from c).

Question 3:

Returning to our initially motivating hovercraft example, if we include the electromechanical dynamics of the propulsive fan used to move the vehicle, the motion of the hovercraft can be modeled using the transfer function

$$G(s) = \frac{12}{s^2(\tau s + 1)}$$

with $\tau > 0$.

a.) Use Nyquist to prove that it is *impossible* to stably control this system using a proportional control law $u(t) = Ke(t)$.

b.) Suppose instead that we use the "proportional+derivative" (PD) control law

$$u(t) = K_p e(t) + K_d \dot{e}(t)$$

where both K_p and K_d are positive. Prove that this control strategy can produce a stable closed-loop system, provided that $\tau K_p < K_d$. Sketch representative Nyquist plots for this system for gains which both do, and do not, satisfy this constraint (2 sketches) and apply the stability criterion in each case. HINT: where is the zero in $L(s)$ as compared to the pole at $-1/\tau$ in the two different cases?

c.) Suppose specifically that $\tau = 0.17$, and for simplicity we use initially $K_d = K_p$ in b). Find the value for K_p which will result in $L(s)$ having maximal phase margin. Determine the resulting closed-loop poles and verify that they are all stable.

d.) For a unit step $y_d(t)$, about how much overshoot would you expect to see in the response for $y(t)$ based on the poles of $T(s)$ in c)? Use Matlab to obtain the exact step response and label on this plot the exact overshoot and settling time. Notice that there is more overshoot than you might have predicted. Why?

e.) Suppose that you were dissatisfied with the achievable maximum phase margin in c), and wanted to increase it by 10°. How would you modify the controller gains? How would these changes affect the magnitude crossover frequency? Quantify how this redesign affects the step response transients in d).