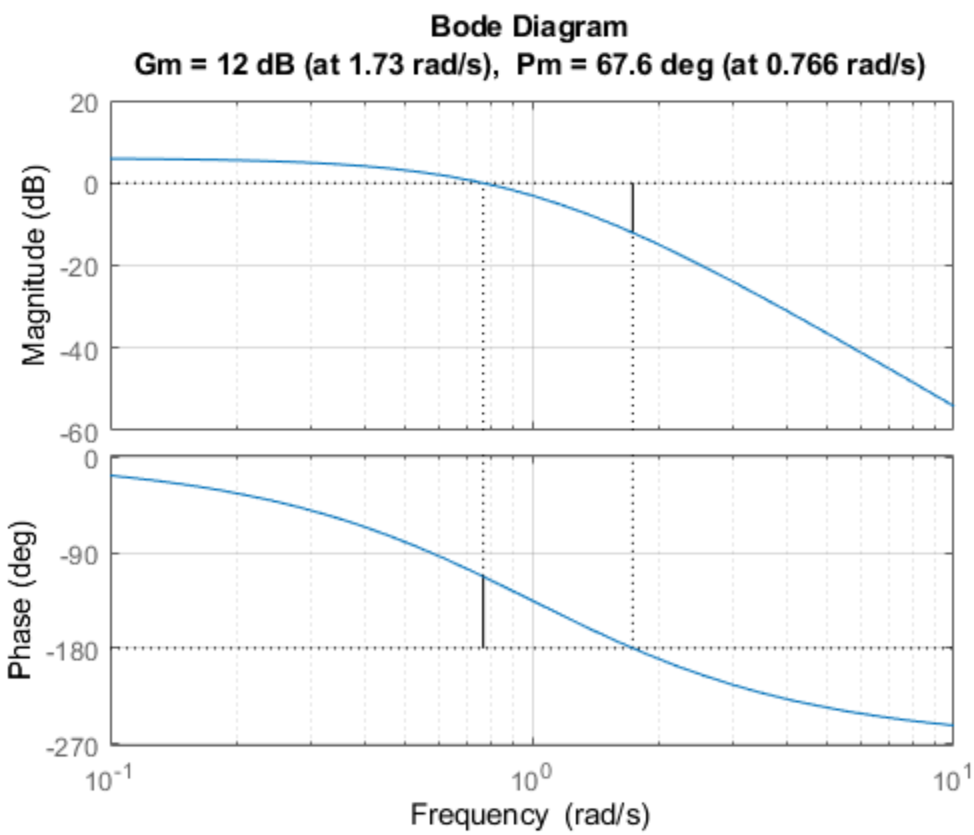
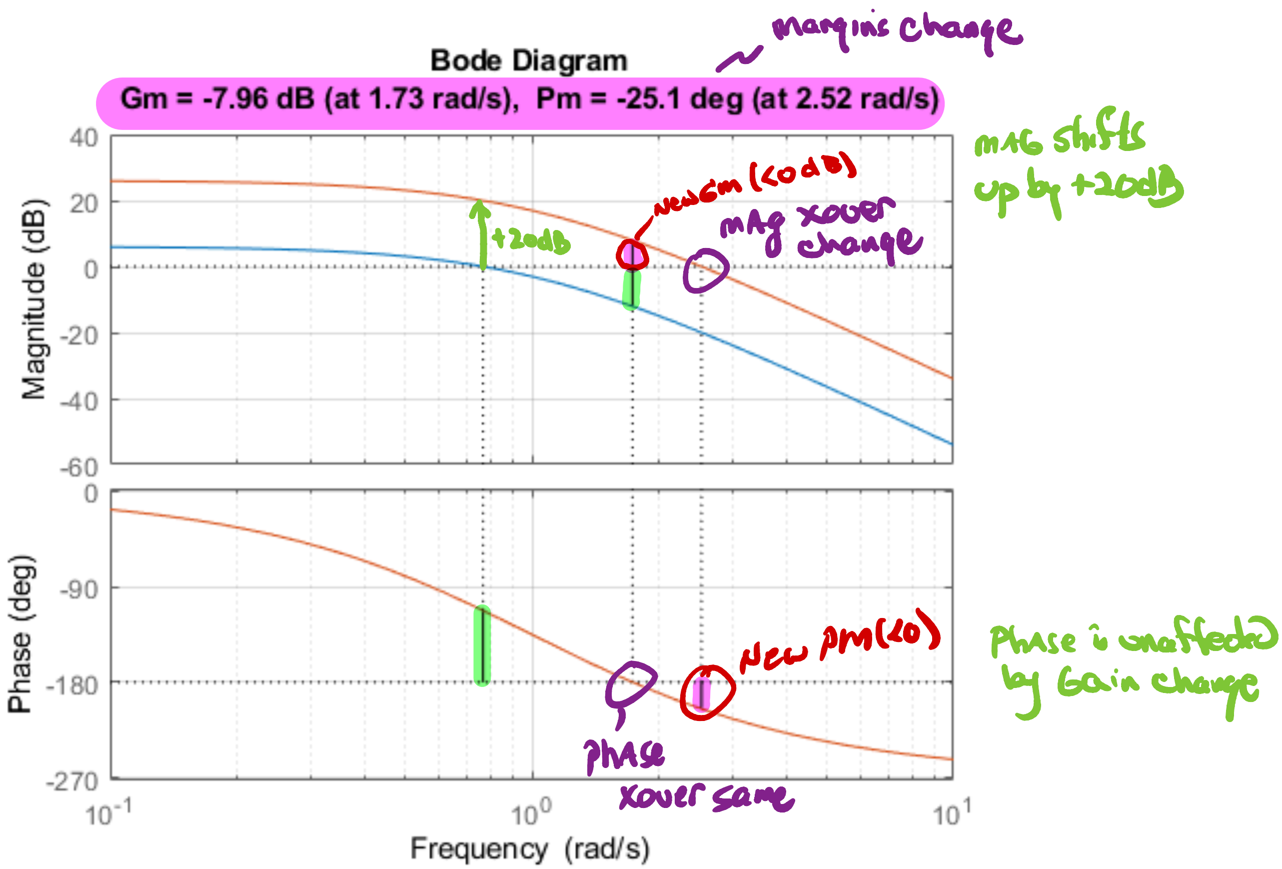
$$G(s) = \frac{2}{(s+1)^3}$$



Bode Diagram
Gm = 12 dB (at 1.73 rad/s), Pm = 67.6 deg (at 0.766 rad/s)
Magnitude (dB)
20
0
-20
-40
-60
Phase (deg)
0
-90
-180
-270
10^-1
10^0
10^1
Frequency (rad/s)

$$G'(s) = \frac{20}{(s+1)^3}$$ (K_B increased by factor of 10)



Poles at origin

Poles at origin will introduce a unique feature to a polar plot.

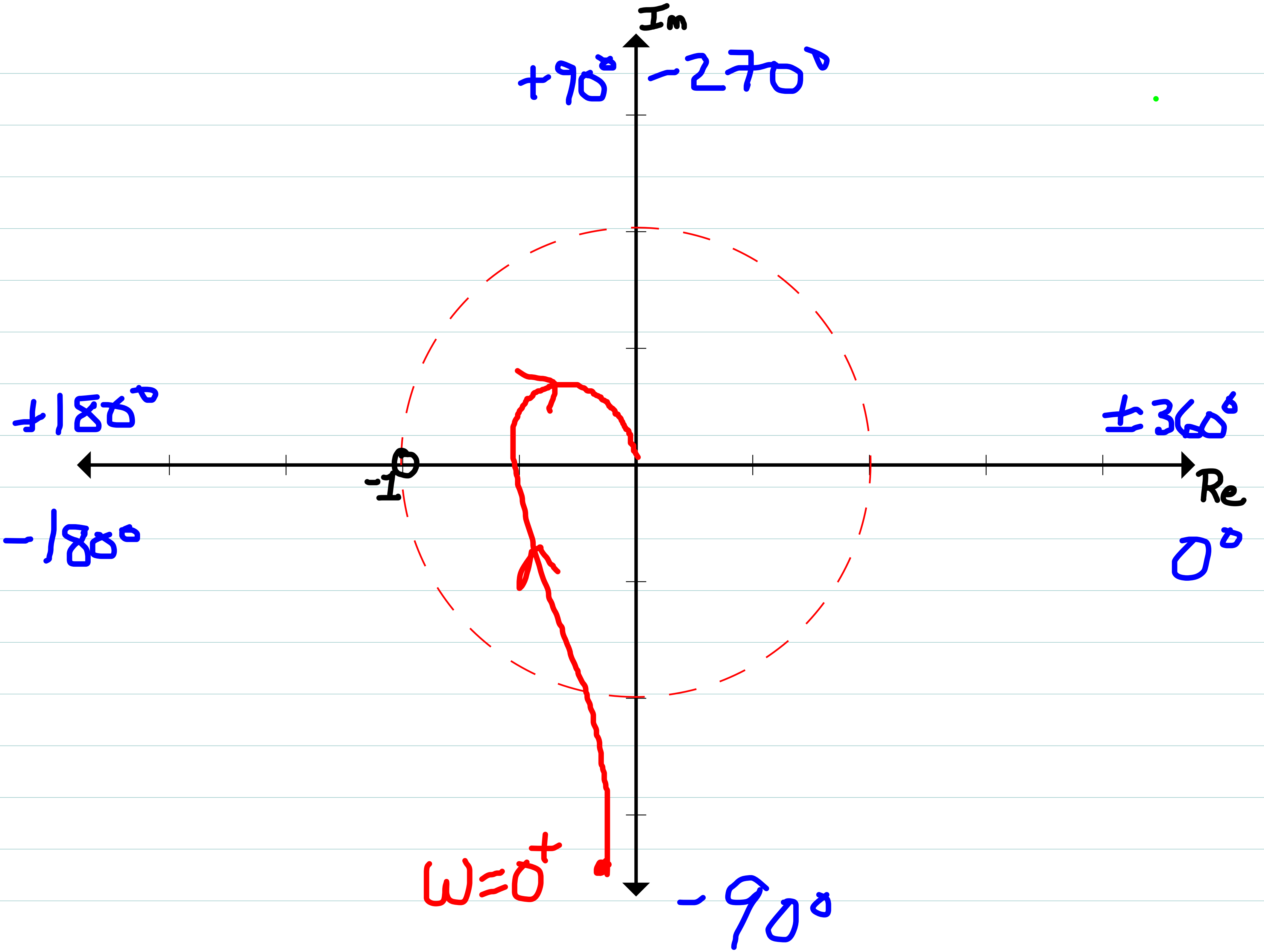
$$\lim_{\omega \to 0} \measuredangle G(j\omega) = \measuredangle K_B - N\,90^\circ$$

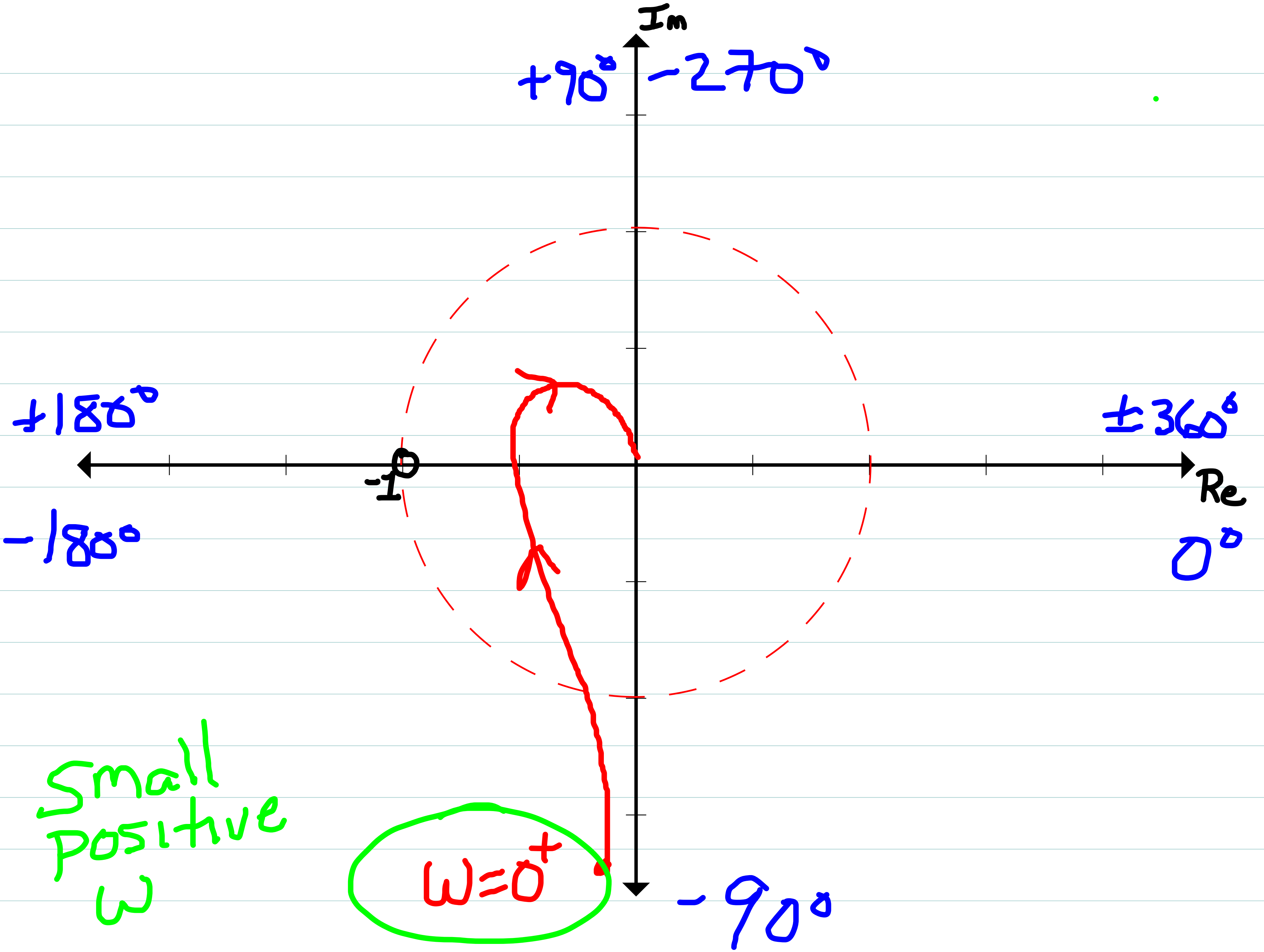
and $\lim_{\omega \to \emptyset} |G(j\omega)| = \infty$ in these cases

$\Rightarrow$ Polar plot will exhibit a "tail" along one of the coordinate axes.

Example:

$$G(s) = \frac{K_B}{s(Ts+1)^2} \qquad T, K_B > \emptyset$$


Im
+90°
-270°
+180°
±360°
-180°
0°
-1
Re
ω=0+
-90°



Im
+90°
-270°
+180°
-1
-180°
±360°
Re
0°
Small positive ω
ω=0+
-90°

Note: Which side of a coordinate axis the tail lies on is sometimes important.

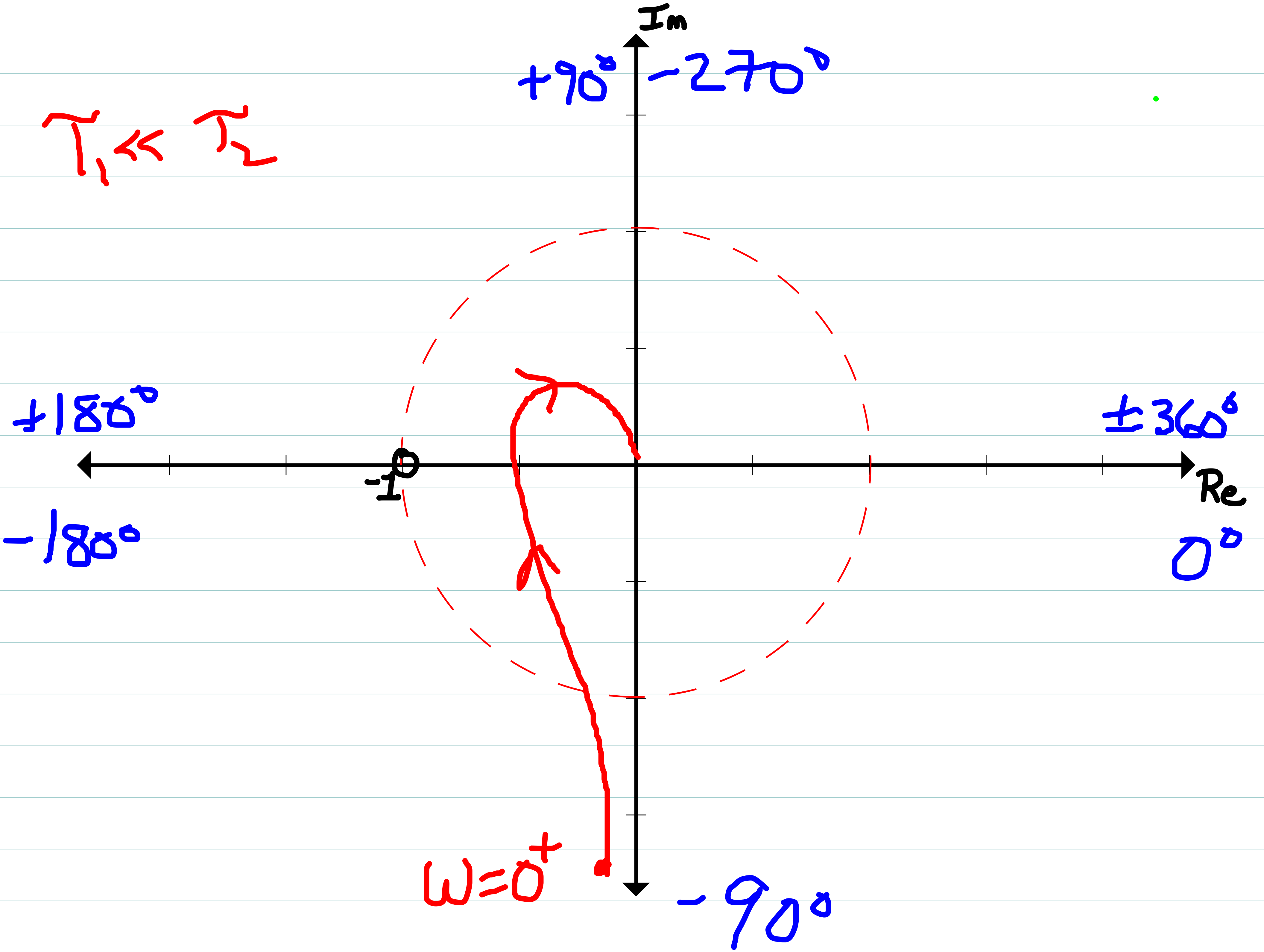
$\Rightarrow$ Determined by asymptotic behavior of phase as $\omega \to \emptyset$.

Example:

$$G(s) = K_B\left[\frac{(T_1 s+1)}{s(T_2 s+1)^3}\right]$$

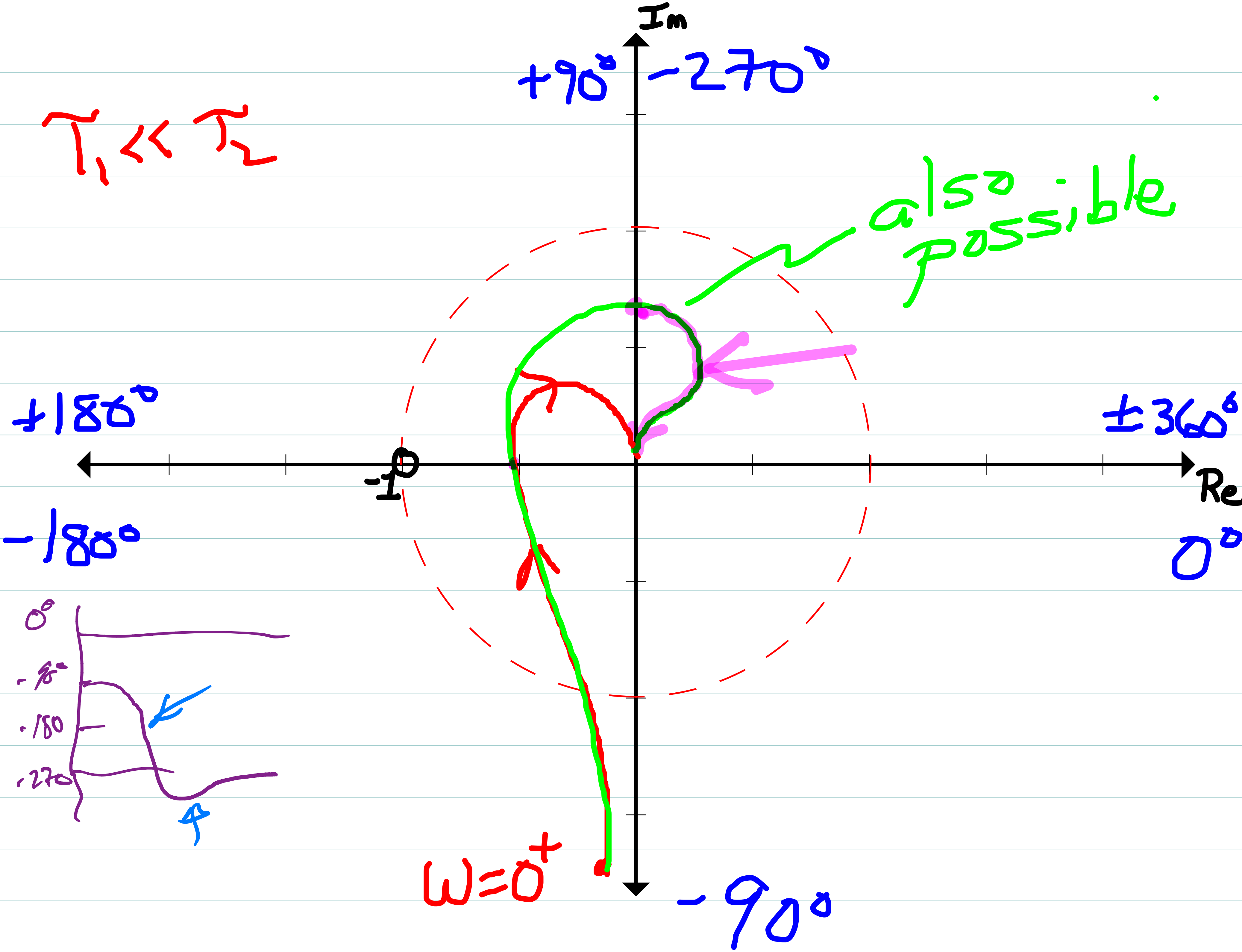
if $T_1 << T_2$ (so $\frac{1}{T_1} >> \frac{1}{T_2}$) then as $\omega \to \emptyset$ phase approaches $-90°$ *from below* (equivalently, phase is decreasing as ω increases from $\emptyset$).

Conversely, if $T_1 >> T_2$, phase approaches $-90°$ *from above* as $\omega \to 0$.

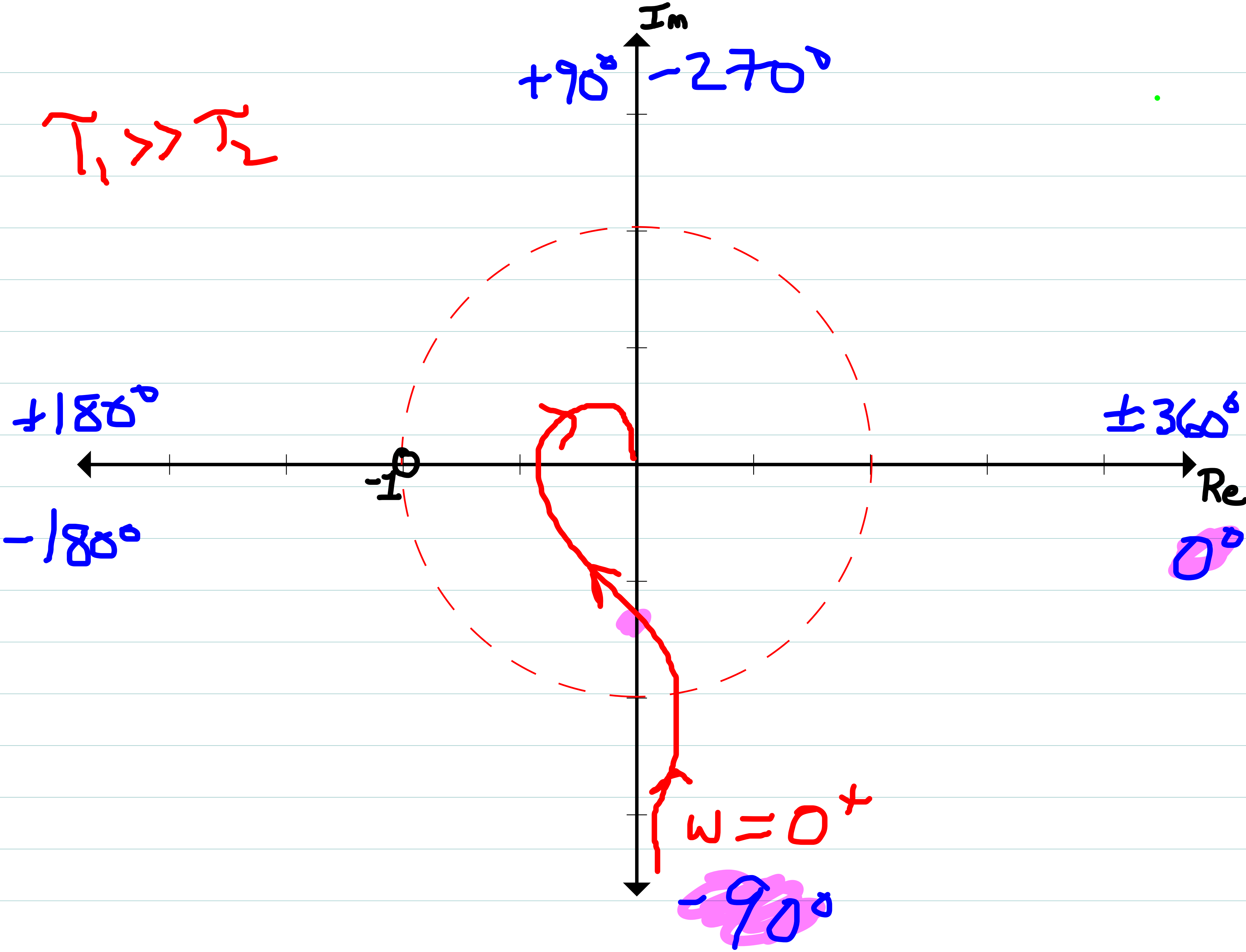


$T_1 \ll T_2$
Im
Re
$+90°$
$-270°$
$+180°$
$-180°$
$\pm 360°$
$0°$
-1
$-90°$
$\omega = 0^+$

$T_1 << T_2$



$\tau_1 >> \tau_2$

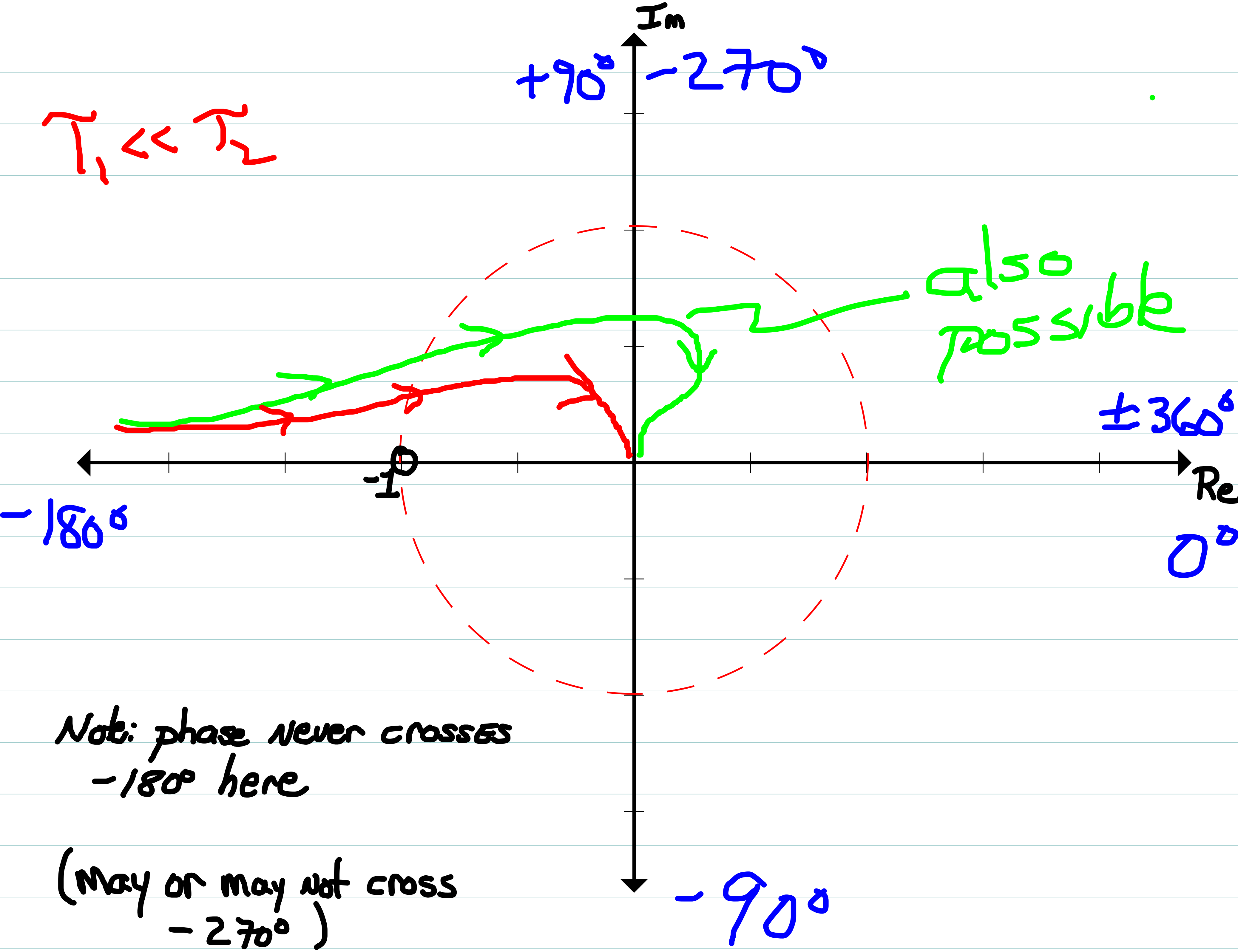


Add'l poles at origin change the coordinate axis the tail lies along.

Example:

$$G(s) = K_B \left[\frac{T_1 s + 1}{s^2 (T_2 s + 1)^2} \right]$$

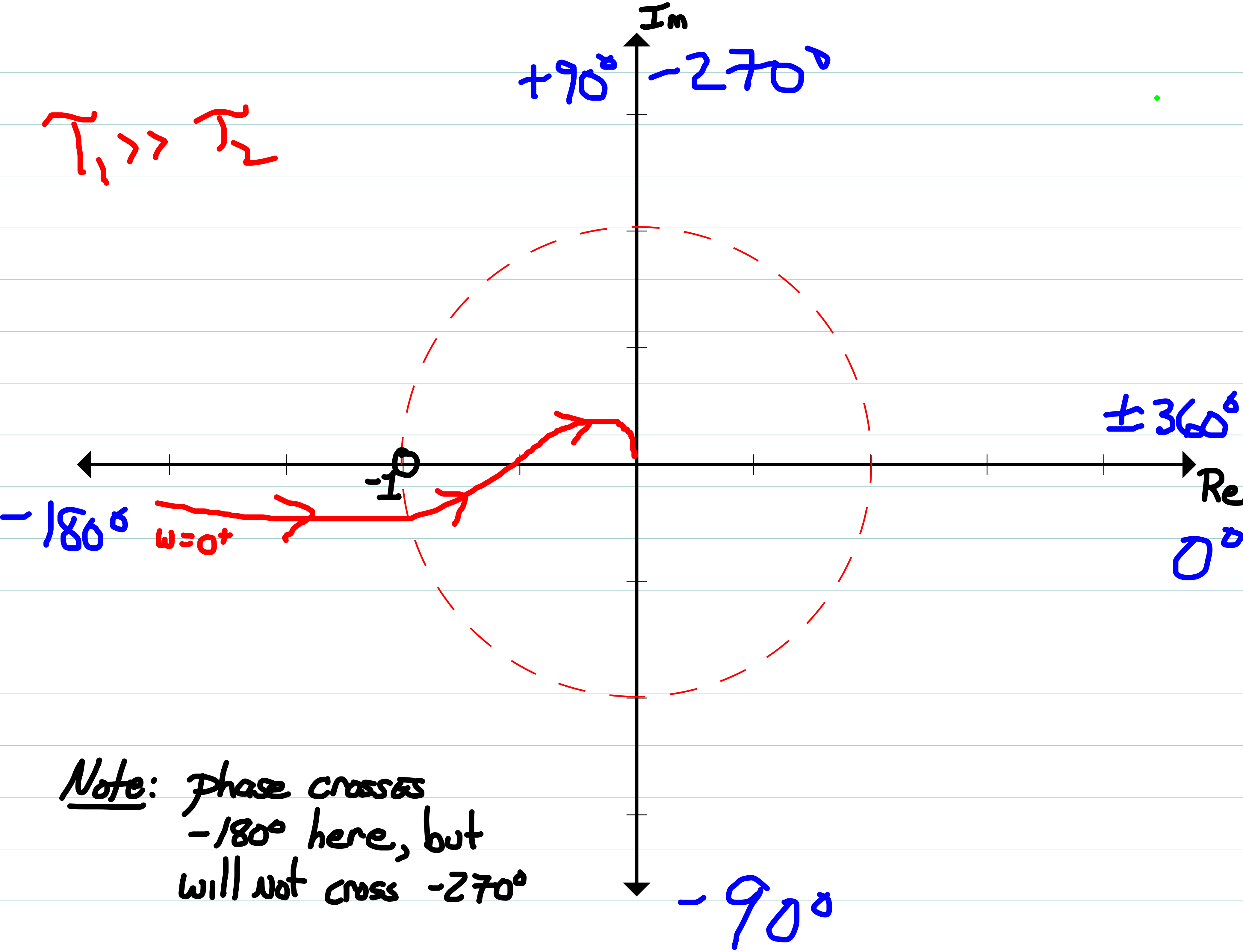
$T_1 << T_2$



Note: phase never crosses
-180° here

(may or may not cross
-270°)

$T_1 >> T_2$



<u>Note</u>: Phase crosses −180° here, but will not cross −270°

A more complicated example

$$G(s) = K_B\left[\frac{(T_2 s+1)^2}{s^2(T_1 s+1)(T_3 s+1)^2}\right]$$

With $T_1 \gg T_2 \gg T_3 > \phi$ $\left(\frac{1}{T_1} \ll \frac{1}{T_2} \ll \frac{1}{T_3}\right)$

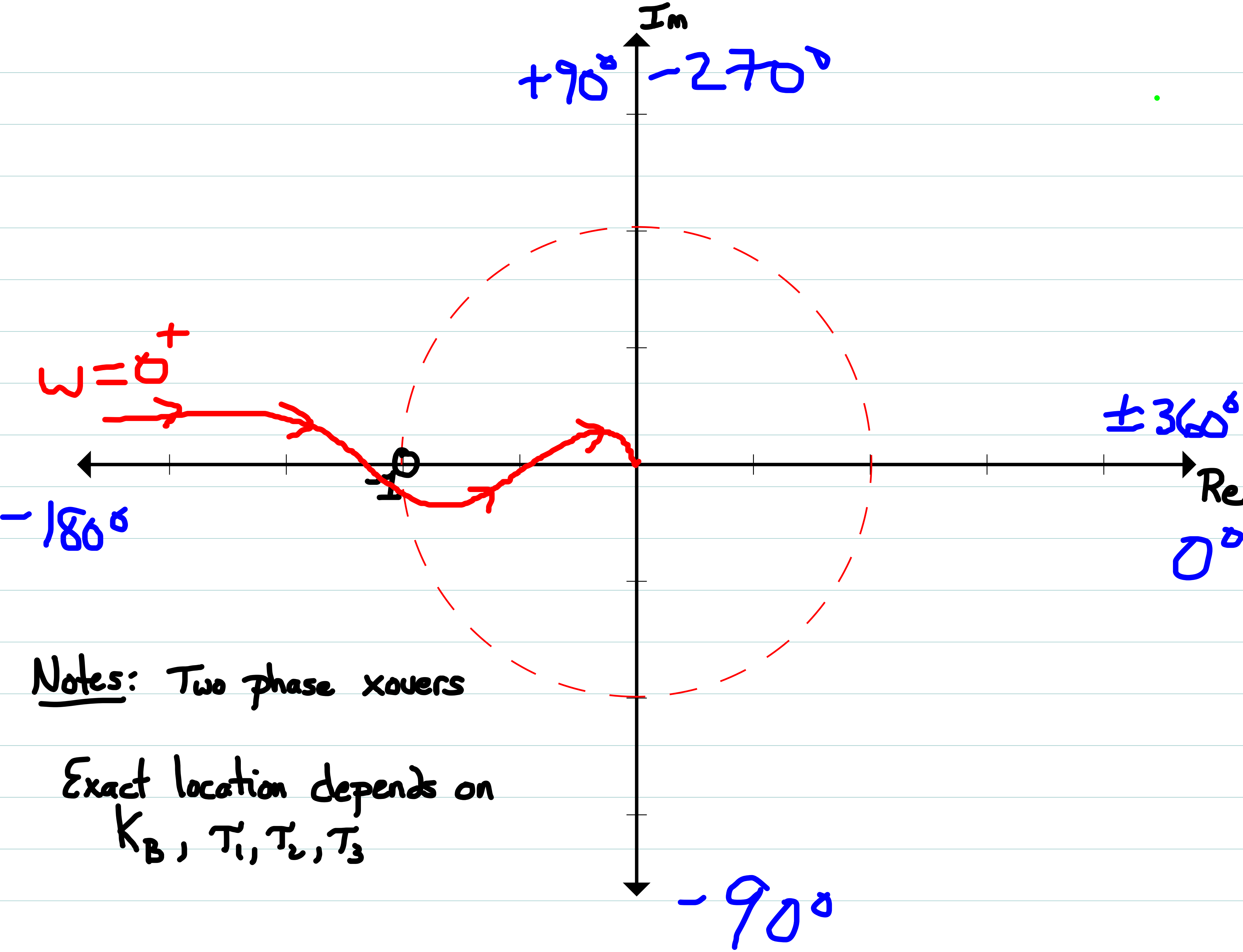
Low freq phase: $-180°$

high freq phase: $-270°$

phase initially decreases from pole at $-1/T_1$

Then increases due to double zero at $-1/T_2$

Then falls again due to double pole at $-1/T_3$



<u>Notes</u>: Two phase xovers

Exact location depends on K_B, T_1, T_2, T_3

Log magnitude - Angle Diagram (Nichols plot)

⇒ Plot $|G(j\omega)|_{dB}$ vs. $\measuredangle G(j\omega)$ as ω varies from $\emptyset$ to ∞

⇒ Angle in deg is horizontal Axis

⇒ Magnitude in dB is vertical Axis

⇒ Plot usually centered so "origin" corresponds to $-180°$ in phase, $\emptyset$ dB in magnitude

Log magnitude - Angle Diagram (Nichols plot)

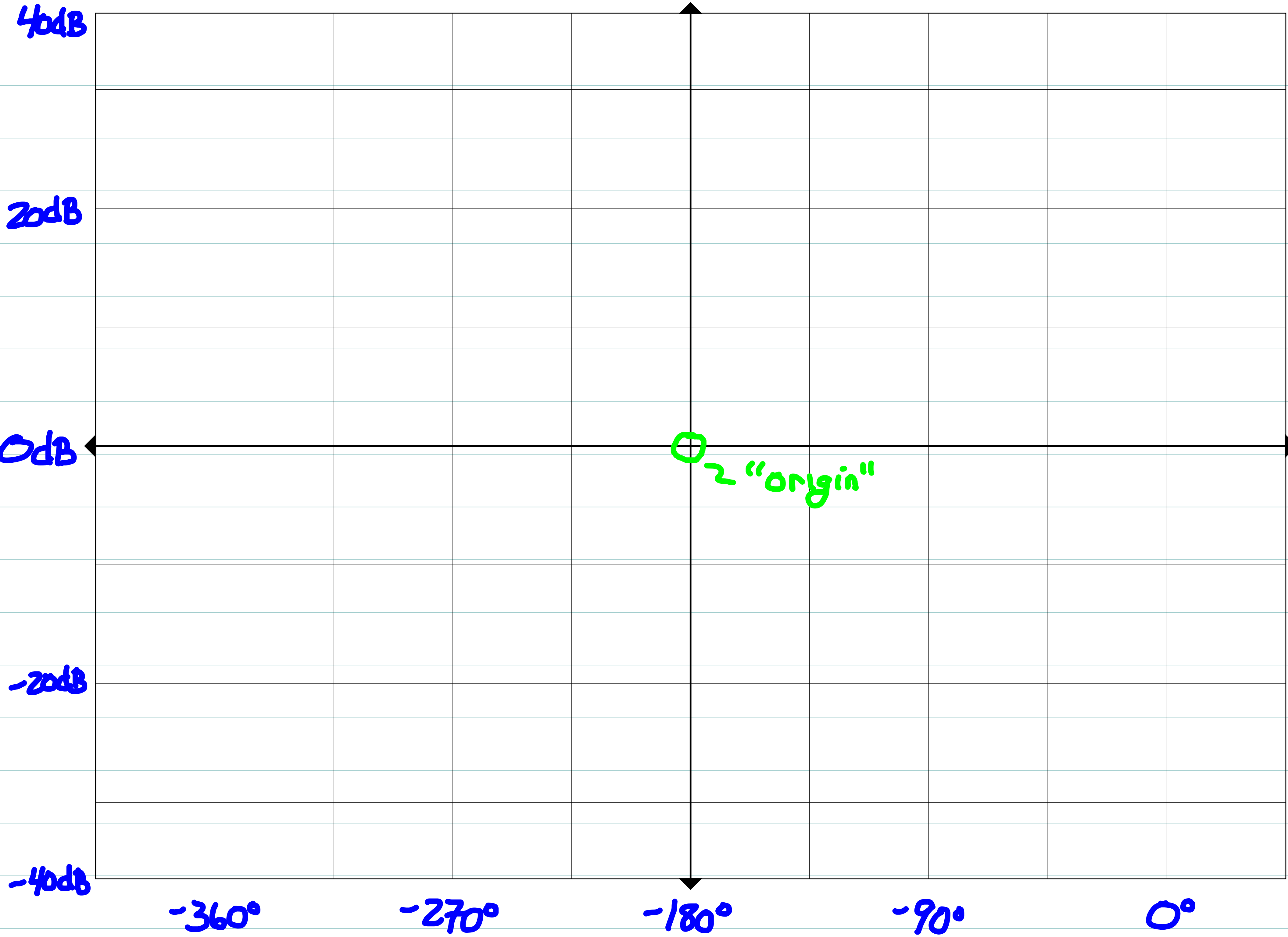
$\Rightarrow$ Plot $|G(j\omega)|_{dB}$ vs. $\measuredangle G(j\omega)$ as ω varies from ϕ to ∞

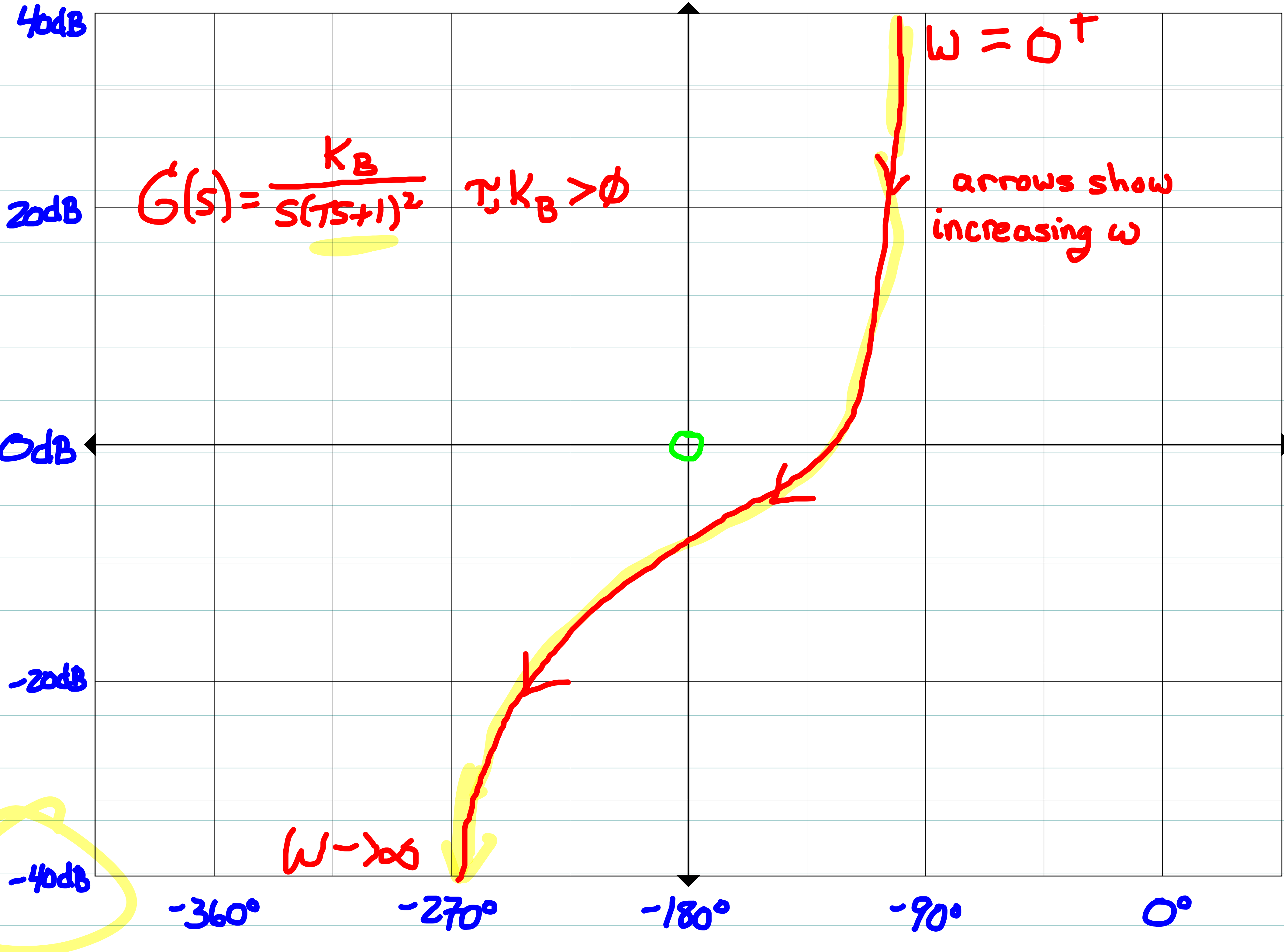
$\Rightarrow$ Angle in deg is horizontal Axis

$\Rightarrow$ Magnitude in dB is vertical Axis

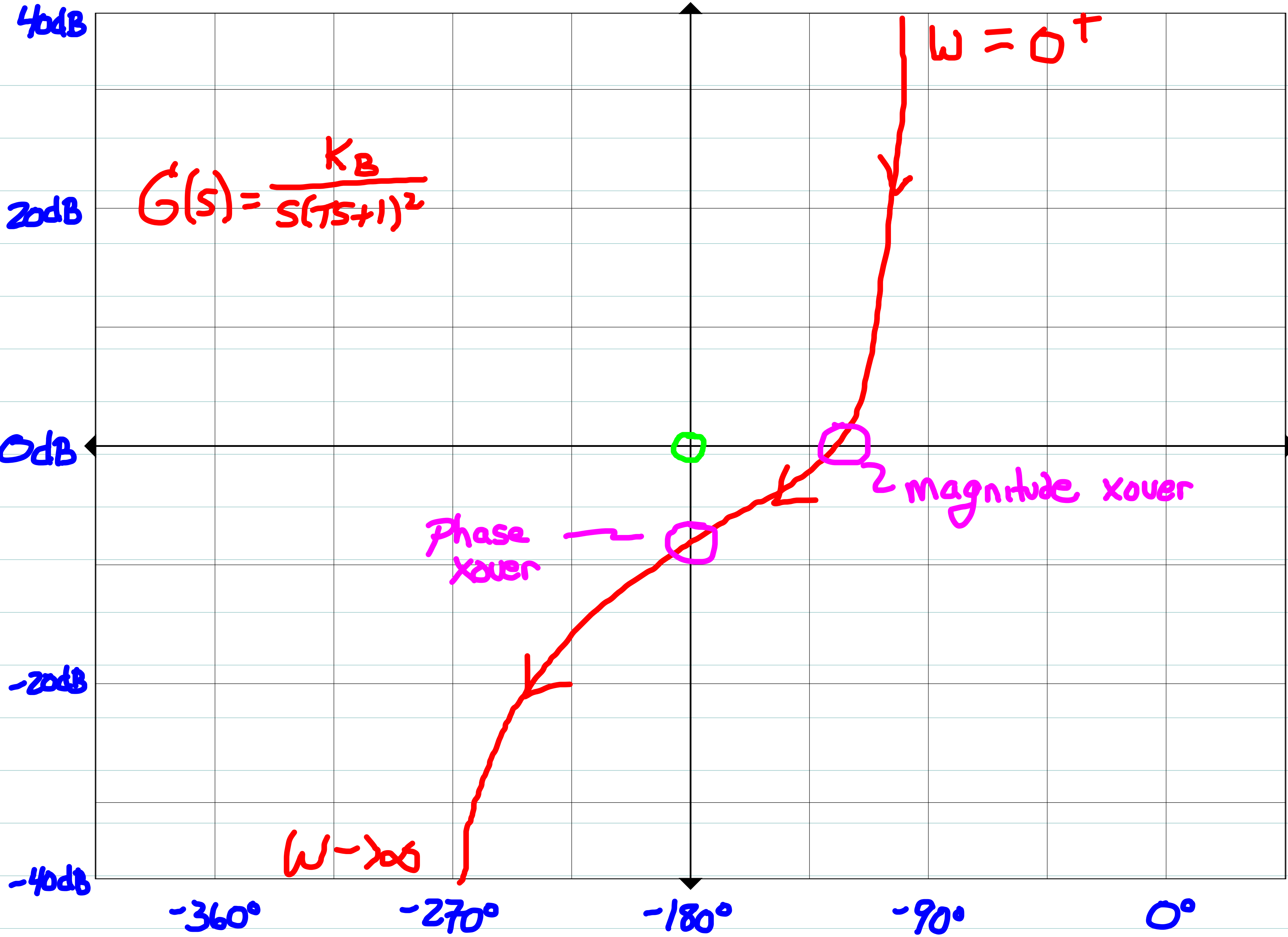
$\Rightarrow$ Plot usually centered so "origin" corresponds to $-180°$ in phase, ϕ dB in magnitude

$\Rightarrow$ "Origin" of plot corresponds to -1 point on polar diagram

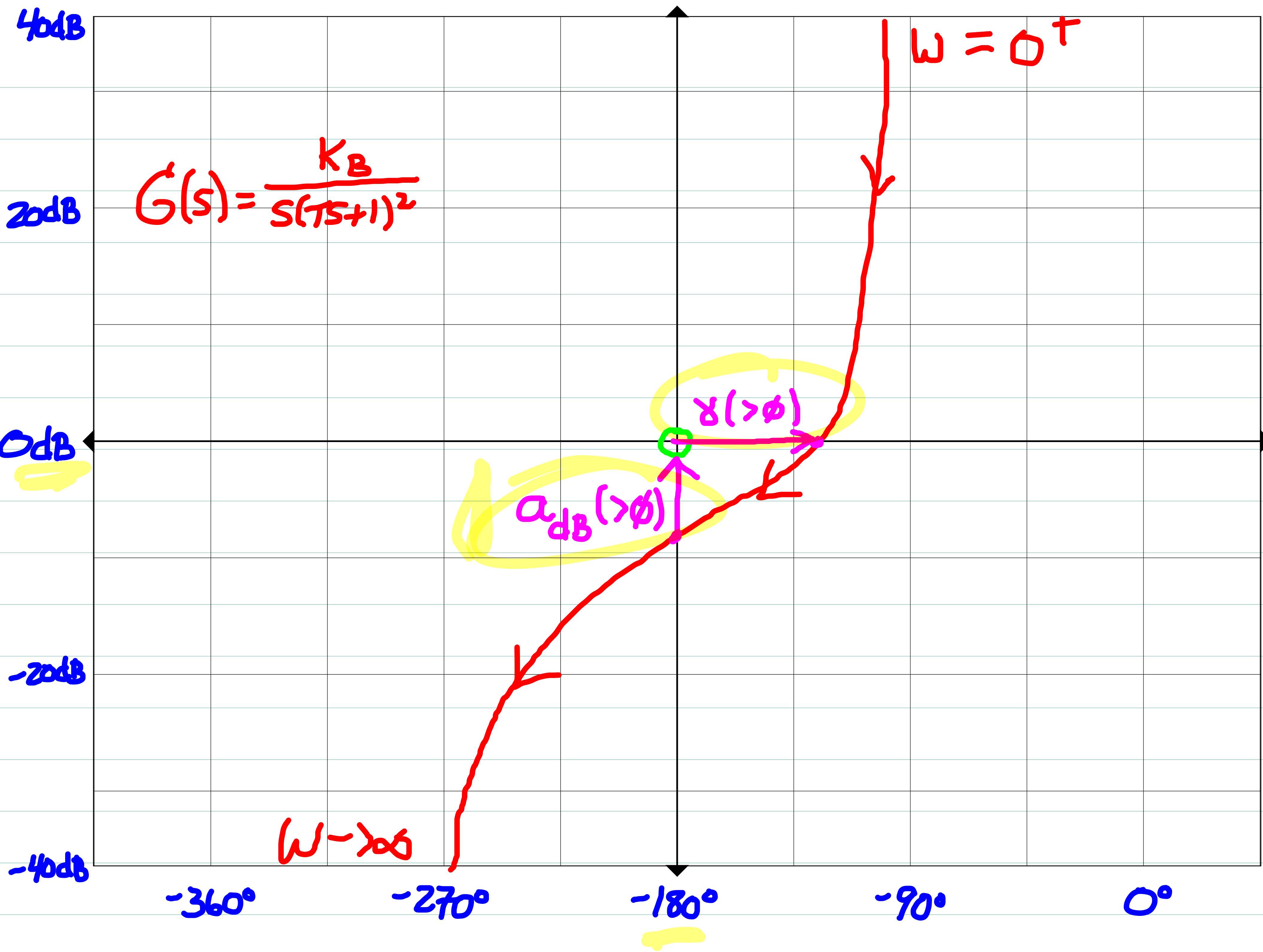

40dB
20dB
0dB
-20dB
-40dB
-360°
-270°
-180°
-90°
0°
"origin"



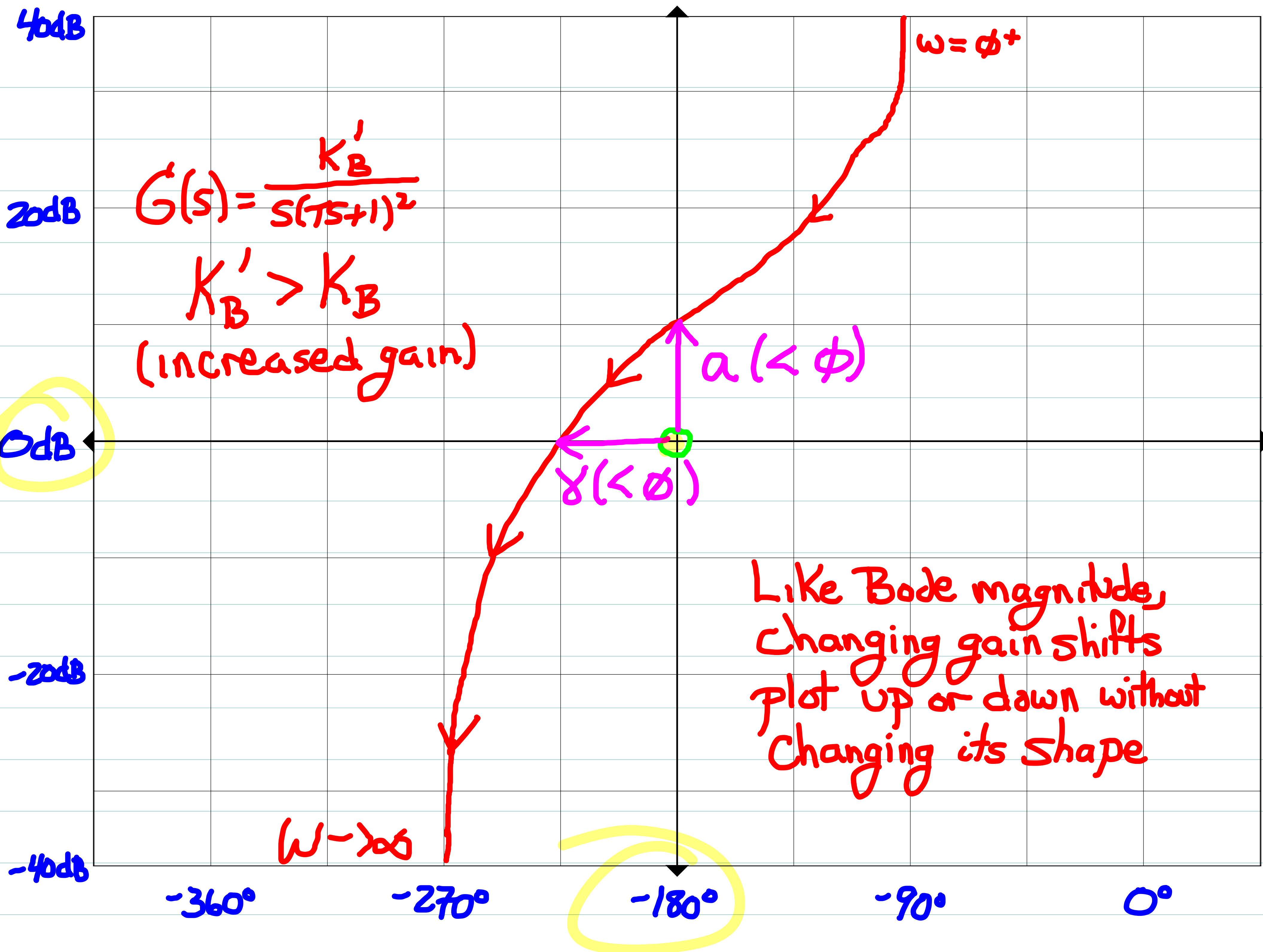
40dB
20dB
0dB
-20dB
-40dB
-360°
-270°
-180°
-90°
0°
$\omega = 0^+$
$G(s) = \dfrac{K_B}{s(\tau s+1)^2}$
$\tau, K_B > \phi$
arrows show increasing ω
$\omega \to \infty$



$G(s) = \frac{K_B}{s(Ts+1)^2}$
$\omega = 0^+$
$\omega \to \infty$
magnitude xover
Phase xover
40dB
20dB
0dB
-20dB
-40dB
-360°
-270°
-180°
-90°
0°



$G(s) = \frac{K_B}{s(Ts+1)^2}$
$\omega = 0^+$
$\omega \to \infty$
$\gamma (>\phi)$
$a_{dB} (>\phi)$
40dB
20dB
0dB
-20dB
-40dB
-360°
-270°
-180°
-90°
0°



40dB
20dB
0dB
-20dB
-40dB
-360°
-270°
-180°
-90°
0°
ω = 0⁺
ω → ∞
G(s) = K'_B / s(Ts+1)²
K'_B > K_B
(increased gain)
a (< 0)
γ (< 0)
Like Bode magnitude, changing gain shifts plot up or down without changing its shape

⇒ Primary use is to easily see margins, measured along orthogonal AXES relative to "origin"

⇒ Phase margin measured along horizontal Axis to magnitude crossover point

⇒ positive if crossing is to right of "origin" negative otherwise

⇒ Gain margin (in dB) measured along vertical Axis to phase crossover point

⇒ Positive if crossing is below "origin" negative otherwise.

⇒ Primary use is to easily see margins, measured along orthogonal axes relative to "origin"

⇒ Phase margin measured along horizontal axis to magnitude crossover point

⇒ <u>positive</u> if crossing is to <u>right</u> of "origin" negative otherwise

⇒ Gain margin (in dB) measured along vertical axis to phase crossover point

⇒ <u>positive</u> if crossing is <u>below</u> "origin" negative otherwise.

⇒ <u>Why</u> is proximity of polar/Nichols to -1 so important??