1.

a) $\psi = 30^\circ$, $\theta = 40^\circ$, $\phi = 10^\circ$

$c\psi = \cos(\psi)$ $s\psi = \sin(\psi)$
$c\theta = \cos(\theta)$ $s\theta = \sin(\theta)$
$c\phi = \cos(\phi)$ $s\phi = \sin(\phi)$

$$\vec{e}_B = {}^B R_I \vec{e}_I, \quad {}^B R_I = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{bmatrix}$$

b) $$\phi = \text{acos}\left(\frac{1}{2}(R_{11}+R_{22}+R_{33}-1)\right) = \boxed{0.8456 \text{ rad}} = 48.45^\circ$$

$$\vec{e} = \frac{1}{2\sin(\phi)}\begin{bmatrix} R_{23}-R_{32} \\ R_{31}-R_{13} \\ R_{12}-R_{21} \end{bmatrix} = \frac{1}{2\sin(\phi)}\begin{bmatrix} -0.0331 \\ 1.2778 \\ 0.7788 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -0.0221 \\ 0.8537 \\ 0.5203 \end{bmatrix}}$$

c) $$\left.\begin{aligned} q_1 &= e_1 \sin(\phi/2) \\ q_2 &= e_2 \sin(\phi/2) \\ q_3 &= e_3 \sin(\phi/2) \\ q_4 &= \cos(\phi/2) \end{aligned}\right\} \Rightarrow \boxed{\vec{q} = \begin{bmatrix} -0.0091 \\ 0.3503 \\ 0.2153 \\ 0.9119 \end{bmatrix}} \quad \vec{\beta} = \begin{bmatrix} q_4 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

d) ${}^B\omega = (0.1, 0.2, 0)$ rad/sec

$$\dot{\vec{\beta}} = \frac{1}{2}\begin{bmatrix} \beta_0 & -\beta_1 & -\beta_2 & -\beta_3 \\ \beta_1 & \beta_0 & -\beta_3 & \beta_2 \\ \beta_2 & \beta_3 & \beta_0 & -\beta_1 \\ \beta_3 & -\beta_2 & \beta_1 & \beta_0 \end{bmatrix}\begin{pmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \Rightarrow \boxed{\dot{\vec{\beta}} = \begin{bmatrix} -0.0346 \\ 0.0242 \\ 0.1019 \\ -0.0005 \end{bmatrix}}$$