Lecture 5 - Orbital Elements

Orbital Elements:

$\vec{r}, \vec{v}$

Orbital elements are an alternate set of coordinates that are used instead of $\vec{r}, \vec{v}$ (Cartesian coordinates)
B/c 6 Cartesian coordinates $\Rightarrow$ 6 orbital elements

We have already been using 3 of the orbital elements: a, e, ν

We have so far just thought of orbits in 2D $\rightarrow$ ie in the orbital plane

The other 3 orbital elements describe the orientation of the orbital plane in 3D space & the orientation of the orbit in the orbital plane.

i = inclination: angle between $\vec{h}$ & $\hat{z}$ ($\vec{h} = \vec{r} \times \vec{v}$)

Ω = longitude of the ascending node: angle from $\hat{x}$ to the ascending node

ascending node is the point on the orbit where the orbital plane crosses the equatorial plane & the s/c is ascending (aka "moving towards $\hat{z}$)

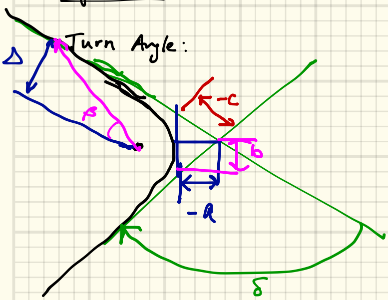
ω = argument of periapsis: angle from the ascending node to periapsis (in the orbital plane) in the direction of spacecraft motion.

$a, e, i, \Omega, \omega, \nu$

All the orbital elements are constant except ν, unless we execute a maneuver.

Hyperbolas (continued)

Turn Angle:



Turn angle: δ

$\sin\left(\frac{\delta}{2}\right) = \frac{a}{c}$ $\qquad e = \frac{c}{a}$

$\boxed{\sin\left(\frac{\delta}{2}\right) = \frac{1}{e}}$

Δ = miss distance

$h = rv\cos\gamma$

At ∞, V is parallel to the asymptotes

$h = rv\cos\gamma = V_\infty r \sin\beta = V_\infty \Delta$ $\qquad \boxed{h = V_\infty \Delta}$

Canonical Units: useful when doing calculations by hand

method of nondimensionalisation

1 AU = Astronomical Unit = distance from the Sun to the Earth

= 149,597,871 km

→ interplanetary trajectories

DU = distance units =

often, radius of the Earth

$\mu = 1 \ DU^3/TU^2$

We pick a DU (some # of km) and we know that $\mu = 1 \ DU^3/TU^2$. So we can solve for TU.

1 DU = 6378 km

$\mu = 1 \ DU^3/TU^2 = 3.986 \times 10^5 \ km^3/s^2$

$$1 \frac{DU^3}{TU^2} \cdot \left(\frac{6378 \text{ km}}{1 \text{ DU}}\right)^3 \cdot \left(\frac{1 \text{ TU}}{X \text{ sec}}\right)^2 = 3.986 \times 10^5 \ \frac{km^3}{s^2}$$

⇒ solve for X. ⇒ 1 TU = X sec

Then, express $\vec{r}$, $\vec{v}$, etc (a) in terms of DU & DU/TU.