1.

a) $\psi = 30^\circ$ $\quad c\psi = \cos(\psi) \quad s\psi = \sin(\psi)$

$\theta = 40^\circ$ $\quad c\theta = \cos(\theta) \quad s\theta = \sin(\theta)$

$\phi = 10^\circ$ $\quad c\phi = \cos(\phi) \quad s\phi = \sin(\phi)$

$$\vec{e}_B = {}^B R_{\mathcal{I}}\, \vec{e}_{\mathcal{I}}, \quad {}^B R_{\mathcal{I}} = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{bmatrix}$$

b) $$\phi = \operatorname{acos}\left(\tfrac{1}{2}(R_{11}+R_{22}+R_{33}-1)\right) = \boxed{0.8456 \text{ rad}} = 48.45^\circ$$

$$\vec{e} = \frac{1}{2\sin(\phi)}\begin{bmatrix} R_{23}-R_{32} \\ R_{31}-R_{13} \\ R_{12}-R_{21} \end{bmatrix} = \frac{1}{2\sin(\phi)}\begin{bmatrix} -0.0331 \\ 1.2778 \\ 0.7788 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -0.0221 \\ 0.8537 \\ 0.5203 \end{bmatrix}}$$

c) $$\left.\begin{aligned} q_1 &= e_1 \sin(\phi/2) \\ q_2 &= e_2 \sin(\phi/2) \\ q_3 &= e_3 \sin(\phi/2) \\ q_4 &= \cos(\phi/2) \end{aligned}\right\} \Rightarrow \boxed{\vec{q} = \begin{bmatrix} -0.0091 \\ 0.3503 \\ 0.2153 \\ 0.9119 \end{bmatrix}} \quad \vec{\beta} = \begin{bmatrix} q_4 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

d) ${}^B\omega = (0.1, 0.2, 0)$ rad/sec

$$\dot{\vec{\beta}} = \frac{1}{2}\begin{bmatrix} \beta_0 & -\beta_1 & -\beta_2 & -\beta_3 \\ \beta_1 & \beta_0 & -\beta_3 & \beta_2 \\ \beta_2 & \beta_3 & \beta_0 & -\beta_1 \\ \beta_3 & -\beta_2 & \beta_1 & \beta_0 \end{bmatrix}\begin{pmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \Rightarrow \boxed{\dot{\vec{\beta}} = \begin{bmatrix} -0.0346 \\ 0.0242 \\ 0.1019 \\ -0.0005 \end{bmatrix}}$$

problem 01

```
clearvars all;
clc;
```

part a

```
% given
angs_deg = [30, 40, 10];
syms psi theta phi;

% solution
angs = deg2rad(angs_deg);
Rx = [
    1, 0, 0;
    0, cos(phi), sin(phi);
    0, -sin(phi), cos(phi);
];
Ry = [
    cos(theta), 0, -sin(theta);
    0, 1, 0;
    sin(theta), 0, cos(theta);
];
Rz = [
    cos(psi), sin(psi), 0;
    -sin(psi), cos(psi), 0;
    0, 0, 1;
];

R_sym = Rx*Ry*Rz;
R = double(subs(R_sym, [psi theta phi], angs));

% display
R_sym, R
```

R_sym =

$$\begin{pmatrix} \cos(\psi)\cos(\theta) & \cos(\theta)\sin(\psi) & -\sin(\theta) \\ \cos(\psi)\sin(\phi)\sin(\theta)-\cos(\phi)\sin(\psi) & \cos(\phi)\cos(\psi)+\sin(\phi)\sin(\psi)\sin(\theta) & \cos(\theta)\sin(\phi) \\ \sin(\phi)\sin(\psi)+\cos(\phi)\cos(\psi)\sin(\theta) & \cos(\phi)\sin(\psi)\sin(\theta)-\cos(\psi)\sin(\phi) & \cos(\phi)\cos(\theta) \end{pmatrix}$$

```
R = 3×3
    0.6634    0.3830   -0.6428
   -0.3957    0.9087    0.1330
    0.6350    0.1661    0.7544
```

part b

```
% solution
phi = acos(1/2*(R(1,1)+R(2,2)+R(3,3)-1));
e = 1/(2*(sin(phi)))*[
```

```
    R(2,3)-R(3,2);
    R(3,1)-R(1,3);
    R(1,2)-R(2,1);
];

% display
phi, e
```

```
phi =
0.8456
e = 3×1
   -0.0221
    0.8537
    0.5203
```

part c

```
% solution
q = [
    e(1)*sin(phi/2);
    e(2)*sin(phi/2);
    e(3)*sin(phi/2);
    cos(phi/2)
];

% display
q
```

```
q = 4×1
   -0.0091
    0.3503
    0.2135
    0.9119
```

part d

```
% given
omega = [0.1, 0.2, 0];

% solution
beta0 = q(4);
beta1 = q(1);
beta2 = q(2);
beta3 = q(3);

beta_dot = 1/2*[
    beta0, -beta1, -beta2, -beta3;
    beta1,  beta0, -beta3,  beta2;
    beta2,  beta3,  beta0, -beta1;
    beta3, -beta1,  beta1,  beta0;
]*[
    0;
```

```
    omega(1);
    omega(2);
    omega(3);
];

% display
beta_dot
```

```
beta_dot = 4×1
   -0.0346
    0.0242
    0.1019
   -0.0005
```

p02

clear history

```
clearvars all;
clc;
```

part a

```
% given
I = diag([10; 20; 30])
```

```
I = 3×3
    10     0     0
     0    20     0
     0     0    30
```

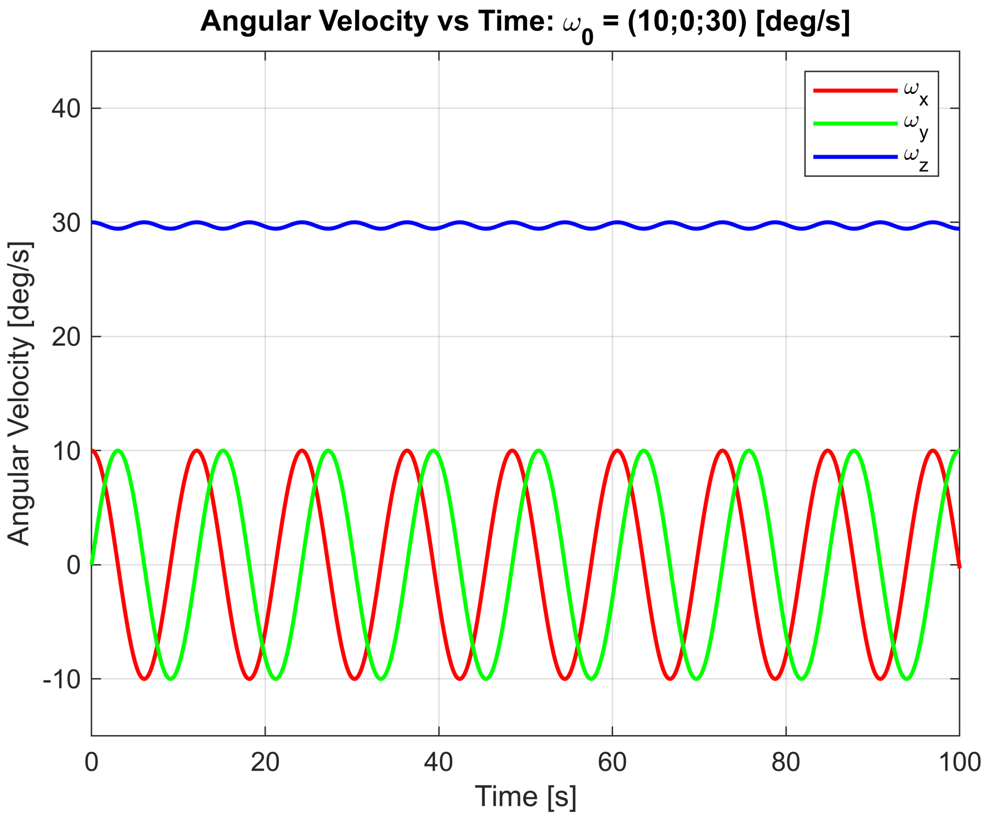
```
omega0_deg = [10; 0; 30];
omega0 = deg2rad(omega0_deg);

% solution
H0 = I*omega0;
H0_mag = norm(H0);
T0 = 1/2*transpose(omega0)*I*omega0;

% display
H0_mag, T0
```

```
H0_mag =
15.8046
T0 =
4.2646
```

part b

```
% solution
dwdt = @(t, omega) inv(I)*(-cross(omega, I*omega));
tspan = [0, 100];
oopts = odeset('RelTol',1e-9,'AbsTol',1e-9);

[t1, w1] = ode45(dwdt, tspan, omega0, oopts);
w1_deg = rad2deg(w1);

% plot
fig = figure;
plot(t1, w1_deg(:,1), 'r', t1, w1_deg(:,2), 'g', t1, w1_deg(:,3), 'b', 'LineWidth',
1.5);
title('Angular Velocity vs Time: \omega_0 = (10;0;30) [deg/s]');
legend('\omega_x','\omega_y','\omega_z');
xlabel('Time [s]');
```

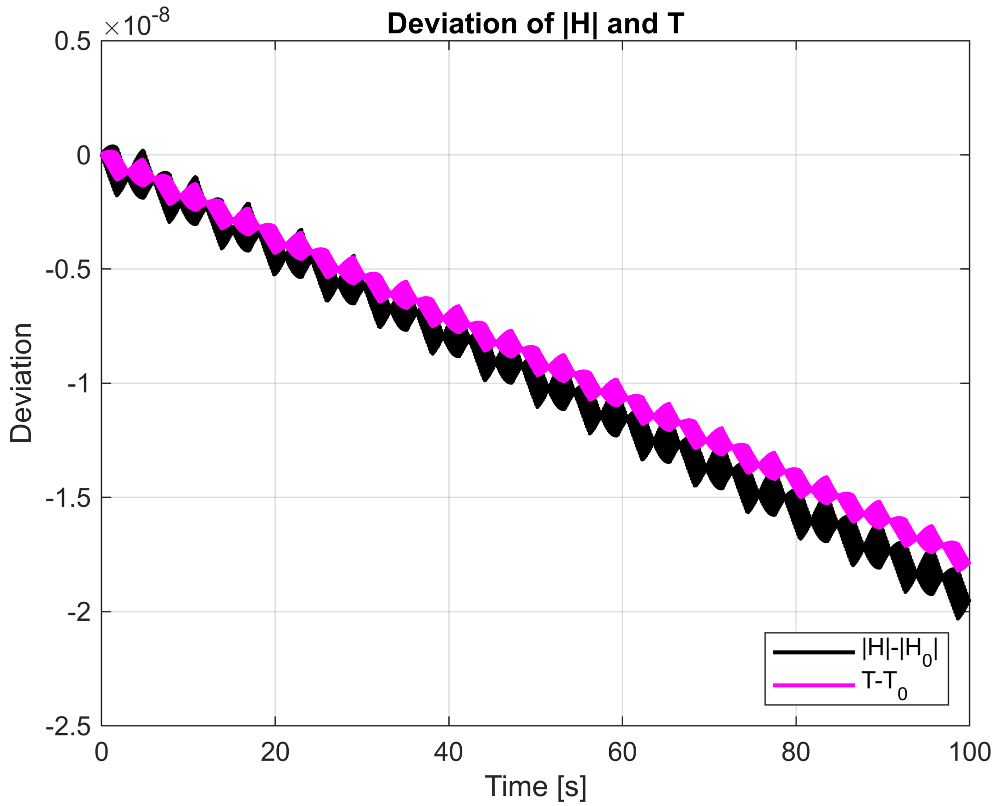
```
ylabel('Angular Velocity [deg/s]');
ylim([-15 45]);
grid on;

% save
saveas(fig, "./images/s02b.png");
```



part c

```
% solution
H1 = transpose(I * transpose(w1));
H1_mag = sqrt(sum(H1.^2,2));
T1 = 1/2*sum(w1.*H1, 2);

dH = H1_mag-H0_mag;
dT = T1-T0;

% plot
fig = figure;
plot(t1, dH, 'k', t1, dT, 'm', 'LineWidth', 1.5);
title('Deviation of |H| and T');
legend('|H|-|H_0|','T-T_0', 'Location', 'southeast');
xlabel('Time [s]');
ylabel('Deviation');
grid on;
```

```
% save
saveas(fig, "./images/s02c.png");
```

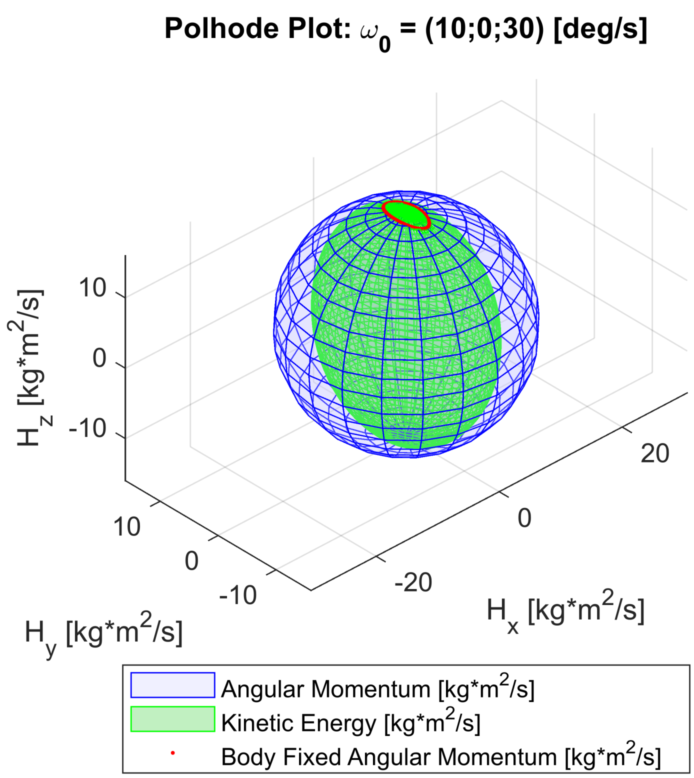


I believe my code is working, as the both the angular momentum deviation and kinetic energy deviation are extremely minimal, being on the order of 10^{-8}.

part d

```
% solution
[Xs, Ys, Zs] = sphere(20);
Xs = H0_mag*Xs;
Ys = H0_mag*Ys;
Zs = H0_mag*Zs;

radii = sqrt(2*T0.*diag(I));
[Xe, Ye, Ze] = ellipsoid(0,0,0, radii(1), radii(2), radii(3), 60);

% plot
fig = figure;
hold on;
% sphere
surf(Xs, Ys, Zs, 'FaceAlpha',0.3, 'EdgeColor','b', 'FaceColor', [0.8 0.8 1]);
% ellipsoid
surf(Xe, Ye, Ze, 'FaceAlpha',0.3, 'EdgeColor','g', 'FaceColor', [0.2 0.8 0.2]);
% polhode points (body-fixed H)
plot3(H1(:,1), H1(:,2), H1(:,3), '.r', 'MarkerSize', 5);
```

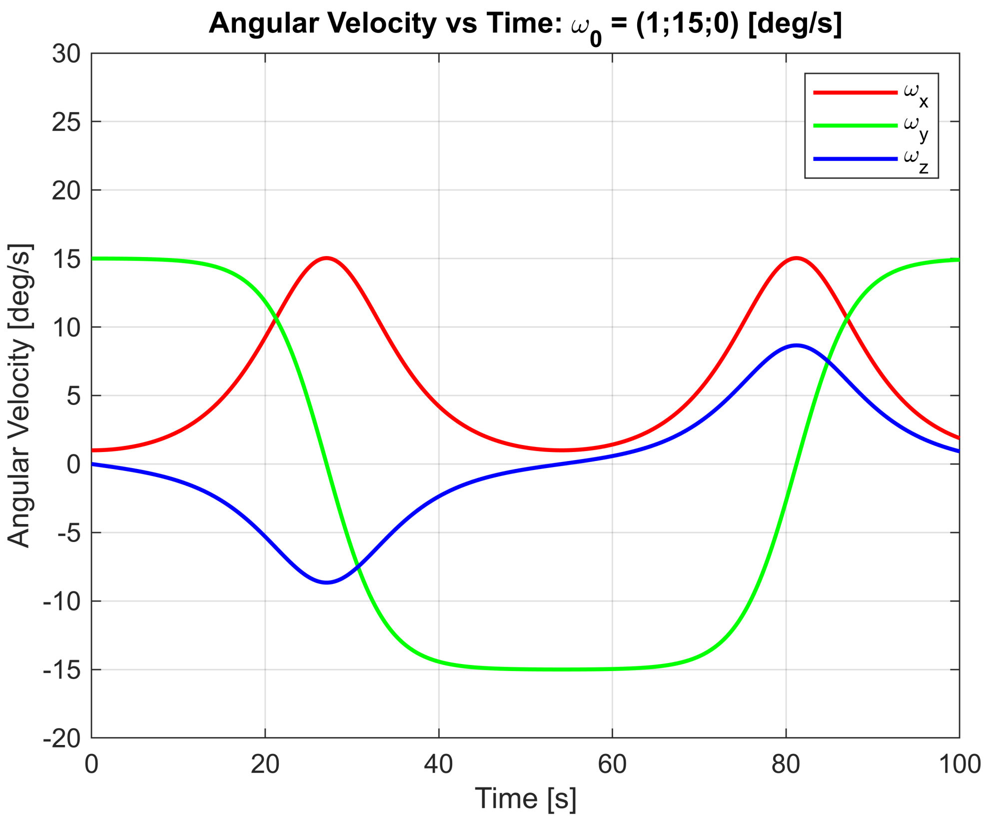
```
title('Polhode Plot: \omega_0 = (10;0;30) [deg/s]');
legend('Angular Momentum [kg*m^2/s]','Kinetic Energy [kg*m^2/s]','Body Fixed
Angular Momentum [kg*m^2/s]', 'Location', 'southoutside');
xlabel('H_x [kg*m^2/s]');
ylabel('H_y [kg*m^2/s]');
zlabel('H_z [kg*m^2/s]');
axis equal;
grid on;
view([-43.3 33.8])
hold off;

% save
saveas(fig, "./images/s02d.png");
```



part e

```
% solution
omega2_deg = [1; 15; 0];
omega2 = deg2rad(omega2_deg);
[t2, w2] = ode45(dwdt, tspan, omega2, oopts);
w2_deg = rad2deg(w2);

% plot
fig = figure;
plot(t2, w2_deg(:,1), 'r', t2, w2_deg(:,2), 'g', t2, w2_deg(:,3), 'b', 'LineWidth',
1.5);
```

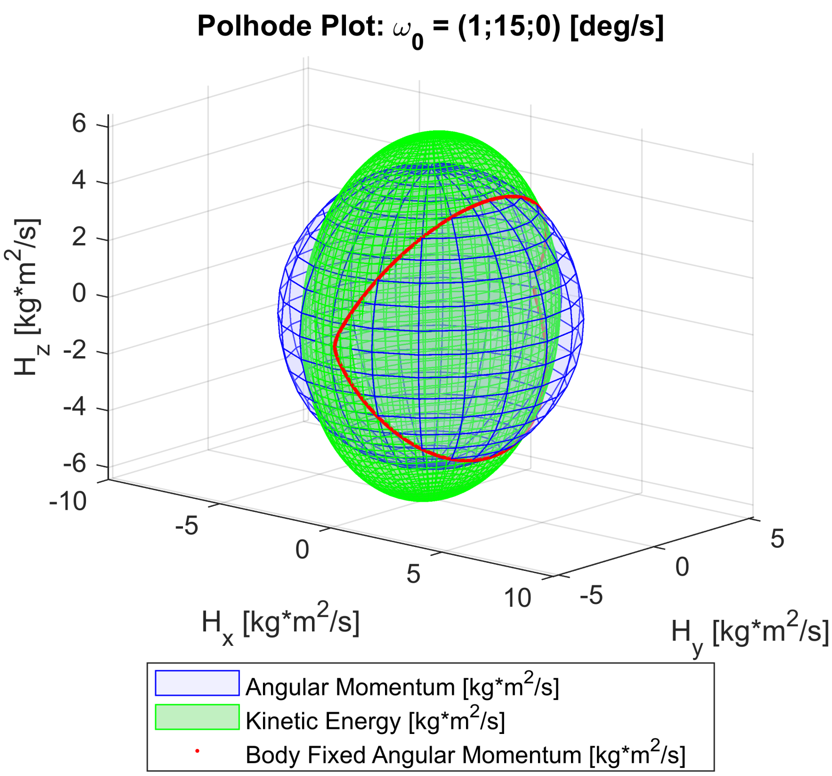
```
title('Angular Velocity vs Time: \omega_0 = (1;15;0) [deg/s]');
legend('\omega_x','\omega_y','\omega_z');
xlabel('Time [s]');
ylabel('Angular Velocity [deg/s]');
ylim([-20 30]);
grid on;

% save
saveas(fig, "./images/s02e.png");
```



part f

```
% solution
H2_0 = I*omega2;
H2_0_mag = norm(H2_0);
T2_0 = 1/2*transpose(omega2)*I*omega2;

H2 = transpose(I*transpose(w2));
H2_mag = sqrt(sum(H2.^2,2));
T2 = 1/2*sum(w2.*H2, 2);

[Xs, Ys, Zs] = sphere(20);
Xs = H2_0_mag*Xs;
Ys = H2_0_mag*Ys;
Zs = H2_0_mag*Zs;
```

```
radii = sqrt(2*T2_0.*diag(I));
[Xe, Ye, Ze] = ellipsoid(0,0,0, radii(1), radii(2), radii(3), 60);

% plot
fig = figure;
hold on;
% sphere
surf(Xs, Ys, Zs, 'FaceAlpha',0.3, 'EdgeColor','b', 'FaceColor', [0.8 0.8 1]);
% ellipsoid
surf(Xe, Ye, Ze, 'FaceAlpha',0.3, 'EdgeColor','g', 'FaceColor', [0.2 0.8 0.2]);
% polhode points (body-fixed H)
plot3(H2(:,1), H2(:,2), H2(:,3), '.r', 'MarkerSize', 5);
title('Polhode Plot: \omega_0 = (1;15;0) [deg/s]');
legend('Angular Momentum [kg*m^2/s]','Kinetic Energy [kg*m^2/s]','Body Fixed
Angular Momentum [kg*m^2/s]', 'Location', 'southoutside');
xlabel('H_x [kg*m^2/s]');
ylabel('H_y [kg*m^2/s]');
zlabel('H_z [kg*m^2/s]');
axis equal;
grid on;
view([40.6 14.7]);
hold off;

% save
saveas(fig, "./images/s02f.png");
```



The spacecraft in part b and d has a much higher kinetic energy in the Z axis than in the X and Y axes, and as well varies each axis in only small oscillations. This is far different from the spacecraft in e and f, who has far more energy-variance within each axis, as well as having its overall spread be much closer together. These two differences result in the Polhode plot of the first spacecraft being confined to a small area high in the positive Z axis, while the second spacecraft's plot ventures all over the place.