Lecture 2: Airfoils

ENAE311H Aerodynamics I

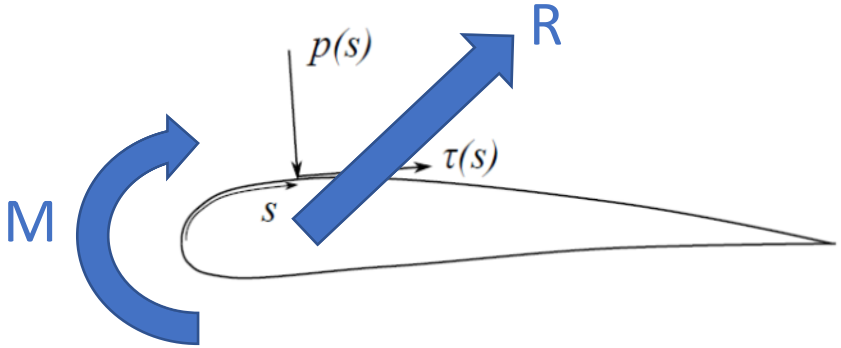
C. Brehm

Definitions

- An airfoil is a body designed to produce a desired reaction force/moment when in motion relative to the surrounding air. On an aircraft, airfoils
  - provide lift to oppose gravity
  - provide forces and moments to stabilize and maneuver the aircraft

- Aerodynamic forces and moments derive from two sources:
  1. The pressure distribution on the body, p
  2. The shear-stress distribution, τ

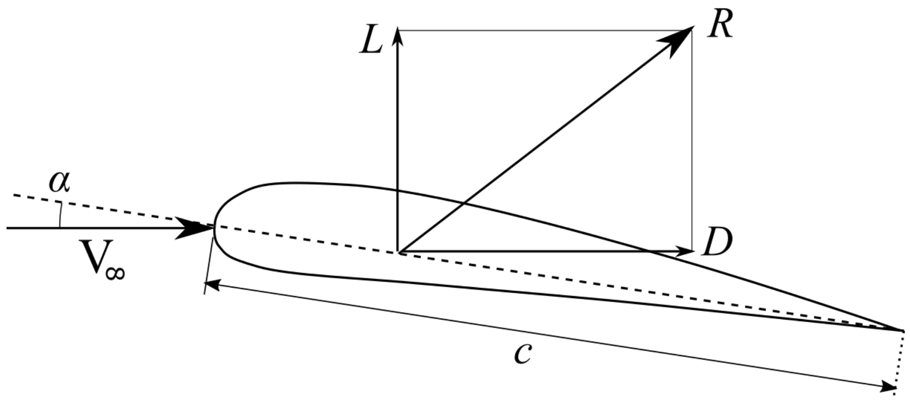
  → together these give the resultant force, R, and moment, M, acting on an airfoil



s is distance from leading edge on 2D airfoil

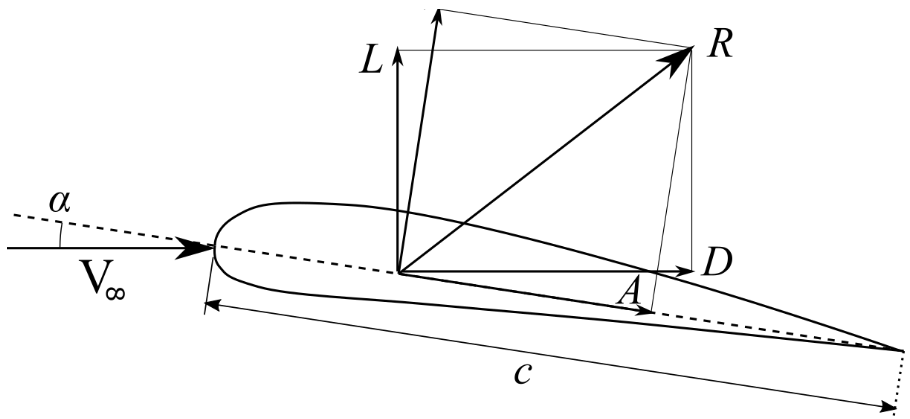
Force components

- It is useful to split the resultant force, $\boldsymbol{R}$, on a 2D airfoil into two components. The coordinate system for this split can be defined relative to either
  - The freestream direction, in which case the components are
    - Lift $(L) \equiv$ force normal to $\boldsymbol{V}_\infty$
    - Drag $(D) \equiv$ force tangential to $\boldsymbol{V}_\infty$



Force components

- It is useful to split the resultant force, $\boldsymbol{R}$, on a 2D airfoil into two components. The coordinate system for this split can be defined relative to either
  - The freestream direction, in which case the components are
    - Lift $(L) \equiv$ force normal to $\boldsymbol{V}_\infty$
    - Drag $(D) \equiv$ force tangential to $\boldsymbol{V}_\infty$
  - The airfoil chord direction, in which case the components are
    - Normal force component $(N) \equiv$ force normal to $\boldsymbol{c}$
    - Axial force component $(A) \equiv$ force tangential to $\boldsymbol{c}$



- Treating as vector quantities, in either case we have $\boldsymbol{L} + \boldsymbol{D} = \boldsymbol{N} + \boldsymbol{A} = \boldsymbol{R}$

- Given the normal and axial force components, the lift and drag can be calculated according to

$$
\begin{aligned}
L &= N\cos\alpha - A\sin\alpha \\
D &= A\cos\alpha + N\sin\alpha
\end{aligned}
$$

where α is the angle of attack, i.e., the angle between the freestream and chord directions.

Force/moment coefficients

- It is standard practice to cast the force components in nondimensional form – this allows them to be compared to reference quantities in a meaningful way. Let us introduce the dynamic pressure:

$$q_\infty \equiv \frac{1}{2}\rho_\infty V_\infty^2$$

- Since pressure has dimensions of force divided by area, by introducing a reference area S, this allows us to write nondimensional force coefficients as follows:

$$\text{Lift coefficient: } C_L \equiv \frac{L}{q_\infty S}$$

$$\text{Drag coefficient: } C_D \equiv \frac{D}{q_\infty S}$$

$$\text{Normal force coefficient: } C_N \equiv \frac{N}{q_\infty S}$$

$$\text{Axial force coefficient: } C_A \equiv \frac{A}{q_\infty S}$$

- Using a reference length, l, we can also define the nondimensional moment coefficient:

$$\text{(Pitching) Moment coefficient: } C_M \equiv \frac{M}{q_\infty S l}$$

- Note that S and l need to be chosen to be appropriate for the particular geometry (e.g., planform area and mean chord length for an airfoil).

Force/moment coefficients

- For a two-dimensional body, we specify the force and moment coefficients per unit span. For a 2D airfoil then:

$$c_l \equiv \frac{L'}{q_\infty c}, \quad c_d \equiv \frac{D'}{q_\infty c}, \quad c_m \equiv \frac{M'}{q_\infty c^2}$$

Here, a primed variable denotes the force or moment per unit span.

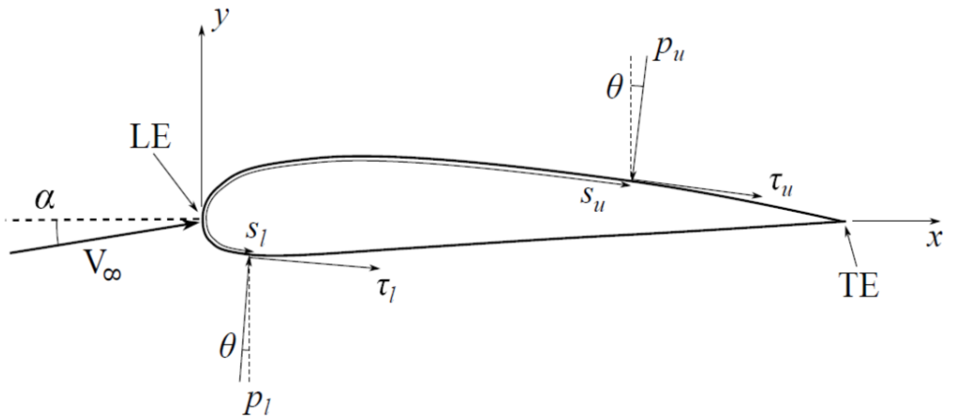
- It is often also useful to express the pressure and surface shear stress in terms of nondimensional pressure and skin-friction coefficients:

$$\text{Pressure coefficient: } C_p \equiv \frac{p-p_\infty}{q_\infty}$$

$$\text{Skin-friction coefficient: } C_f \equiv \frac{\tau}{q_\infty}$$

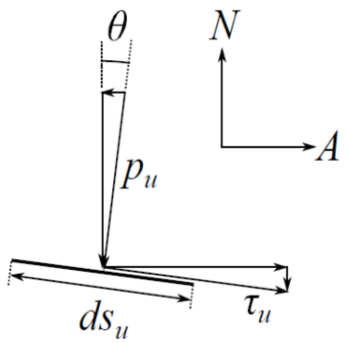
Calculating forces

- Assume we are given the pressure and shear-stress distributions on an airfoil (along with its surface profile $y_u(x)$ and $y_l(x)$). We consider a coordinate system aligned with the airfoil chord:



θ is angle of surface normal relative to vertical (positive for clockwise)

- Consider first an element, ds_u, on the upper surface. The pressure and shear stress will contribute to the normal and axial forces as:



$$
\begin{aligned}
dN' &= -p_u ds_u \cos\theta - \tau_u ds_u \sin\theta \\
dA' &= -p_u ds_u \sin\theta + \tau_u ds_u \cos\theta
\end{aligned}
$$

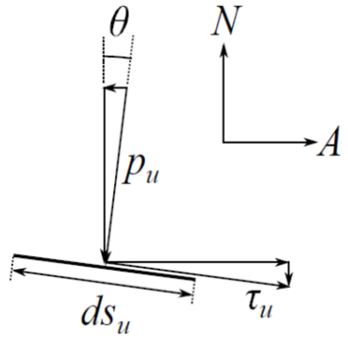
- Similarly, for an element on the lower surface:

$$
\begin{aligned}
dN' &= p_l ds_l \cos\theta - \tau_l ds_l \sin\theta \\
dA' &= p_l ds_l \sin\theta + \tau_l ds_l \cos\theta
\end{aligned}
$$

Calculating forces

- We can now integrate p and τ along each surface of the airfoil to obtain the normal and axial forces:

$$\begin{aligned}
N' &= -\int_{LE}^{TE} (p_u \cos\theta + \tau_u \sin\theta) ds_u + \int_{LE}^{TE} (p_l \cos\theta - \tau_l \sin\theta) ds_l \\
A' &= \int_{LE}^{TE} (-p_u \sin\theta + \tau_u \cos\theta) ds_u + \int_{LE}^{TE} (p_l \sin\theta + \tau_l \cos\theta) ds_l
\end{aligned}$$



- It is generally easier, however, to integrate w.r.t. x than s. Noting from geometry that

$$dx = ds\cos\theta, \quad dy = -ds\sin\theta$$

and also that $dy = \frac{dy}{dx} dx$, we can then write

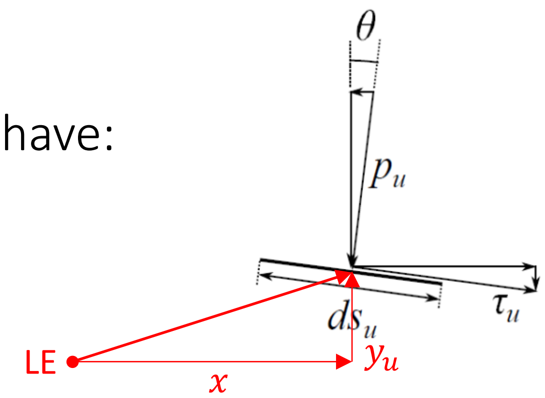
$$\begin{aligned}
N' &= -\int_0^c \left(p_u - \tau_u \frac{dy_u}{dx} \right) dx + \int_0^c \left(p_l + \tau_l \frac{dy_l}{dx} \right) dx \\
&= \int_0^c (p_l - p_u) dx + \int_0^c \left(\tau_u \frac{dy_u}{dx} + \tau_l \frac{dy_l}{dx} \right) dx \\
A' &= \int_0^c \left(p_u \frac{dy_u}{dx} - p_l \frac{dy_l}{dx} \right) dx + \int_0^c (\tau_u + \tau_l) dx.
\end{aligned}$$

Convert to L' and D' via our earlier coordinate transformations

Calculating pitching moment

- Unlike the forces, the pitching moment depends on the point about which the moment is taken. Here we consider the moment about the leading edge and choose the convention that pitch up (increasing α) is a positive M:

- If we consider contributions to the moment from a small length (area) element, we have:



$$dM'_u = (p_u \cos\theta + \tau_u \sin\theta) x ds_u + (-p_u \sin\theta + \tau_u \cos\theta) y_u ds_u$$

$$dM'_l = (-p_l \cos\theta + \tau_l \sin\theta) x ds_l + (p_l \sin\theta + \tau_l \cos\theta) y_l ds_l$$

- Integrating from LE to TE:

$$M'_{LE} = \int_{LE}^{TE} [(p_u \cos\theta + \tau_u \sin\theta)x + (-p_u \sin\theta + \tau_u \cos\theta) y_u] ds_u + \int_{LE}^{TE} [(-p_l \cos\theta + \tau_l \sin\theta)x + (p_l \sin\theta + \tau_l \cos\theta) y_l] ds_l$$

- And with our expressions for dx and dy:

$$M'_{LE} = \int_0^c \left[p_u - p_l - \tau_u \frac{dy_u}{dx} - \tau_l \frac{dy_l}{dx} \right] x dx + \int_0^c \left[\left(p_u \frac{dy_u}{dx} + \tau_u \right) y_u + \left(-p_l \frac{dy_l}{dx} + \tau_l \right) y_l \right] dx$$

Formulae for force/moment coefficients

- If we divide our expressions for N', A', and M' through by $q_\infty c$ or $q_\infty c^2$, and make use of our definitions of the pressure and skin-friction coefficients, we can write for the force and moment coefficients:

$$c_n = \frac{1}{c}\left[\int_0^c (C_{p_l} - C_{p_u})dx + \int_0^c \left(C_{f_u}\frac{dy_u}{dx} + C_{f_l}\frac{dy_l}{dx}\right)dx\right]$$

$$\boxed{c_a = \frac{1}{c}\left[\int_0^c \left(C_{p_u}\frac{dy_u}{dx} - C_{p_l}\frac{dy_l}{dx}\right)dx + \int_0^c (C_{f_u} + C_{f_l})dx\right]}$$

$$c_{m_{LE}} = \frac{1}{c^2}\left\{\int_0^c \left[C_{p_u} - C_{p_l} - C_{f_u}\frac{dy_u}{dx} - C_{f_l}\frac{dy_l}{dx}\right]xdx + \int_0^c \left[\left(C_{p_u}\frac{dy_u}{dx} + C_{f_u}\right)y_u + \left(-C_{p_l}\frac{dy_l}{dx} + C_{f_l}\right)y_l\right]dx\right\}$$

Convert to c_l and c_d via our coordinate transformations

Center of pressure

- Although it is the distributed pressure and shear stress on an airfoil that contribute to the net forces and moment, it is possible to represent the resultant forces as acting through a single point to produce the same moment – this point we call the "center of pressure".
- In fact, in general there are an infinite number of points for which this will hold, but if we restrict ourselves to points along the chord line (so A' doesn't contribute to the moment), this reduces to finding the location x_{cp} such that

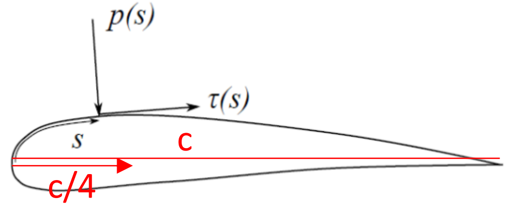
$$\boxed{M'_{LE} = -x_{cp} N'}$$

- If the angle of attack is small, we also have

$$x_{cp} \approx -\frac{M'_{LE}}{L'}$$

- Note that, if the moment were calculated about x_{cp}, the result would be zero.
- For a two-dimensional airfoil, x_{cp} is usually close to the quarter-chord point. For transforming between effective origins, we can use, for example,

$$M'_{LE} = -\frac{c}{4}N' + M'_{c/4} \qquad \text{or} \qquad M'_{LE} \approx -\frac{c}{4}L' + M'_{c/4}$$



1. Consider an infinitely thin flat plate with a 1 m chord at an angle of attack of 15° to an oncoming flow. The pressure distributions on the upper and lower surfaces are given by $p_u = 2 \times 10^4(x-1) + 2.7 \times 10^4$ and $p_l = 1 \times 10^4(x-1) + 1.1 \times 10^5$, where x is the distance from the leading edge along the chord; the shear stress distributions are $\tau_u = 144x^{-0.3}$ and $\tau_l = 360x^{-0.3}$. Here, the units of p and τ are N m^{-2}. Calculate the normal and axial forces, the lift and drag, moments about the leading edge and quarter chord, all per unit span, as well as the center of pressure.

2. A series of experiments is performed on a two-dimensional airfoil in which the lift, drag and moment coefficients (the latter about the quarter chord) are measured over a range of angles of attack from 0 to 10°. The lift coefficient curve is found to be well approximated by the equation

$$c_l = 0.2 + 6\alpha, \tag{1}$$

where α is the angle of attack in radians. The drag is found to be well approximated by

$$c_d = 0.006 + 0.3\alpha^2 \tag{2}$$

while $c_{m,c/4}$ increases linearly from -0.04 for α=0 to -0.03 for α=10°. Make a plot of x_{cp}/c as a function of α for this airfoil.