Alternate Design Perspectives

Our correlation between phase margin/crossover and the poles of $T(s)$ [hence its transient response characteristics] is approximate and tenuous at best.

It would be nice if we could specifically target the desired closed-loop poles, and design $H(s)$ to obtain them.

There are, in fact, techniques for this, although in using them we give up many of the insights afforded by the freq. response design methods . . .

(Everything is a trade-off! There are no magic bullets in this game!)

Recall the Characteristic Equation:

All closed-loop poles satisfy: $1 + L(s) = \emptyset$

$$\boxed{\Rightarrow L(s) = -1}$$

Let $L_0(s) = [L(s)]_{K=1}$ (K = compensator gain – real!)

Then s is a CL pole if: $K L_0(s) = -1$

which requires $\boxed{L_0(s) \text{ to be real.}}$

For any such s: $K = \dfrac{-1}{L_0(s)}$

is the gain which would make this s a CL pole

Moreover: $L_0(s)$ is real (hence s a possible CL pole)

if: $\measuredangle L_0(s) = \ell(180°)$ (ℓ = any integer)

In particular:

$L_0(s) = (1+2\ell)\,180°$ (odd multiple of 180°)

$\Rightarrow L_0(s)$ is a <u>negative</u> real number

$\Rightarrow$ corresponding $K = \dfrac{-1}{L_0(s)}$ is positive

and:

$L_0(s) = 2\ell(180°)$ (even multiple of 180°)

$\Rightarrow$ corresponding K is <u>negative</u>

"Angle condition", $K > 0$

If we restrict ourself initially to $K > 0$, we need

$$\boxed{\measuredangle L(s) = \measuredangle L_0(s) = (1+2\ell)180^\circ}$$

for s to be a CL pole. This is the "angle condition".

Any value of s satisfying this condition will be a CL pole for an appropriate positive value of K.

Suppose that we want a specific CL pole, s_{Des}.

We need $\measuredangle L(s_{Des}) = (1+2\ell)180^\circ$

But recall: $\measuredangle L(s) = \measuredangle G(s) + \measuredangle H(s)$ for any $s \in \mathbb{C}$

Thus, to make s_{Des} a CL pole we need

$$(1+2\ell)180^\circ = \measuredangle G(s_{Des}) + \measuredangle H(s_{Des})$$

Hence, must design compensator $H(s)$ so that:

$$\boxed{\measuredangle H(s_{Des}) = (1+2\ell)180^\circ - \measuredangle G(s_{Des})}$$

Similar to Bode design approach, define:

$$\boxed{\varphi_{req} = (1+2\ell)180^\circ - \measuredangle G(s_{Des})}$$

(choose ℓ to get φ_{req} in range $[-180^\circ, +180^\circ]$)

Then choose poles/zeros in $H(s)$ so that

$$\boxed{\measuredangle H(s_{Des}) = \varphi_{req}}$$

Example:

Suppose $G(s) = \frac{3}{s(s+2)}$

and we want $T(s)$ to have a pole at $s_{Des} = -3+3j$

$$\measuredangle G(s_{Des}) = 116.56°$$

(In Matlab: angle(evalfr(G, -3+3j)))

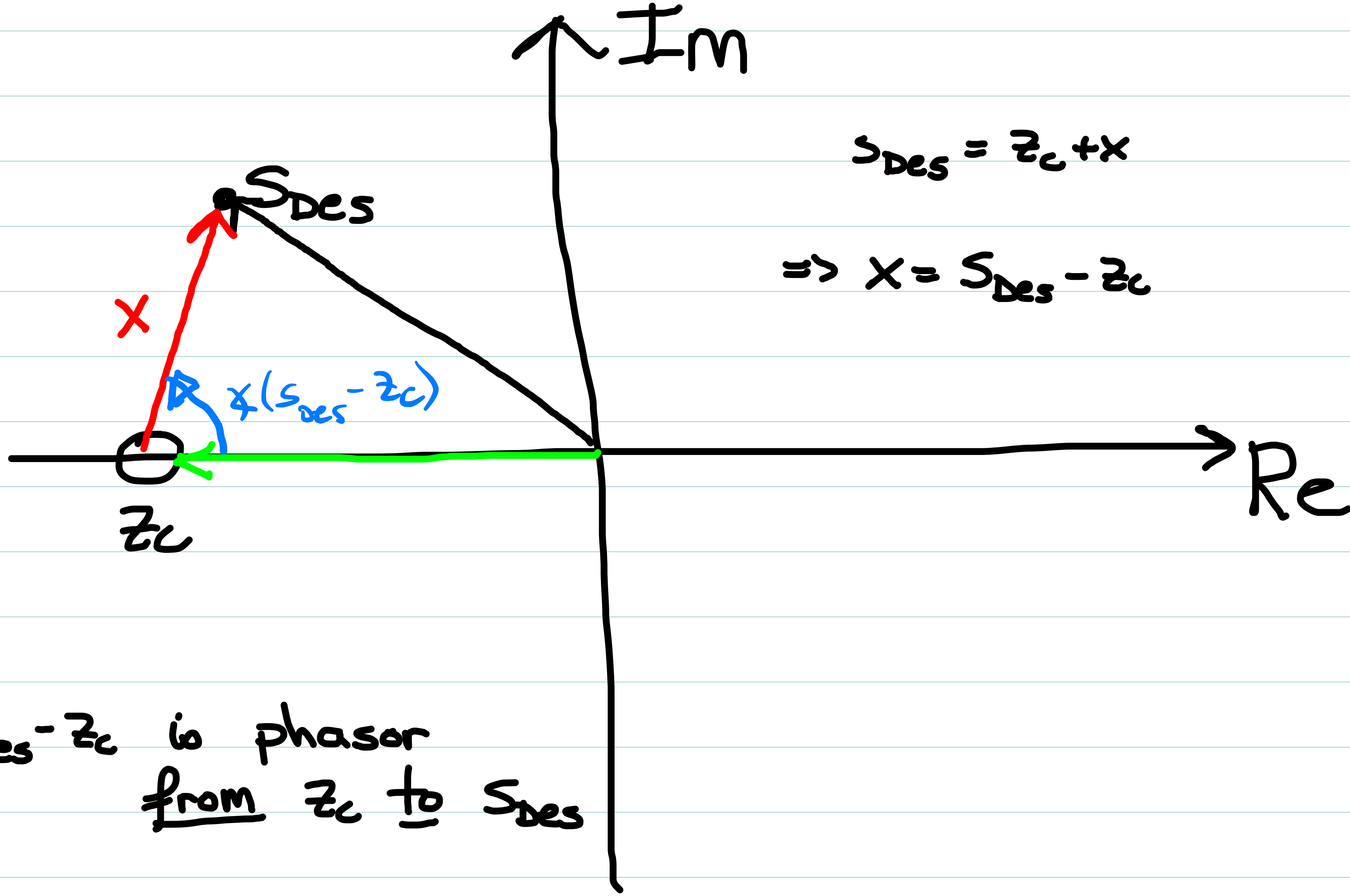
$$\Rightarrow \varphi_{req} = 180° - 116.56° = 63.43° \text{ here}$$

Assume initially: $H(s) = K(s - z_c)$, $K > 0$

(Not implementable: for illustration only!)

Then we need $\measuredangle(s_{Des} - z_c) = 64.43°$

Visualization – Phasor Interpretation

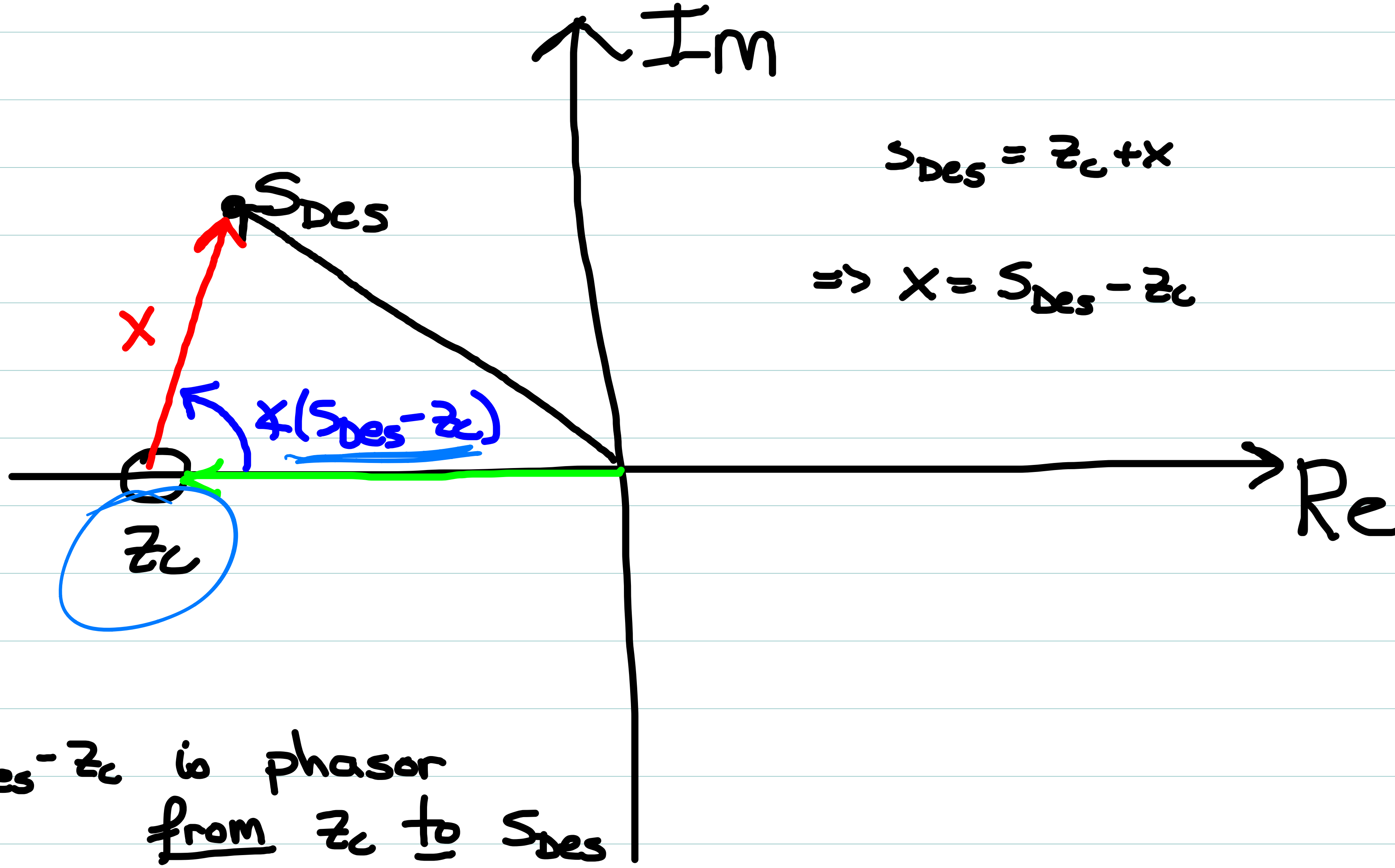


$s_{Des} = z_c + x$

$\Rightarrow x = s_{Des} - z_c$

$s_{Des} - z_c$ is phasor
<u>from</u> z_c <u>to</u> s_{Des}

Visualization – Phasor Interpretation



$s_{Des} = z_c + x$

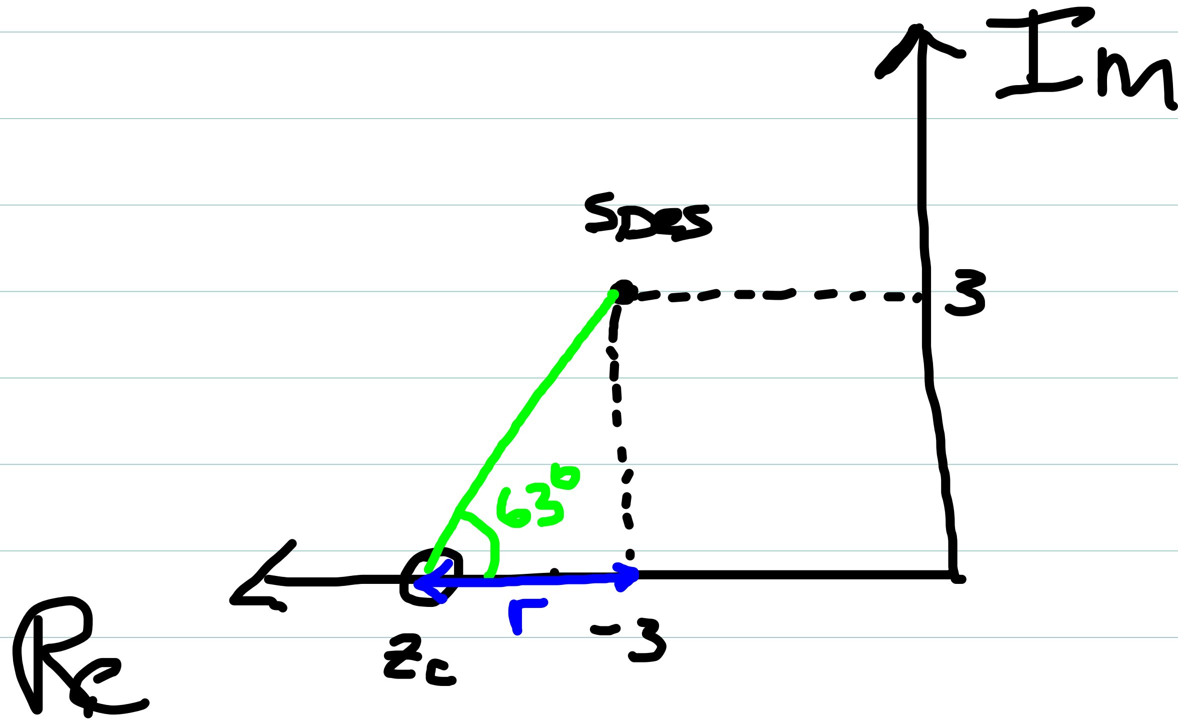
$\Rightarrow x = s_{Des} - z_c$

$s_{Des} - z_c$ is phasor <u>from</u> z_c <u>to</u> s_{Des}

<u>Note</u>: unlike Bode designs we can get <u>up to +180°</u> at s_{Des} from a <u>single</u> zero.

Example cont'd

If we need $\measuredangle(s_{Des} - z_c) = 63.43°$ at $s_{Des} = -3+3j$:



$$\tan(63.43°) = \frac{3}{r} \Rightarrow r = 1.5$$

$$\Rightarrow z_c = -4.5$$

Example, cont'd

So $H(s) = K(s+4.5)$ and then

$$L_o(s) = \frac{3(s+4.5)}{s(s+2)}$$

$$|L_o(s_{Des})| = 3/4$$ (Matlab: abs(evalfr(Lo, -3+3j)))

so $K = 4/3$.

Check:

$$T(s) = \frac{4(s+4.5)}{s^2+2s+4(s+4.5)} = \frac{4(s+4.5)}{s^2+6s+18} \checkmark$$

roots at $-3 \pm 3j$

Notes

1.) To be implementable $H(s)$ needs a pole. Choose pole p_c so that

$$\measuredangle(s_{Des} - p_c) \approx 5^\circ$$

Then $\measuredangle H(s_{Des}) = \measuredangle(s_{Des} - z_c) - \measuredangle(s_{Des} - p_c) = \measuredangle(s_{Des} - z_c) - 5^\circ$

Add $+5^\circ$ to φ_{req} to account for required pole.

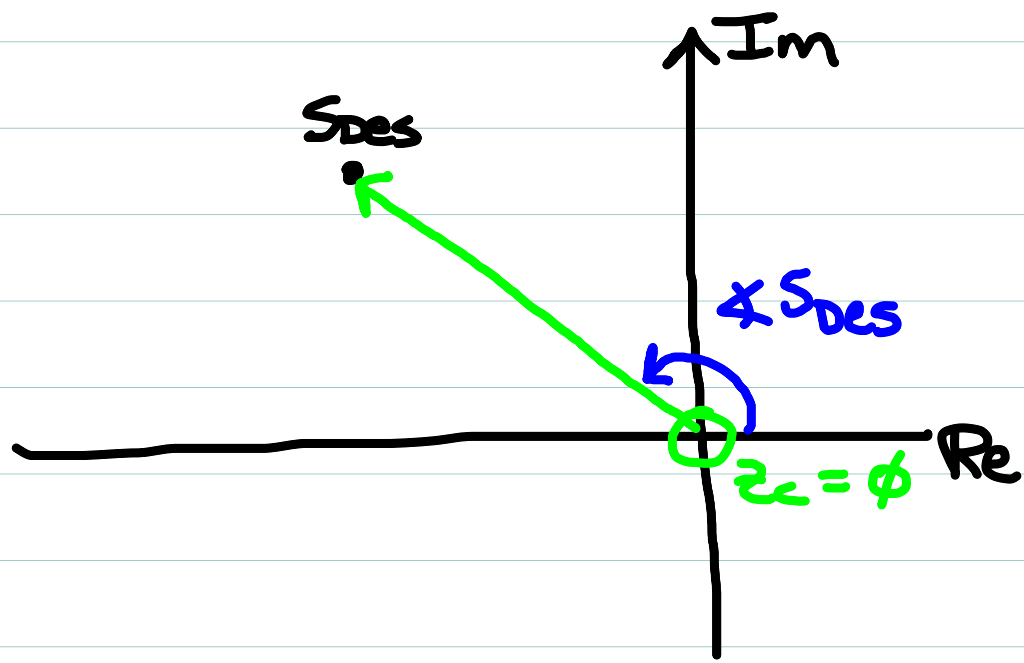
("β-minimizing" principle is quite messy here).

2.) Keep $\varphi_{req} < 90^\circ$, pref. below $60^\circ - 70^\circ$, or else zero will be closer to imag axis than s_{Des}, creating substantial additional overshoot. "Split" large φ_{req} over multiple zeros if necessary.

Notes (cont).

3.) Do not choose z_c in RHP! (We'll see why later)

$\Rightarrow$ places practical limit on maximum angle contribution from a zero



(effictive max for $z_c \leq \phi$)

Notes (cont)

4.) Design method guarantees s_{Des} is a CL pole, but says nothing about location of other CL poles.

$\boxed{\text{These might actually be unstable!}}$

Suppose: $G(s) = \dfrac{2}{s^2(s+1)}$, $s_{Des} = -2 + 0j$

$G(-2) = -1/2 \Rightarrow \varphi_{req} = \emptyset \Rightarrow H(s) = K > \emptyset$ sufficient

$K = \dfrac{-1}{-1/2} = 2$ and here

$$T(s) = \frac{4}{s^3 + s^2 + 4}$$

With poles at $\boxed{-2}$, $\boxed{\frac{1}{2} \pm \frac{4}{3} j}$ ← unstable!

To use these ideas effectively as a design tool, we must have <u>some</u> idea where the <u>other</u> poles of $T(s)$ will be; i.e. at least if they are stable.

Requires us to more generally understand <u>all</u> <u>possible</u> solutions of $1+L(s)=\emptyset$

Or, equivalently, the "locus" of points in the complex plane which satisfy the angle condition(s):

$$\measuredangle L_o(s) = (1+2\ell)180^\circ \qquad (\text{if } K>\emptyset)$$

$$\measuredangle L_o(s) = (2\ell)180^\circ \qquad (\text{if } K>\emptyset).$$

"Root Locus" Method for CL pole prediction

Set up: $L(s) = K\left[\frac{N(s)}{D(s)}\right]$

$\Rightarrow$ $\mathrm{Deg}\{N(s)\} = m$; m zeros z_i such that $N(z_i) = \emptyset$

$$N(s) = (s-z_1)(s-z_2)\cdots(s-z_m) = \prod_{i=1}^{m}(s-z_i)$$

$\Rightarrow$ $\mathrm{Deg}\{D(s)\} = n$; n poles p_k such that $D(p_k) = \emptyset$

$$D(s) = (s-p_1)(s-p_2)\cdots(s-p_n) = \prod_{k=1}^{n}(s-p_k)$$

$\Rightarrow$ $n \geq m$: no more zeros than poles

$\Rightarrow$ Characteristic equation: s is a CL pole if

$$1 + L(s) = \emptyset$$

Basic Observations

$$1+L(s)=\phi \Rightarrow 1+K\left[\frac{N(s)}{D(s)}\right]=\phi$$

$$\Rightarrow D(s)+KN(s)=\phi$$

This is an n^{th} order polynomial equation to define CL poles:

$\boxed{\Rightarrow \text{There are } n \text{ CL poles, same number as OL poles}}$

Consider limit as $K \to \phi$. Then CE becomes: $D(s)=\phi$

$\Rightarrow$ same eq'n as defines OL poles.

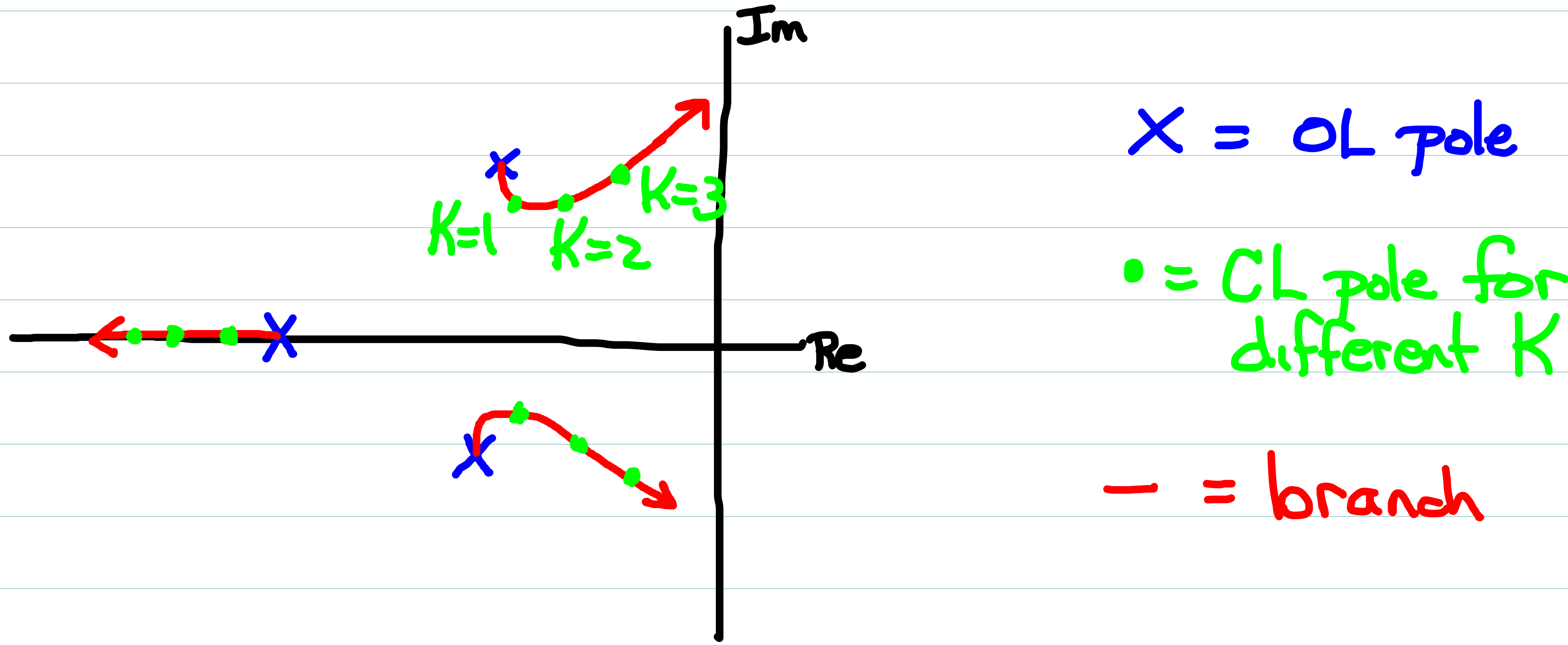
$\boxed{\Rightarrow \text{In low gain limit, } K\to\phi\text{, the CL poles are same as OL poles}}$

Varying K

⇒ As K changes, the CL pole locations migrate away from OL poles

⇒ Each CL pole location traces out a continuous curve starting at an OL pole. These curves are called branches.

⇒ Since there are n CL poles, there are n branches



Symmetry

⇒ Recall that complex roots of polynomial equations occur in <u>conjugate pairs</u>.

⇒ If $s \in \mathbb{C}$ satisfies $1 + L(s) = \emptyset$, so also $\bar{s}$ satisfies $1 + L(\bar{s}) = \emptyset$.

⇒ CL pole locations are symmetric about <u>real</u> Axis.

⇒ $\boxed{\text{Branches of CL pole loci are symmetric ("mirror image") about real Axis.}}$

⇒ Can we predict branch behavior as $|K|$ increases?

High gain limit: $|K| \to \infty$

Recall CL poles satisfy $D(s) + KN(s) = \emptyset$

Equivalently if $K \neq \emptyset$:

$$N(s) + \left[\frac{1}{K}\right] D(s) = \emptyset$$

and as $|K| \to \infty$ we have: $N(s) = \emptyset$

$\Rightarrow$ As $|K| \to \infty$, the CL poles coincide with OL zeros!

$\Rightarrow$ Branches terminate at OL zeros!

$\Rightarrow$ OL zeros "attract" CL poles to them in high gain limit

$\Rightarrow$ RHP zeros in $L(s)$ are dangerous!

High gain limit, cont

⇒ n CL poles (branches), but only $m \leq n$ OL zeros.

⇒ What happens to other $n-m$ CL poles (branches)?

⇒ *The remaining $n-m$ branches asymptote to infinity*

⇒ But how? Depends on sign of K. Suppose for simplicity we take $K > 0$.

⇒ Recall "angle condition" for $K > 0$:

if s is a possible CL pole, then

$$\measuredangle L(s) = (1+2\ell)180^\circ \quad \text{(odd multiple of } 180^\circ\text{)}.$$