ENAE311H Homework 1

Due: Tuesday, September 10th by 5pm (upload online)

1. Consider an infinitely thin flat plate with a 1 m chord at an angle of attack of 15° to an oncoming flow. The pressure distributions on the upper and lower surfaces are given by $p_u = 2 \times 10^4(x-1) + 2.7 \times 10^4$ and $p_l = 1 \times 10^4(x-1) + 1.1 \times 10^5$, where x is the distance from the leading edge along the chord; the shear stress distributions are $\tau_u = 144x^{-0.3}$ and $\tau_l = 360x^{-0.3}$. Here, the units of p and τ are N m^{-2}. Calculate the normal and axial forces, the lift and drag, moments about the leading edge and quarter chord, all per unit span, as well as the center of pressure.

2. A series of experiments is performed on a two-dimensional airfoil in which the lift, drag and moment coefficients (the latter about the quarter chord) are measured over a range of angles of attack from 0 to 10°. The lift coefficient curve is found to be well approximated by the equation

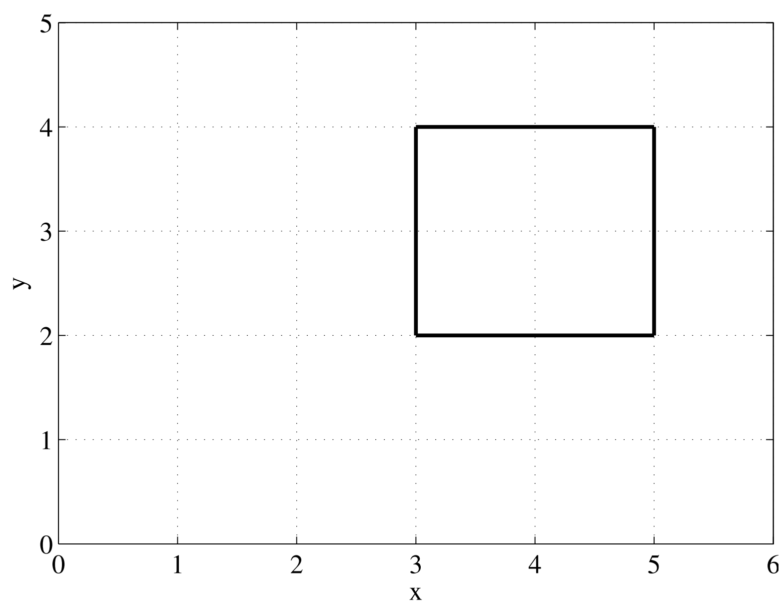
$$c_l = 0.2 + 6\alpha, \tag{1}$$

where α is the angle of attack in radians. The drag is found to be well approximated by

$$c_d = 0.006 + 0.3\alpha^2 \tag{2}$$

while $c_{m,c/4}$ increases linearly from -0.04 for α=0 to -0.03 for α=10°. Make a plot of x_{cp}/c as a function of α for this airfoil.

3. Given the pressure distribution $p(x,y) = x^2 + 2y^2 + 50$ acting on the body shown below, calculate the following:
a) The pressure force per unit depth on each face of the body.
b) The magnitude and direction of the net force per unit depth on the body.
c) The magnitude and direction of the net moment per unit depth on the body acting about the origin of the coordinate system.
d) The location of the object's center of pressure with respect to the origin (Note: in two dimensions, the center of pressure as we defined it in class is not unique - for this problem, calculate the x location based on the moment contributions from the two y-normal faces, and vice versa).



4. Consider the steady flow of viscous incompressible fluid through a smooth, square pipe. The friction between the pipe wall and the fluid will result in a drop in pressure, Δp, from one end of the pipe to the other that will depend on the length and width of the pipe, l and d, the density and coefficient of viscosity of the fluid, ρ and μ, and the flow velocity, V, i.e., $\Delta p = f(l, d, \rho, \mu, V)$. Use the Buckingham Pi theorem to show that:

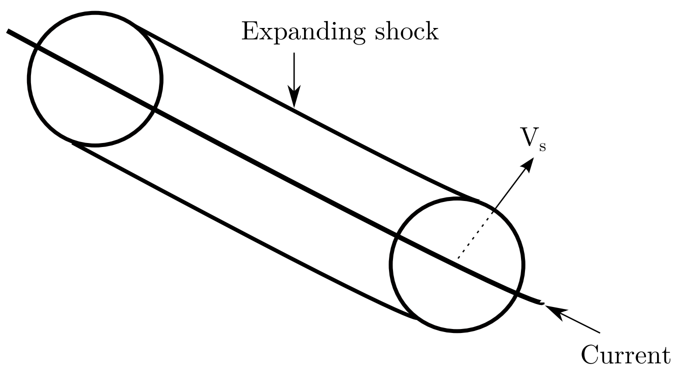
$$\frac{\Delta p}{\rho V^2} = F(Re, l/d), \tag{3}$$

where Re is the Reynolds number, $Re = \rho d V/\mu$ (note that we could also define the Reynolds number based on the pipe length rather than the pipe width). If we extended the analysis to include pipe roughness (with a characteristic roughness height ϵ), how would the above equation be modified?

5. a) Consider a long cylindrical wire that suddenly has a large electrical current passed through it, such that the wire explodes, depositing an energy per unit depth, E, into the surrounding air. This causes a cylindrically expanding shock wave to propagate outwards from the initial position of the wire. If E is large, the initial propagation rate of this shock is independent of the pressure and temperature of the gas (this is known as the strong-shock limit, which we will return to later in the course); the only gas property that is important then is the density, ρ. Using dimensional analysis, show that in this case the shock velocity, V_s, decays with time t as

$$V_s = c\left(\frac{E}{\rho}\right)^{1/4} t^{-1/2}, \tag{4}$$

where c is a constant.



b) In fact, other thermodynamic properties of the quiescent air enter into the problem through the specific heats at constant volume and pressure, c_v and c_p. In this case, use the Buckingham Pi theorem (you can assume the results from part a) to show that

$$\frac{V_s^4 t^2 \rho}{E} = F(\gamma), \tag{5}$$

where $\gamma = c_p/c_v$.