University of Maryland at College Park

DEPT. OF AEROSPACE ENGINEERING

ENAE 432: Aerospace Control Systems

Problem Set #11

Question 1:

The transfer function for the dynamics of a particular system is given by

$$G(s) = \frac{6}{(s+3)^2}$$

To accurately track at least constant y_d, you decide try the PI compensator $H(s) = K_p + (K_i/s)$

a.) Relative to the system poles there are 3 possible locations for the location of the zero in the compensator: i.) to the left of the plant poles at -2; ii.) between the poles at -2 and the origin, and finally iii.) to the right of the origin. Sketch by hand, as accurately as possible, the locus of possible closed-loop poles as $K > 0$ increases in *each* of these three cases. Determine the real axis portions of the locus, the asymptotes and their intercept.

b.) One of the possibilities above will result in the closed-loop system being unstable for any K. Identify which case (i.-iii.) and explain why.

c.) The other two cases show that the closed-loop system with this compensator can be stable for high K provided that the zero is placed appropriately. Identify a simple root-locus derived constraint on the location of the zero which will guarantee a stable closed-loop system for large values of K. Verify that this condition is also equivalent to ensuring that $L(s)$ has positive phase margin for any K.

d.) Determine values of K_p and K_i so that $T(s)$ is an ideal second-order transfer function without zeros, whose poles have damping ratio $\sqrt{2}/2$ and the fastest possible settling time.

Question 2:

For the system

$$G(s) = \frac{5(s-1)}{s-6}$$

a.) Use a root locus argument to show that it is possible to stabilize this system using a compensator that has only a gain and a single *unstable* pole (but *without* using unstable cancellation!) Sketch the resulting locus. What constraint must the compensator pole satisfy to ensure $T(s)$ can be stable for a nontrivial range of controller gain?

b.) Specify the complete details for the design of such a compensator $H(s)$ that ensures $T(s)$ has double real poles at -2. Show the complete resulting root locus for your design.

c.) Determine the input $u(t)$ your controller would produce for this system when $y_d(t)$ is a unit step and the compensator in b) is used. Show (analytically and numerically) that $u(t)$ is bounded, find its peak magnitude and its (finite) steady-state value.

d.) Determine the equations that describe both the ZOH and the Tustin discretization of the compensator you designed in b).

Question 3:

a.) Find the discrete ZOH state-space equations that correspond to the compensator

$$H(s) = \frac{15(3s+1)^5}{(s+1)^3(s^2+2s+10)}$$

where the sample interval is $T_s = 0.04$ seconds (25 Hz sample rate).

Use the Matlab command

```
[Ah,Bh,Ch,Dh] = ssdata(canon(H))
```

to get a state-space model for $H(s)$ (Matlab will choose the so-called "block modal" form for this). Then use the `expm` function to find the `Ad` matrix, and finally carry out rest of the linear algebra needed to determine the remaining components of the discretization.

b.) Repeat a) but using the Tustin discretization. You may use the `c2d` function for this, instead of attempting the computation manually.

Question 4:

Suppose that

$$G(s) = \frac{2}{s^2(s^2+3)}$$

a.) Use a root locus sketch to argue that this system cannot be stabilized with a proportional controller. HINT: calculate the angle of departure from the complex poles, and from the poles at the origin, in addition to the usual asymptotes, etc.

b.) Suppose that we desire to have closed-loop poles at $-1 \pm j$, and any other closed-loop poles should be repeated real at -4. Show that a compensator of the form

$$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

can accomplish this, and determine the required $H(s)$ in ZPK format. Compute the corresponding $T(s)$ and determine its poles to verify the success of your design. Where are the zeros of $T(s)$?

c.) Repeat b), but instead using

$$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

d.) Plot the step responses of $T(s)$ for the two possible control strategies above. Are either particularly satisfactory by the usual metrics? Why do you suppose this is the case, given that we'd expect the closed-loop poles we designed for to have satisfactory transients?

e.) Apart from the observations in d), is there any practical reason to prefer the design in c) over that in b)? Explain your reasoning.