ENAE 404 - 0101: Homework 01

2BP

Due on February 11th, 2025 at 11:59 PM

Dr. Barbee, 09:30

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Problem 1:

Considering the orbit of Didymos from HW00:

1. Plot the specific energy of the orbit as a function of time.
2. Using the subplot function, plot the specific angular momentum magnitude and x, y, z components as a function of time.
3. Explain why the previous two plots indicate that your 2BP propogator is working properly.

Solution

Part A

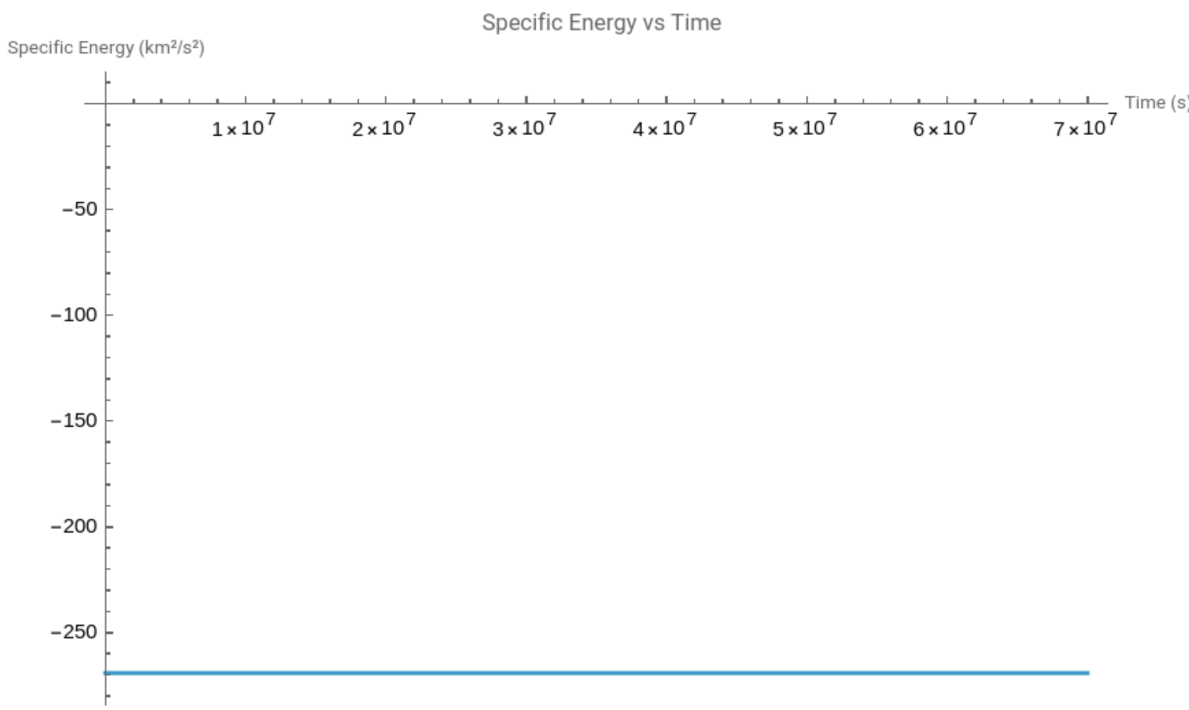

Figure 1: Didymos' Specific Energy vs. Time

```
(* Interpolated solutions for position and velocity *)
rDsol[t_] := Evaluate[rD[t] /. solDidymos[[1]]];
vDsol[t_] := Evaluate[vD[t] /. solDidymos[[1]]];

(* Specific energy: kinetic plus potential *)
energy[t_] := 1/2 Norm[vDsol[t]]^2 - muSun/Norm[rDsol[t]];

(* --- Plot the specific energy as a function of time --- *)
energyPlot = Plot[
    energy[t], {t, 0, tmaxDidymos},
    PlotRange -> All,
    AxesLabel -> {"Time (s)", "Specific Energy (km\.b2/s\.b2)"},
    AxesOrigin -> {0,0},
    PlotLabel -> "Specific Energy vs Time",
    ImageSize -> Large
```

Problem 1 continued on next page...

```
];

(* --- Display the plot --- *)
Print[energyPlot];
```

Part B

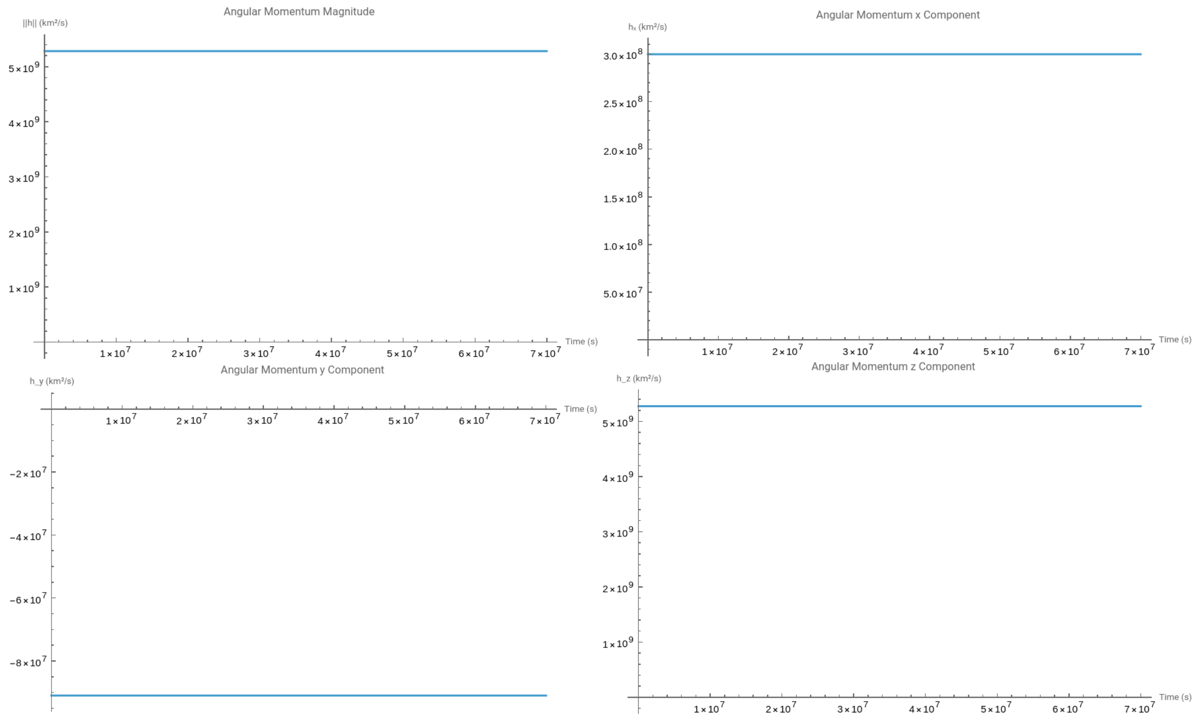

Figure 2: Didymos' Components of Specific Angular Momentum vs. Time

```
(* Specific angular momentum vector and its components *)
h[t_] := Cross[rDsol[t], vDsol[t]];
hMag[t_] := Norm[h[t]];
hX[t_] := h[t][[1]];
hY[t_] := h[t][[2]];
hZ[t_] := h[t][[3]];

(* --- Plot the angular momentum quantities in subplots --- *)
hMagPlot = Plot[
    hMag[t], {t, 0, tmaxDidymos},
    PlotRange -> All,
    AxesLabel -> {"Time (s)", "||h|| (km\.b2/s)"},
    AxesOrigin -> {0,0},
    PlotLabel -> "Angular Momentum Magnitude",
    ImageSize -> Large
];

hXPlot = Plot[
    hX[t], {t, 0, tmaxDidymos},
    PlotRange -> All,
    AxesLabel -> {"Time (s)", " h    (km\.b2/s)"},
    AxesOrigin -> {0,0},
    PlotLabel -> "Angular Momentum x Component",
```

```
    ImageSize -> Large
];

hYPlot = Plot[
    hY[t], {t, 0, tmaxDidymos},
    PlotRange -> All,
    AxesLabel -> {"Time (s)", "h_y (km\.b2/s)"},
    AxesOrigin -> {0,0},
    PlotLabel -> "Angular Momentum y Component",
    ImageSize -> Large
];

hZPlot = Plot[
    hZ[t], {t, 0, tmaxDidymos},
    PlotRange -> All,
    AxesLabel -> {"Time (s)", "h_z (km\.b2/s)"},
    AxesOrigin -> {0,0},
    PlotLabel -> "Angular Momentum z Component",
    ImageSize -> Large
];

(* Arrange the angular momentum plots in a 2x2 grid *)
angularMomentumGrid = GraphicsGrid[
    {
        {hMagPlot, hXPlot},
        {hYPlot, hZPlot}
    },
    Spacings -> {2, 2}
];

(* --- Display the plots --- *)
Print[angularMomentumGrid];
```

Problem 2:

For what value(s) of the true anomaly is the flight path angle zero?

1. For a circle?
2. For an ellipse?
3. For a hyperbola?
4. For a parabola?

Solution

Part A

$$\forall \nu \in \mathbb{R} \quad \square$$

Part B

$$\nu = 0^\circ, 180^\circ \quad \square$$

Part C

$$\nu = 0^\circ \quad \square$$

Part D

$$\nu = 0^\circ \quad \square$$

Problem 3:

The computer in Luke Skywalker's X-Wing is on the fritz. He sees Earth outside his window, and he knows his current altitude is 6×10^3 km, his velocity is $8.5 \, \frac{\text{km}}{\text{s}}$, and his flight path angle is $0.5°$. For this problem and all other problems involving Earth orbits in this assignment, use $\mu = 3.986 \times 10^5 \, \frac{\text{km}^3}{\text{s}^2}$ and Earth Radius $= 6378$ km.

1. What type of conic is Luke's current orbit?
2. What is the semi-major axis of his orbit?
3. What is the specific angular momentum magnitude of his orbit?
4. What is the eccentricity of his orbit?
5. What is the radius of periapsis of his orbit?

Solution

Part A

$$\begin{aligned}
\epsilon &= \frac{v^2}{2} - \frac{\mu}{r} \\
\epsilon &= 3.923 \, \frac{\text{kJ}}{\text{kg}} \\
\epsilon &> 0 \quad \square
\end{aligned}$$

$\therefore$ The orbit is hyperbolic.

Part B

$$\begin{aligned}
\epsilon &= \frac{-\mu}{2a} \implies a = \frac{-\mu}{2\epsilon} \\
a &= -50\,803 \, \text{km} \quad \square
\end{aligned}$$

Part C

$$\begin{aligned}
h &= rv \cos \gamma \\
h &= 105\,209 \, \frac{\text{km}^2}{\text{s}} \quad \square
\end{aligned}$$

Part D

$$\begin{aligned}
p &= a\left(1 - e^2\right) = \frac{h^2}{\mu} \\
\implies e &= \sqrt{1 - \frac{h^2}{a\mu}} \\
e &= 1.244 \quad \square
\end{aligned}$$

Part E

$$\begin{aligned}
r_p &= a\left(1 - e\right) \\
r_p &= 12\,377 \, \text{km} \quad \square
\end{aligned}$$

Problem 4:

Consider an Earth-orbiting satellite with a semi major axis of 2×10^4 km and an eccentricity of 0.4.

1. Calculate the radius of the satellite at a true anomaly of 30°.
2. Calculate the radius of the satellite at a true anomaly of 330°.
3. Calculate the velocity of the satellite at a true anomaly of 30°.
4. Calculate the velocity of the satellite at a true anomaly of 330°.
5. Calculate the flight path angle of the satellite at a true anomaly of 30°.
6. Calculate the flight path angle of the satellite at a true anomaly of 330°.
7. What is the radius of apoapsis of this orbit?
8. What is the velocity at apoapsis of this orbit?

Solution

Part A

$$\begin{aligned} r &= \frac{a\left(1-e^2\right)}{1+e\cos\nu} \\ r &= 12\,478\,\mathrm{km} \quad \square \end{aligned}$$

Part B

$$\begin{aligned} &r\left(30^\circ\right) = r\left(330^\circ\right) \\ \implies &r = 12\,478\,\mathrm{km} \quad \square \end{aligned}$$

Part C

$$\begin{aligned} v &= \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}} \\ v &= 6.63\,\frac{\mathrm{km}}{\mathrm{s}} \quad \square \end{aligned}$$

Part D

$$\begin{aligned} &r\left(30^\circ\right) = r\left(330^\circ\right) \\ \implies &v\left(30^\circ\right) = v\left(330^\circ\right) \\ &\implies v = 6.63\,\frac{\mathrm{km}}{\mathrm{s}} \quad \square \end{aligned}$$

Part E

$$\begin{aligned} r_p &= a\left(1-e\right) \\ r_p &= 12 \times 10^3\,\mathrm{km} \end{aligned}$$

Problem 4 continued on next page. . .

$$v_p = \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}}$$
$$v_p = 6.819 \, \frac{\text{km}}{\text{s}}$$
$$\gamma = \arccos\left(\frac{r_p v_p}{r v}\right)$$
$$\gamma = 8.43^\circ \quad \square$$

Part F

$$r\left(30^\circ\right) = r\left(330^\circ\right)$$
$$\implies v\left(30^\circ\right) = v\left(330^\circ\right)$$
$$\implies \gamma\left(30^\circ\right) = \gamma\left(330^\circ\right)$$
$$\implies \gamma = 8.43^\circ \quad \square$$

Part G

$$r_a = a\left(1 + e\right)$$
$$r_a = 28 \times 10^3 \, \text{km} \quad \square$$

Part H

$$v_a = \sqrt{\frac{2\mu}{r_a} - \frac{\mu}{a}}$$
$$v_a = 2.923 \, \frac{\text{km}}{\text{s}} \quad \square$$

Problem 5:

Consider an Earth-centered orbit with a radius of periapsis of 1×10^4 km and accentricity of 1.

1. What is the velocity at periapsis?
2. What is the radius of apoapsis?
3. What type of conic is this orbit?

Solution

Part A

$$v_p = \lim_{a \to \infty} \sqrt{\frac{2\mu}{r_p} - \frac{\mu}{a}}$$
$$v_p = 9.929\,\frac{\text{km}}{\text{s}}$$
$$v = 8.929\,\frac{\text{km}}{\text{s}} \quad \square$$

Part B

For parabolic orbits, the craft will escape the gravitational pull of the planet, and thus there is neither an apoapsis nor a radius of apoapsis.

$$r_a = \infty \quad \square$$

Part C

$$e = 1$$

$\therefore$ The orbit is parabolic.