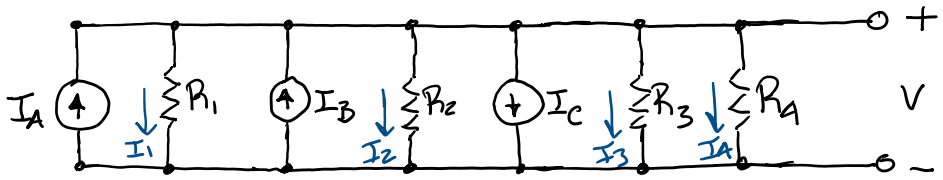
DC circuit Analysis



How many nodes? Only 2.

Apply KCL to the top node

$$I_A - I_1 + I_B - I_2 - I_C - I_3 - I_4 = 0$$

$$\Rightarrow I_A + I_B - I_C = I_1 + I_2 + I_3 + I_4$$

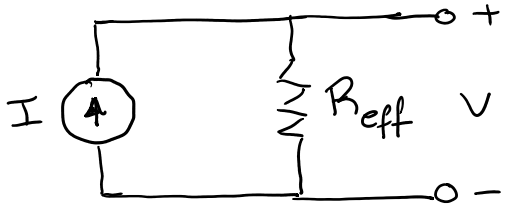
Apply Ohm's Law

$$\underbrace{I_A + I_B - I_C}_{\text{net current flow, } I} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \frac{V}{R_4}$$

$$= V \underbrace{\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right]}_{R_{eff}^{-1}}$$

$$= \frac{V}{R_{eff}}$$

Equivalent circuit



$$I = \frac{V}{R_{eff}}$$

Resisters in parallel

Resisters in parallel

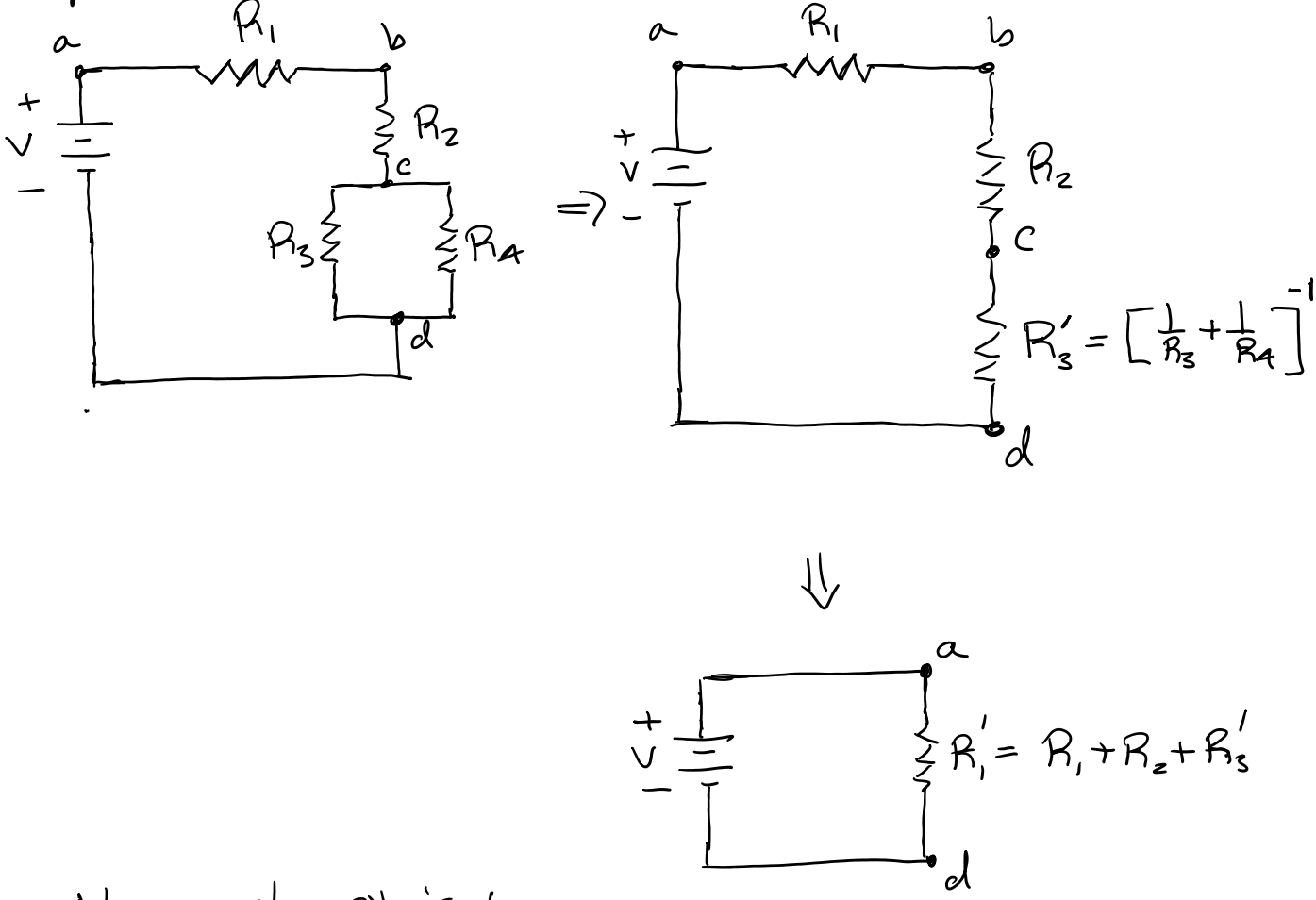
$$R_{eff} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \right]^{-1}$$

Can also show for resisters in series

$$R_{eff} = R_1 + R_2 + \cdots + R_N$$

This can be used for any circuit that is a hierarchy of node pairs.

Example:



Now apply Ohm's law,

$$V = IR_1' \Rightarrow I = \frac{V}{R_1'}$$

To find voltage across nodes, say nodes c and d,

To find voltage across nodes, say nodes c and d, apply KVL,

$$V_{cD} = V - V_{ab} - V_{bc}$$

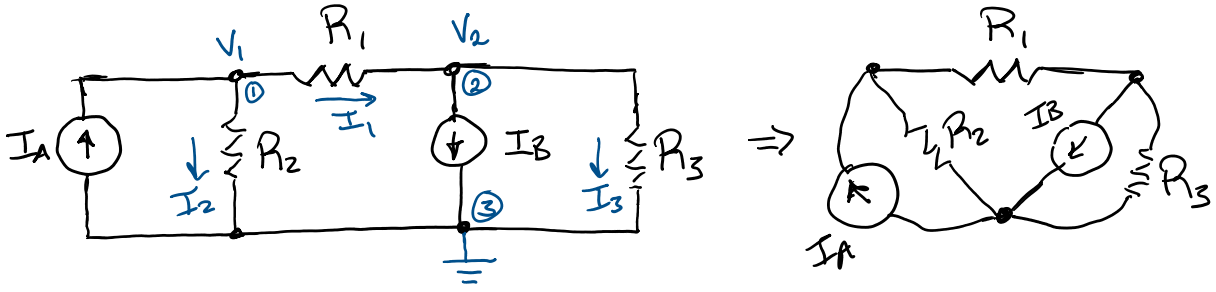
To find current flow through R_3 and R_4, just apply Ohm's law

$$V_{cd} = I_{R_3} R_3 \Rightarrow I_{R_3} = \frac{V_{cd}}{R_3}$$

Nodal Analysis

1. Identify a reference node in the circuit. Call this the "ground node". The voltage at the ground node will be assigned to be zero.
2. Apply KCL to each node in the circuit, except for one.
3. Solve the resulting system of equations.

Example:



3 nodes

Choose a ground node; choose node 3

Apply KCL to node 1

$$I_A - I_1 - I_2 = 0$$

Apply Ohm's Law

$$I_A - \frac{V_1 - V_2}{R_1} - \frac{V_1 - 0}{R_2} = 0$$

$$\Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_1 - \frac{1}{R_1} V_2 = I_A$$

Apply KCL to node 2,

$$I_1 - I_B - I_3 = 0$$

Apply Ohm's law

$$\frac{V_1 - V_2}{R_1} - I_B - \frac{V_2 - 0}{R_3} = 0$$

$$\Rightarrow \left(\frac{1}{R_1} + \frac{1}{R_3}\right) V_2 - \frac{1}{R_1} V_1 = -I_B$$

Now have 2 eq'n and 2 unknowns that we can solve for V_1 and V_2

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_1} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_A \\ -I_B \end{bmatrix}$$