Lecture 21: Polhode Plots

From last lecture, we knew that even though $T_c = 0$, the $\dot{\vec{w}} \neq 0$ (if $\vec{w}$ has nonzero components in more than 1 direction).

Ex. if $\vec{w} = [w_1, 0, 0] \Rightarrow \dot{\vec{w}} = 0$

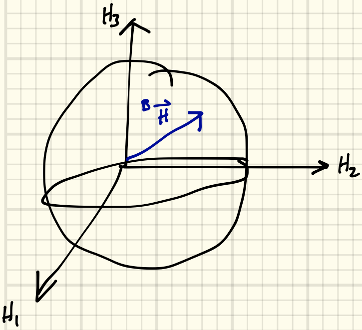
if $\vec{w} = [w_1, w_2, 0] \Rightarrow \dot{\vec{w}} \neq 0$

But, $\vec{w}$ can't be any random value, it must satisfy constraints on conservation of angular momentum magnitude & kinetic energy.

Our goal is to graphically investigate how $\vec{w}$ is changing.

Known: $H^2 = H_1^2 + H_2^2 + H_3^2$

$$\Rightarrow 1 = \frac{H_1^2}{H^2} + \frac{H_2^2}{H^2} + \frac{H_3^2}{H^2} \leftarrow \text{eqn for a sphere}$$



B/c H is conserved, ${}^B\vec{H}$ must point to a point on the surface of this sphere.

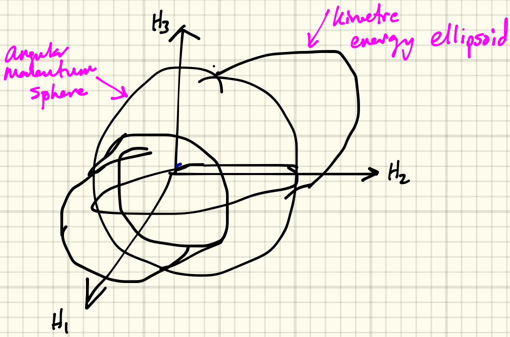
$\vec{H}$ contains information about $[I]$ & $\vec{w}$.

We know $[I]$ is constant in the body-fixed frame.

We can also rewrite the kinetic energy expression in terms of H:

$$1 = \frac{H_1^2}{2I_1T} + \frac{H_2^2}{2I_2T} + \frac{H_3^2}{2I_3T} \rightarrow \text{Eqn for ellipsoid: } 1 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$$

$\Rightarrow {}^B\vec{H}$ must also point to a point on the surface of the kinetic energy ellipsoid.



${}^B\vec{H}$ must point to an intersection between the KE ellipsoid & H sphere.

Polhode Plots

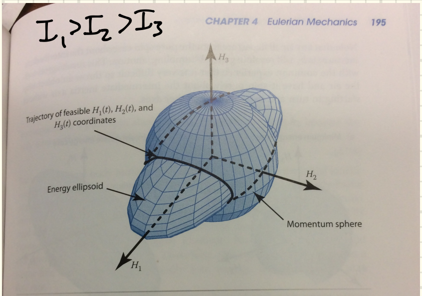


Fig. 4.10 General intersection of the momentum sphere and the energy ellipsoid.

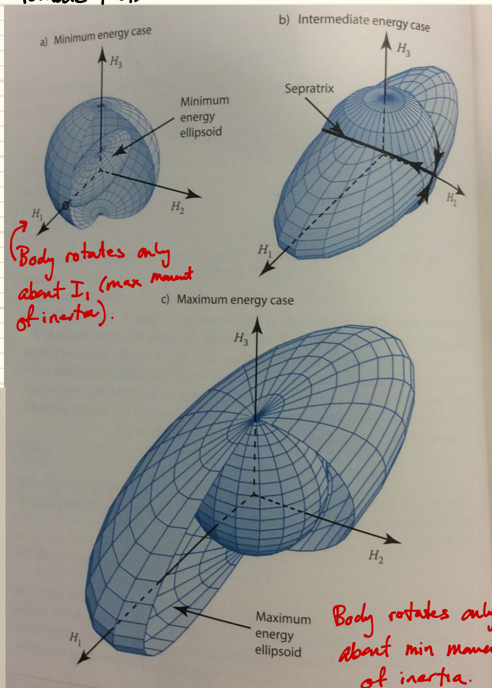


Fig. 4.11 Special cases of kinetic energy ellipsoid and momentum sphere intersections.

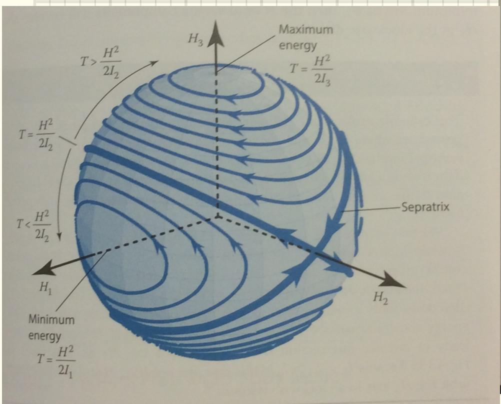


Fig. 4.12 Family of energy ellipsoid and momentum sphere intersections.

Review: $^I\vec{H}$ = constant

$^B H$ = constant (sphere), but $^B\vec{H} \neq$ constant

$^B[I]$ = constant b/c rigid body

$^B\dot{\vec{\omega}} \neq 0$ even though $\vec{L} = 0$ if $\vec{\omega}$ has nonzero components in more than 1 direction.

In the body-fixed frame, ω is transferred between components

Since $^I\vec{H}$ = constant & $^I\dot{\vec{\omega}} \neq$ constant $\Rightarrow$ $^I[I] \neq$ constant

$\Rightarrow$ the body is rotating

Polhode plots assume that the body-fixed frame is a principal axes

$^B\vec{H}$ must point to an intersection between the angular momentum sphere and KE ellipsoid.