Lecture 19: Flow through Converging-Diverging Nozzles and Supersonic Wind Tunnels

ENAE311H Aerodynamics I

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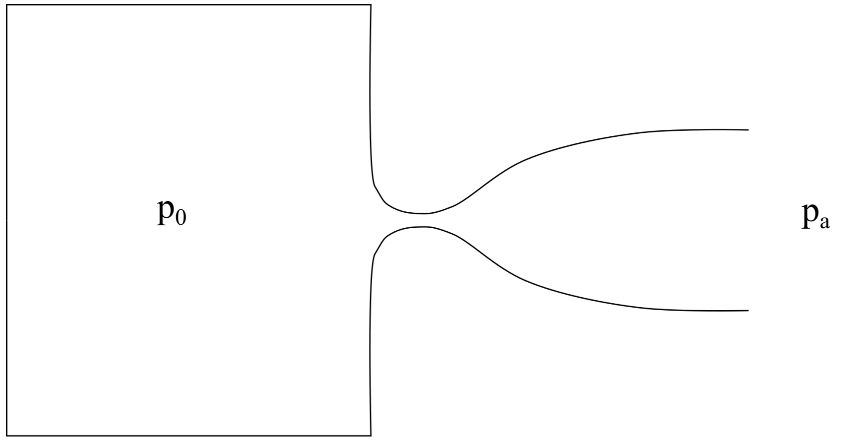
Subsonic solutions

Imagine we have a converging-diverging nozzle, supplied by a large reservoir (at pressure p_0) and exhausting into an ambient atmosphere (of pressure p_a).



If p_a is matched to p_0, there won't be any flow through the nozzle. If we begin to lower p_a, however, a subsonic flow will establish itself through the nozzle (including at the throat). Note the following:

- The nozzle exit pressure must match p_a, which fixes the Mach number throughout the rest of the nozzle via the area/Mach-number relationship (flow is isentropic throughout).
- However, A^* is now an imagined reference area ($A^* < A_t$) and is not achieved within the nozzle.

This situation will continue until the flow becomes exactly sonic at the throat, at which point

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = 0.528 \quad \text{for } \gamma = 1.4.$$

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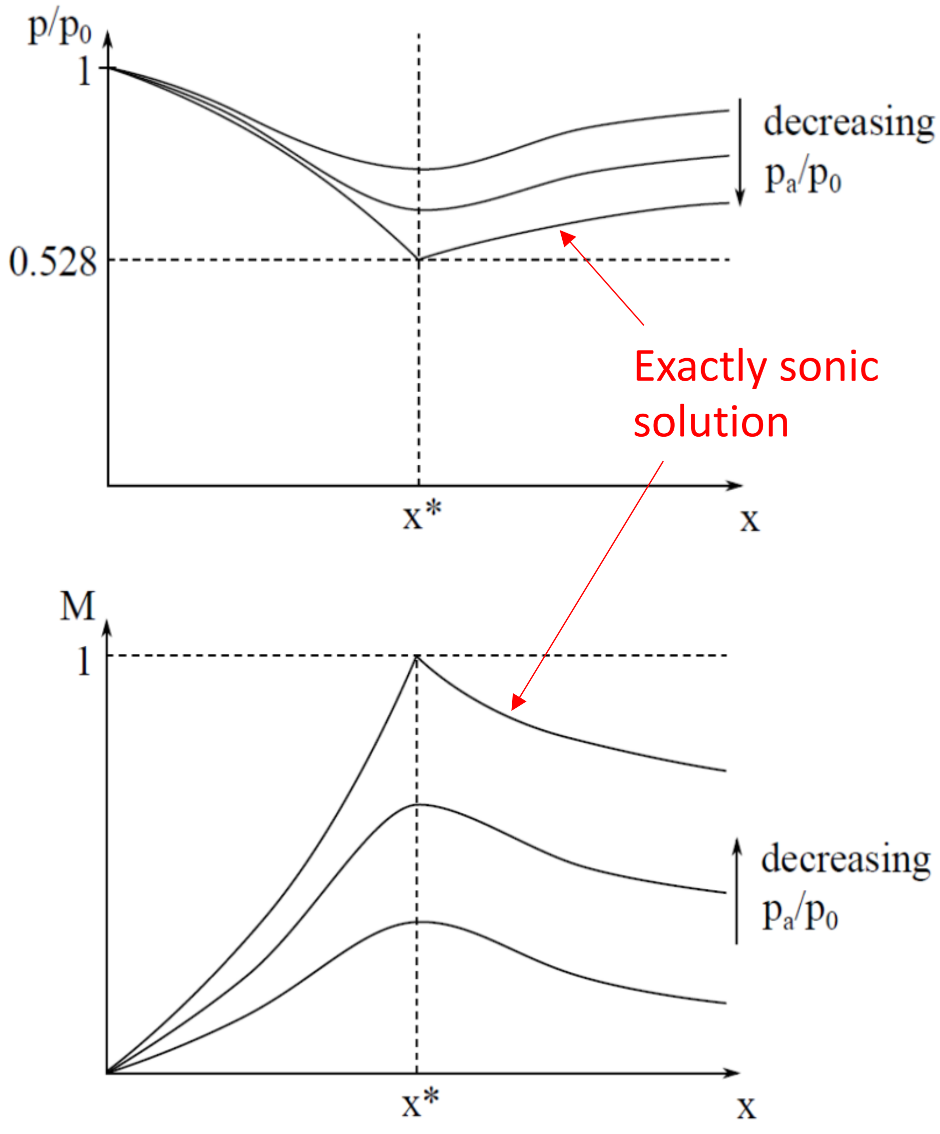


Choked solutions

Now imagine we decrease p_a further from that exactly choked point. There is no longer a solution for which the flow remains fully subsonic throughout the nozzle, so we must have supersonic flow downstream of the throat.

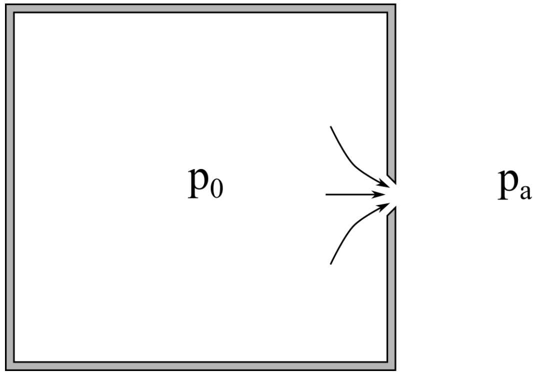
Note, however, that now there is no way for information to travel upstream from the exit to the throat, so any further reduction in p_a will not affect the flow between the reservoir and the throat

→ we say that the throat is "choked".

For a choked throat, the (maximum) mass flux through the nozzle (independent of p_a) is

$$\dot{m} = \rho^* u^* A^* = \rho^* a^* A_t$$

Note that we can also infer from these results the behavior of a pressurized reservoir discharging to an ambient atmosphere through an orifice (with minimum area at the exit). Provided $p_a < 0.528 p_0$, the flow at the exit will be sonic/choked, and the mass flux will be given by the relation above.



In particular, we will have

$$\dot{m}_{max} \propto \frac{p_0 A_t}{\sqrt{T_0}}.$$

Supersonic solutions

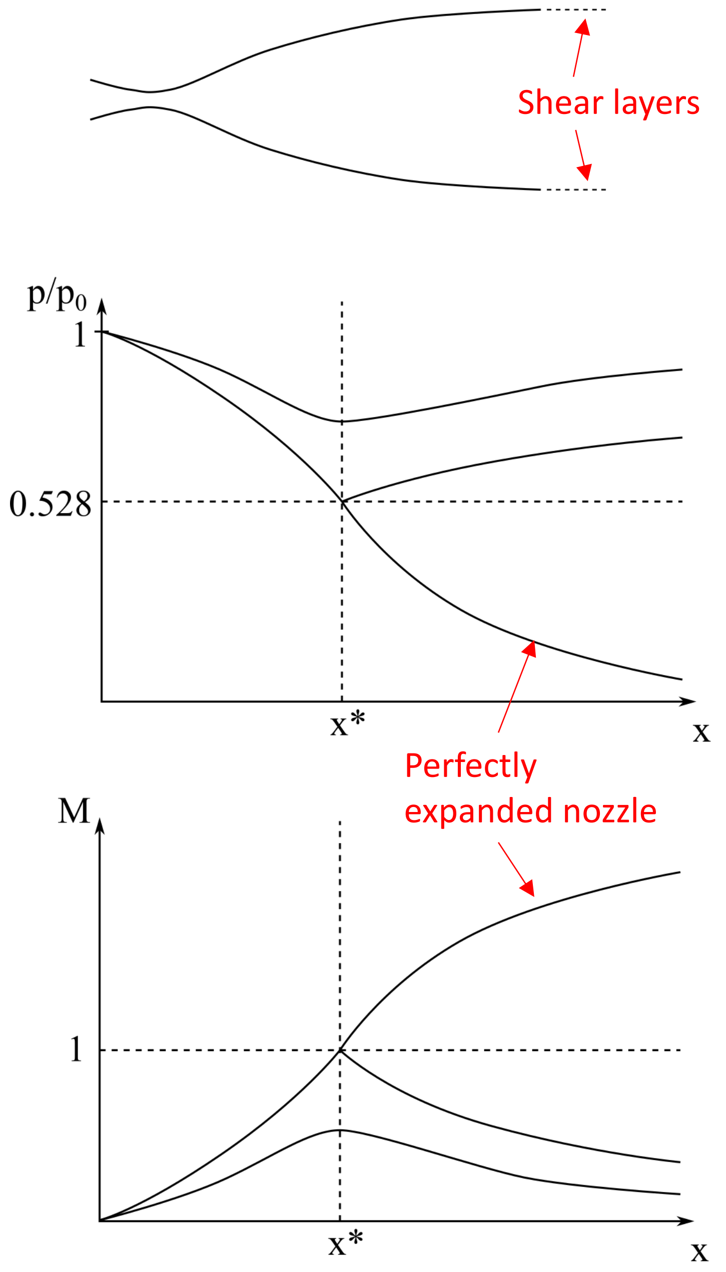
Returning to the converging/diverging nozzle, let us explore the range of solutions available with a sonic throat.

- If the ratio of ambient to reservoir pressure is exactly the value corresponding to the isentropic solution

$$\frac{p_a}{p_0} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\gamma/(\gamma-1)}$$

with M matching the value for A_e/A^* in the area/Mach-number relationship, then the flow will be smoothly varying and isentropic throughout the nozzle:

→ nozzle is "perfectly expanded"



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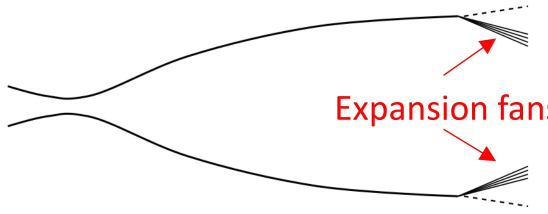

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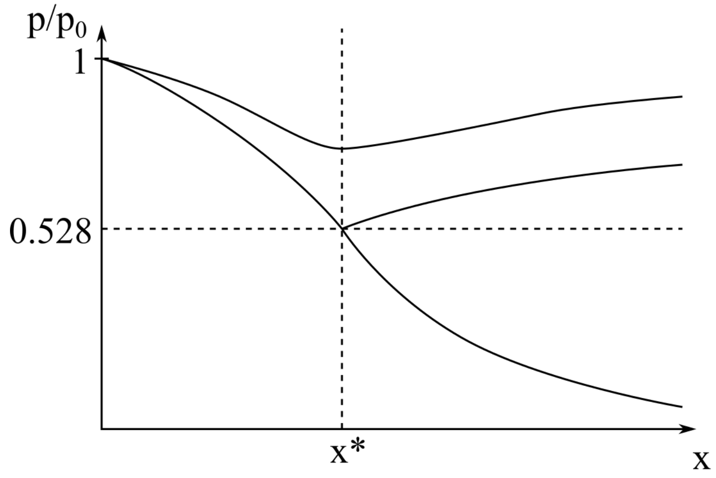
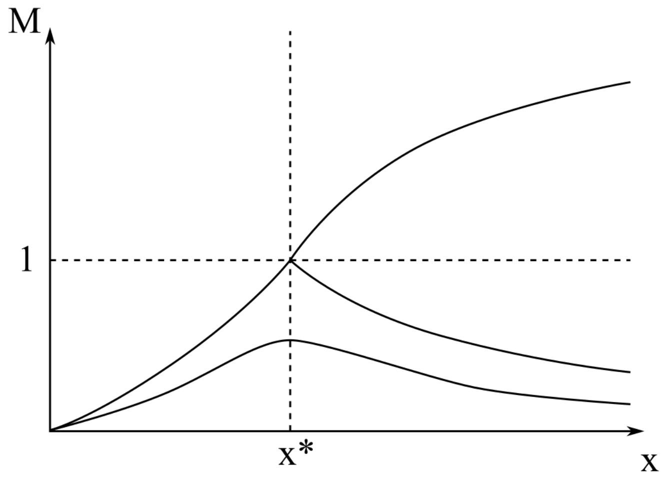
- If p_a is below this perfectly expanded value, expansion waves will form at the nozzle exit, with shear layers deflected outwards

→ nozzle is "under-expanded"





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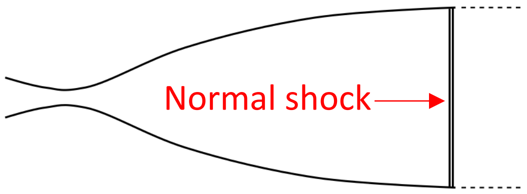

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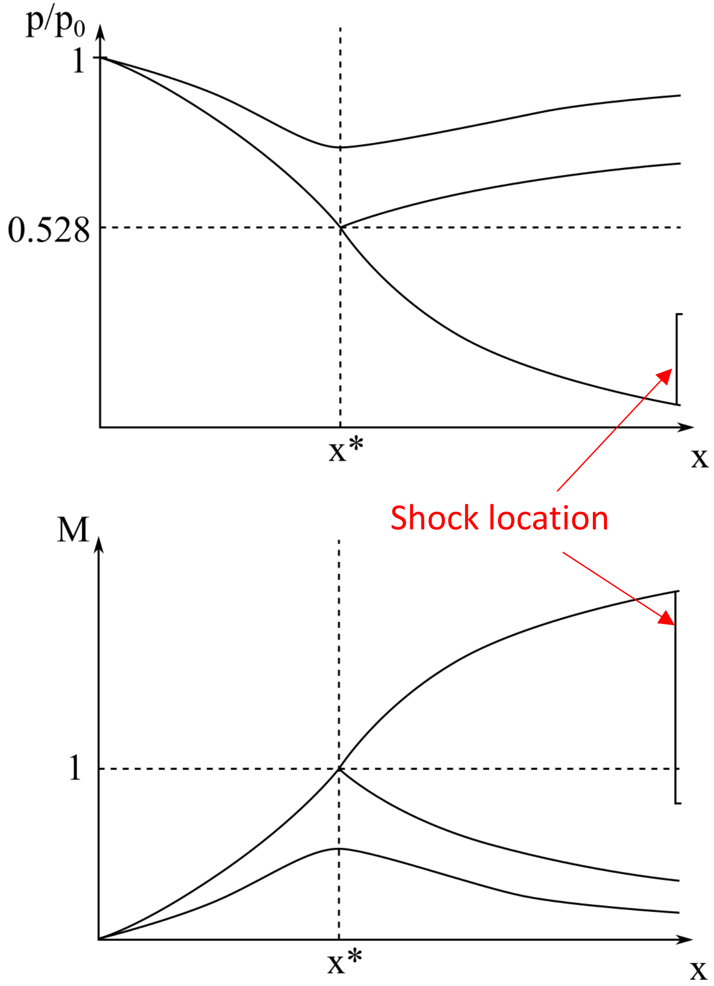
- If p_a is between the perfectly expanded and exactly sonic values, then a shock(s) will need to be present to bring the isentropic solution back up to the ambient pressure

→ nozzle is "over-expanded"

- The shock can form exactly at the nozzle exit (if the ambient pressure is exactly that post-normal-shock value), inside, or outside the nozzle.



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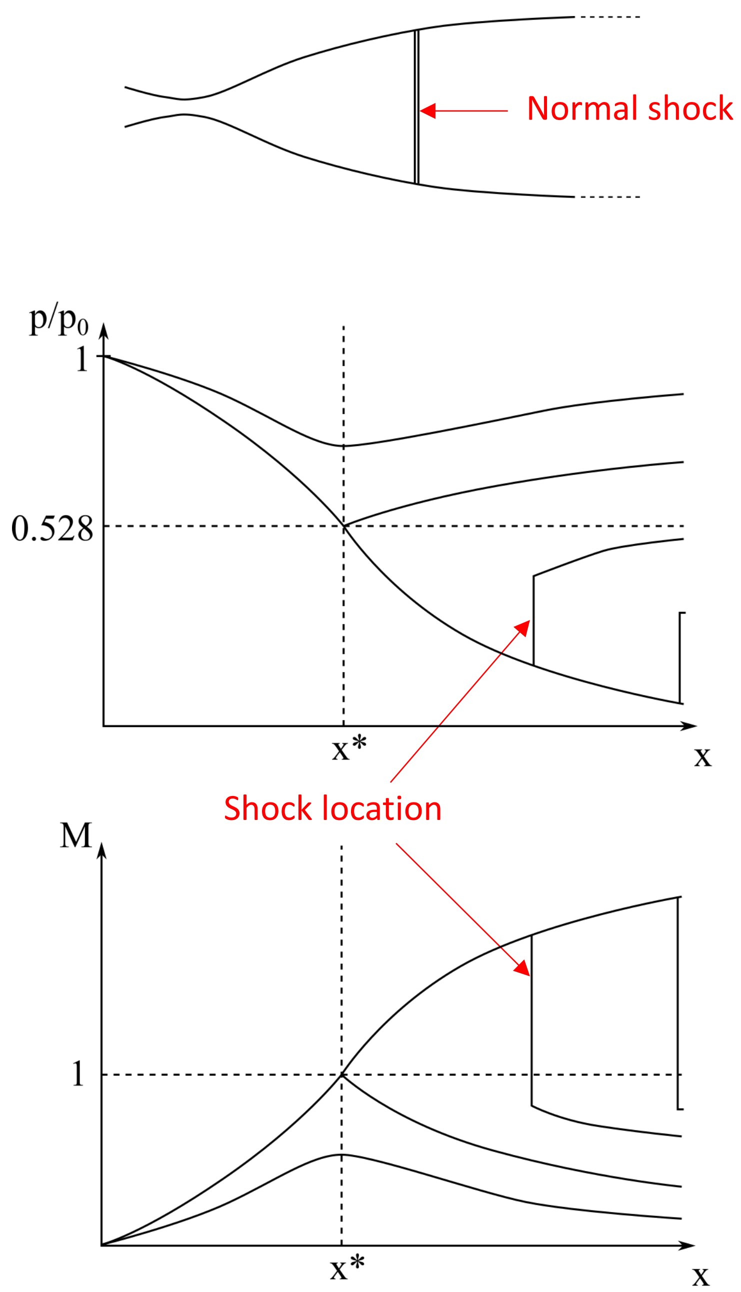

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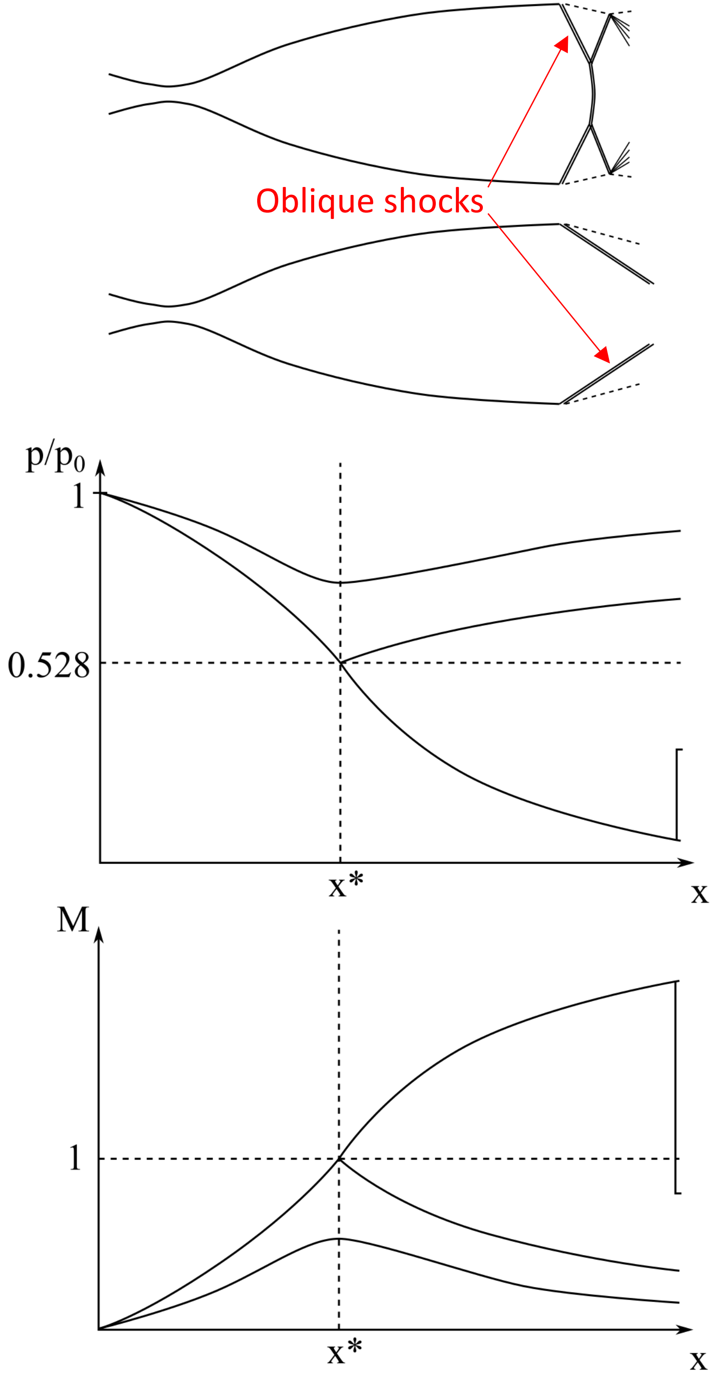

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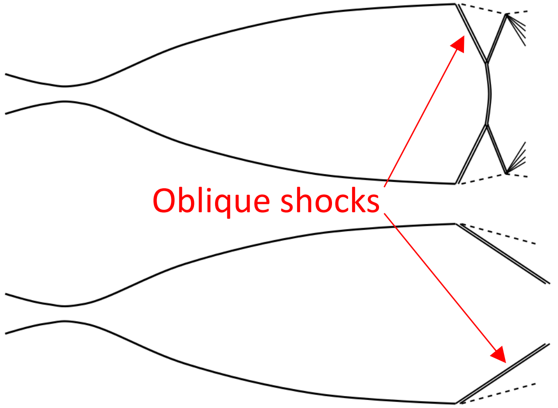

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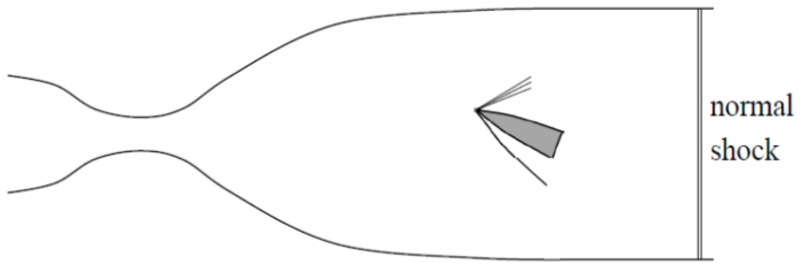
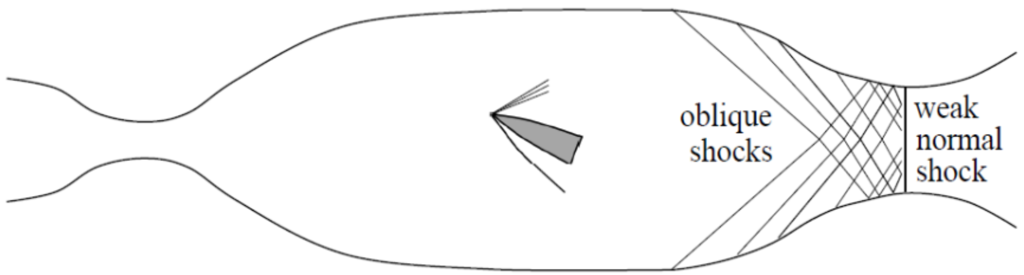
Flow through supersonic wind tunnels

- In a closed-circuit, continuous-flow supersonic wind tunnel, dissipative effects lead to stagnation-pressure losses, requiring a continual power input to maintain the pressure difference across the nozzle.
- It is highly undesirable to maintain supersonic flow for any longer than necessary, so we typically desire to decelerate the flow to subsonic conditions downstream of the test section.
- One option to achieve this is through a normal shock:



However, the shock is unstable (and sensitive to model introduction), and pressure losses are large.

- A better option is a supersonic diffuser, which is less sensitive and (generally) has lower pressure losses:



Supersonic diffusers

Consider the flow then through a supersonic diffuser.

If there were no losses in the test section, the flow would be isentropic and the diffuser throat area would be the same as that of the supersonic nozzle.

In a real tunnel, however, the stagnation pressure at the second throat will be lower than at the first. For a steady flow, however, the mass flux will be the same and we thus have

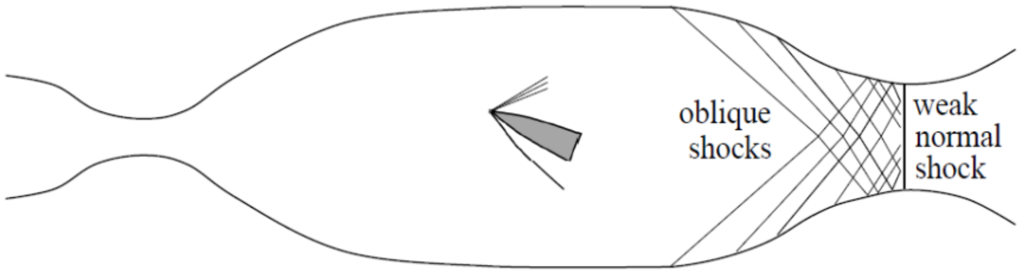
$$\rho_1^* a_1^* A_{t1} = \rho_2^* a_2^* A_{t2}.$$

We can to good approximation assume the flow to be adiabatic, so $T_2^* = T_1^*$, $a_2^* = a_1^*$, and also

$$\frac{\rho_1^*}{\rho_2^*} = \frac{p_1^*}{T_1^*}\frac{T_2^*}{\rho_2^*} = \frac{p_1^*}{p_2^*}.$$

Thus, from above

$$\frac{A_{t2}}{A_{t1}} = \frac{p_1^*}{p_2^*}.$$



Note, however, that

$$p^* = p_0 \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma+1)}$$

We can thus conclude

$$\frac{A_{t2}}{A_{t1}} = \frac{p_{01}}{p_{02}}.$$

Since $p_{02} < p_{01}$, we must therefore have that $A_{t2} > A_{t1}$.