Lecture 11: More maneuvers

Tangential Maneuvers:

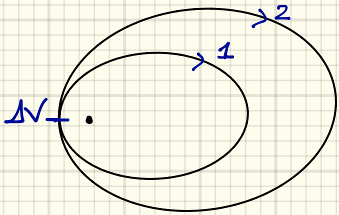
EX | Initial orbit: $r_p = 8000$ km, $r_a = 10{,}000$ km

Final orbit: $r_p = 8000$ km, $r_a = 12{,}000$ km

Assume orbiting Earth

Both orbits have the same orbital plane $\Rightarrow (\Omega, i)$

$\omega_1 = \omega_2$



B/c orbits intersect at r_p, that is where the burn will occur.

$V_{p2} > V_{p1}$

$\Delta V = V_{p2} - V_{p1}$

$$\frac{V^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \Rightarrow V_p = \sqrt{\frac{2\mu}{r_p} - \frac{2\mu}{2a}}$$

Orbit 1: $2a = 18{,}000$ km $= r_{a_1} + r_{p_1}$

orbit 2: $2a = 20{,}000$ km

$$V_{p_1} = \sqrt{\frac{2\mu}{r_{p_1}} - \frac{2\mu}{18000}}$$

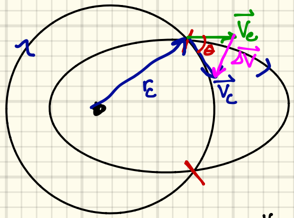
$$V_{p_2} = \sqrt{\frac{2\mu}{r_{p_2}} - \frac{2\mu}{20{,}000}}$$

Non tangential, in-plane:

Same orbital plane

EX | Initial orbit: $r_p = 8000$ km, $r_a = 10{,}000$ km

Final orbit: Circle : $r_c = 9000$ km



$\vec{V_e} + \vec{\Delta V} = \vec{V_c}$

Non-tangential maneuver: change velocity magnitude & direction

Law of Cosines:

$$\Delta V^2 = V_1^2 + V_2^2 - 2V_1V_2\cos\theta$$

$V_1 = V_e$ $\qquad V_2 = V_c = \sqrt{\frac{\mu}{r_c}}$

$$\boxed{\Delta V^2 = V_e^2 + V_c^2 - 2V_eV_c\cos(-\gamma_1)}$$

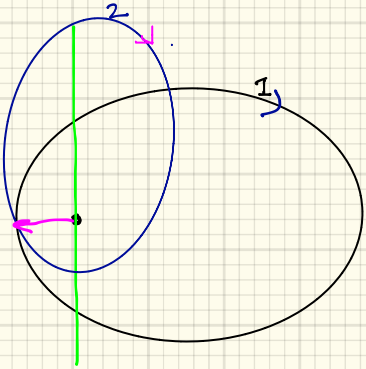
$$\frac{V_e^2}{2} - \frac{\mu}{r_c} = -\frac{\mu}{(r_a + r_p)} \Rightarrow V_e = \sqrt{\frac{2\mu}{r_c} - \frac{2\mu}{r_a + r_p}}$$

Find θ using FPA: $\theta = \gamma_2 - \gamma_1$ $\qquad \gamma_2 = \gamma_c = 0$

$$h = \sqrt{\mu p} = rV\cos\gamma \Rightarrow \sqrt{\mu p_1} = r_c V_e \cos\gamma_1$$

Ex| Initial orbit: $r_p = 8000$ km, $r_a = 10{,}000$ km $\omega_1 = 90°$ $\Omega_1 = \Omega_2$, $i_1 = i_2$

Final orbit: $p = 8000$ km, $e = 0.3$ $\omega_2 = 0°$



$\omega_1 + \nu_1 = \omega_2 + \nu_2$

$90 + 0 = 0 + 90$

The two orbits intersect at periapsis of orbit 1 $\Rightarrow r_{p_1} = 8000$ km

Velocities are not in the same direction

$\Rightarrow$ law of cosines

$$\Delta v^2 = v_1^2 + v_2^2 - 2 v_1 v_2 \cos\theta$$

$v_1^2 = v_{p_1}^2$ = velocity at periapsis of orbit 1.

$$\frac{v_{p_1}^2}{2} - \frac{\mu}{r_{p_1}} = \frac{-\mu}{r_{p_1} + r_{a_1}}$$ $\Rightarrow$ Solve for v_{p_1}.

v_2^2 = velocity of orbit 2 at the semi-latus rectum of orbit 2 = r_{p_1}

$$\frac{v_2^2}{2} - \frac{\mu}{r_{p_1}} = \frac{-\mu}{2a_2}$$

$\uparrow$ solve for v_2

Need a_2: $p = a(1-e^2)$

$p_2 = a_2(1-e_2^2)$

$$\frac{8000}{(1-0.3^2)} = a_2$$

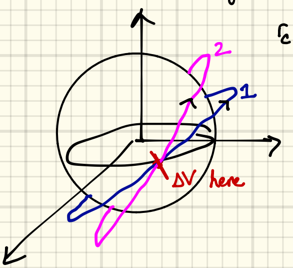
$\theta = \gamma_2 - \gamma_1$, $\gamma_1 = 0$ (b/c at periapsis)

$$h = \sqrt{\mu p_2} = r_{p_1} v_2 \cos\gamma_2$$

Non-tangential, out-of-plane: want to change Ω, i

Difficult to visualize intersection points when $\Omega_1 \neq \Omega_2$. Focus primarily on changing inclination

2 circular orbits $\rightarrow$ only difference between orbits is $i_1 \neq i_2$ $(\Omega_1 = \Omega_2)$



$r_{c_1} = r_{c_2}$

Orbits intersect at the ascending & descending nodes

$\Rightarrow$ burn occur at ascending node

v_1 & v_2 are not in the same direction $\Rightarrow$ law of cosines

$$\Delta v^2 = v_1^2 + v_2^2 - 2 v_1 v_2 \cos\theta$$

$v_1 = v_2$ b/c $r_{c_1} = r_{c_2}$ $\rightarrow$ $v = \sqrt{\dfrac{\mu}{r_c}}$

v, r_c

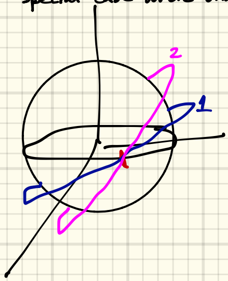
$$\Delta v^2 = 2v^2(1-\cos\theta)$$ $\theta = \Delta i$

Ex| Change i, a, e

Initial orbit: $r_c = 8000\,km$, $i = 10°$ $\quad \Omega_1 = \Omega_2$

Final orbit: $r_p = 8000\ km$, $r_a = 12000\ km$, $\omega = 0$, $i = 15°$

Special case where orbits intersect at the asc. node, and $\gamma_1 = \gamma_2 = 0$ at that point



These orbits do not intersect at the descending node

$\Delta v^2 = v_1^2 + v_2^2 - 2 v_1 v_2 \cos\theta$

$v_1 = \sqrt{\frac{\mu}{r_c}}$

$v_2 \neq v_1 = \sqrt{\frac{2\mu}{r_p} - \frac{2\mu}{r_a + r_p}}$

$\gamma_1 = \gamma_2 = 0$ at intersection

$\Rightarrow \theta = \Delta i$