Homework Assignment 2: 2BP

ENAE 404: Space Flight Dynamics

Include units on your answers and axis labels on your plots. Be sure to assign answers to specific problems during your gradescope submission. Include your code at the end of your solution packet.

1. Given the following position and velocity vectors, calculate the Keplerian orbital elements, assuming Earth is the central body. Do not use a computer code to do this. Vectors in units of km and km/s.

$$\mathbf{r} = 3634.1\hat{\mathbf{x}} + 5926\hat{\mathbf{y}} + 1206.6\hat{\mathbf{z}} \tag{1}$$

$$\mathbf{v} = -6.9049\hat{\mathbf{x}} + 4.3136\hat{\mathbf{y}} + 2.6163\hat{\mathbf{z}} \tag{2}$$

2. Write a Matlab (or other) function to convert from Cartesian coordinates to orbital elements.
    1. Using subplot, plot the osculating orbital elements for the orbit of Didymos from HW0.
    2. Describe why your plots make sense (reference both the time variation of the orbital elements as well as the plot of the orbit in 3D space).
3. Write a Matlab (or other) function to convert from orbital elements to Cartesian coordinates. Propagate the following orbit (about Earth) for one period. State the period of the orbit, plot the orbit in 3D (use the 'axis equal' command), plot the deviation of the energy as compared to the initial energy ($E_i - E_0$), and plot the osculating orbital elements.
    - a=20,000km
    - e=0.4
    - i=100°
    - Ω=30°
    - ω=15°
    - ν=15°
4. Sketch the following orbits in 2D and 3D. Assume that none of the spacecraft impact Earth.
    - In the 2D orbit, label: periapsis, angular momentum vector, ascending node, descending node, spacecraft location, portion of the orbit in the southern hemisphere
    - In the 3D orbit, label: angular momentum vector, ascending node, periapsis

Spacecraft ID	e	i (deg)	Ω (deg)	ω (deg)	ν (deg)
A	0.3	60	30	160	30
B	0.3	60	330	90	10
C	0.5	120	30	30	180

5. Consider a spacecraft on a hyperbolic trajectory that will fly by Mars. The trajectory's semi-major axis is -11,000km and it's eccentricity is 1.8. Calculate the turn angle, miss distance, hyperbolic excess speed, and radius of periapsis of the flyby.

   (a) What is the turn angle?

   (a) _____

   (b) Hyperbolic excess speed?

   (b) _____

   (c) Miss distance?

   (c) _____

   (d) Radius of periapsis?

   (d) _____

6. Give the orbital elements for an Earth-orbiting spacecraft crossing the $\hat{y}$ axis in a retrograde, equatorial, circular orbit at an altitude of 1DU. All angles in degrees.

   (a) What is the semi-major axis (in DU)?

   (a) _____

(b) Eccentricity?

(b) _____

(c) Inclination?

(c) _____

(d) Longitude of the ascending node?

(d) _____

(e) Argument of periapsis?

(e) _____

(f) True anomaly?

(f) _____

(g) True longitude at epoch?

(g) _____

7. Match the following orbits to the descriptions below. Example solution: g) A,C.

Spacecraft ID	e	i (deg)	Ω (deg)	ω (deg)	ν (deg)
A	1	60	180	160	30
B	2	160	260	90	10
C	0.5	20	210	30	180
D	0.2	90	110	210	270

(a) This is a retrograde orbit.

(a) _____

(b) This spacecraft currently has a positive flight path angle.

(b) _____

(c) This spacecraft is currently in the southern hemisphere.

(c) _____

(d) This spacecraft is currently at apoapsis.

(d) _____

(e) This orbit has a periapsis in the southern hemisphere.

(e) _____

(f) This orbit has a line of nodes that is collinear with $\hat{x}$.

(f) _____

8. Given an elliptical orbit about the Earth with an eccentricity of 0.3 and a radius of periapsis of 8000km, calculate the time of flight:

   (a) from $\nu = 20^\circ$ to $\nu = 30^\circ$

(a) _____

   (b) from $\nu = 300^\circ$ and $\nu = 20^\circ$

(b) _____