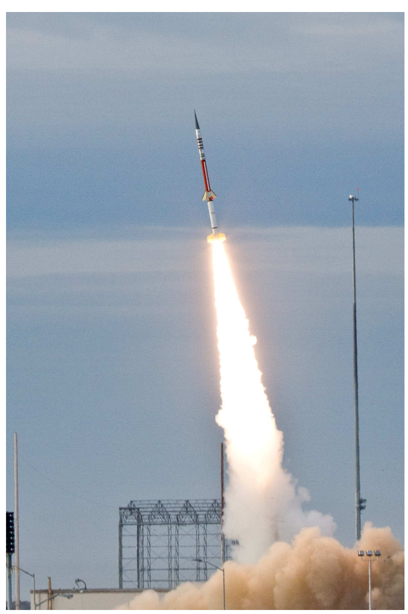
ENAE311H Homework 2

Due: Friday, September 27th by 5pm (online submission)

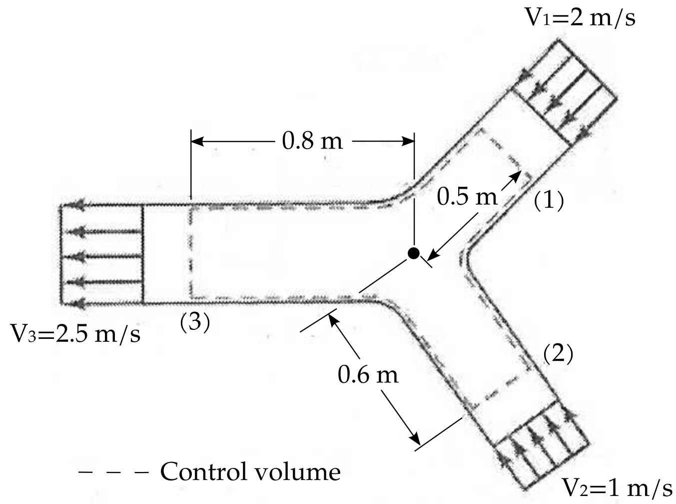
1. The SR-71 is reputed to have flown at Mach 3.6 at an altitude of approximately 80,000 feet (where the ambient conditions are T=221 K, ρ=0.044 kg m^{-3}). An experiment is performed on a 1/10-scale reproduction of the aircraft in a wind tunnel, where the free-stream conditions are T_∞=120 K, V_∞=790.5 m s^{-1}, and ρ_∞=0.2 kg m^{-3}. Are the two flows dynamically similar? (Assume $a=\sqrt{\gamma RT}$, with γ=1.4 and R=287 J kg^{-1} K^{-1}, and $\mu \propto T^{2/3}$)

Dryden Flight Research Center EC95-42883-4 Photographed 1995
SR-71B over snow-capped mountains. NASA photo

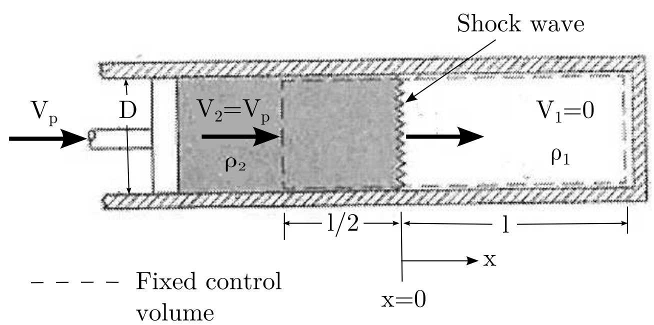
2. During launch, a Terrier sounding rocket reaches a velocity of 400 m s^{-1} at an altitude of 40,000 feet (T=217 K, ρ=0.30 kg m^{-3}). A 1/5-scale reproduction is being tested in a wind tunnel where the static pressure is 75 kPa. Calculate the velocity, temperature and density of the wind-tunnel flow in order to ensure dynamical similarity with flight (again assume $a \propto \sqrt{T}$ and $\mu \propto T^{2/3}$).



3. Water flows in the branching pipe shown in the figure below with uniform velocity at each inlet and outlet. The fixed control volume coincides with the system at time $t = 20\,\text{s}$.
a) Sketch the boundary of the system at $t = 20.1\,\text{s}$.
b) Write down expressions for the mass of fluid that entered and exited the control volume during this $0.1\,\text{s}$ time interval (in terms of the fluid density and areas of the branches).



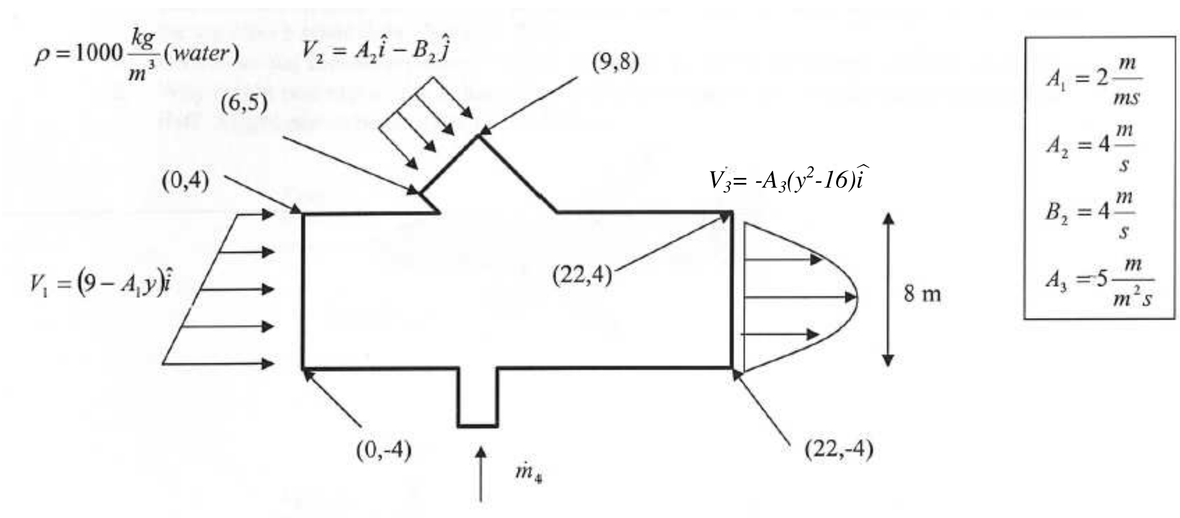
4. A plunger is pushed into a closed, air-filled pipe of diameter D with velocity V_p as shown below. The flow within the control volume consists of two distinct regions: one of zero velocity and density ρ_1; the other with a uniform fluid velocity V_p and density ρ_2. The two regions are separated by an interface (a shock wave) that travels to the right with constant speed $V_s > V_p$. Determine the mass within the control volume shown as a function of time from t=0 (when the shock is at x=0) until the shock reaches the end of the pipe (x=l).



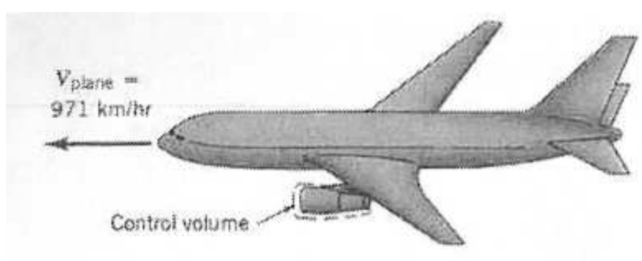
5. The expression for the mass flow rate across a surface is:

$$\dot{m} = \iint_S \rho \vec{v} \cdot d\vec{A}, \tag{1}$$

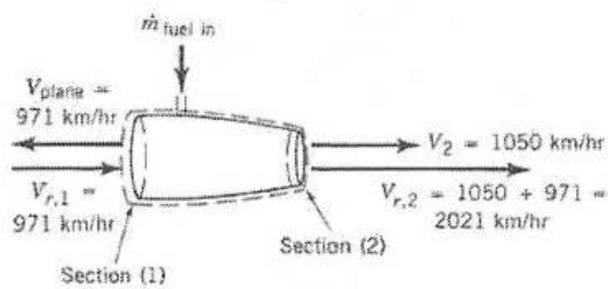
where ρ is the fluid density, $\vec{v}$ is the fluid velocity, and $d\vec{A}$ is the differential surface area element (with outward normal). For the steady flow geometry shown on the following page:
a) Calculate the mass flow rate (per unit depth) across faces 1, 2, and 3.
b) How is the sign of the mass flow rate related to the orientation of the velocity vector and the surface?
c) How would you determine the mass flow rate (per unit depth) at face 4?



6. An airplane moves through the air at 971 km/hr, as shown in the figure below. The frontal area of the jet engine is $0.80\,m^2$ and the density of the air entering the engine is $0.736\,kg/m^3$. A stationary observer estimates that, relative to the earth, the engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is $0.558\,m^2$ and the exhaust gas density is $0.515\,kg/m^3$.
a) Estimate the mass flow rate of fuel in kg/hr.
b) Now assume that the velocity distribution of the exhaust gas is parabolic (i.e., similar to face 3 in question 4) instead of flat and that the ground-based observer has estimated the peak exhaust velocity. Assuming that the ratio of fuel mass flow to air intake mass flow is the same as in part (a), how does the air intake mass flow change? Why?
c) Why might we expect the exhaust gas velocity distribution to be somewhat parabolic in shape rather than flat?



(a)



(b)