PHYS 313

HW 06: Assignment 6

Due on March 13th, 2025 at 11:59 PM

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Problem 3.2:

In one sentence, justify Earnshaw's Theorem.

Solution

Problem 3.3:

Find the general solution to Laplace's equation in spherical coordinates, for the case where V depends only on r. Do the same for cylindrical coordinates, assuming V depends only on s.

Solution

Problem 3.4:

1. Show that the average electric field over a spherical surface, due to charges outside the sphere, is the same as the field at the center.
2. What is the average due to charges inside the sphere?

Solution

Problem 3.7:

Find the force on the charge $+q$ in the below image, noting that the xy plane is a grounded conductor.

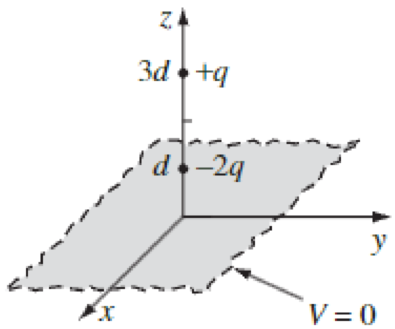

Figure 1: Diagram for Problem 3.7

Solution

Problem 3.8:

1. Using the law of consines, show that the following equations are equivalent:

$$\mathrm{V}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{\mathscr{r}} + \frac{q'}{\mathscr{r}'}\right) \tag{1}$$

$$\mathrm{V}(r,\theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\sqrt{r^2 + a^2 - 2ra\cos()(\theta)}} - \frac{q}{\sqrt{R^2 + \left(\frac{ra}{R}\right)^2 - 2ra\cos()(\theta)}}\right] \tag{2}$$

   Where r and θ are the usual spherical polar coordinates, with the z axis along the line through q. In this form, it is obvious that $V = 0$ on the sphere $r = R$.

2. Find the induced surface charge on the sphere, as a function of θ. Integrate this to get the total induced charge. (What *should* it be?)

3. Calculate the energy of this configuration.

Solution

Problem 3.13:

Find the potential in the infinite slot of Ex3.3 if the boundary at $x = 0$ consists of two metal strips: one, from $y = 0$ to $y = \frac{a}{2}$, is held at a constant potential V_0, and the other, from $y = \frac{a}{2}$ to $y = a$, is at potential $-V_0$.

Solution

Similar to the answer in Ex3.3, the configuration retains its independence from z. We again have to solve Laplace's equation but subjected to different boundary conditions:

$$\frac{\partial^2}{\partial x^2}(V) + \frac{\partial^2}{\partial y^2}(V) = 0, \begin{cases} V = 0 & y = 0 \\ V = 0 & y = a \\ V = V_0 & 0 < y < \frac{a}{2}, x = 0 \\ V = -V_0 & \frac{a}{2} < y < a, x = 0 \\ V \to 0 & x \to \infty \end{cases}.$$

This can be accomplished using a similar technique as Griffiths, as follows:

$$\begin{aligned}
Y\frac{\mathrm{d}^2}{\mathrm{d}x^2}(X) + X\frac{\mathrm{d}^2}{\mathrm{d}y^2}(Y) &= 0 \\
\frac{1}{X}\frac{\mathrm{d}^2}{\mathrm{d}x^2}(X) + \frac{1}{Y}\frac{\mathrm{d}^2}{\mathrm{d}y^2}(Y) &= 0 \\
\frac{\mathrm{d}^2}{\mathrm{d}x^2}(X) = k^2X, \quad \frac{\mathrm{d}^2}{\mathrm{d}y^2}(Y) &= -k^2Y \\
\mathrm{X}(x) = Ae^{kx} + Be^{-kx}, \quad \mathrm{Y}(y) &= C\sin(ky) + D\cos(ky) \\
\mathrm{V}(x,y) &= \left(Ae^{kx} + Be^{-kx}\right)(C\sin(ky) + D\cos(ky)) \\
\text{condition (v)} &\implies A = 0 \\
\therefore \mathrm{V}(x,y) &= e^{-ky}(C\sin(ky) + D\cos(ky)) \\
\text{condition (i)} &\implies D = 0 \\
\therefore \mathrm{V}(x,y) &= Ce^{-ky}\sin(ky) \\
\text{condition (ii)} &\implies \sin(ka) = 0 \\
\therefore k &= \frac{n\pi}{a}, \quad n = \{1,2,3,\dots\} \\
\mathrm{V}(x,y) &= \sum_{n=1}^{\infty} C_n e^{-n\pi\frac{x}{a}}\sin(n\pi\frac{y}{a})
\end{aligned}$$

Here is where we diverge from Griffiths' Ex3.3. We want to fulfill our conditions (iii) and (iv) as follows:

$$\begin{aligned}
\mathrm{V}(0, 0 < y < \frac{a}{2}) &= \sum_{n=1}^{\infty} C_n e^{-n\pi\frac{x}{a}}\sin(n\pi\frac{y}{a}) = V_0, \\
\mathrm{V}(0, \frac{a}{2} < y < a) &= \sum_{n=1}^{\infty} C_n e^{-n\pi\frac{x}{a}}\sin(n\pi\frac{y}{a}) = -V_0.
\end{aligned}$$