1.

a) $\varphi = 30°$
   $\theta = 40°$
   $\phi = 10°$

   $c\varphi = \cos(\varphi) \quad s\varphi = \sin(\varphi)$
   $c\theta = \cos(\theta) \quad s\theta = \sin(\theta)$
   $c\phi = \cos(\phi) \quad s\phi = \sin(\phi)$

   $$\vec{e}_B = {}^B R_I \, \vec{e}_I, \quad {}^B R_I = \begin{bmatrix} c\theta c\varphi & c\theta s\varphi & -s\theta \\ s\phi s\theta c\varphi - c\phi s\varphi & s\phi s\theta s\varphi + c\phi c\varphi & s\phi c\theta \\ c\phi s\theta c\varphi + s\phi s\varphi & c\phi s\theta s\varphi - s\phi c\varphi & c\phi c\theta \end{bmatrix}$$

b) $\phi = a\cos\left(\frac{1}{2}(R_{11} + R_{22} + R_{33} - 1)\right) = \boxed{0.8456 \text{ rad}} = 48.45°$

   $$\vec{e} = \frac{1}{2\sin(\phi)} \begin{bmatrix} R_{23} - R_{32} \\ R_{31} - R_{13} \\ R_{12} - R_{21} \end{bmatrix} = \frac{1}{2\sin(\phi)} \begin{bmatrix} -0.0331 \\ 1.2778 \\ 0.7788 \end{bmatrix}$$

   $$= \boxed{\begin{bmatrix} -0.0221 \\ 0.8537 \\ 0.5203 \end{bmatrix}}$$

c) $q_1 = e_1 \sin(\phi/2)$
   $q_2 = e_2 \sin(\phi/2)$
   $q_3 = e_3 \sin(\phi/2)$
   $q_4 = \cos(\phi/2)$

   $\Rightarrow \quad \vec{q} = \begin{bmatrix} -0.0091 \\ 0.3503 \\ 0.2153 \\ 0.9119 \end{bmatrix} \qquad \vec{\beta} = \begin{bmatrix} q_4 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$

d) ${}^B \omega = (0.1, 0.2, 0) \text{ rad/sec}$

   $$\dot{\vec{\beta}} = \frac{1}{2} \begin{bmatrix} \beta_0 & -\beta_1 & -\beta_2 & -\beta_3 \\ \beta_1 & \beta_0 & -\beta_3 & \beta_2 \\ \beta_2 & \beta_3 & \beta_0 & -\beta_1 \\ \beta_3 & -\beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{pmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \Rightarrow \dot{\vec{\beta}} = \begin{bmatrix} -0.0346 \\ 0.0242 \\ 0.1019 \\ -0.0005 \end{bmatrix}$$

# problem 01

```matlab
clearvars all;
clc;
```

## part a

```matlab
% given
angs_deg = [30, 40, 10];
syms psi theta phi;

% solution
angs = deg2rad(angs_deg);
Rx = [
    1, 0, 0;
    0, cos(phi), sin(phi);
    0, -sin(phi), cos(phi);
];
Ry = [
    cos(theta), 0, -sin(theta);
    0, 1, 0;
    sin(theta), 0, cos(theta);
];
Rz = [
    cos(psi), sin(psi), 0;
    -sin(psi), cos(psi), 0;
    0, 0, 1;
];

R_sym = Rx*Ry*Rz;
R = double(subs(R_sym, [psi theta phi], angs));

% display
R_sym, R
```

R_sym =

$$\begin{pmatrix} \cos(\psi)\cos(\theta) & \cos(\theta)\sin(\psi) & -\sin(\theta) \\ \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \cos(\phi)\cos(\psi) + \sin(\phi)\sin(\psi)\sin(\theta) & \cos(\theta)\sin(\phi) \\ \sin(\phi)\sin(\psi) + \cos(\phi)\cos(\psi)\sin(\theta) & \cos(\phi)\sin(\psi)\sin(\theta) - \cos(\psi)\sin(\phi) & \cos(\phi)\cos(\theta) \end{pmatrix}$$

R = 3×3
```
    0.6634    0.3830   -0.6428
   -0.3957    0.9087    0.1330
    0.6350    0.1661    0.7544
```

## part b

```matlab
% solution
phi = acos(1/2*(R(1,1)+R(2,2)+R(3,3)-1));
e = 1/(2*(sin(phi)))*[
```

```
    R(2,3)-R(3,2);
    R(3,1)-R(1,3);
    R(1,2)-R(2,1);
];

% display
phi, e
```

```
phi =
0.8456
e = 3×1
   -0.0221
    0.8537
    0.5203
```

## part c

```
% solution
q = [
    e(1)*sin(phi/2);
    e(2)*sin(phi/2);
    e(3)*sin(phi/2);
    cos(phi/2)
];

% display
q
```

```
q = 4×1
   -0.0091
    0.3503
    0.2135
    0.9119
```

## part d

```
% given
omega = [0.1, 0.2, 0];

% solution
beta0 = q(4);
beta1 = q(1);
beta2 = q(2);
beta3 = q(3);

beta_dot = 1/2*[
    beta0, -beta1, -beta2, -beta3;
    beta1,  beta0, -beta3,  beta2;
    beta2,  beta3,  beta0, -beta1;
    beta3, -beta1,  beta1,  beta0;
]*[
    0;
```

```
    omega(1);
    omega(2);
    omega(3);
];

% display
beta_dot
```

```
beta_dot = 4×1
   -0.0346
    0.0242
    0.1019
   -0.0005
```

# p02

## clear history

```
clearvars all;
clc;
```

## part a

```
% given
I = diag([10; 20; 30])
```

```
I = 3×3
    10     0     0
     0    20     0
     0     0    30
```

```
omega0_deg = [10; 0; 30];
omega0 = deg2rad(omega0_deg);

% solution
H0 = I*omega0;
H0_mag = norm(H0);
T0 = 1/2*transpose(omega0)*I*omega0;

% display
H0_mag, T0
```

```
H0_mag =
15.8046
T0 =
4.2646
```

## part b

```
% solution
dwdt = @(t, omega) inv(I)*(-cross(omega, I*omega));
tspan = [0, 100];
oopts = odeset('RelTol',1e-9,'AbsTol',1e-9);

[t1, w1] = ode45(dwdt, tspan, omega0, oopts);
w1_deg = rad2deg(w1);

% plot
fig = figure;
plot(t1, w1_deg(:,1), 'r', t1, w1_deg(:,2), 'g', t1, w1_deg(:,3), 'b', 'LineWidth',
1.5);
title('Angular Velocity vs Time (\omega_0 = [10;0;30] deg/s)');
legend('\omega_x','\omega_y','\omega_z');
xlabel('Time (s)');
```
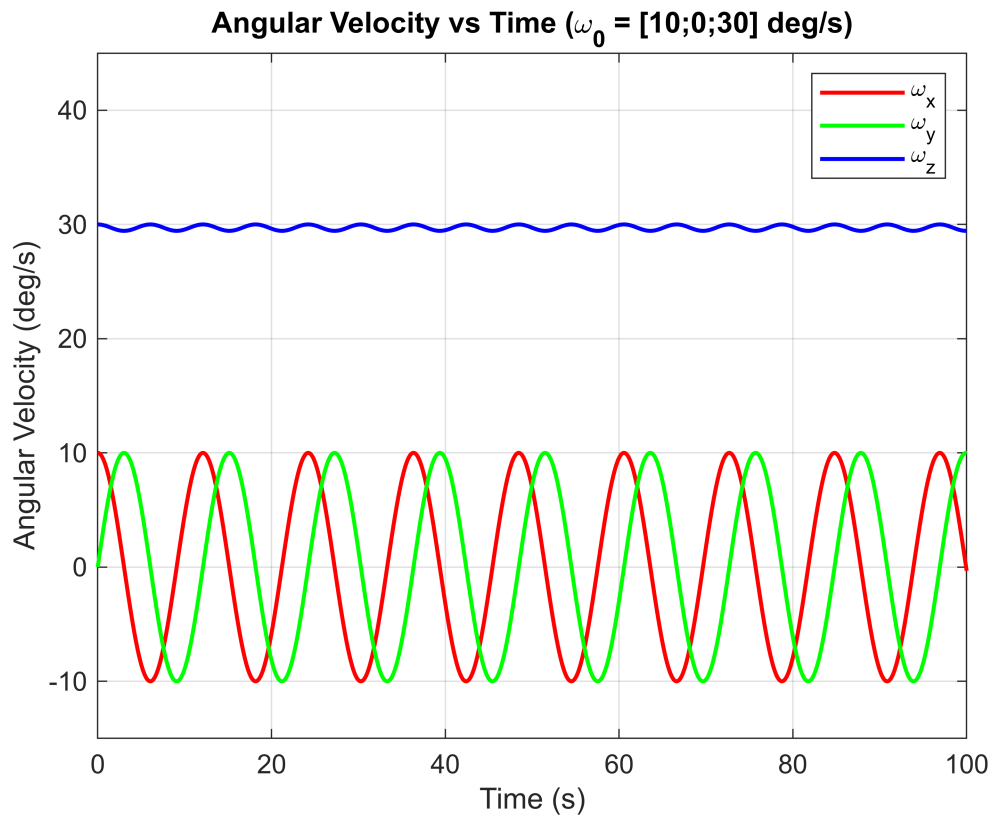
```
ylabel('Angular Velocity (deg/s)');
ylim([-15 45]);
grid on;

% save
saveas(fig, "./images/s02b.png");
```

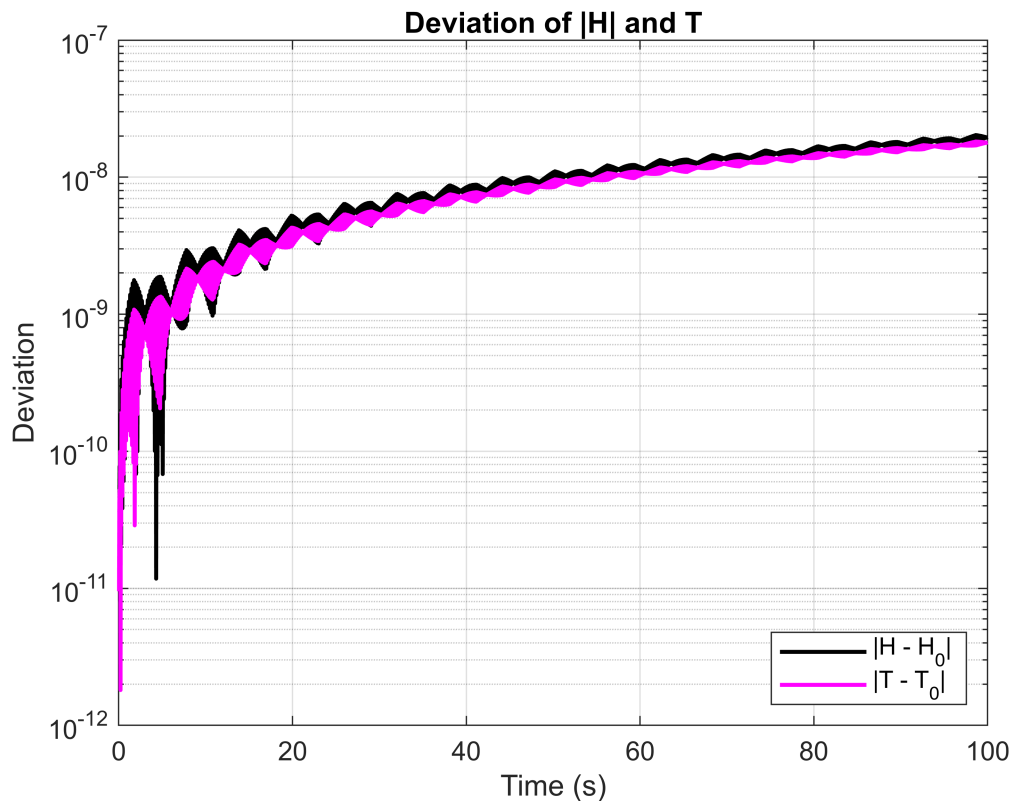**Angular Velocity vs Time ($\omega_0$ = [10;0;30] deg/s)**



## part c

```
% solution
H1 = transpose(I * transpose(w1));
H1_mag = sqrt(sum(H1.^2,2));
T1 = 1/2*sum(w1.*H1, 2);

dH = abs(H1_mag-H0_mag);
dT = abs(T1-T0);

% plot
fig = figure;
plot(t1, dH, 'k', t1, dT, 'm', 'LineWidth', 1.5);
title('Deviation of |H| and T');
legend('|H - H_0|','|T - T_0|', 'Location', 'southeast');
xlabel('Time (s)');
ylabel('Deviation');
yscale("log");
grid on;
```

2

```
% save
saveas(fig, "./images/s02c.png");
```


Deviation of |H| and T

I believe my code is working, as the both the angular momentum deviation and kinetic energy deviation are extremely minimal, being on the order of $10^{-8}$.

## part d

```
% solution
[Xs, Ys, Zs] = sphere(20);
Xs = H0_mag*Xs;
Ys = H0_mag*Ys;
Zs = H0_mag*Zs;

radii = sqrt(2*T0.*diag(I));
[Xe, Ye, Ze] = ellipsoid(0,0,0, radii(1), radii(2), radii(3), 60);

% plot
fig = figure;
hold on;
% sphere
surf(Xs, Ys, Zs, 'FaceAlpha',0.3, 'EdgeColor','b', 'FaceColor', [0.8 0.8 1]);
% ellipsoid
surf(Xe, Ye, Ze, 'FaceAlpha',0.3, 'EdgeColor','g', 'FaceColor', [0.2 0.8 0.2]);
% polhode points (body-fixed H)
```
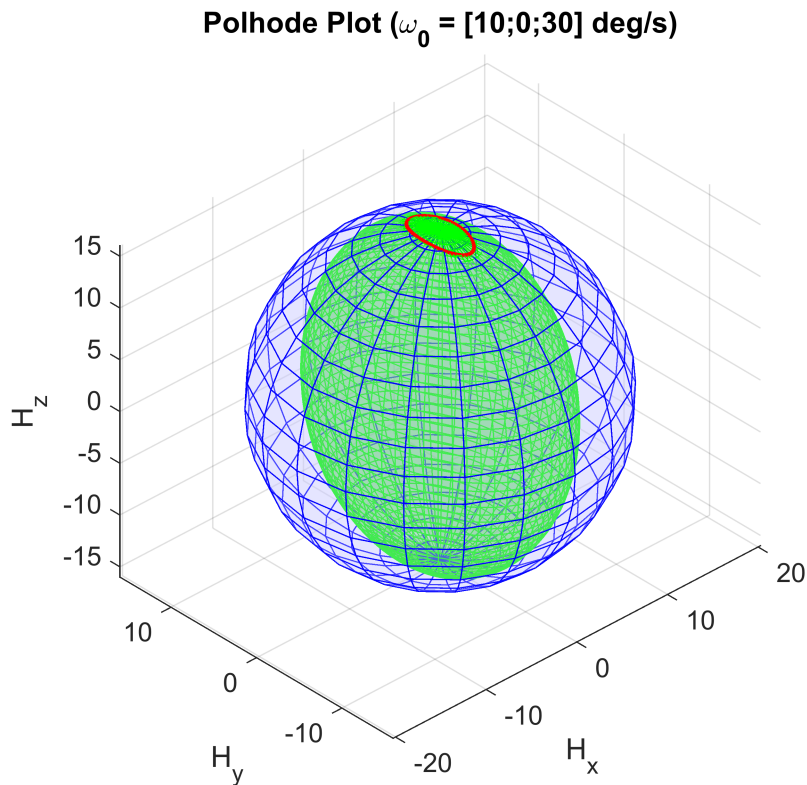
3

```
plot3(H1(:,1), H1(:,2), H1(:,3), '.r', 'MarkerSize', 5);
title('Polhode Plot (\omega_0 = [10;0;30] deg/s)');
xlabel('H_x');
ylabel('H_y');
zlabel('H_z');
axis equal;
grid on;
view([-43.3 33.8])
hold off;

% save
saveas(fig, "./images/s02d.png");
```



**Polhode Plot ($\omega_0$ = [10;0;30] deg/s)**

## part e

```
% solution
omega2_deg = [1; 15; 0];
omega2 = deg2rad(omega2_deg);
[t2, w2] = ode45(dwdt, tspan, omega2, oopts);
w2_deg = rad2deg(w2);

% plot
fig = figure;
plot(t2, w2_deg(:,1), 'r', t2, w2_deg(:,2), 'g', t2, w2_deg(:,3), 'b', 'LineWidth',
1.5);
title('Angular Velocity vs Time (\omega_0 = [1;15;0] deg/s)');
```
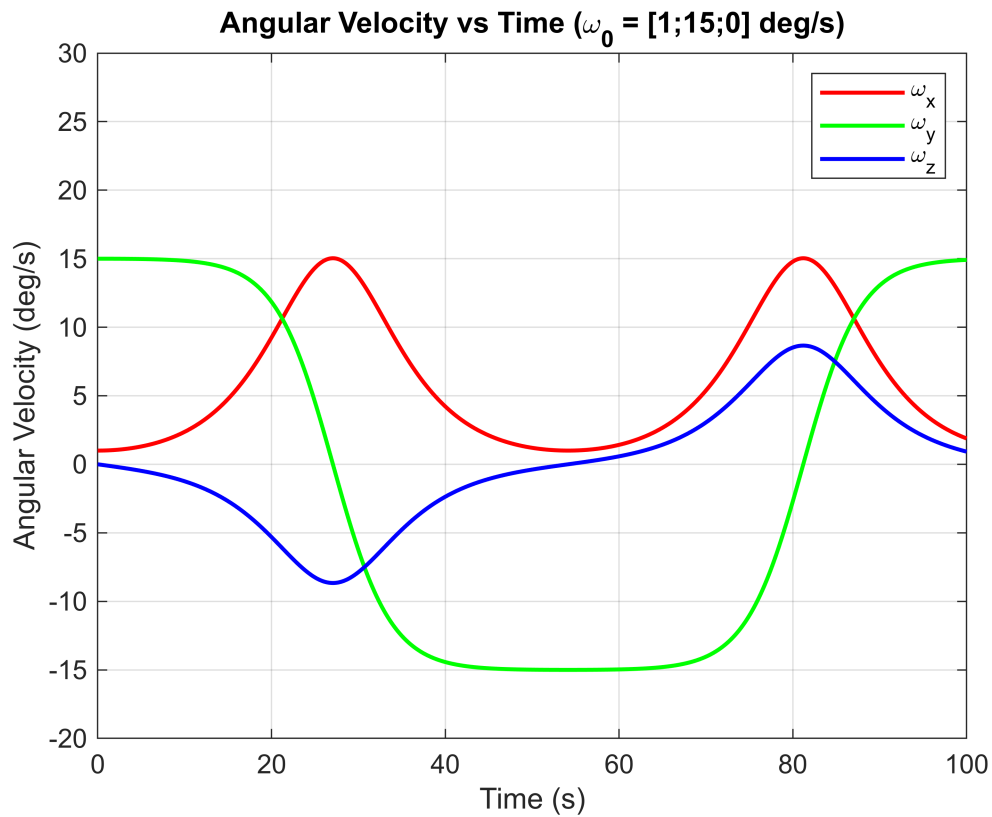
```
legend('\omega_x','\omega_y','\omega_z');
xlabel('Time (s)');
ylabel('Angular Velocity (deg/s)');
ylim([-20 30]);
grid on;

% save
saveas(fig, "./images/s02e.png");
```



Angular Velocity vs Time ($\omega_0$ = [1;15;0] deg/s)

## part f

```
% solution
H2_0 = I*omega2;
H2_0_mag = norm(H2_0);
T2_0 = 1/2*transpose(omega2)*I*omega2;

H2 = transpose(I*transpose(w2));
H2_mag = sqrt(sum(H2.^2,2));
T2 = 1/2*sum(w2.*H2, 2);

[Xs, Ys, Zs] = sphere(20);
Xs = H2_0_mag*Xs;
Ys = H2_0_mag*Ys;
Zs = H2_0_mag*Zs;

radii = sqrt(2*T2_0.*diag(I));
```

5

```
[Xe, Ye, Ze] = ellipsoid(0,0,0, radii(1), radii(2), radii(3), 60);

% plot
fig = figure;
hold on;
% sphere
surf(Xs, Ys, Zs, 'FaceAlpha',0.3, 'EdgeColor','b', 'FaceColor', [0.8 0.8 1]);
% ellipsoid
surf(Xe, Ye, Ze, 'FaceAlpha',0.3, 'EdgeColor','g', 'FaceColor', [0.2 0.8 0.2]);
% polhode points (body-fixed H)
plot3(H2(:,1), H2(:,2), H2(:,3), '.r', 'MarkerSize', 5);
title('Polhode Plot (\omega_0 = [1;15;0] deg/s)');
xlabel('H_x');
ylabel('H_y');
zlabel('H_z');
axis equal;
grid on;
view([40.6 14.7]);
hold off;

% save
saveas(fig, "./images/s02f.png");
```



Polhode Plot ($\omega_0$ = [1;15;0] deg/s)
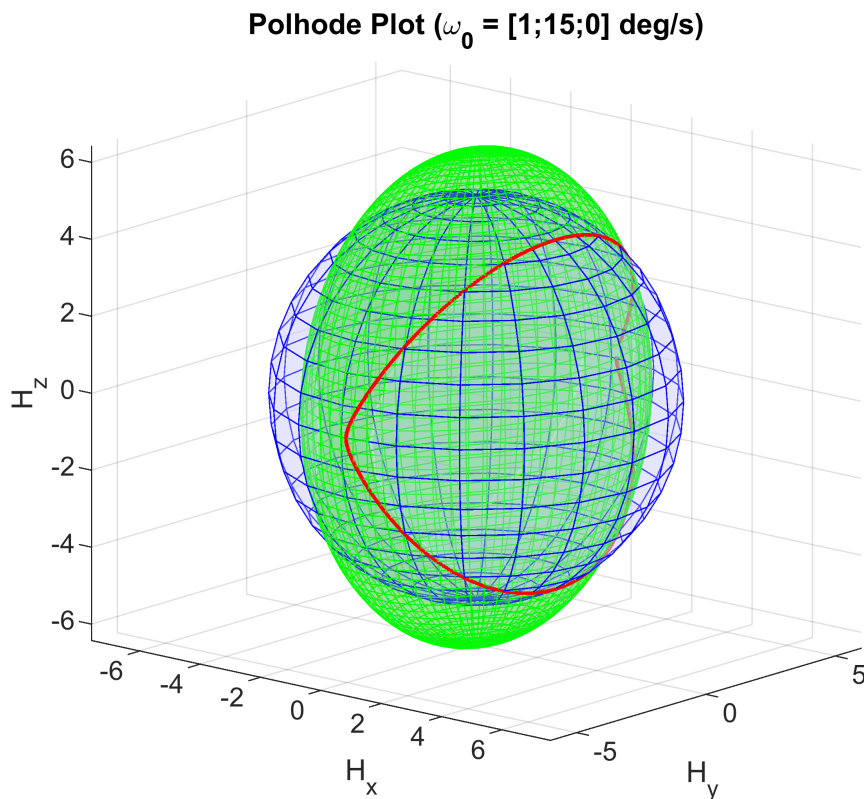
The spacecraft in part b and d has a much higher kinetic energy in the Z axis than in the X and Y axes, and as well varies each axis in only small oscillations. This is far different from the spacecraft in e and f, who has far more energy-variance within each axis, as well as having its overall spread be much closer together. These two

differences result in the Polhode plot of the first spacecraft being confined to a small area high in the positive Z axis, while the second spacecraft's plot ventures all over the place.