Case Study

Consider system from HW #8: $G(s) = \dfrac{1}{10s^2(s+1)}$

We had 2 designs

Case 1: "low bandwidth" $\omega_r = 0.1,\ \gamma = 45°$

$$H_1(s) = \frac{0.036(28s+1)}{(3.57s+1)}$$

Case 2: "Higher bandwidth" $\omega_r = 1,\ \gamma = 45°$

$$H_2(s) = \frac{2.426(2.41s+1)^2}{(.41s+1)^2}$$

Step response characteristics:

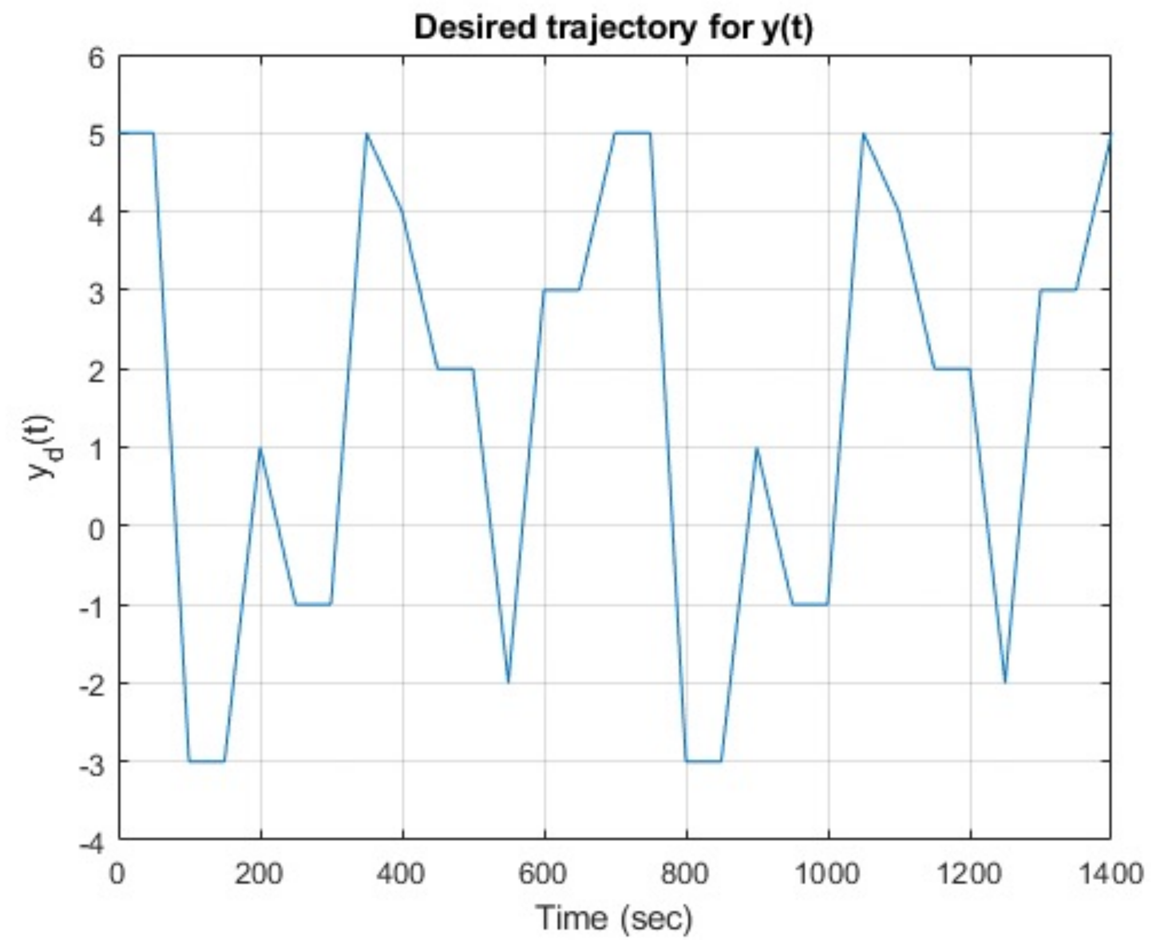
Case 1 gives about $t_s = 67.2$ sec, %OS = 33%

Case 2 gives about $t_s = 10.2$ sec, %OS = 34%

$e_{ss}(t) = 0$ in both cases (since $L(s)$ has 2 poles at 0)

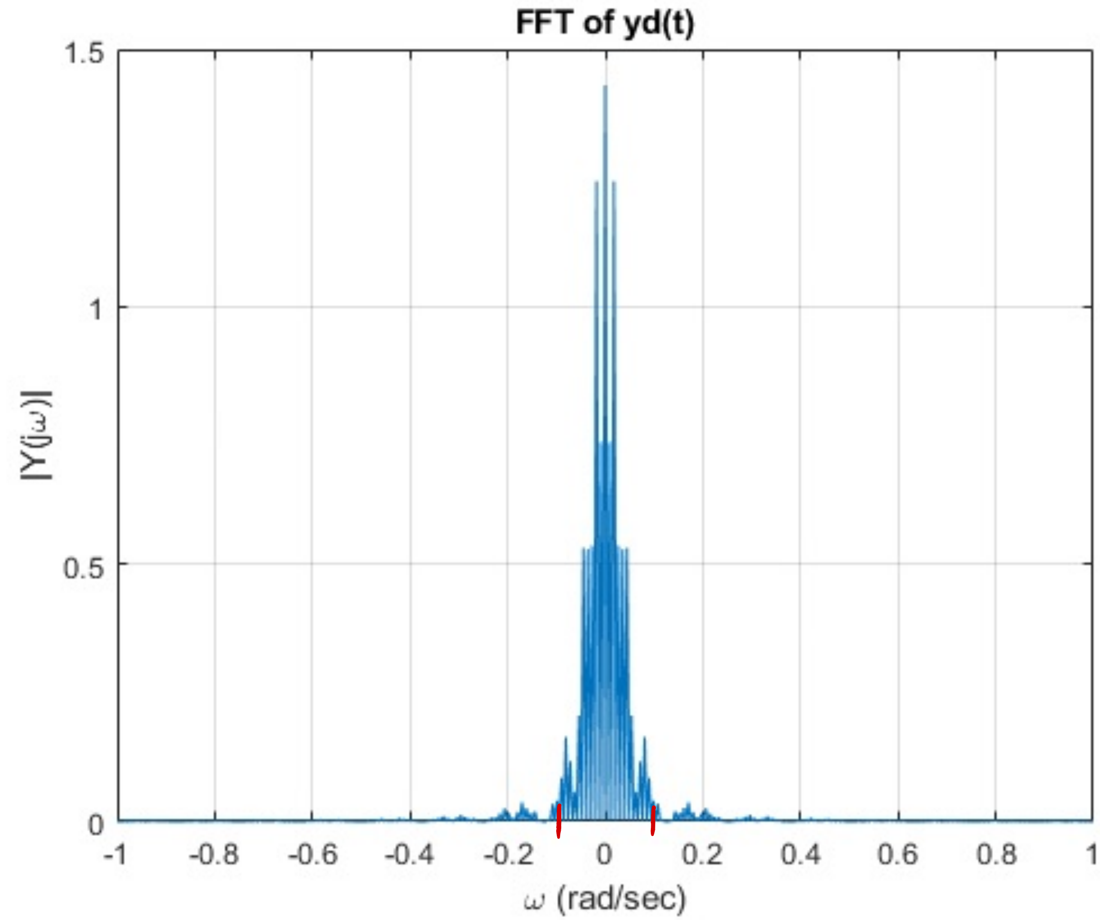
More complicated $y_d(t)$

Suppose instead $y_d(t)$ looks like:



Perhaps representative of a path followed by a drone navigating a cluttered environment.

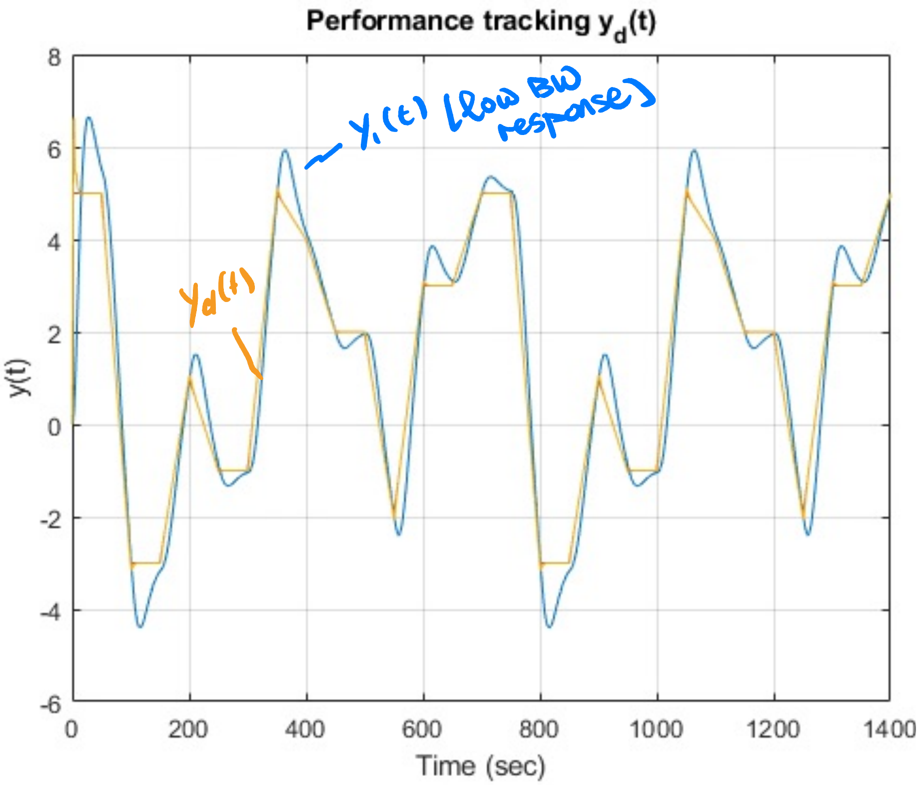
Frequency content of $y_d(t)$



Note that $y_d(t)$ has significant freq. content near and below 0.1 rad/sec, but almost none near or above 0.5 rad/sec

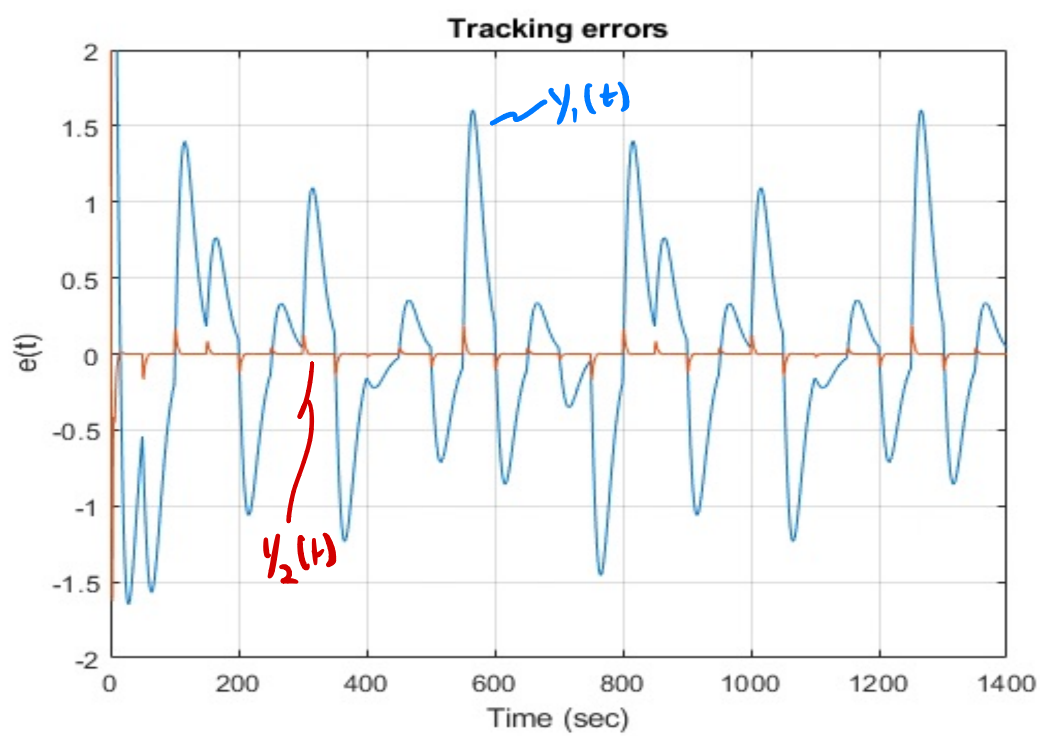
$\Rightarrow$ Expect low BW ($\omega_r = 0.1$) design to have poor tracking, But high BW ($\omega_r = 1$) design to have good tracking.

Tracking performance

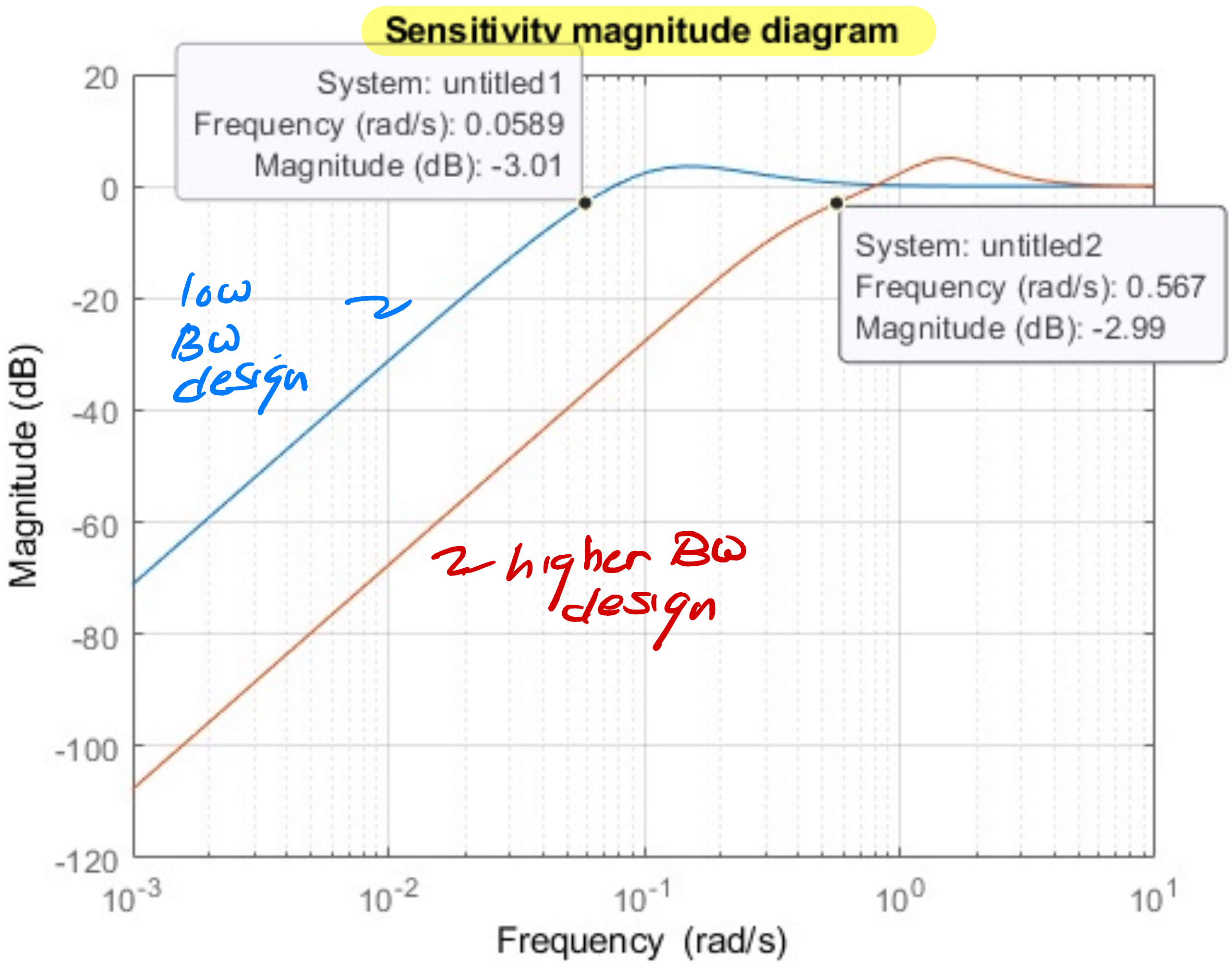


High BW response $y_2(t)$ almost identical to $y_d(t)$ at this scale

High BW response has atleast 10 times less tracking error!



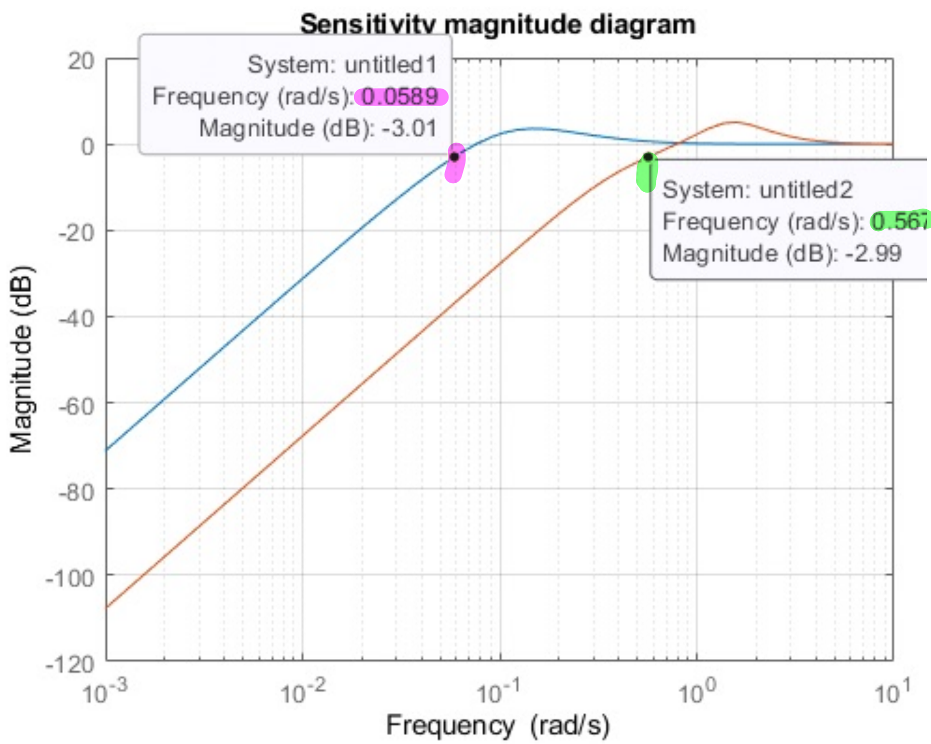
Importance of "bandwidth"

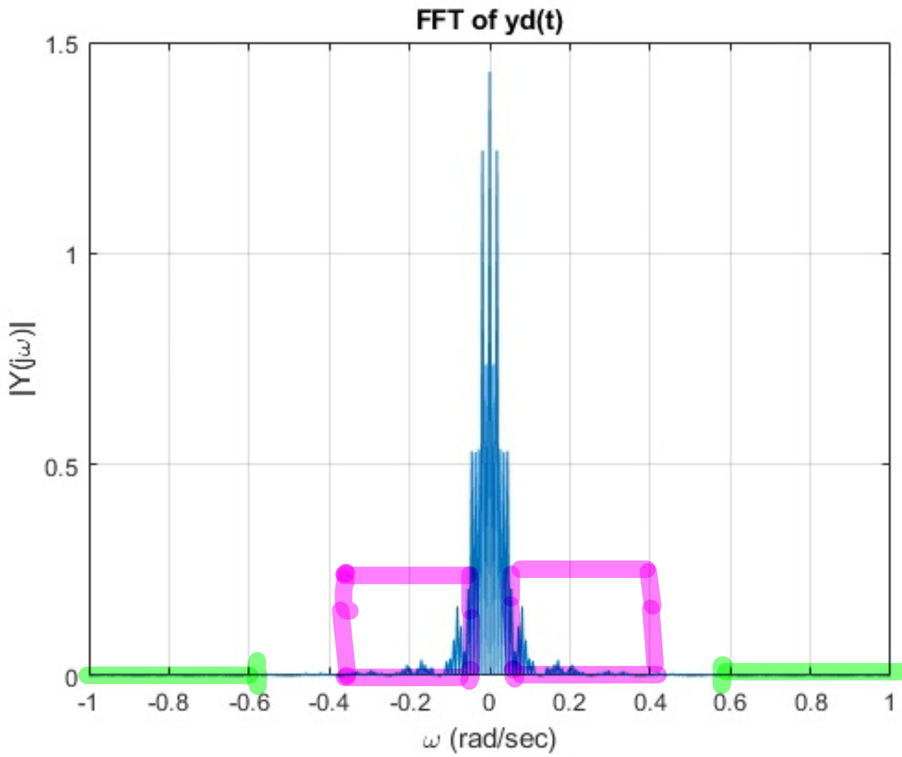


Recall $\omega_\gamma/2.5 \leq \omega_B \leq \omega_\gamma$ typically

here $\omega_B \approx \omega_\gamma/1.7$ in both cases

Importance of "bandwidth"



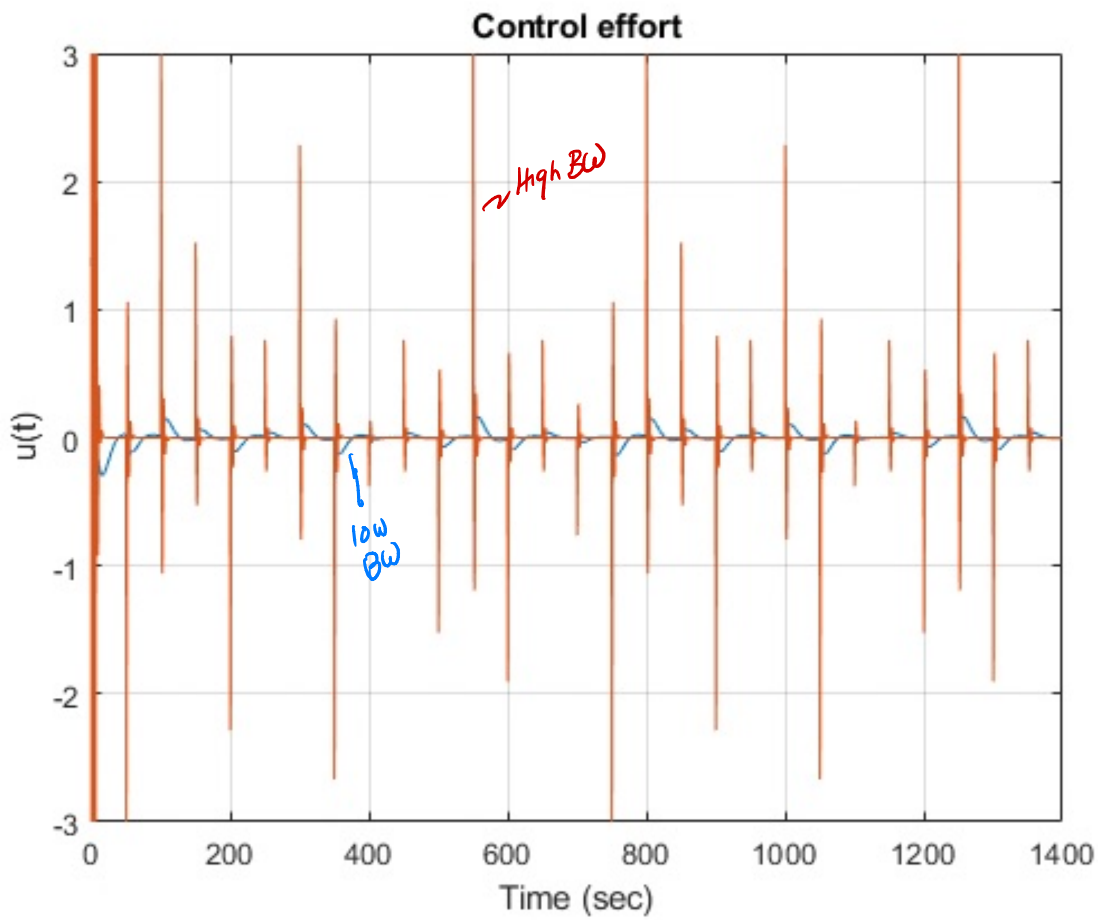


$y_d(t)$ has significant freq content between 0.06 and 0.4 rad/sec which low BW design fails to track well

However, $y_d(t)$ has very little freq content above 0.6 rad/sec, so high BW design does a good job tracking

(Remember: need $|S(j\omega)| \ll 0\,dB$ to track freq ω with minimal steady-state error)

Control Effort:

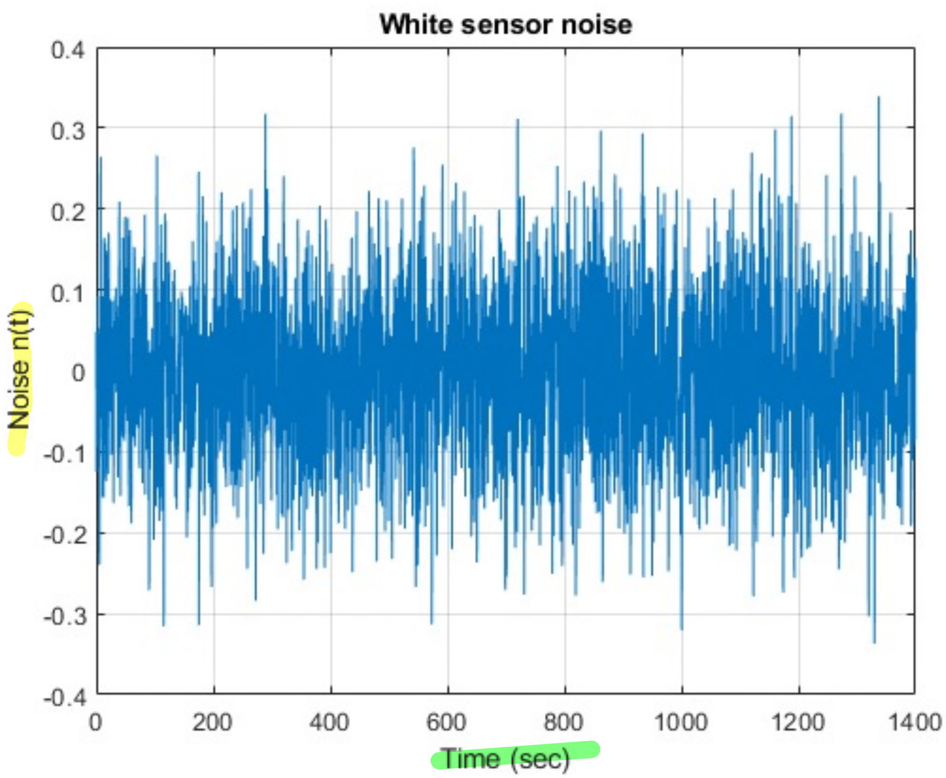


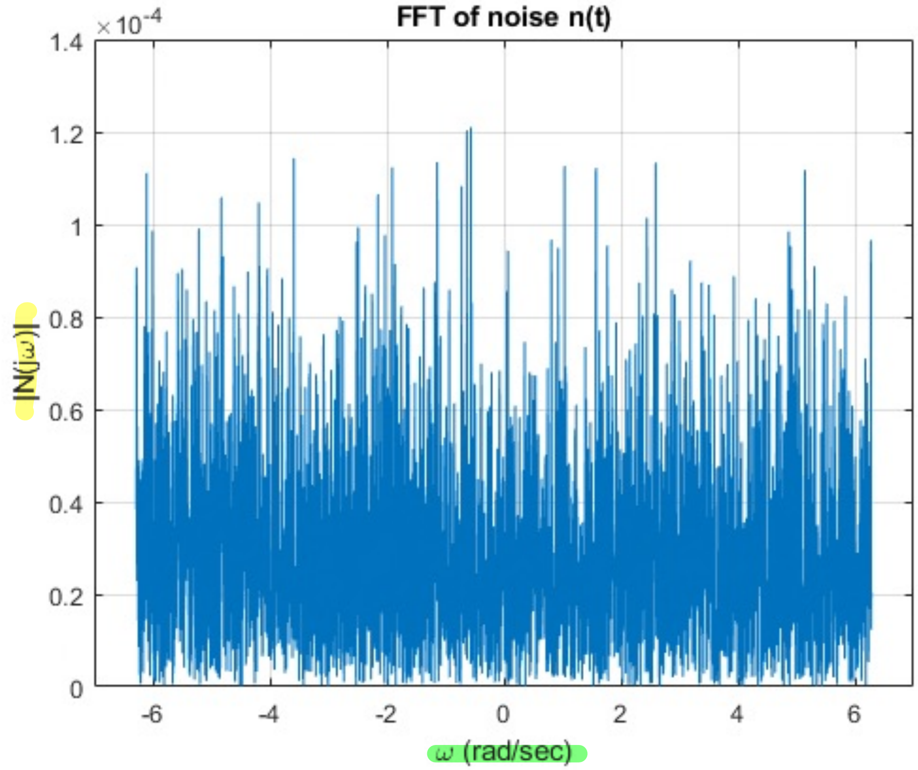
Of couse, significantly more force/moment must be applied to the vehicle to make it accurately execute those sharp corners in $y_d(t)$.

High BW inputs $u(t)$ about 20x larger than low BW.

Noise Sensitivity

Consider a "white" (extremely broadband) noise corrupting sensor measurements (worst case model): $y_m(t) = y(t) + n(t)$





Significant amplitudes <u>across all fequencies</u>.

Remember that each freq. in $n(t)$ will show up in $y(t)$ with amplitude multiplied by $|T(j\omega)|$

Recall: $Y(s) = T(s)Y_d(s) + S_i(s)D(s) + T(s)N(s)$

look at this term

If $n(t) = \sum_k A_k \sin(\omega_k t)$ (i.e. many diff't sinusoids in $n(t)$)

then $y(t)$ will contain

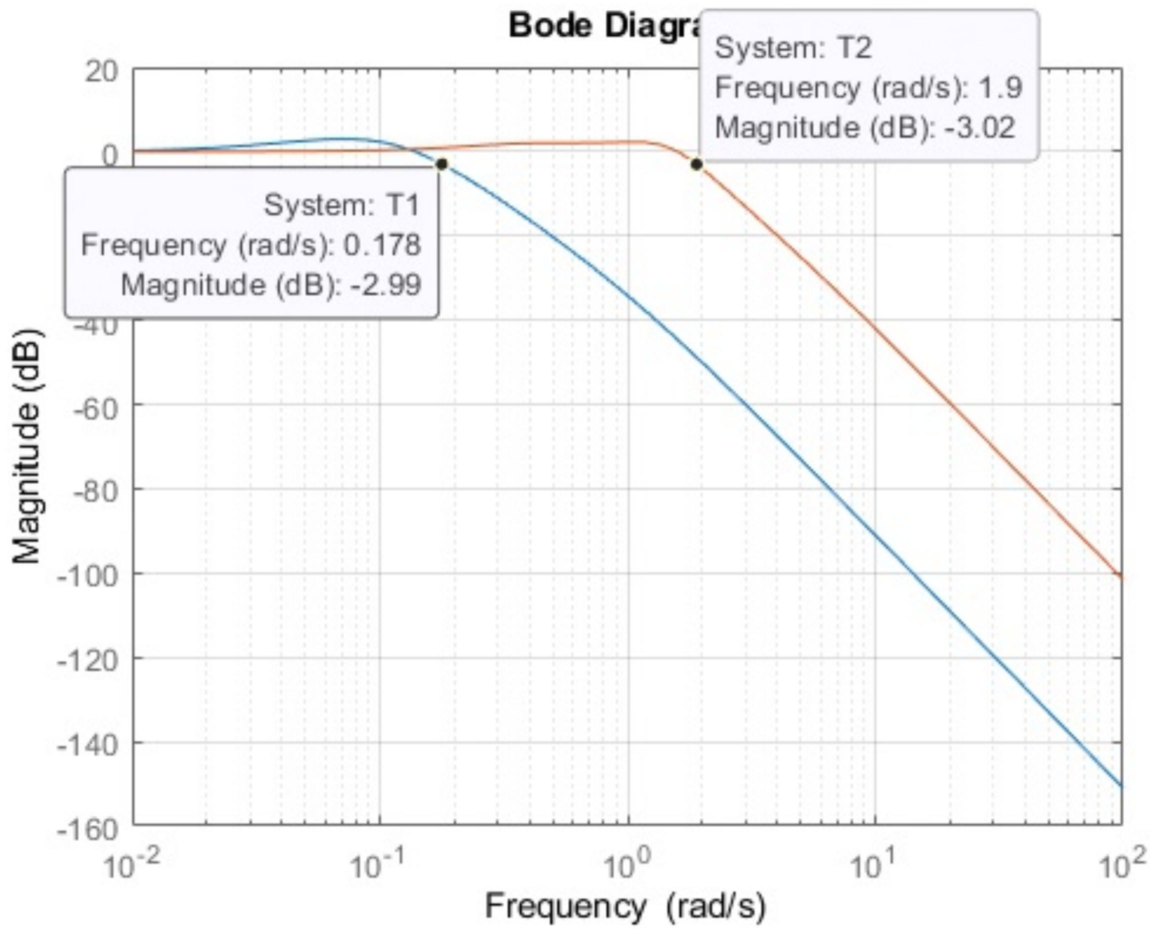
$$\sum_k A_k |T(j\omega_k)| \sin(\omega_k t + \measuredangle T(j\omega_k))$$ (by linearity)

which will be non-negligible wherever $|T(j\omega)| \approx 1$

Want $|T(j\omega)| << 0$ dB to "reject" noise at freq ω

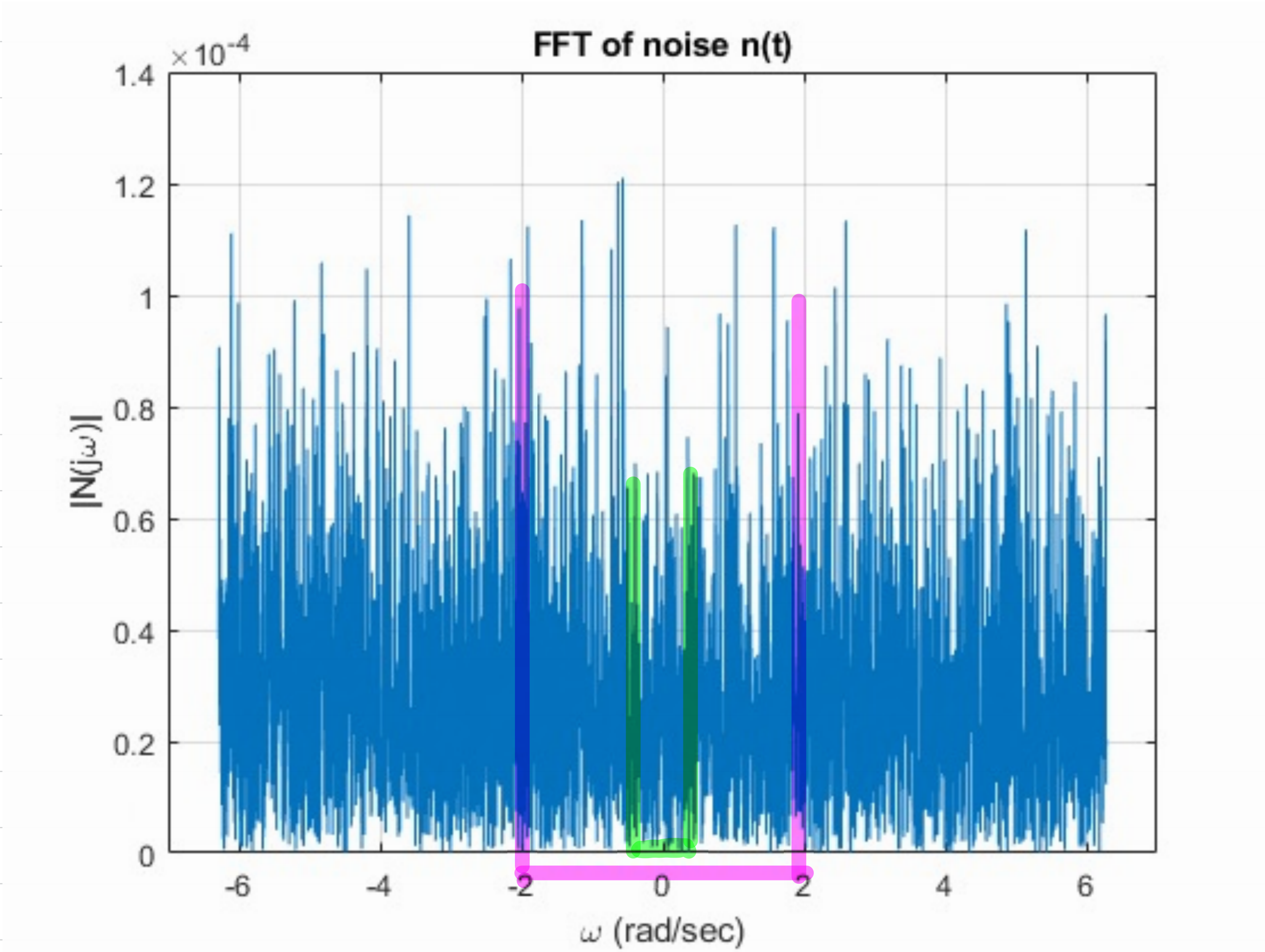
If $|T(j\omega)| \simeq 1$ at noise freqs ω_k, those components of noise will be "PASSED through" unfiltered to $y(t)$ creating significant unwanted motion.

CL Bode magnitudes



High BW design will "pass through" ($T(j\omega) \approx 1$) a larger range of freqs (up to ~2 rad/sec) compared to low BW design (which passes only up to ~0.2 rad/sec)

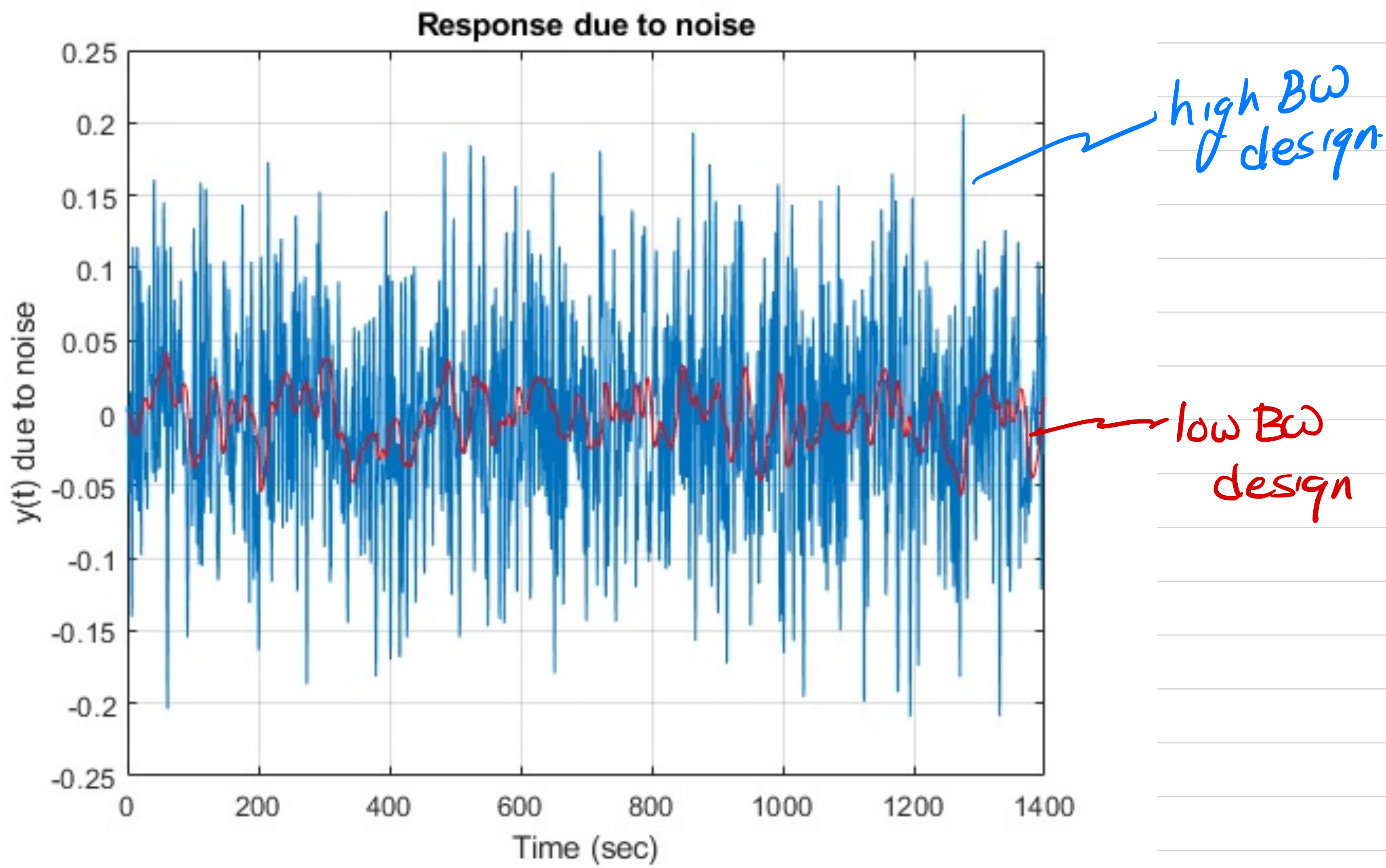
$\Rightarrow$ More effect of noise on high BW design



freqs passed through by low BW T(s)

freqs passed through by high BW T(s)

Noise effect on y(t)



Higher BW design shows 3x greater impact of Noise on y(t).