"Phasor" Notation

Observation: Complex number add'n/sub'n follows same rules as 2D (planar) vectors

$z_1 = a_1 + b_1 j, \quad z_2 = a_2 + b_2 j$ $\qquad \underline{V}_1 = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \quad \underline{V}_2 = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$

$z_3 = z_1 + z_2$ $\qquad \underline{V}_3 = \underline{V}_1 + \underline{V}_2$

$= (a_1 + a_2) + (b_1 + b_2) j$ $\qquad = \begin{bmatrix} a_1 + a_2 \\ b_1 + b_2 \end{bmatrix}$

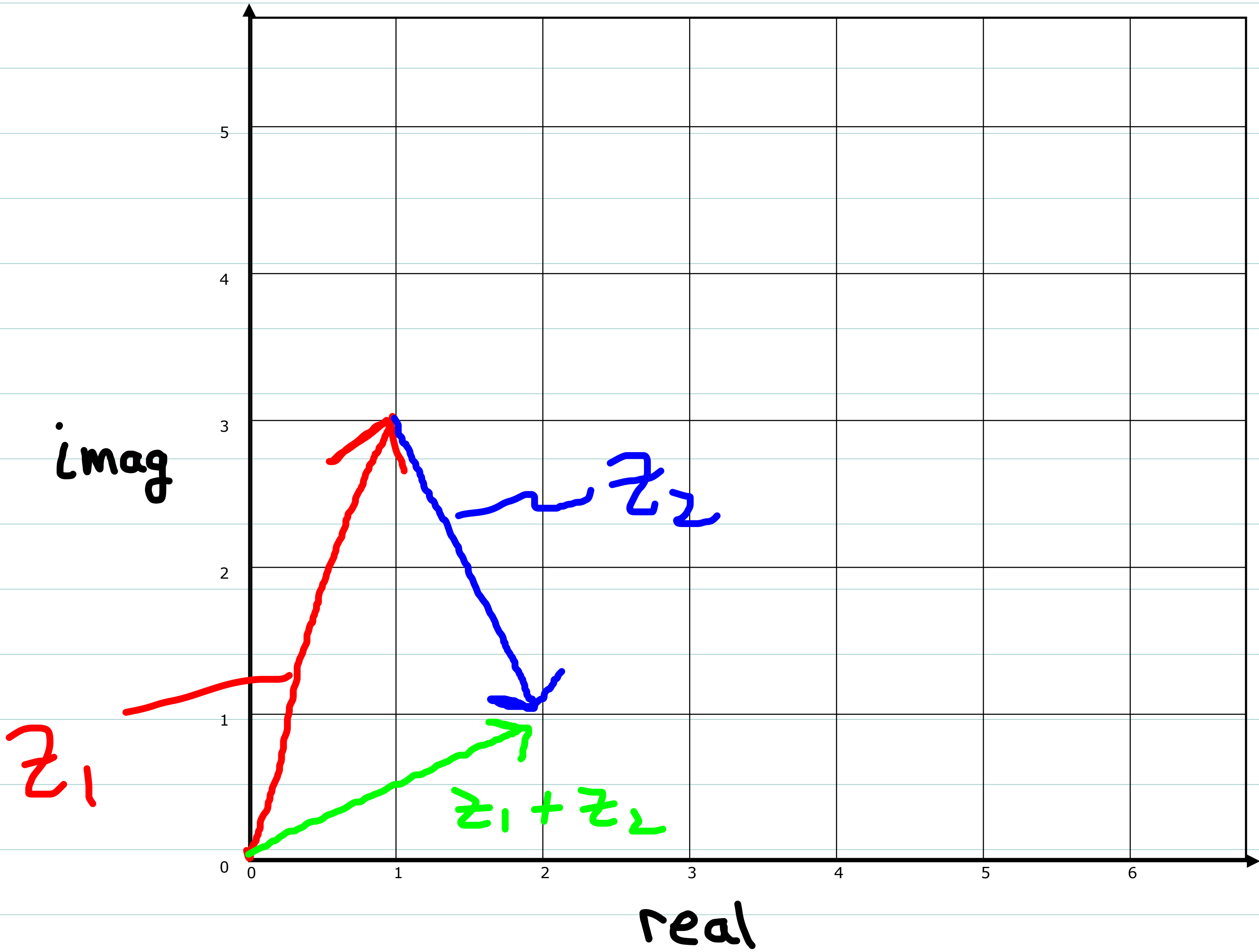
i.e. identify real part with 1st component of 2D vector; imag part with 2nd component.

⇒ Can interpret complex numbers as planar vectors

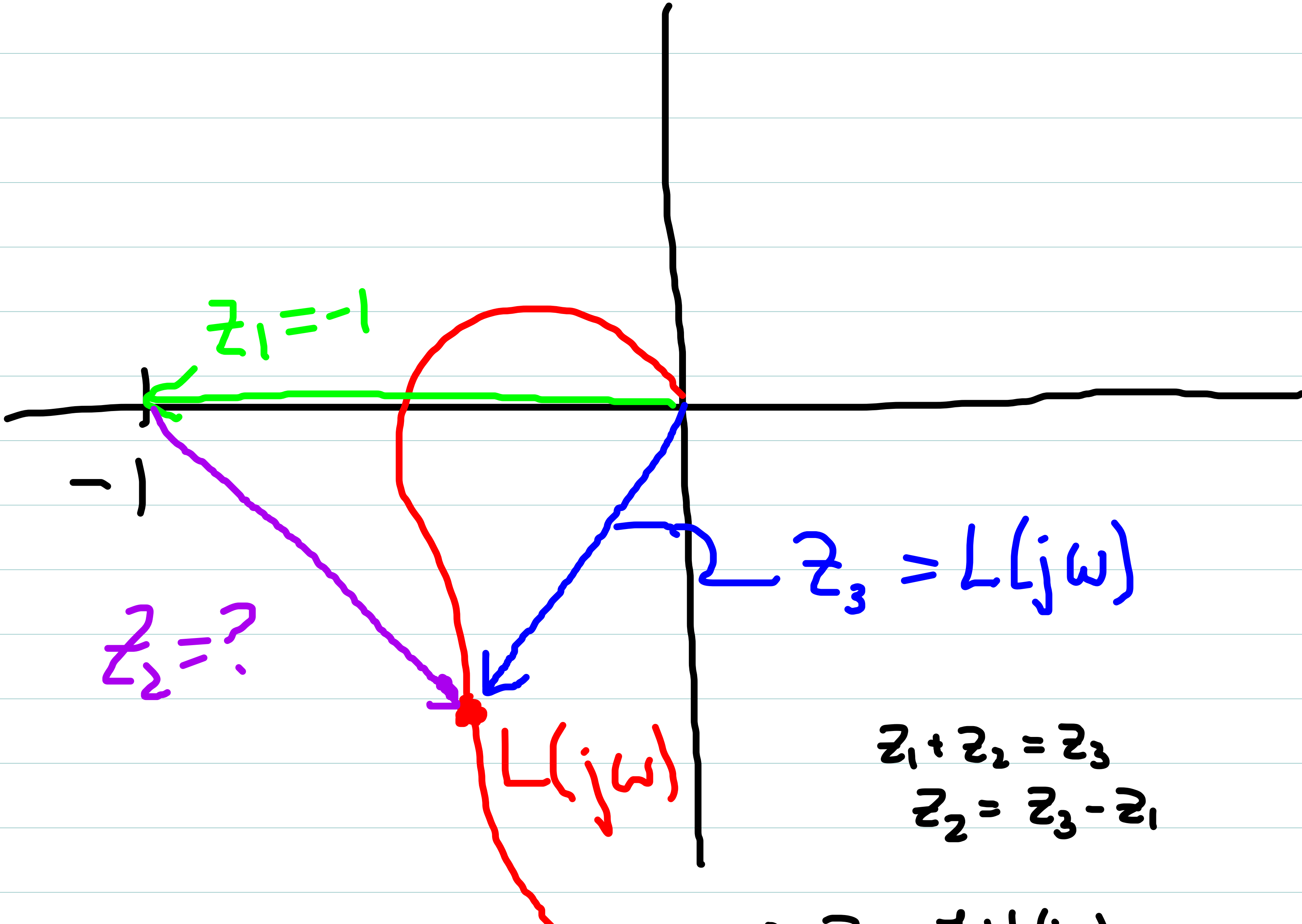
⇒ Can use vector graphical add'n tricks for complex numbers

Example

$z_1 = 1 + 3j, \; z_2 = 1 - 2j \Rightarrow z_3 = z_1 + z_2 = 2 + j$



Important Application



$z_1 + z_2 = z_3$

$z_2 = z_3 - z_1$

$\Rightarrow z_2 = 1 + L(j\omega)$

Thus:

Complex number $1+L(j\omega)$ can be graphically visualized as the phasor from -1 to $L(j\omega)$ on polar plot.

$\Rightarrow$ $|1+L(j\omega)|$ is the distance from -1 to polar plot at freq ω.

$\Rightarrow$ Good robustness requires this doesn't get too small!

$\Rightarrow$ But note: $|1+L(j\omega)| = |S(j\omega)|^{-1}$

$\Rightarrow$ Thus, good robustness requires $|S(j\omega)|$ not get too big.

$\Rightarrow$ Good designs have $|S(j\omega)|$ which do not exhibit a large **peak**!

$$\left[\max_{\omega} |S(j\omega)|\right]^{-1} = \min_{\omega} |1 + L(j\omega)|$$

$=$ smallest distance from -1 to the polar/Nyquist plot

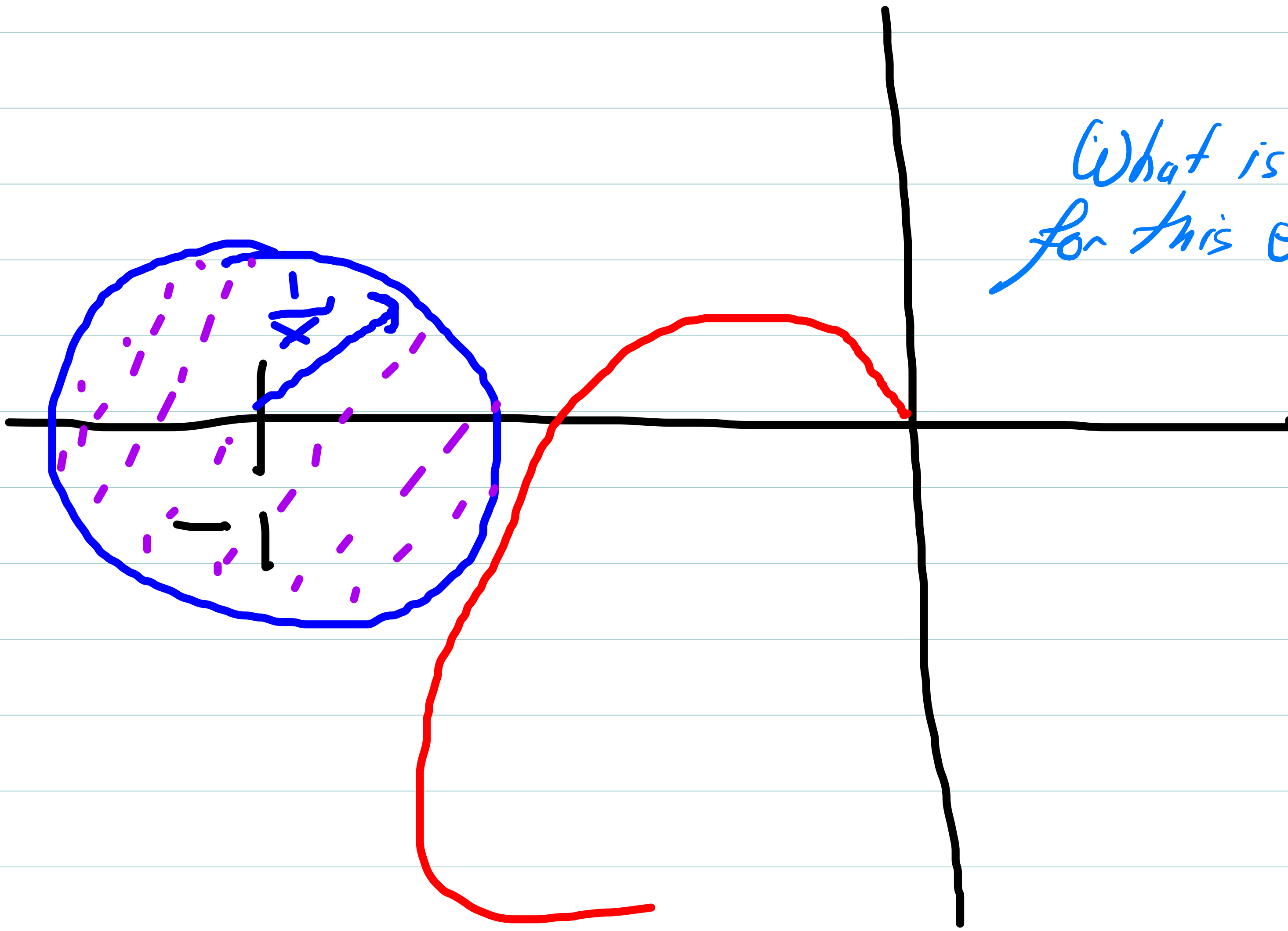
We do not want this to be too small, hence we need to ensure $\max_{\omega} |S(j\omega)|$ is not too big.

What is an appropriate target for $\max_{\omega} |S(j\omega)|$?

Now:

$$|S(j\omega)|_{max} < X \Rightarrow |1 + L(j\omega)| > \frac{1}{X} \text{ for all } \omega \geq 0$$

$\Rightarrow$ Polar (Nyquist) diagram of $L(j\omega)$ cannot enter a disk of radius $\frac{1}{X}$ centered at -1



What is a "good" size for this exclusion disk?

This property guarantees certain minimum phase + gain margins

for example, can show: $|S(j\omega)|_{max} < 2$ (+6 dB) $\Rightarrow |1+L(j\omega)| > 1/2$

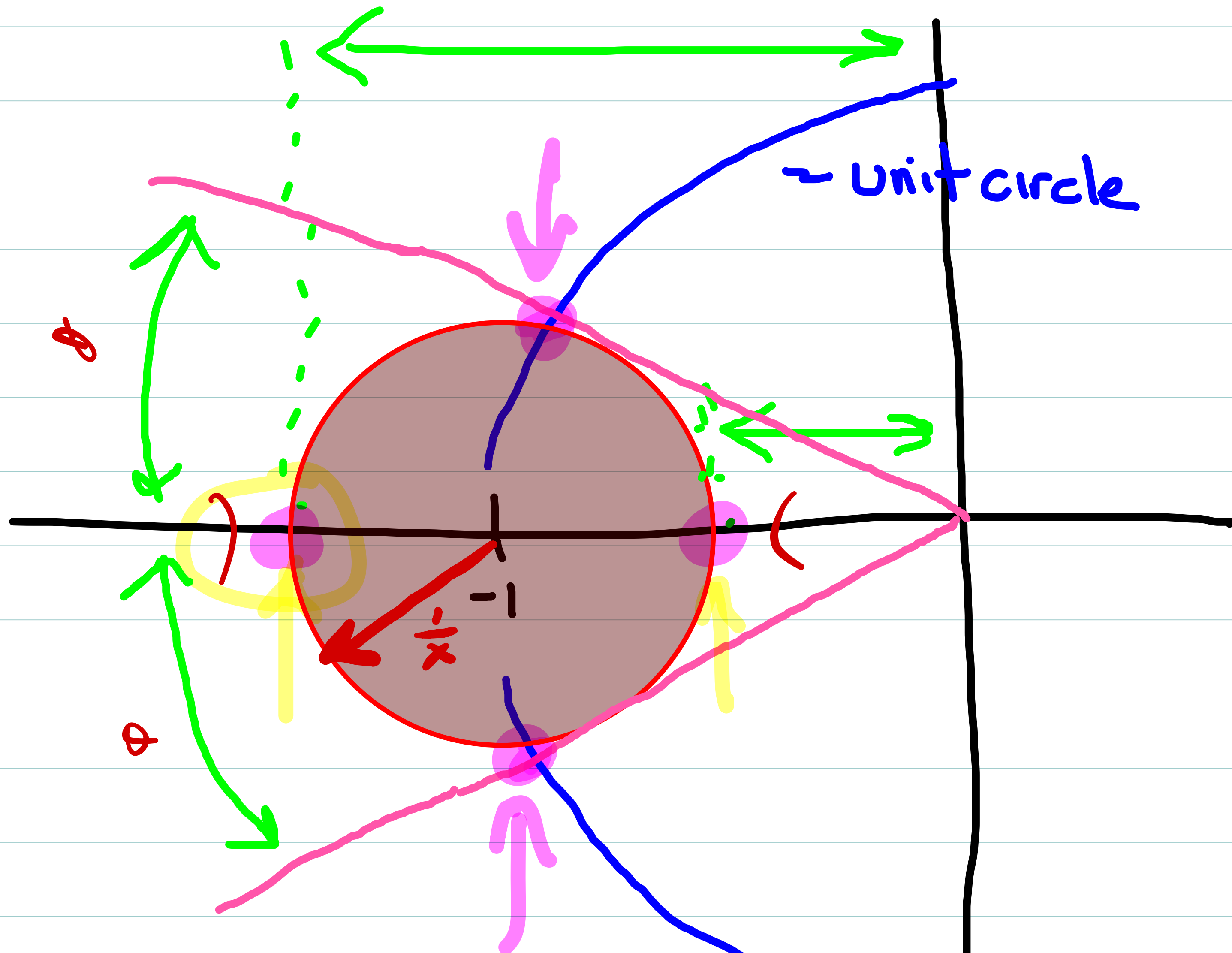
$\Rightarrow a < 2/3$ (−3.5 dB), $a > 2$ (+6 dB)

$\Rightarrow |\gamma| > 29°$

(Note that these are pretty close to the common industry standard req'ts: $|a|_{dB} \geq 6$, $|\gamma| \geq 30°$)

However, a specific set of gain, phase margins does not conversely guarantee a bound on $|S(j\omega)|_{max}$ (as shown in previous example!)

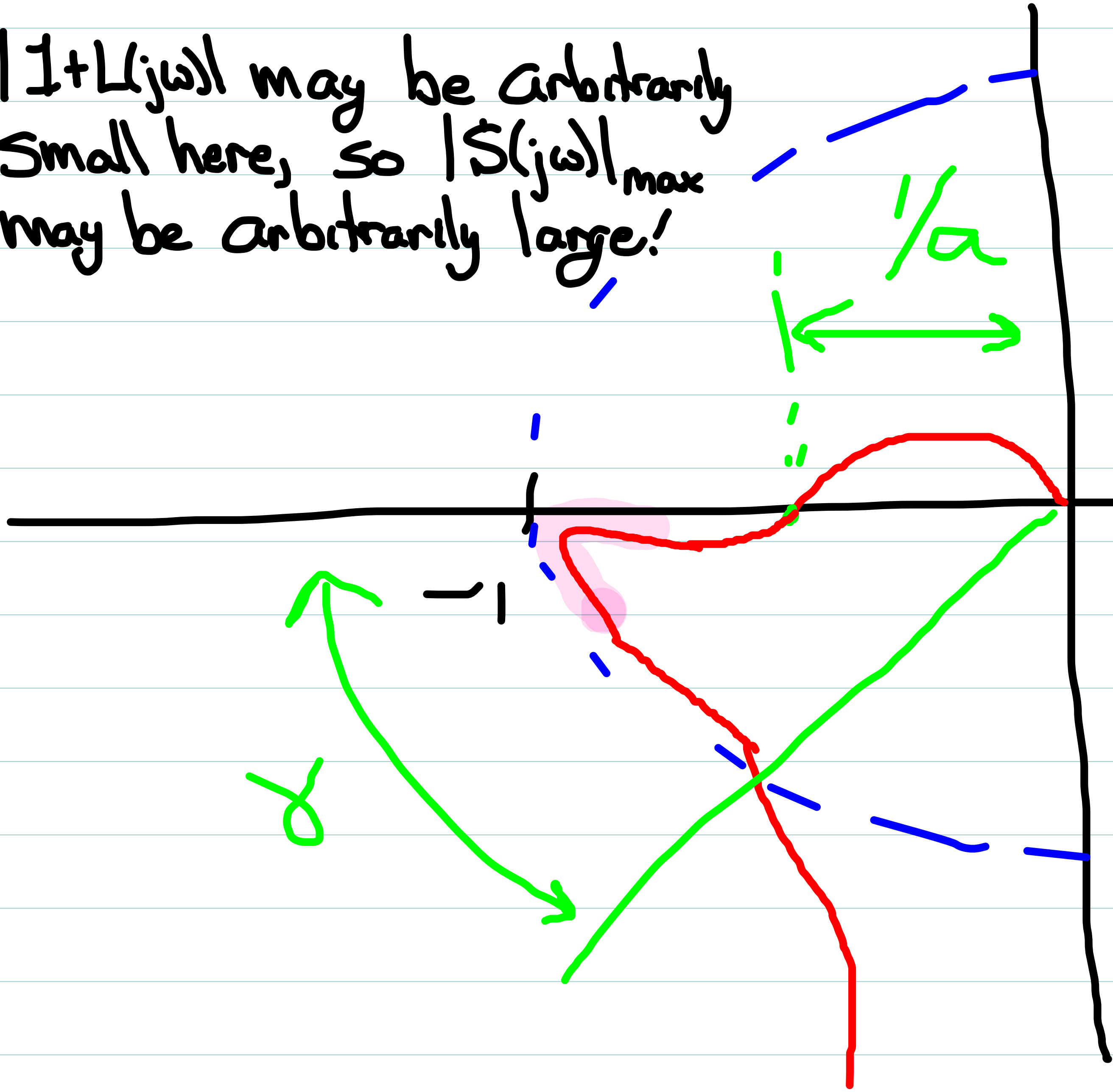
$\Rightarrow |S(j\omega)|_{max}$ (peak of sensitivity diagram) is a superior measure of robustness, and $|S(j\omega)|_{max} \lesssim +6$ dB is a good nominal target.



$$x = \max_{\omega} |S(j\omega)|$$

$$\frac{1}{x} = \min_{\omega} |1 + L(j\omega)|$$

$|1+L(j\omega)|$ may be arbitrarily small here, so $|S(j\omega)|_{max}$ may be arbitrarily large!



= range of possible $\tilde{G}(j\omega)$, i.e. "true" polar plot lies somewhere in shaded region.

[Not a lot of room to tolerate model error if peak of $|S(j\omega)|$ is large]