Lecture 17: Oblique and Curved Shocks

ENAE311H Aerodynamics I

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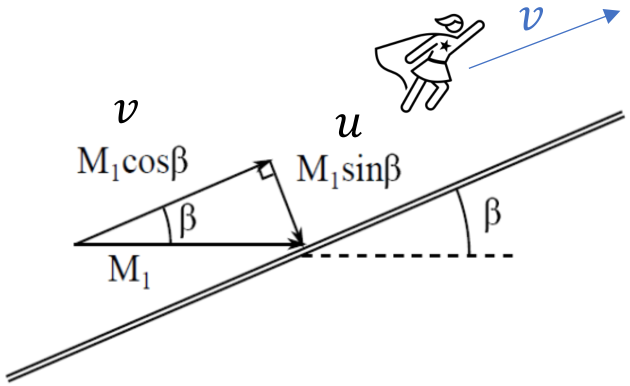
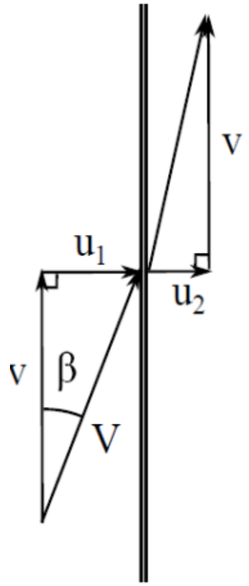
Oblique shocks of finite strength



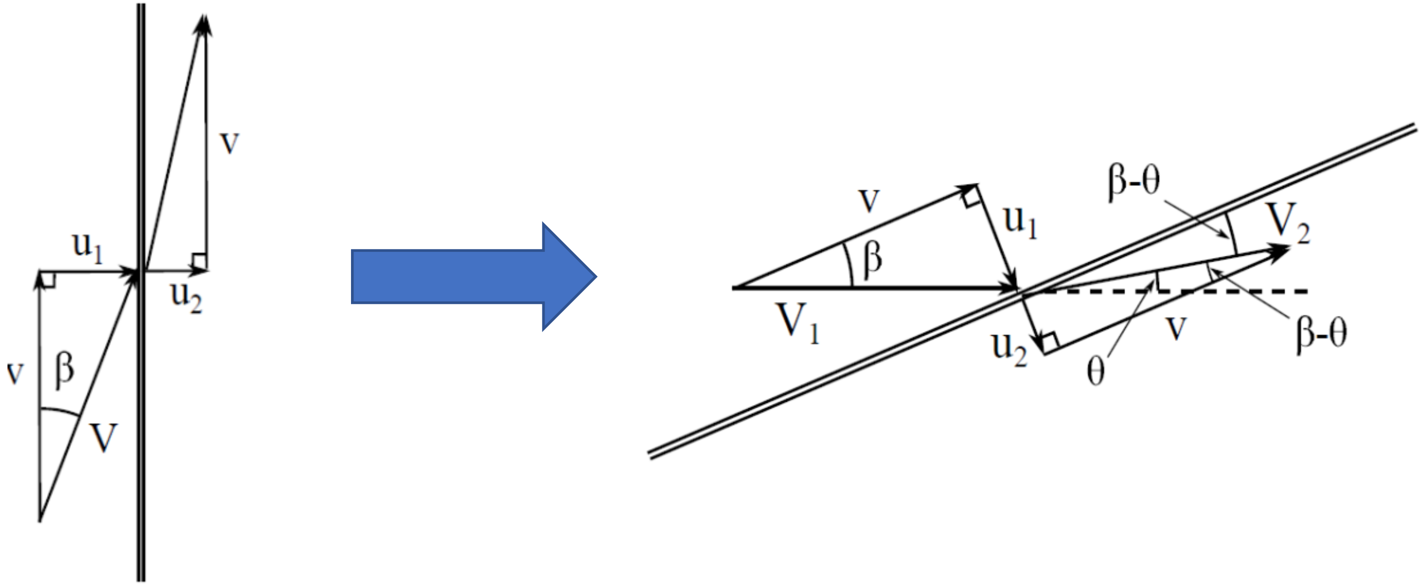


$$
\begin{aligned}
\frac{\rho_2}{\rho_1} &= \frac{(\gamma+1)M_{n1}^2}{2+(\gamma-1)M_{n1}^2} \\
\frac{p_2}{p_1} &= 1+\frac{2\gamma}{\gamma+1}(M_{n1}^2-1) \\
\frac{T_2}{T_1} &= \left[1+\frac{2\gamma}{\gamma+1}(M_{n1}^2-1)\right]\frac{2+(\gamma-1)M_{n1}^2}{(\gamma+1)M_{n1}^2}.
\end{aligned}
$$

Flow deflection through an oblique shock

We have seen how the thermodynamic properties vary through a shock – let's now see what happens to the flow velocity.

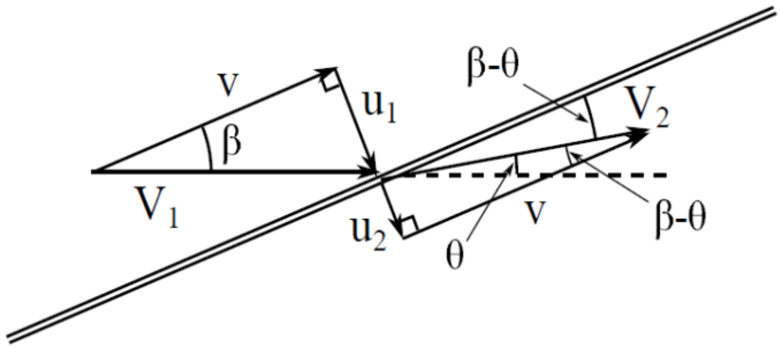
Consider again our transformed picture of the oblique shock (moving in a reference frame along the shock). Note again that the normal velocity component decreases, whereas the tangential component (which is zero in the transformed frame) is unaffected. If we now transform back to our original reference frame (by re-superimposing the tangential velocity component), the normal component will still decrease and the tangential component (now finite) is unchanged.



The flow is thus deflected towards the shock (though the flow angle remains less than the shock angle).

Flow deflection through an oblique shock

Consider the geometry shown to the right. We note the following:



$$\tan\beta = \frac{u_1}{v},$$

and

$$\tan(\beta-\theta) = \frac{u_2}{v}.$$

Using the continuity equation (on the normal velocity components), we can then write

$$\frac{\tan(\beta-\theta)}{\tan\beta} = \frac{u_2}{u_1} = \frac{\rho_1}{\rho_2} = \frac{2+(\gamma-1)M_1^2\sin^2\beta}{(\gamma+1)M_1^2\sin^2\beta}.$$

Now, we know

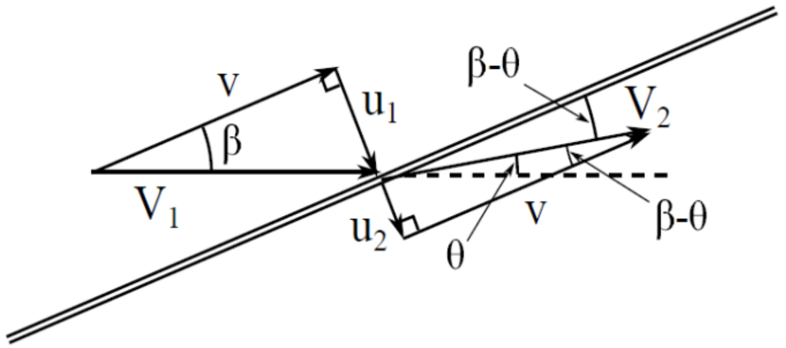
$$\tan(\beta-\theta) = \frac{\tan\beta-\tan\theta}{1+\tan\beta\tan\theta},$$

And thus can write our above expression as

$$\frac{1}{\tan\beta}\frac{\tan\beta-\tan\theta}{1+\tan\beta\tan\theta} = \frac{2+(\gamma-1)M_1^2\sin^2\beta}{(\gamma+1)M_1^2\sin^2\beta}.$$

Flow deflection through an oblique shock

Simplifying, we arrive at



$$\boxed{\tan\theta = 2\cot\beta\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}.}$$

This expression is known as the $\theta - \beta - M$ relation, and gives the deflection angle in terms of the Mach number and shock angle (unfortunately, there is no simple relation that gives the shock angle in terms of the deflection angle).

To determine the Mach number behind the shock, we first use our normal shock relation to determine the normal component of the post-shock Mach number

$$M_{n2}^2 = \frac{2 + (\gamma - 1)M_{n1}^2}{2\gamma M_{n1}^2 - (\gamma - 1)}.$$

From the geometry above, we see

$$M_{n2} = M_2\sin(\beta - \theta),$$

And thus

$$\boxed{M_2^2\sin^2(\beta - \theta) = \frac{2 + (\gamma - 1)M_1^2\sin^2\beta}{2\gamma M_1^2\sin^2\beta - (\gamma - 1)}.}$$

Flow deflection through an oblique shock

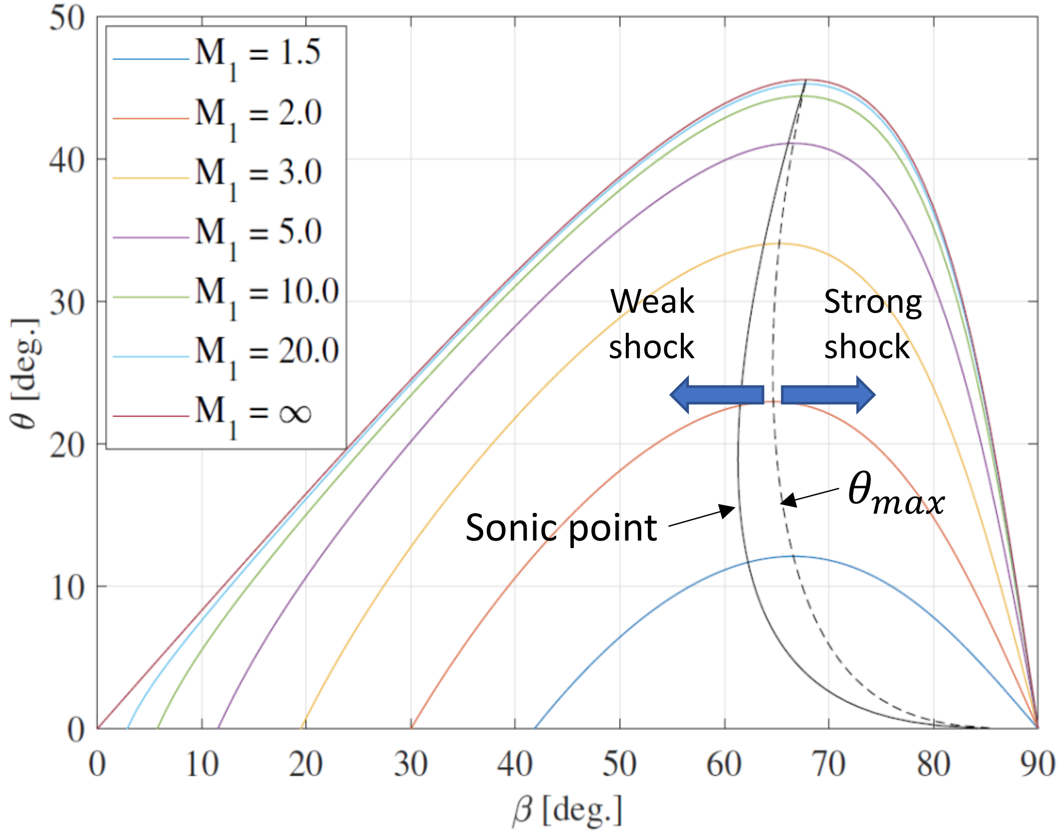
For a given M, the $\theta - \beta - M$ relation has two zeros – one at $\beta = \pi/2$ and one at $\beta = \text{asin}(1/M_1)$. In between, θ is everywhere positive, and thus must reach a maximum, θ_{max}.

The shock angle at this point, $\beta_{\theta max}$, is given by

$$\sin^2 \beta_{\theta max} = \frac{1}{4\gamma M_1^2}\left\{(\gamma+1)M_1^2 - 4 + \sqrt{(\gamma+1)[(\gamma+1)M_1^4 + 8(\gamma-1)M_1^2 + 16]}\right\}.$$

This maximum divides the curve into two: a strong-shock branch ($\beta > \beta_{\theta max}$) and a weak-shock branch ($\beta < \beta_{\theta max}$).

The post-shock flow is always subsonic on the strong-shock branch, and primarily supersonic on the weak-shock branch, but with a small portion on the latter where $M_2 < 1$. The sonic shock angle, β^*, at which $M_2 = 1$, is given by



$$\sin^2 \beta^* = \frac{1}{4\gamma M_1^2}\left\{(\gamma+1)M_1^2 - (3-\gamma) + \sqrt{(\gamma+1)[(\gamma+1)M_1^4 - 2(3-\gamma)M_1^2 + \gamma + 9]}\right\}.$$

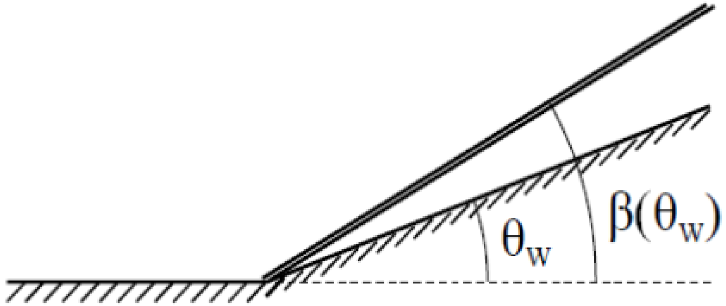
Flow over compression corners and wedges

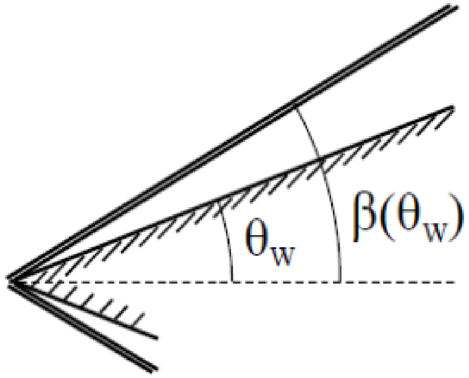
If we place a sharp compression corner or a wedge in a supersonic flow, provided the compression angle θ_w is less than θ_{max} for the given Mach number M_1, an attached oblique shock will form at the corner/vertex.

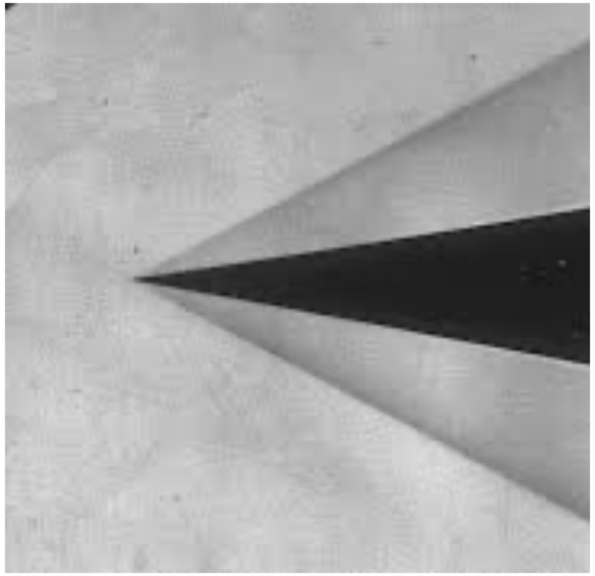
Since the flow angle downstream of the shock has to match the corner/wedge angle, the shock angle will be that corresponding to θ_w and M_1 in the $\theta - \beta - M$ relation. Note that the flow will typically choose the weak solution rather than the strong one.

Streamlines downstream of the shock will be straight and the flow conditions uniform.

Note that if the shock is attached in the case of the wedge, the top and bottom surfaces cannot communicate with one another, so for an asymmetric wedge the top and bottom shock angles will be different (to match the local flow angle).



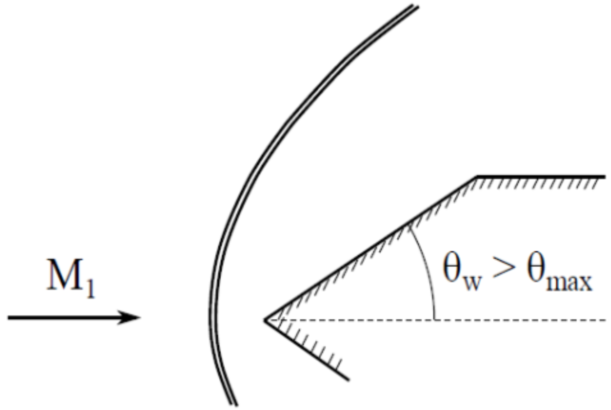


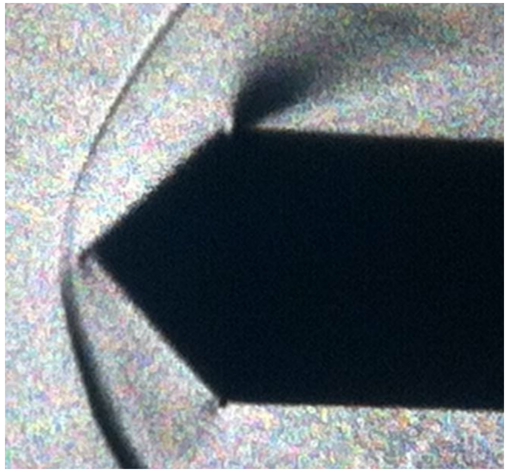


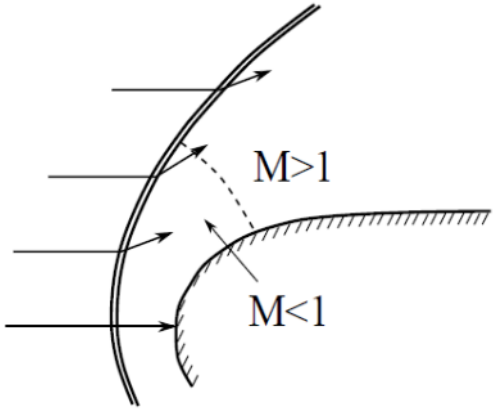
Flow over blunt bodies

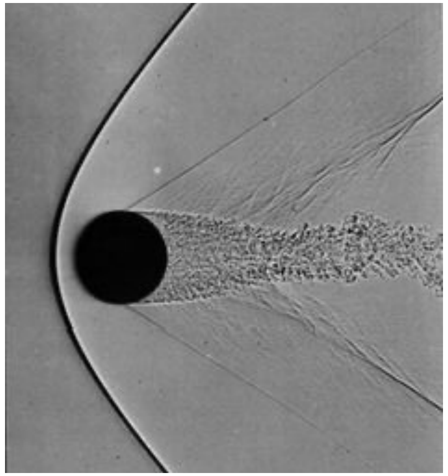
If the wedge angle is increased to above θ_{max}, and attached solution is no longer possible. Instead, a detached shock forms in front of the wedge with a finite shock stand-off distance.

The flowfield in this case will be qualitatively the same as that over a blunt body.









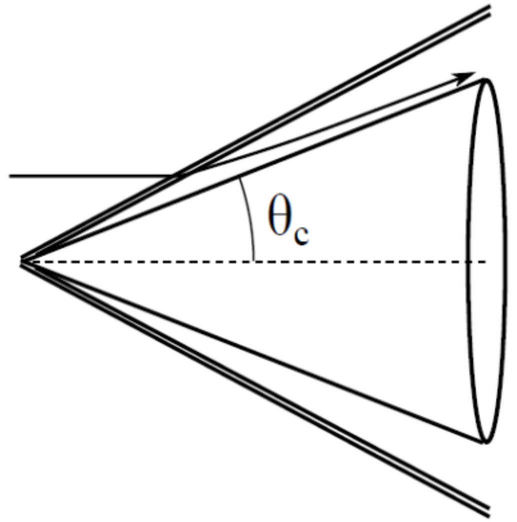
The shock will traverse all points from a normal shock to a Mach wave in the farfield. A finite subsonic region will form near the nose, with the flow accelerating to supersonic conditions downstream. The maximum-deflection will occur inside the subsonic region.

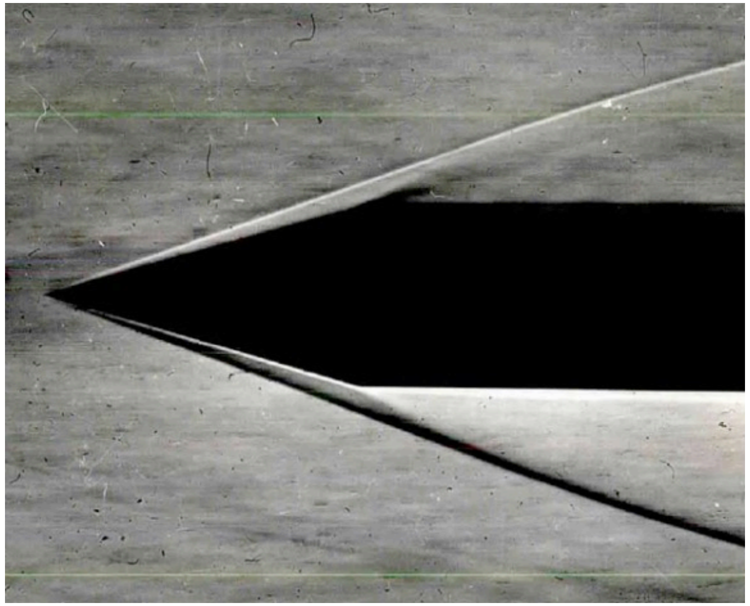
Flow over cones

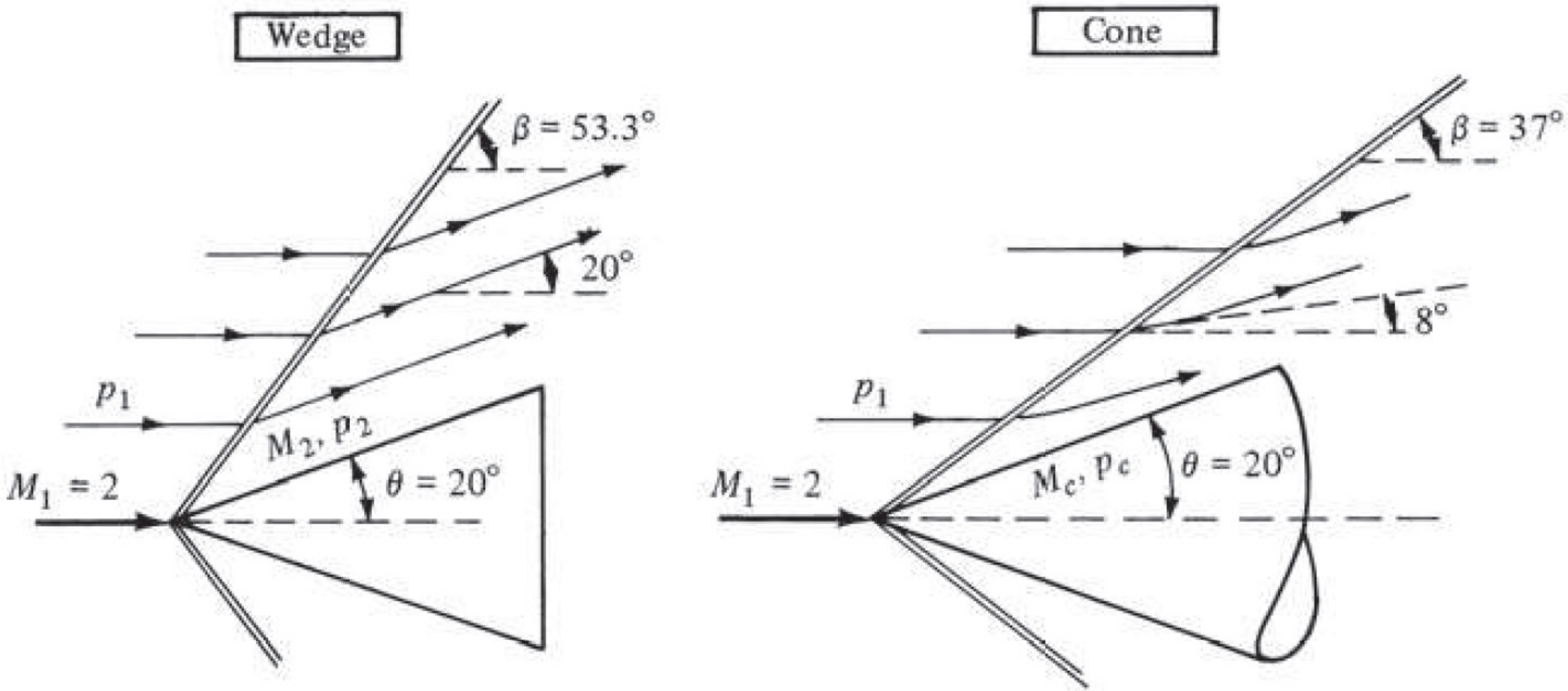
For a slender, infinite cone in supersonic flow, the shock will again be straight (since there is no length scale to scale any curvature).

However, the flow conditions cannot be uniform (this would violate continuity), so the streamlines curve towards the cone surface, with an additional post-shock compression taking place. Conditions are uniform along rays drawn from the cone vertex.

Because of 3-D relieving effects, the disturbance strength (shock angle and pressure jump) produced by a cone is much less than that of a wedge of the same half angle. Also, although the flow will eventually detach with increasing θ_c, this will happen at a much larger angle than for the equivalent symmetrical wedge.

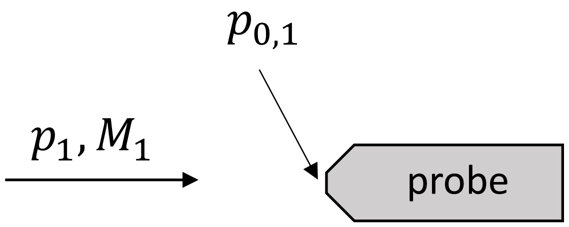





Wedge
$\beta = 53.3°$
$20°$
p_1
M_2, p_2
$\theta = 20°$
$M_1 = 2$
Cone
$\beta = 37°$
$8°$
p_1
M_c, p_c
$\theta = 20°$
$M_1 = 2$

Pitot probes in compressible flow

Subsonic:



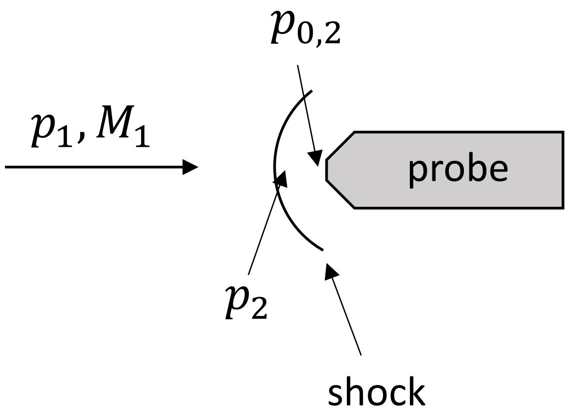
isentropic

$$\frac{p_{0,1}}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\gamma/(\gamma-1)}$$

$$M_1 = \left\{ \frac{2}{\gamma - 1} \left[\left(\frac{p_{0,1}}{p_1} \right)^{(\gamma-1)/\gamma} - 1 \right] \right\}^{1/2}.$$

$$V_1 = M_1 a_1.$$

Supersonic:



across shock

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1).$$

behind shock

$$\frac{p_{0,2}}{p_2} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{p_{0,2}}{p_1} = \left[\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma - 1)} \right]^{\gamma/(\gamma-1)} \left[1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1) \right].$$