Lecture 9: Coordinate Frames & Hohmann transfers

Coordinate Systems:

Earth-Centered Inertial: (ECI)

x: vernal equinox

y: completes the RH'd system

z: perpendicular to equatorial plane

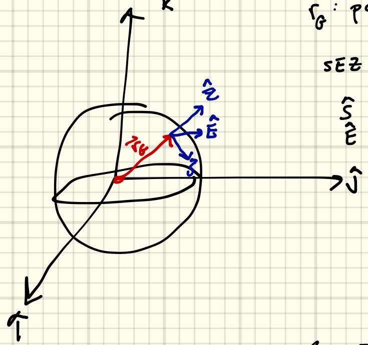
Perifocal Frame: PQW

Earth-Centered Earth-Fixed: (ECEF)

Same as ECI, but the frame rotates with the Earth.

in this case, $\hat{x}$ always points towards the greenwich meridian.

Topocentric Horizon: SEZ



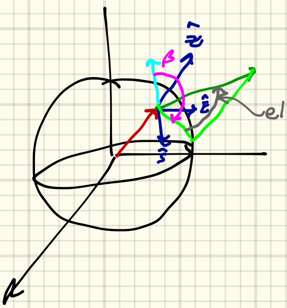
$\vec{r}_G$: position of the ground station

SEZ origin is at $\vec{r}_G$

$\hat{S}$ points South

$\hat{E}$ points East

$\hat{Z} \parallel \vec{r}_G$



β = azimuth = measured clockwise from North to the projection in the SE plane.

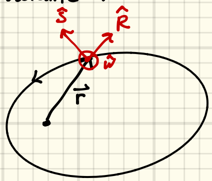
el: elevation: measured from the SE projection to the vector

Heliocentric Ecliptic: origin = sun

$\hat{x}$: vernal equinox

x, y plane is the ecliptic

Satellite: RSW



$\hat{R}$ is along the position vector

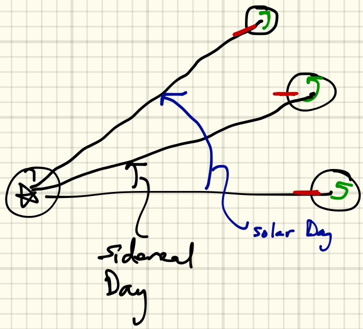
$\hat{W}$ is along $\vec{h}$

$\hat{S}$ completes the RH'd system

Time:

Solar Day: Amount of time between 2 sunrises
Slightly more time than is required for Earth to rotate 360°.

Sidereal Day: time required for Earth to rotate 360° about its axis.



Julian Date: Interval of time (measured in Days) from Jan 1 4713 B.C. at Noon
- often used when calculating the position of planetary bodies
- day starts at noon
- Need a lot of sigfigs, b/c values are in the millions

Maneuvers: ΔV is the magnitude of the velocity change from orbit A to orbit B.

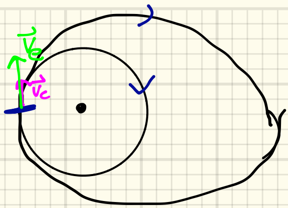
We solve for both ΔV magnitude & also the direction of the burn.

Assume instantaneous maneuvers.

Larger $\Delta V \Rightarrow$ more fuel.

To transfer between 2 orbits using a single, instantaneous maneuver, the 2 orbits must intersect.

At the intersection point, change the velocity vector from its initial value to the value on orbit B at that point.



Velocity of this ellipse at periapsis is greater than the velocity of the circle (From $\mathcal{E}$ eqn $a_{ellipse} > a_{circle}$)

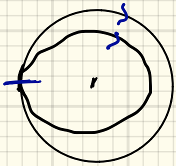
Velocity of the ellipse at periapsis and the circle are in the same direction b/c γ is the same ($\gamma = 0$) for both orbits at that point.

Initially on the circular orbit.

Execute a maneuver to get on the elliptical orbit, need to increase velocity

$$\Delta V = V_e - V_c$$

This type of maneuver where the velocity vector direction does not change is called a "tangential" maneuver.



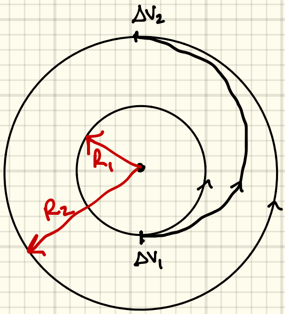
Transfer from circle to ellipse:

S/C needs to slow down

Velocity vectors are in the same direction = tangential maneuver

$$\Delta V = V_c - V_e$$

Hohmann Transfer: Cheapest maneuver between 2 circular orbits:



Half an ellipse connects R_1 & R_2

ΔV_1 & ΔV_2 are both tangential maneuvers

$$\Delta V_1 = V_{tp} - V_{c1}$$

V_{tp} = velocity of the transfer orbit at periapsis

$$V_{c1} = \sqrt{\frac{\mu}{R_1}}$$

$$\mathcal{E} = \frac{V_{tp}^2}{2} - \frac{\mu}{R_1} = -\frac{\mu}{R_1 + R_2} \Rightarrow V_{tp} = \sqrt{\frac{2\mu}{R_1} - \frac{2\mu}{R_1 + R_2}}$$

$$\boxed{\Delta V_1 = \sqrt{\frac{2\mu}{R_1} - \frac{2\mu}{R_1 + R_2}} - \sqrt{\frac{\mu}{R_1}}}$$

→ increase S/C velocity by this amount

$$\Delta V_2 = V_{c2} - V_{ta}$$

$$V_{c2} = \sqrt{\frac{\mu}{R_2}}$$

V_{ta} = velocity of transfer orbit at apoapsis

$$\mathcal{E} = \frac{V_{ta}^2}{2} - \frac{\mu}{R_2} = -\frac{\mu}{R_1 + R_2} \Rightarrow V_{ta} = \sqrt{\frac{2\mu}{R_2} - \frac{2\mu}{R_1 + R_2}}$$

$$\boxed{\Delta V_2 = \sqrt{\frac{\mu}{R_2}} - \sqrt{\frac{2\mu}{R_2} - \frac{2\mu}{R_1 + R_2}}}$$

$$\boxed{\Delta V_{tot} = \Delta V_1 + \Delta V_2}$$