Example #1

$$G(s) = (10s+1)(s/10+1)$$

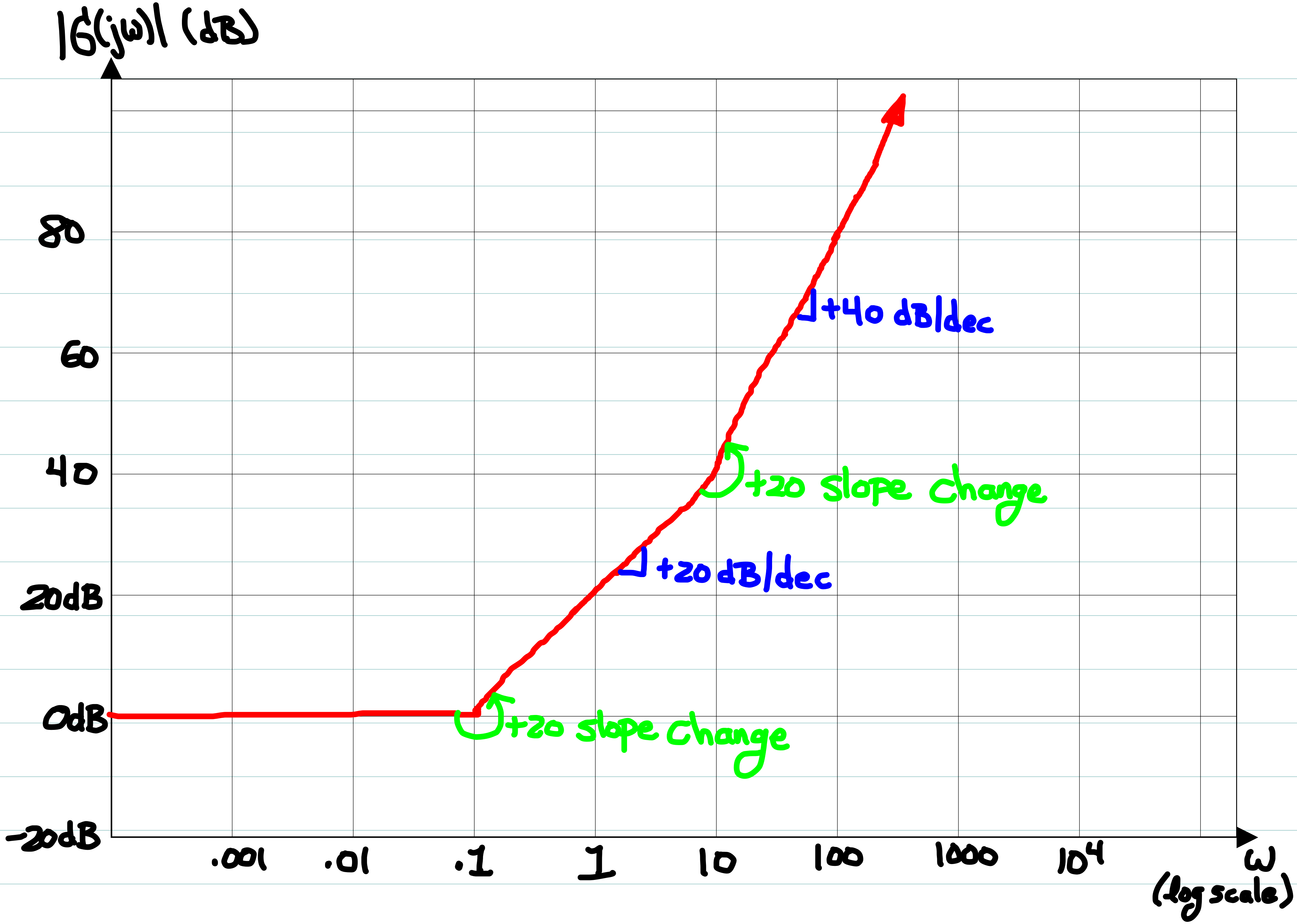
No poles; zeros at $z_1 = -10$, $z_2 = 1/10$

$|G(j\omega)|_{dB}$ will show +20 dB/dec changes at

$\omega = 1/10$ and $\omega = 10$

Below $\omega = 1/10$ the graph will be constant at 0 dB.

Graph bends up by +20dB/dec at $\omega = 1/10$, and again at $\omega = 10$.



|G(jω)| (dB)
80
60
40
20dB
0dB
-20dB
.001
.01
.1
1
10
100
1000
10^4
ω
(log scale)
+40 dB/dec
+20 slope change
+20 dB/dec
+20 slope change

Example #2

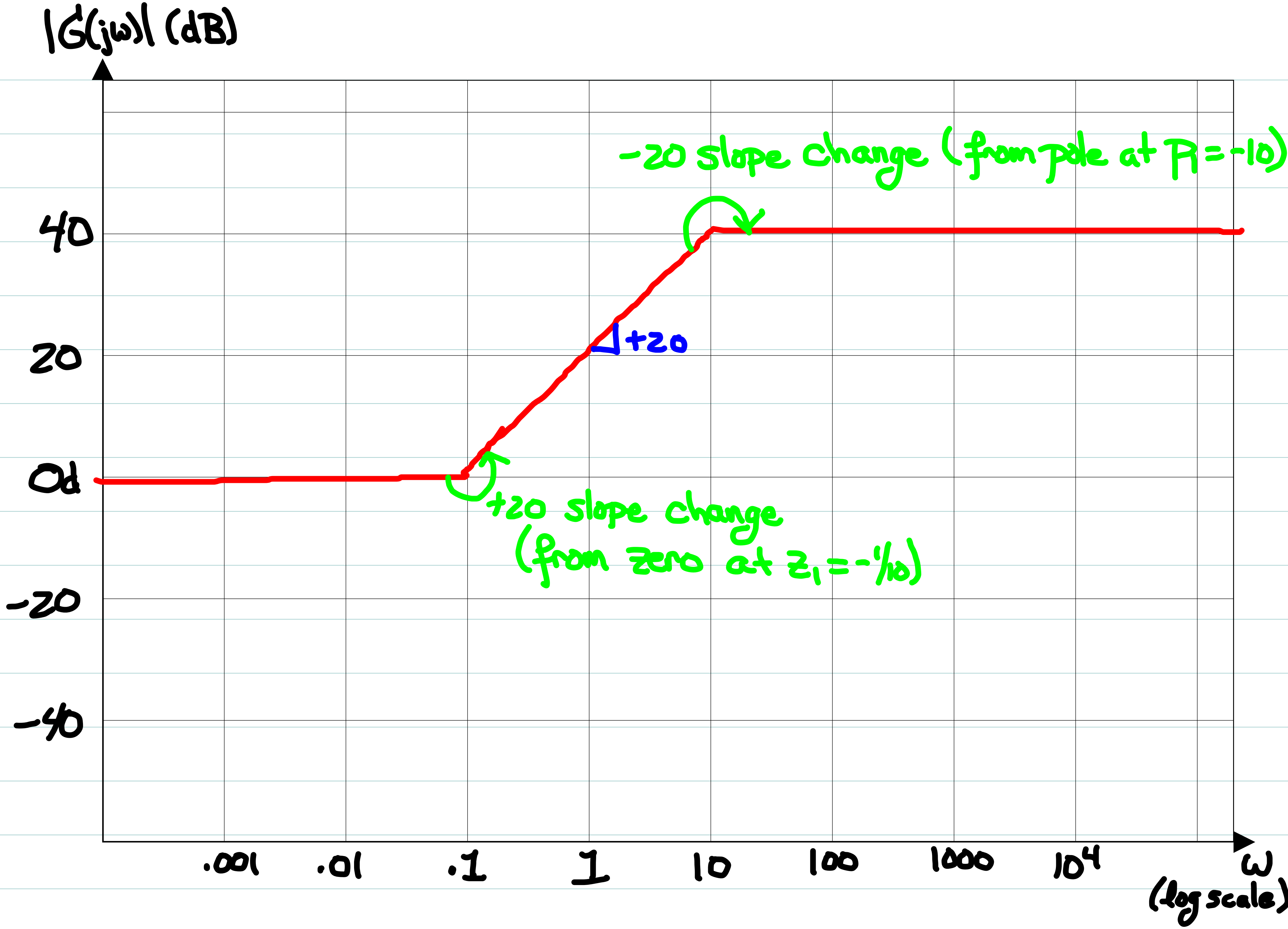
$$G(s) = \frac{(10s+1)}{(s/10+1)}$$

Zero at $z_1 = -1/10$, pole at $p_1 = -10$

Corners at $\omega = 1/10$ and $\omega = 10$ again

But now: at $\omega = 1/10$ slope <u>increases</u> by +20 dB/dec

at $\omega = 10$ slope <u>decreases</u> by −20 dB/dec



|G(jω)| (dB)
40
20
0d
-20
-40
-20 slope change (from pole at P₁=-10)
+20
+20 slope change
(from zero at z₁=-1/10)
.001
.01
.1
1
10
100
1000
10⁴
ω
(log scale)

Gain effect is additive also, and <u>constant</u> for all ω:

$$|K_B(1+j\omega\tau)|_{dB} = |K_B|_{dB} + |1+j\omega\tau|_{dB}$$

$\Rightarrow$ entire graph shifts <u>up or down</u> by $|K_B|_{dB} = 20\log|K_B|$

shifts <u>up</u> if $|K_B|_{dB} > \emptyset$

shifts <u>down</u> if $|K_B|_{dB} < \emptyset$

Gain effect is additive also, and <u>constant</u> for all ω:

$$|K_B(1+j\omega\tau)|_{dB} = |K_B|_{dB} + |1+j\omega\tau|_{dB}$$

$\Rightarrow$ entire graph shifts <u>up or down</u> by $|K_B|_{dB} = 20\log|K_B|$

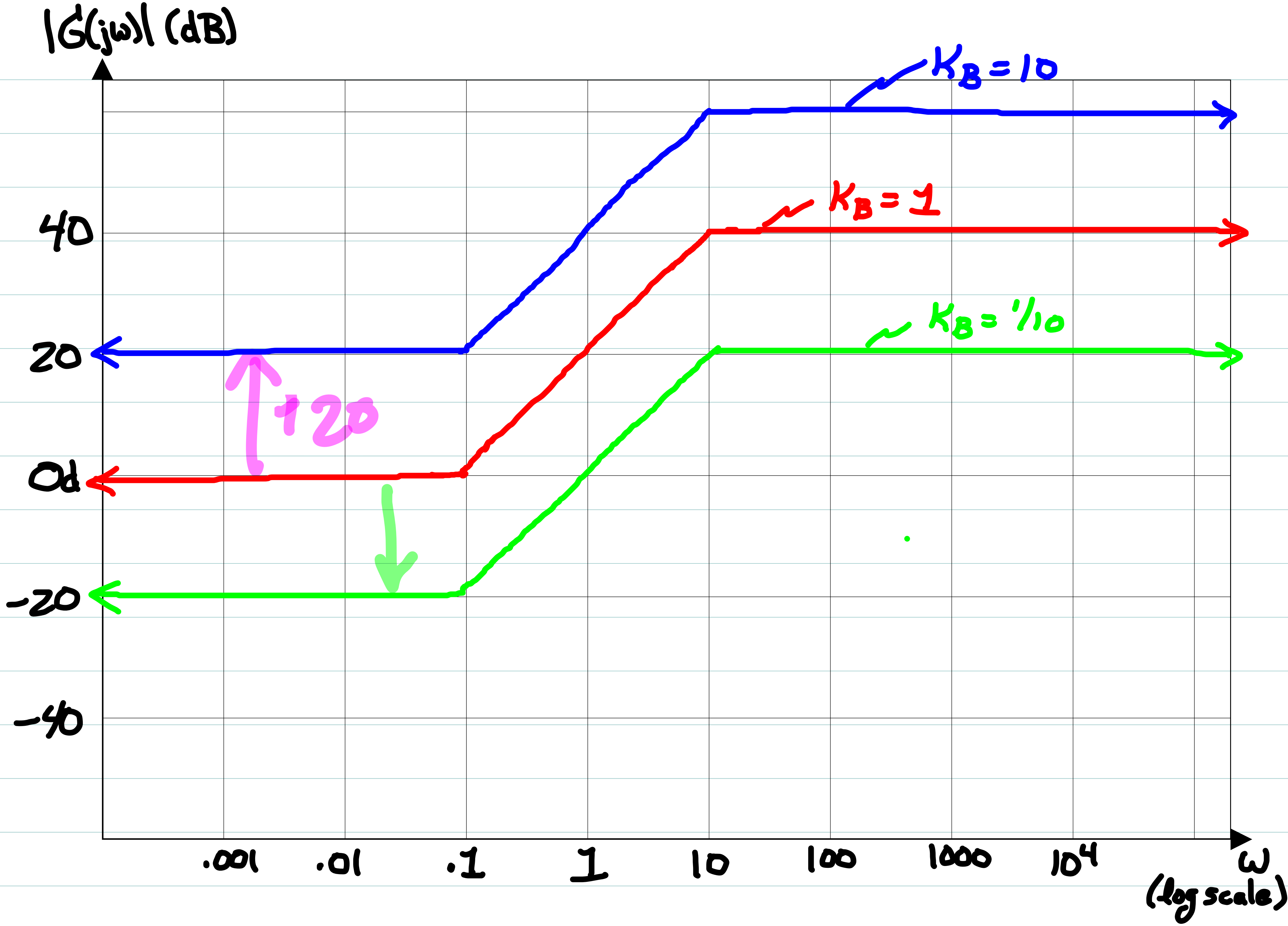
shifts <u>up</u> if $|K_B|_{dB} > 0 \Rightarrow |K_B| > 1$

Shifts <u>down</u> if $|K_B|_{dB} < 0 \Rightarrow |K_B| < 1$

Remember the <u>sign</u> of K_B has <u>No effect</u> on the magnitude diagram!

<u>Example #3:</u>

$$G(s) = K_B\left[\frac{(10s+1)}{(s/10+1)}\right]$$



|G(jω)| (dB)
40
20
0d
-20
-40
.001
.01
.1
1
10
100
1000
10^4
ω
(log scale)
K_B = 10
K_B = 1
K_B = 1/10
+20

Repeated factors

$(1+j\omega\tau)^{\ell}$, ℓ integer ≥ 1

$$|(1+j\omega\tau)^{\ell}|_{dB} = 20 \log |1+j\omega\tau|^{\ell}$$
$$= (20\ell) \log |1+j\omega\tau|$$

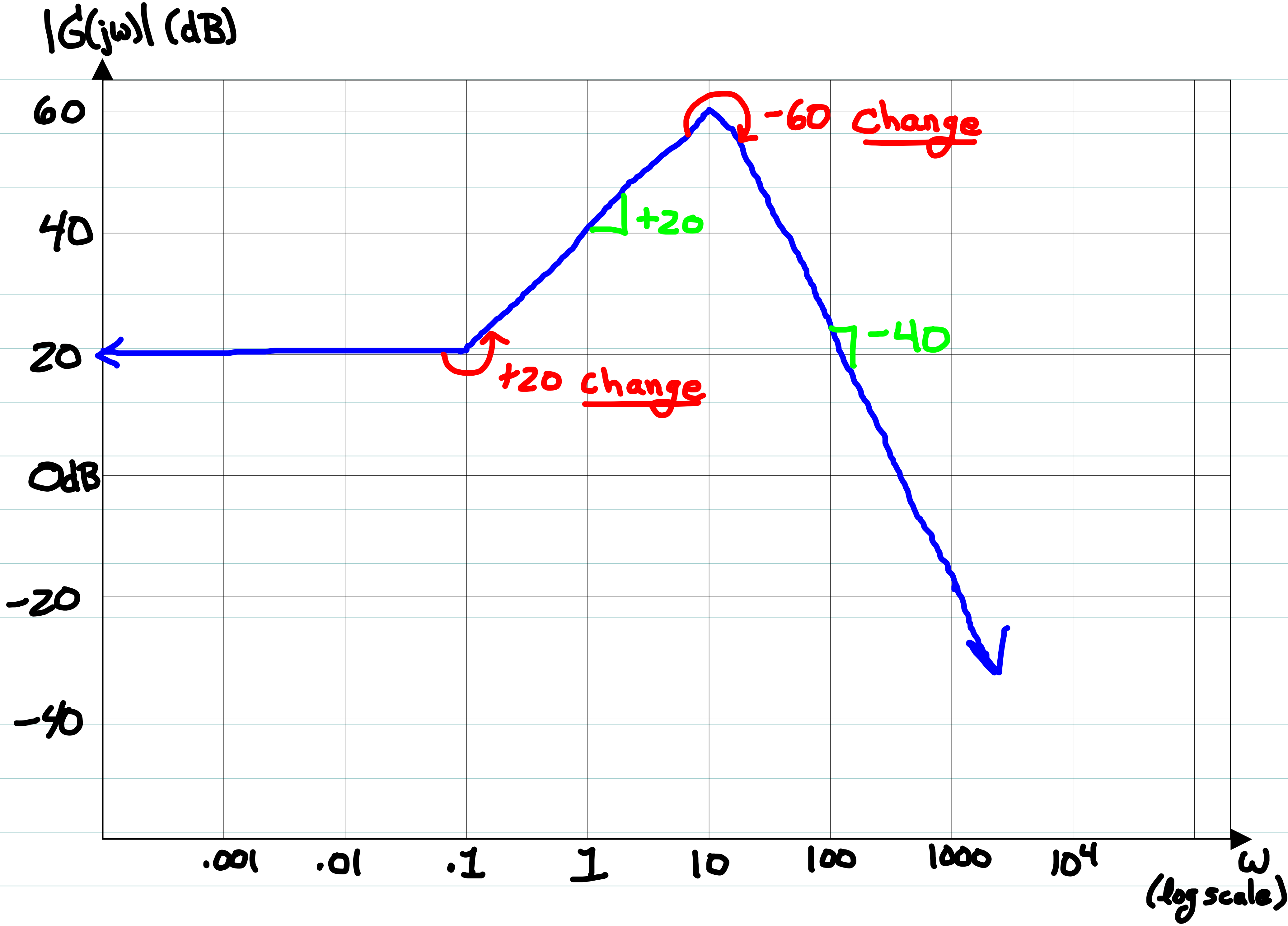
$\Rightarrow$ slope change is $\pm 20\ell$ at $\omega = |1/\tau|$

(positive for zero, negative for pole)

Example #4:

$$G(s) = 10 \left[\frac{(10s+1)}{(s/10+1)^3} \right]$$

+20 slope change at $\omega = 1/10$, $\underline{-60}$ change at $\omega = 10$.



|G(jω)| (dB)
60
40
20
0dB
-20
-40
-60 change
+20
-40
+20 change
.001
.01
.1
1
10
100
1000
10^4
ω
(log scale)

Summary (so far)

⇒ Poles p_k and zeros z_i cause changes in $|G(j\omega)|_{dB}$ graph at corner frequencies $|p_k|$ and $|z_i|$

⇒ Slope of graph changes at these corners

⇒ Zero corners "bend up", i.e. change slope by +20 dB/dec

⇒ Pole corners "bend down", i.e. change slope by −20 dB/dec

⇒ If $|K_B| \neq 1$, entire graph is raised or lowered by $|K_B|_{dB}$

Poles/zeros at origin

Poles at origin (type $N > \emptyset$) or zeros at origin ($N < \emptyset$)

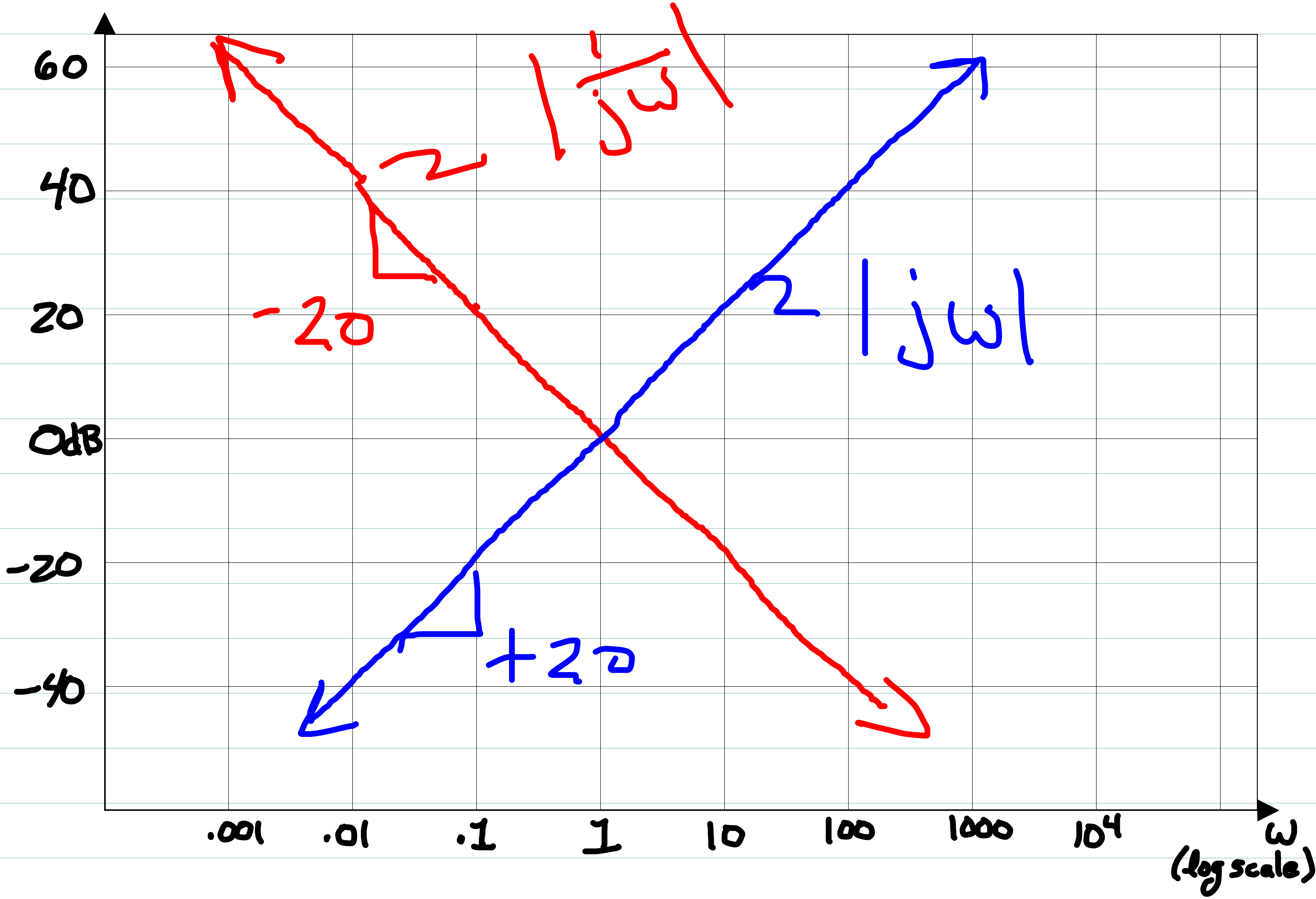
have corner frequencies at $\omega = 0$

$\Rightarrow$ infinitely far to left on horizontal frequency Axis.

These factors do not produce "visible" corners, instead contribute a constant slope of $-20N$ dB/dec for <u>all</u> freqs.

Note also: $|(j\omega)^N| = 1$ at $\omega = 1$ for any N

so graph of $|(j\omega)^N|_{dB}$ will pass through 0 dB at $\omega = 1$



60
40
20
0dB
-20
-40
.001
.01
.1
1
10
100
1000
10^4
ω
(log scale)
∠ |jω|
-20
∠ |jω|
+20

For $G(s)$ with poles/zeros at origin:

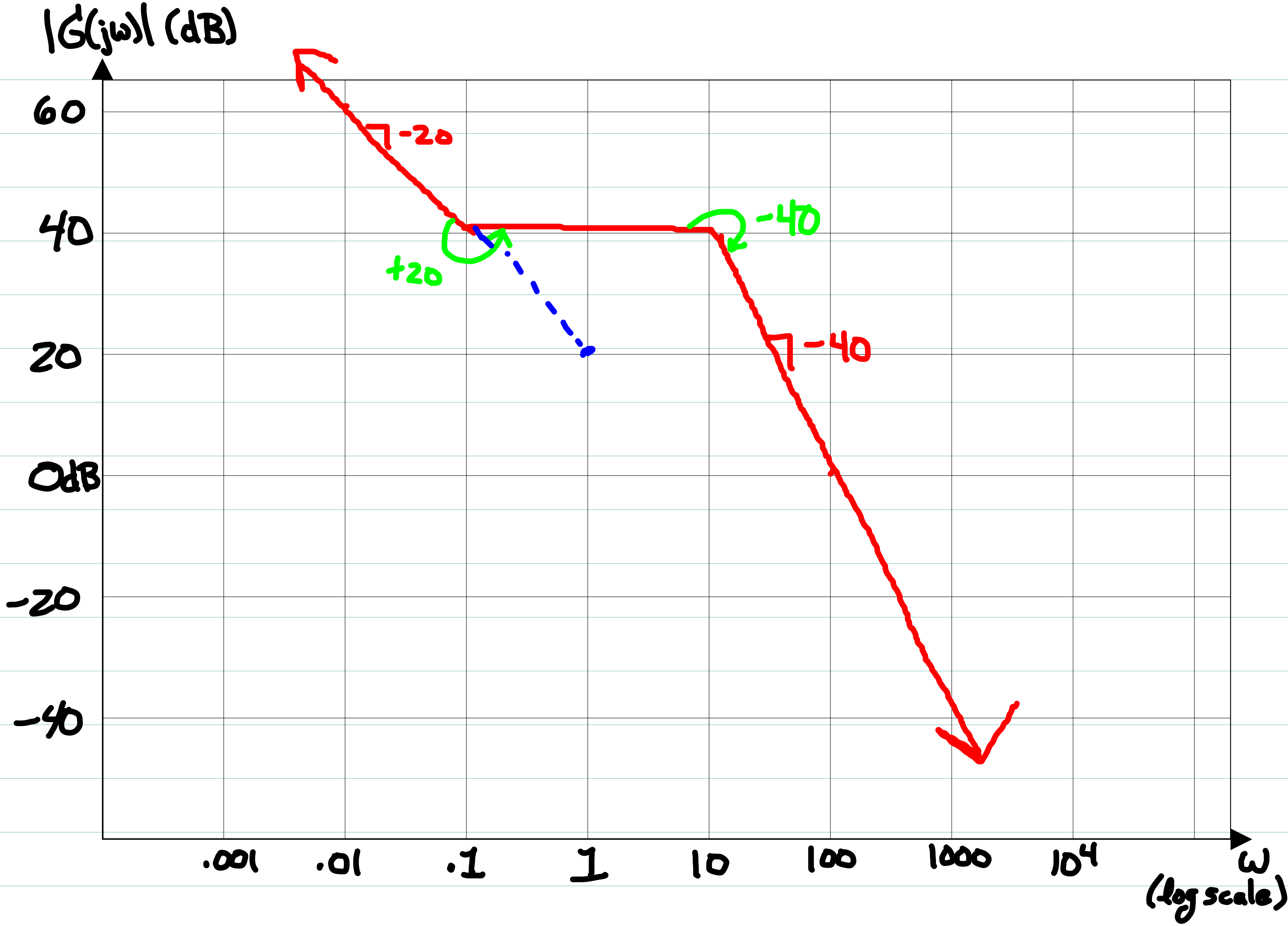
$\Rightarrow$ Start diagram by sketching effect of these poles at low frequencies

$\Rightarrow$ Note if $|K_B| \neq 1$, then this low frequency asymptote will pass through $|K_B|_{dB}$ at $\omega = 1$

$\Rightarrow$ Then add bends due to nonzero z_i and p_K as usual.

<u>Example:</u>

$$G(s) = 10\left[\frac{(10s+1)}{s(s/10+1)^2}\right]$$



|G(jω)| (dB)
60
40
20
0dB
-20
-40
.001
.01
.1
1
10
100
1000
10^4
ω
(log scale)
-20
+20
-40
-40

What about phase?

Recall:

$$\angle G(j\omega) = \angle K_B - N\angle(j\omega) + \sum_{i=1}^{m} \angle\left(1 - \frac{j\omega}{z_i}\right) - \sum_{k=N+1}^{n} \left(1 - \frac{j\omega}{p_k}\right)$$

$$\angle K_B = \begin{cases} \emptyset & K_B > \emptyset \\ -180 & K_B < \emptyset \end{cases} \quad \text{for all } \omega \geq \emptyset$$

$$\angle(j\omega) = 90^\circ \quad \text{for all } \omega \geq \emptyset$$

So, low frequency phase is <u>constant</u> at

$-90N$ if $K_B > \emptyset$

$-180 - 90N$ if $K_B < \emptyset$

Other poles/zeros will cause "bends" at higher freqs.

Phase response from other poles|zeros

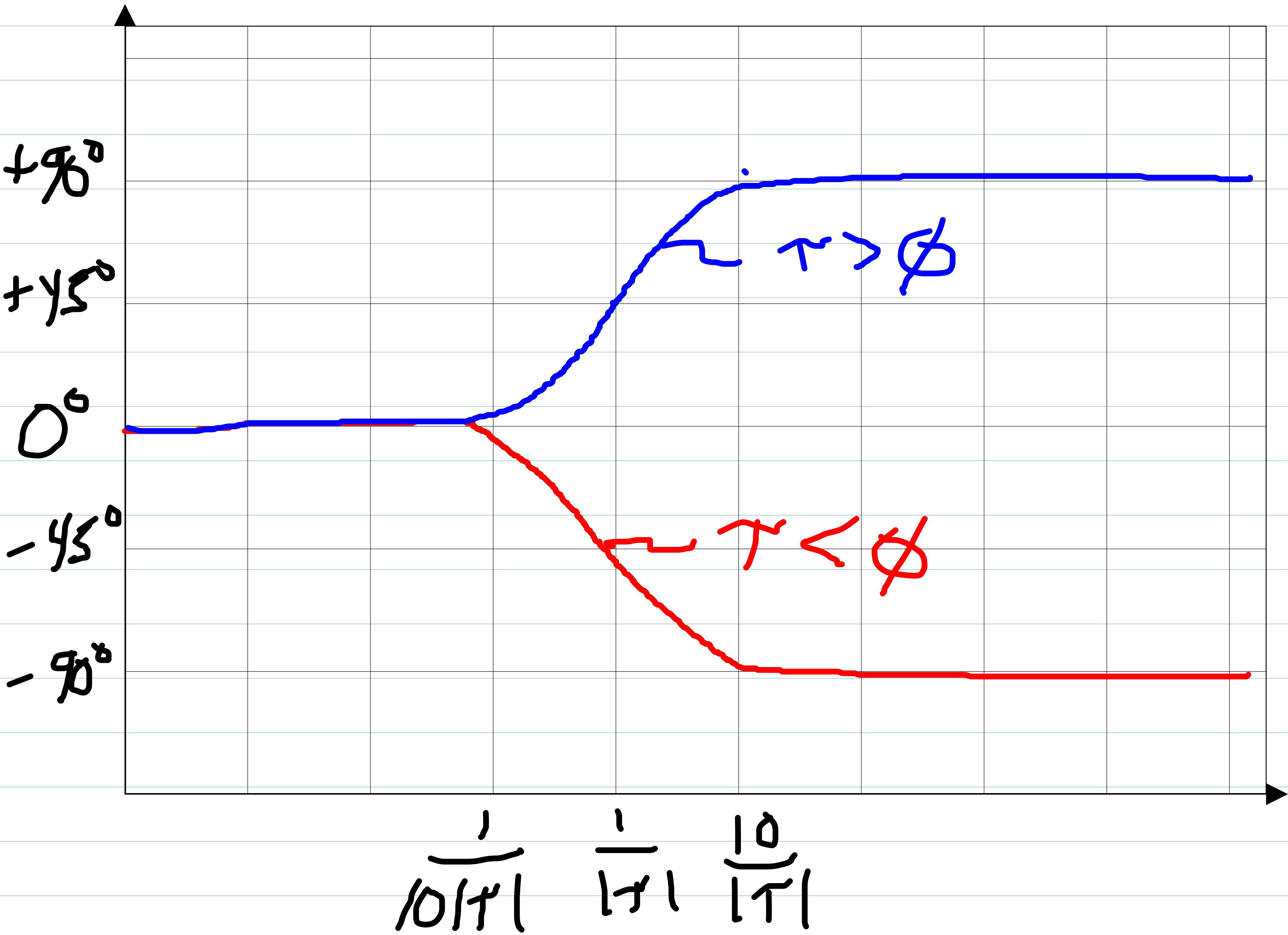
Consider again in generic form $(1+j\omega\tau)$ with
$\tau = -1/z_i$ or $\tau = -1/p_k$

$$\measuredangle(1+j\omega\tau) = \tan^{-1}\omega\tau$$

$$= \begin{cases} \emptyset & \text{if } \omega \ll 1/|\tau| \\ +45^\circ & \text{if } \omega = 1/|\tau| \\ +9\emptyset^\circ & \text{if } \omega \gg 1/|\tau| \end{cases}$$

above is for $\tau > 0$. If instead $\tau < 0$

$$\measuredangle(1+j\omega\tau) = -\tan^{-1}\omega|\tau| = \begin{cases} \emptyset & \text{if } \omega \ll 1/|\tau| \\ -45^\circ & \text{if } \omega = 1/|\tau| \\ -90^\circ & \text{if } \omega \gg 1/|\tau| \end{cases}$$



+90°
+45°
0°
-45°
-90°
τ > 0
τ < 0
1/(10|τ|)
1/|τ|
10/|τ|

Observations

⇒ Phase change due to a single factor occurs in a 2 decade band of frequencies centered at the magnitude corner frequency $1/|\tau|$

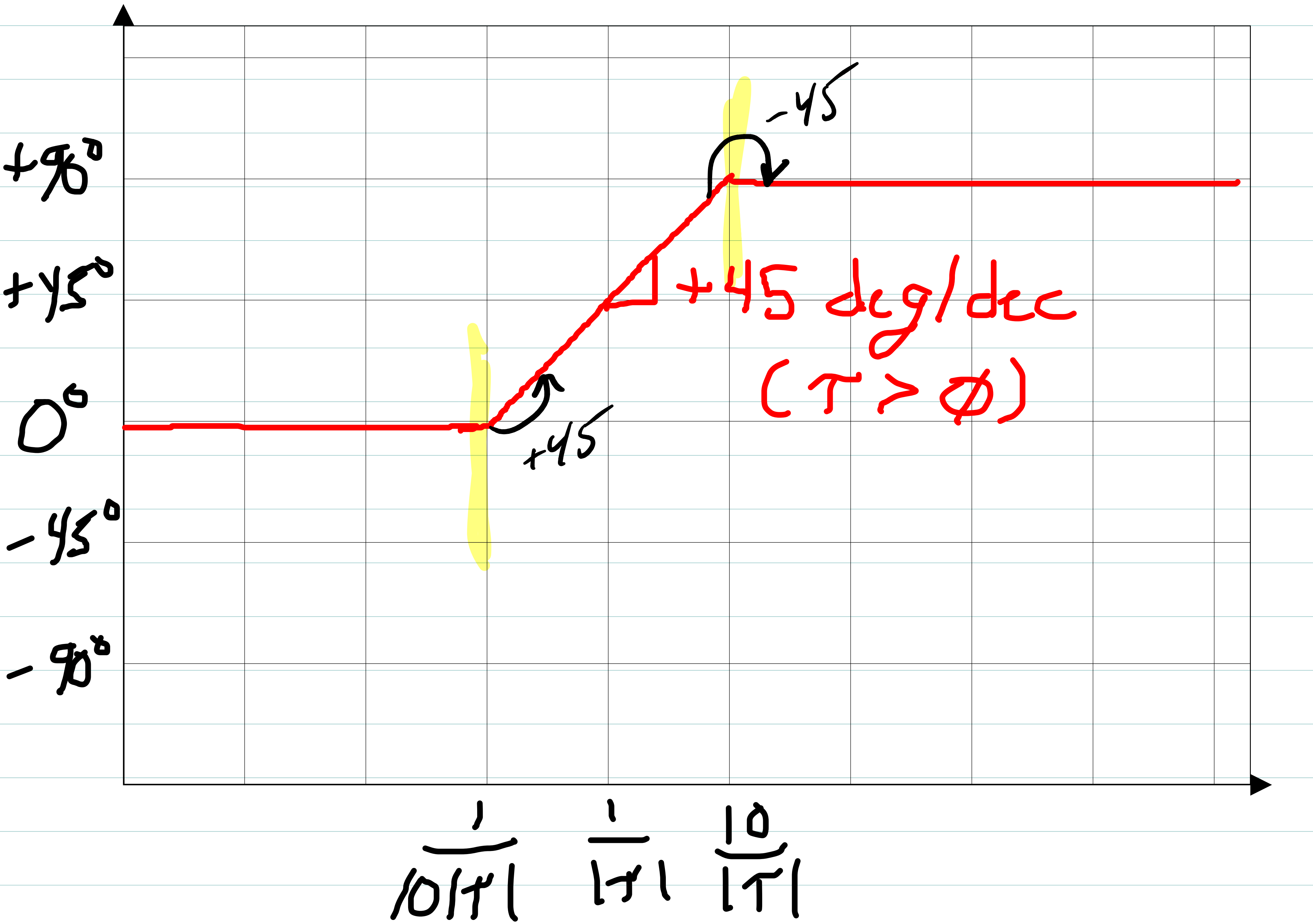
i.e. in band $\frac{1}{10|\tau|} \leq \omega \leq 10/|\tau|$

⇒ Phase is <u>constant</u> outside this band

low freq phase $\approx 0°$

h.f. phase $\approx \pm 90°$ ($+90°$ if $\tau > 0$, $-90°$ if $\tau < 0$

⇒ Phase change is approximate <u>linear</u> across band with slope $\pm 45°/\text{dec}$



+90°
+45°
0°
-45°
-90°
-45
+45
+45 deg/dec
(τ > 0)
1/(10|τ|)
1/|τ|
10/|τ|

Sign of phase change depends on:

$\Rightarrow$ whether factor is pole or zero

$\Rightarrow$ whether factor is RHP ($\tau < \phi$) or LHP ($\tau > \phi$)

Suppose all factors are LHP, $z_i < \phi$ $p_k < \phi$

then all $\tau = {}^{-1}/_{z_i}$ or ${}^{-1}/_{p_k}$ are positive.

This is called the "minimum phase" case

Then:

$\Rightarrow$ zeros cause $+90°$ phase change over band $\frac{|z_i|}{10}$ to $10|z_i|$

$\Rightarrow$ poles cause $-90°$ change over $\frac{|p_k|}{10}$ to $10|p_k|$

(Minimum Phase Systems)

<u>Slopes</u> of phase change are $+45°/\text{dec}$ (zeros) or $-45°/\text{dec}$ (poles) in these bands

Note phase changes in minimum phase cases mirror those for magnitude changes:

$\Rightarrow$ zeros cause <u>positive</u> slope changes
$\Rightarrow$ poles cause <u>negative</u> slope changes.

Graphical addition is again straightforward, but requires a little care:

$\Rightarrow$ slopes are nonzero only in a 2 decade band

$\Rightarrow$ bands from different factors may overlap.

Example:

$$G(s) = \frac{10s+1}{s(s+1)(s/10+1)}$$

Low freq. phase $-90°$

Phase changes:

$+45°/\text{dec}$ in $.01$ to 1

$-45°/\text{dec}$ in $.1$ to 10

$-45°/\text{dec}$ in 1 to 100

Net:

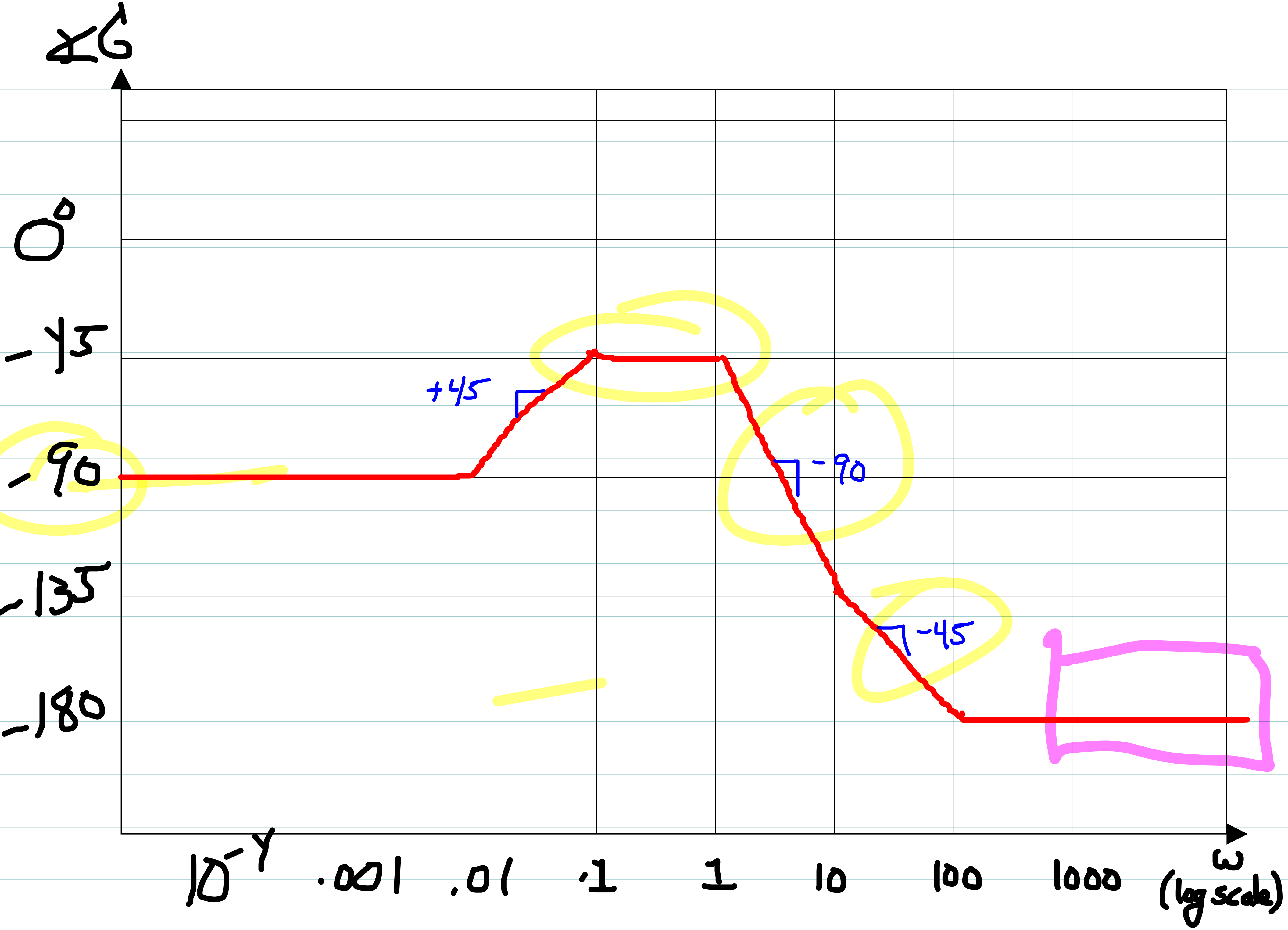
$+45°/\text{dec}$ in $.01$ to $.1$

$0°/\text{dec}$ in $.1$ to 1

$-90°/\text{dec}$ in 1 to 10

$-45°/\text{dec}$ in 10 to 100

Constant for $\omega > 100$.



∡G
0°
-45
-90
-135
-180
+45
-90
-45
10⁻⁴
.001
.01
.1
1
10
100
1000
ω
(log scale)

Repeated factors

Repeated factors $(1+j\omega T)^{\ell}$ multiply the phase changes by ℓ, just like magnitudes.

Example:

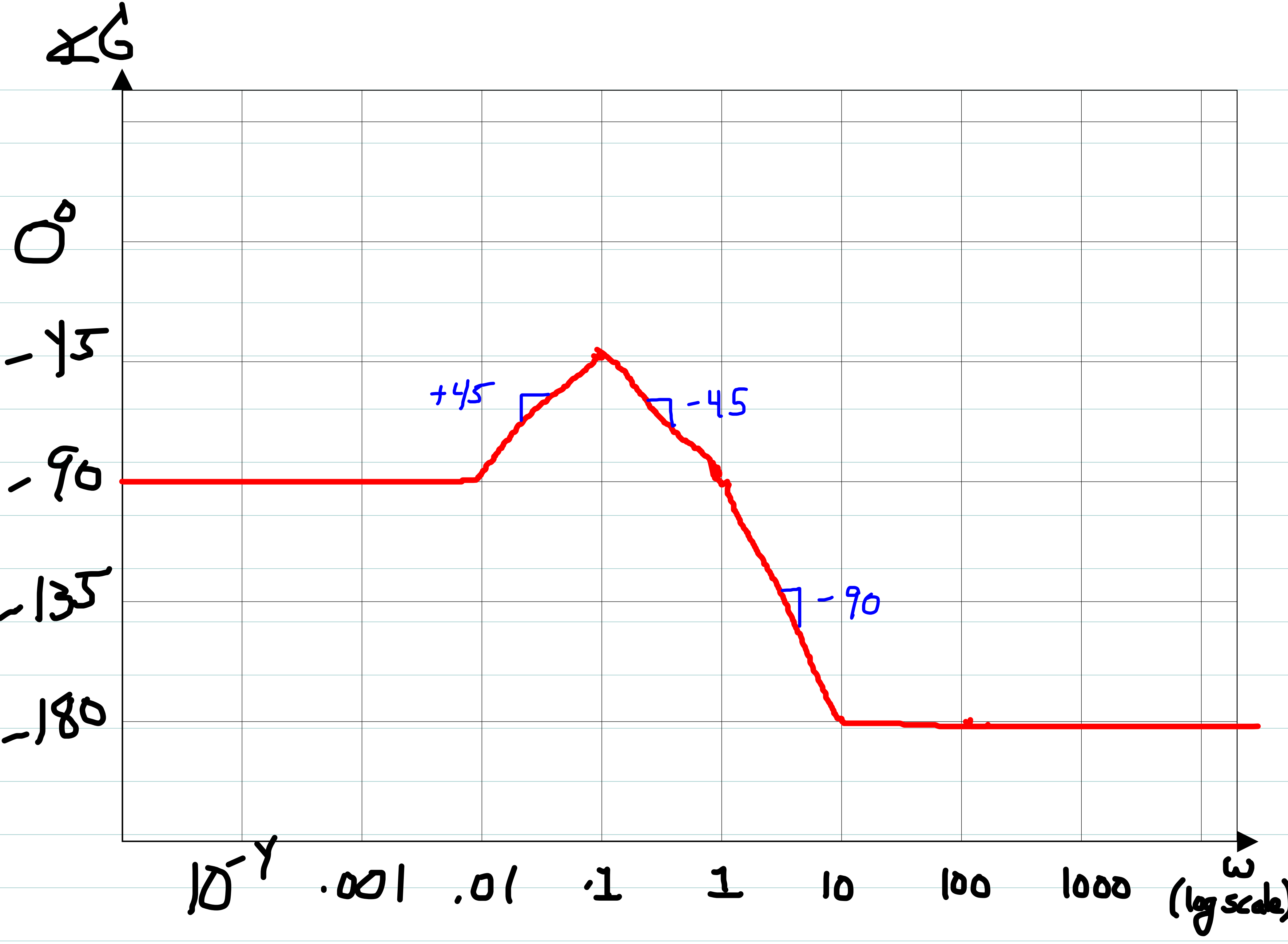
$$G(s) = \frac{10s+1}{s(s+1)^2}$$

Changes:

+45°/dec in .01 to 1
−90°/dec in .1 to 10

Net:

+45°/dec in .01 to .1
−45°/dec in .1 to 1
−90°/dec in 1 to 10



∡G
0°
-45
-90
-135
-180
+45
-45
-90
10^-4
.001
.01
.1
1
10
100
1000
ω
(log scale)

Summary (minimum phase)

$\Rightarrow$ Low freq. phase is $\measuredangle K_B - N\,90°$

$\Rightarrow$ high freq. phase is $\measuredangle K_B - 90°(n-m)$

"relative degree" #poles − #zeros

$\Rightarrow$ Note low and high freq. phases are constant (slope is zero).

$\Rightarrow$ Recall typically $n > m$ for a physical system so high freq. phase is typically negative for a minimum phase system.

$\Rightarrow$ zeros cause $+90°$ change at rate of $+45°/\text{dec}$ in 2 decade band centered at $|z_i|$

$\Rightarrow$ poles cause $-90°$ change at rate of $-45°/\text{dec}$ in 2 decade band centered at $|p_k|$.

Can be tricky to accurately sketch phase

=> Overlapping change regions for multiple factors

=> No standard formula for phase change of underdamped factors

=> Helps to 1^{st} make a table of slope changes over frequency ranges as above

=> Generally, straight-line phase sketch is less accurate than magnitude sketch.

=> Still sufficiently accurate to give us a good general idea of phase behavior.

=> We'll use Matlab when greater accuracy is required.