ENRE 447 - 0101
Homework 03:

Due on March 03th, 2025 at 03:30 PM

Dr. Groth, 03:30 PM

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Problem 1:

The following data represent times to failure (in hours) for 20 mechanical devices.

4	7	8	12	19	27	50	65	66	69
71	73	75	91	107	115	142	166	184	192

1. Find the MTTF and Variance of the failure times.
2. Develop an empirical probability distribution and histogram for the data.

Solution

Part A

$$\sum_{i=1}^{20} t_i = 4 + 7 + \cdots + 184 + 192 = 1543$$

$$\bar{t} = \frac{1543}{20} \approx 77.15$$

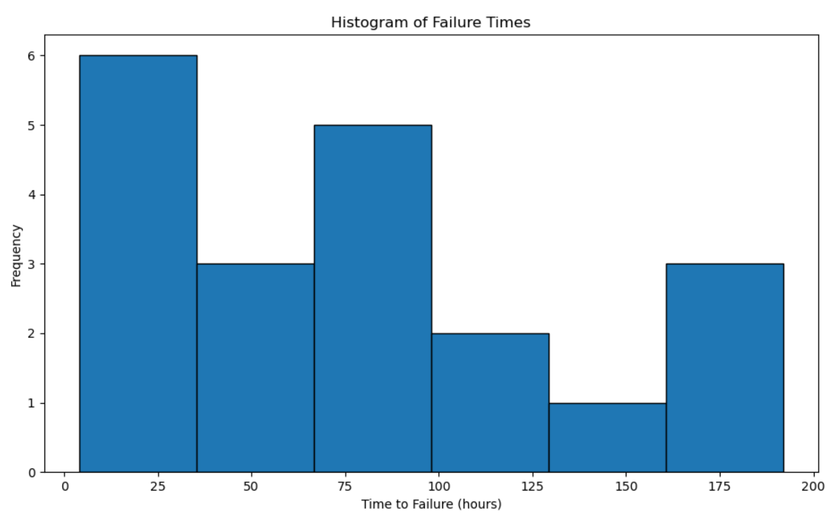
$$\text{MTTF} = 77.15 \text{ hours.} \qquad \square$$

$$\sigma^2 = \frac{1}{20}\sum_{i=1}^{20} t_i^2 - \bar{t}^2$$

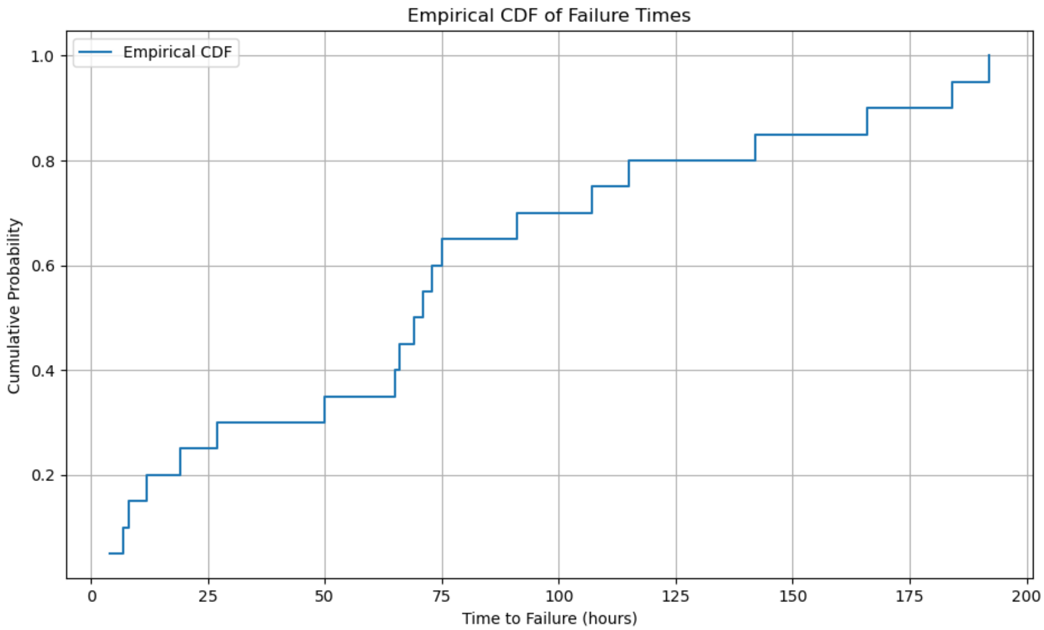
$$\frac{1}{20}\sum_{i=1}^{20} t_i^2 \approx 9229.75, \quad \bar{t}^2 \approx 5952.12$$

$$\sigma^2 \approx 9229.75 - 5952.12 = 3277.63 \text{ (hours)}^2. \qquad \square$$

Part B



Problem 1 continued on next page. . .

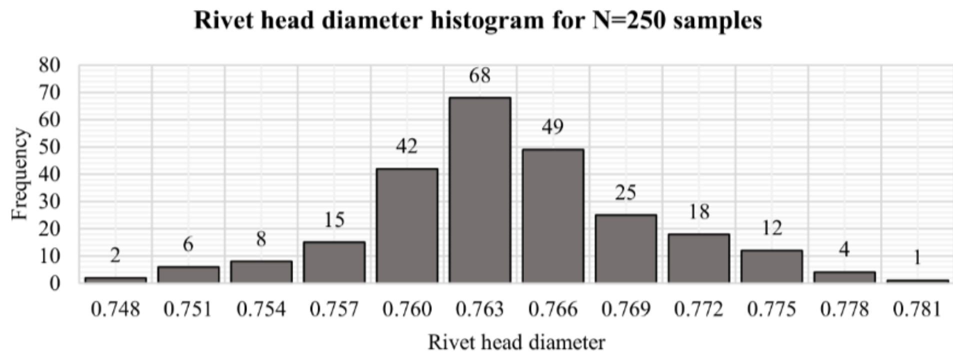



```python
import numpy as np
import matplotlib.pyplot as plt

data = np.array(
    [4, 7, 8, 12, 19, 27, 50, 65, 66, 69, 71, 73, 75, 91, 107, 115, 142, 166, 184, 192]
)
data_sorted = np.sort(data)
N = len(data_sorted)

# Histogram
plt.hist(data, bins="auto", edgecolor="black")
plt.xlabel("Time to Failure (hours)")
plt.ylabel("Frequency")
plt.title("Histogram of Failure Times")
plt.show()

# CDF
cdf = np.arange(1, N + 1) / float(N)

plt.figure(figsize=(8, 5))
plt.step(data_sorted, cdf, where='post', label='Empirical CDF')
plt.xlabel('Time to Failure (hours)')
plt.ylabel('Cumulative Probability')
plt.title('Empirical CDF of Failure Times')
plt.grid(True)
plt.legend()
plt.show()
```

Listing 1: Python code for HW03 P01B

Problem 2:

The histogram below shows the distribution of the diameters of the heads of rivets manufactured by a company. Compute the mean and variance of the diameter.



Diameter:	0.748	0.751	0.754	0.757	0.760	0.763	0.766	0.769	0.772	0.775	0.778	0.781
Frequency:	2	6	8	15	42	68	49	25	18	12	4	1

Solution

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{12} t_i \Pr(t_i) = \frac{2(0.748) + 6(0.751) + \cdots + 4(0.778) + 1(0.781)}{250} \approx 0.7642. \qquad \square$$

$$\overline{x^2} = \frac{1}{N}\sum_{i=1}^{12} t_i \Pr(t_i)^2 = \frac{2(0.748^2) + 6(0.751^2) + \cdots + 1(0.781^2)}{250} \approx 0.58404$$

$$\sigma^2 = \overline{x^2} - \bar{x}^2 \approx 0.58404 - (0.7642)^2 \approx 0.58404 - 0.58395 \approx 0.00009 \text{ (units)}^2. \qquad \square$$

Problem 3:

The sample mean life of ten car batteries is 102.5 months, with a standard deviation of 9.45 months. What are the 80% confidence limits for the mean and standard deviation of a pdf that represents these batteries?

Solution

$$\begin{aligned}
t_{0.10,9} &\approx 1.383 \\
\text{Margin of Error} &= 1.383 \cdot \frac{9.45}{\sqrt{10}} \approx 1.383 \cdot 2.988 \approx 4.14 \\
\text{CI for } \mu : &\quad 102.5 \pm 4.14, \quad (102.5 - 4.14,\ 102.5 + 4.14) \approx (98.36,\ 106.64). \qquad \square \\
s^2 &= (9.45)^2 \approx 89.3025 \\
\chi^2_{0.90,9} &\approx 14.684, \quad \chi^2_{0.10,9} \approx 5.226 \\
\text{Lower variance limit} &= \frac{9 \cdot 89.3025}{14.684} \approx 54.73 \\
\text{Upper variance limit} &= \frac{9 \cdot 89.3025}{5.226} \approx 153.80 \\
\text{CI for } \sigma : &\quad \left(\sqrt{54.73}, \sqrt{153.80}\right) \approx (7.40,\ 12.40). \qquad \square
\end{aligned}$$

Problem 4:

The frequency distribution of time to establish the root causes of a failure by a group of experts is observed and given below. Test whether a normal distribution with known $\sigma = 10$ is an appropriate model for these data.

Time Interval (hour)	Obs. Freq.
$45 - 55$	7
$55 - 65$	18
$65 - 75$	35
$75 - 85$	28
$85 - 95$	12

Solution

$$
\begin{aligned}
\bar{x} &= \frac{7\left(\frac{45+55}{2}\right) + \cdots + 12\left(\frac{85+95}{2}\right)}{7 + \cdots + 12} = \frac{350 + 1080 + 2450 + 2240 + 1080}{100} = \frac{7200}{100} = 72 \\
P(45 \leq X \leq 55) &= \Phi\left(\frac{55-72}{10}\right) - \Phi\left(\frac{45-72}{10}\right) \\
&\approx \Phi(-1.7) - \Phi(-2.7) \approx 0.0446 - 0.0035 = 0.0411 \\
E_1 &= 100 \cdot 0.0411 \approx 4.11 \\
P(55 \leq X \leq 65) &\approx 0.1974, \quad E_2 \approx 19.74 \\
P(65 \leq X \leq 75) &\approx 0.3759, \quad E_3 \approx 37.59 \\
P(75 \leq X \leq 85) &\approx 0.2853, \quad E_4 \approx 28.53 \\
P(85 \leq X \leq 95) &\approx 0.0861, \quad E_5 \approx 8.61 \\
\chi^2 &= \frac{(7-4.11)^2}{4.11} + \frac{(18-19.74)^2}{19.74} + \frac{(35-37.59)^2}{37.59} + \frac{(28-28.53)^2}{28.53} + \frac{(12-8.61)^2}{8.61} \\
&\approx \frac{(2.89)^2}{4.11} + \frac{(-1.74)^2}{19.74} + \frac{(-2.59)^2}{37.59} + \frac{(-0.53)^2}{28.53} + \frac{(3.39)^2}{8.61} \\
&\approx 2.03 + 0.15 + 0.18 + 0.01 + 1.34 \approx 3.71 \\
\mathrm{df} &= 5 - 1 - 1 = 3 \\
\chi^2_{0.05,3} &\approx 7.815, \quad 3.71 < 7.815. \qquad \square
\end{aligned}
$$

Problem 5:

A sample of 50 digits using a random number generator yielded the following data. Is there any reason to doubt the digits are uniformly distributed? Use a Chi-Square test and significance level of 0.05 to test.

Digit:	0	1	2	3	4	5	6	7	8	9
Frequency:	4	8	8	4	10	3	2	2	4	5

Solution

$$
\begin{aligned}
E &= \frac{50}{10} = 5 \\
\chi^2 &= \sum_{i=0}^{9} \frac{(O_i - 5)^2}{5} \\
&= \frac{(4-5)^2 + (8-5)^2 + \cdots + (4-5)^2 + (5-5)^2}{5} \\
&= \frac{1 + 9 + \cdots + 1 + 0}{5} = \frac{69}{5} \approx 13.8 \\
\text{df} &= 10 - 1 = 9 \\
\chi^2_{0.05,9} &\approx 16.92, \quad 13.8 < 16.92. \qquad \square
\end{aligned}
$$

Problem 6:

Consider the following time to failure data with the ranked value of t_i. Use the K-S test to test the hypothesis that the data fit a normal distribution.

Event	1	2	3	4	5	6	7	8	9	10
Time to Failure (hour)	10.3	12.4	13.7	13.9	14.1	14.2	14.4	15.0	15.9	16.1

Solution

$$\begin{aligned}
\bar{t} &= \frac{10.3 + 12.4 + \cdots + 15.9 + 16.1}{10} = \frac{140.0}{10} = 14.0 \\
s &\approx 1.686 \\
\text{For each } t_i, \quad F(t_i) &= \Phi\left(\frac{t_i - 14.0}{1.686}\right) \\
\text{Empirical CDF at } t_i &= \frac{i}{10} \\
D &= \max_{1 \le i \le 10} \left| F(t_i) - \frac{i}{10} \right| \approx 0.129 \\
D_{0.05} &\approx \frac{1.36}{\sqrt{10}} \approx 0.430 \\
0.129 &< 0.430. \qquad \square
\end{aligned}$$