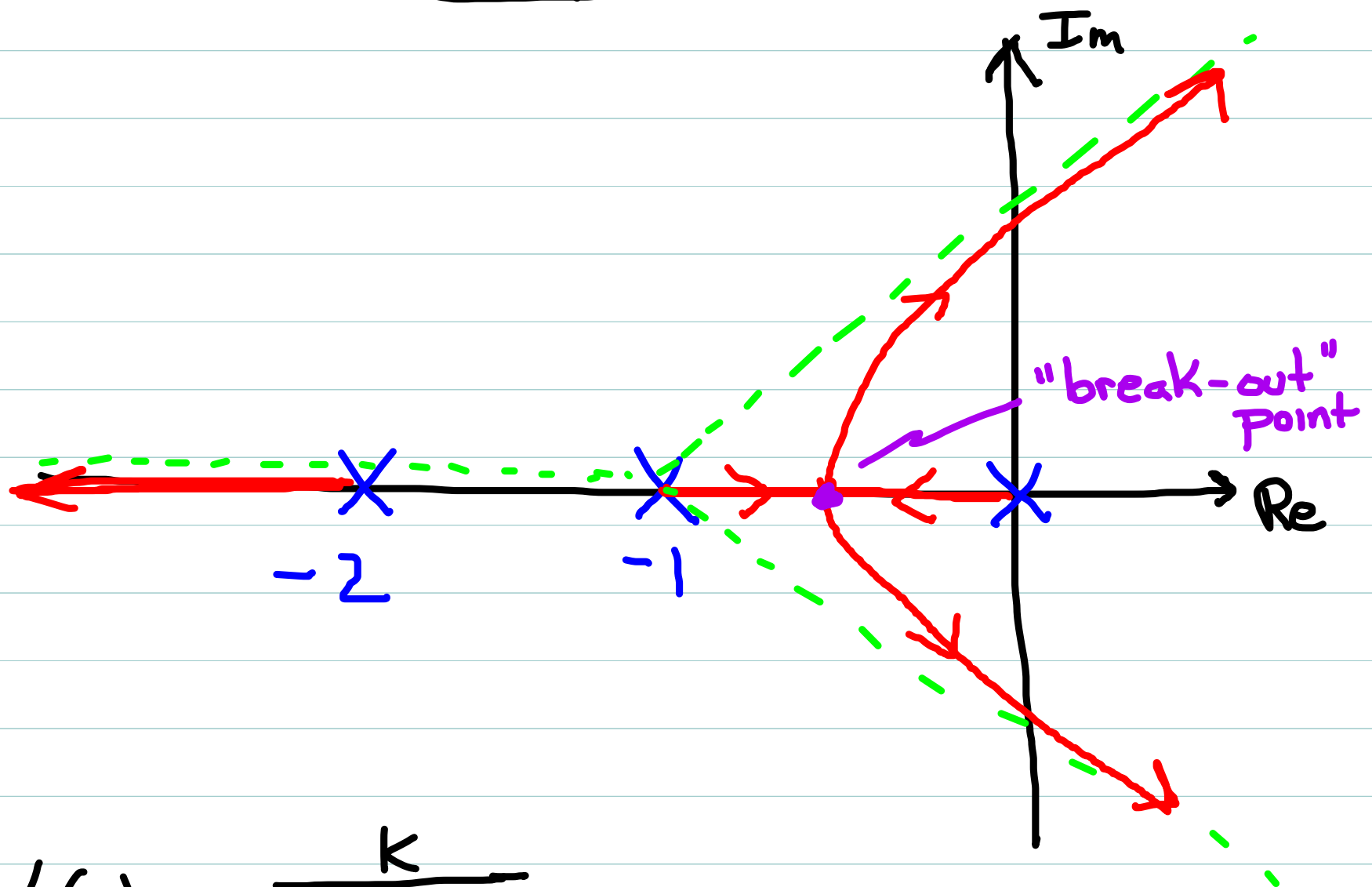


## Example #5, cont



$$L(s) = \frac{K}{s(s+1)(s+2)}$$

## High Gain Instability

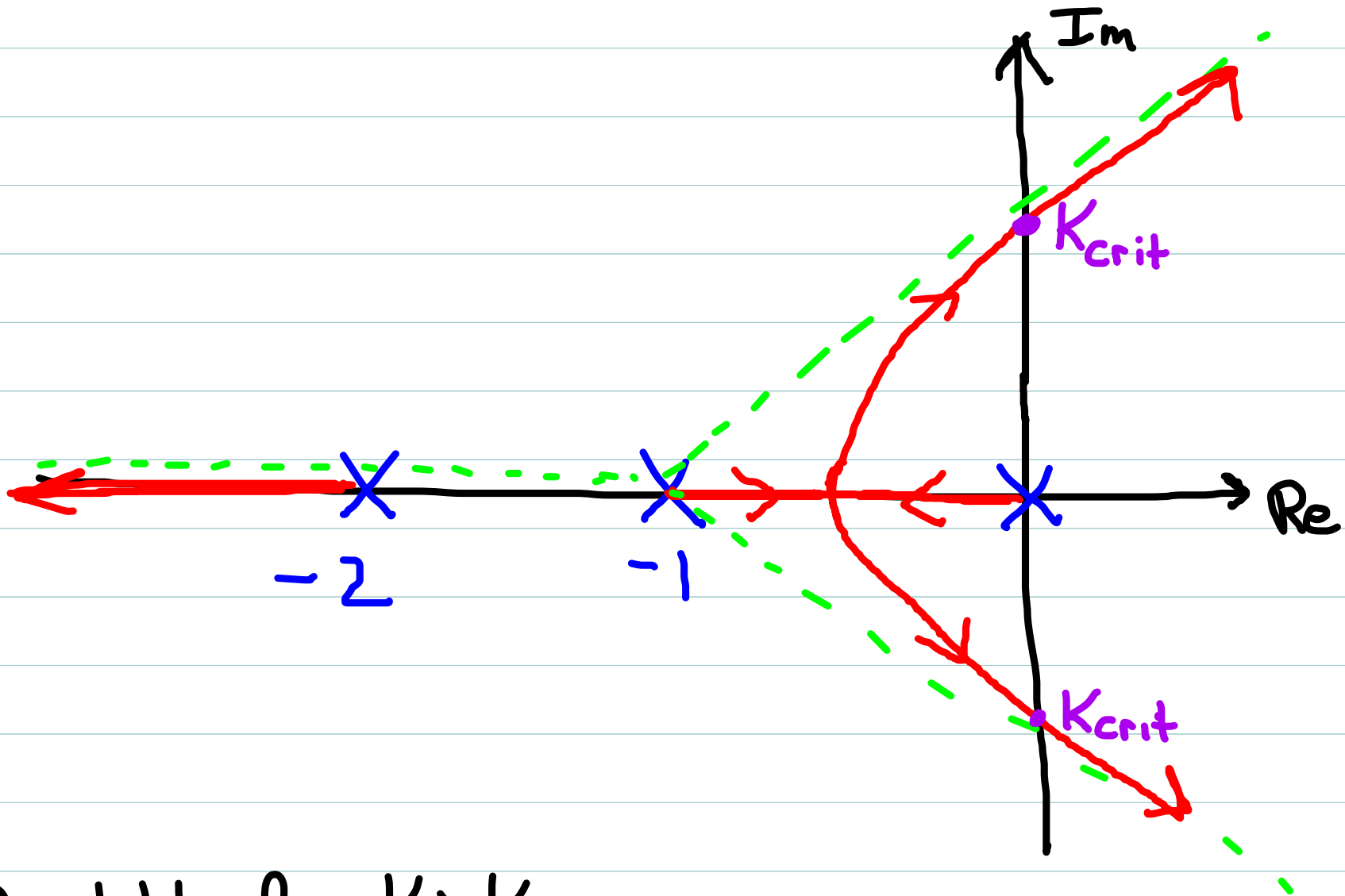
Since this example has Asymptotes in RHP, we can see the CL system will be unstable for sufficiently high gains  $K$ .

Whether this is a problem or not depends on the gain we want/need to get the desired CL poles

$T(s)$  is not automatically unstable b/c the root locus branches in RHP!

Such a locus only tells us  $T(s)$  will be unstable for some values of  $K$ .

## Example #5, cont



Unstable for  $K > K_{crit}$

## Example #6

$$L(s) = \frac{K(s+3)}{(s+1)(s+2)}$$

$$\Rightarrow n=2, m=1, n-m=1$$

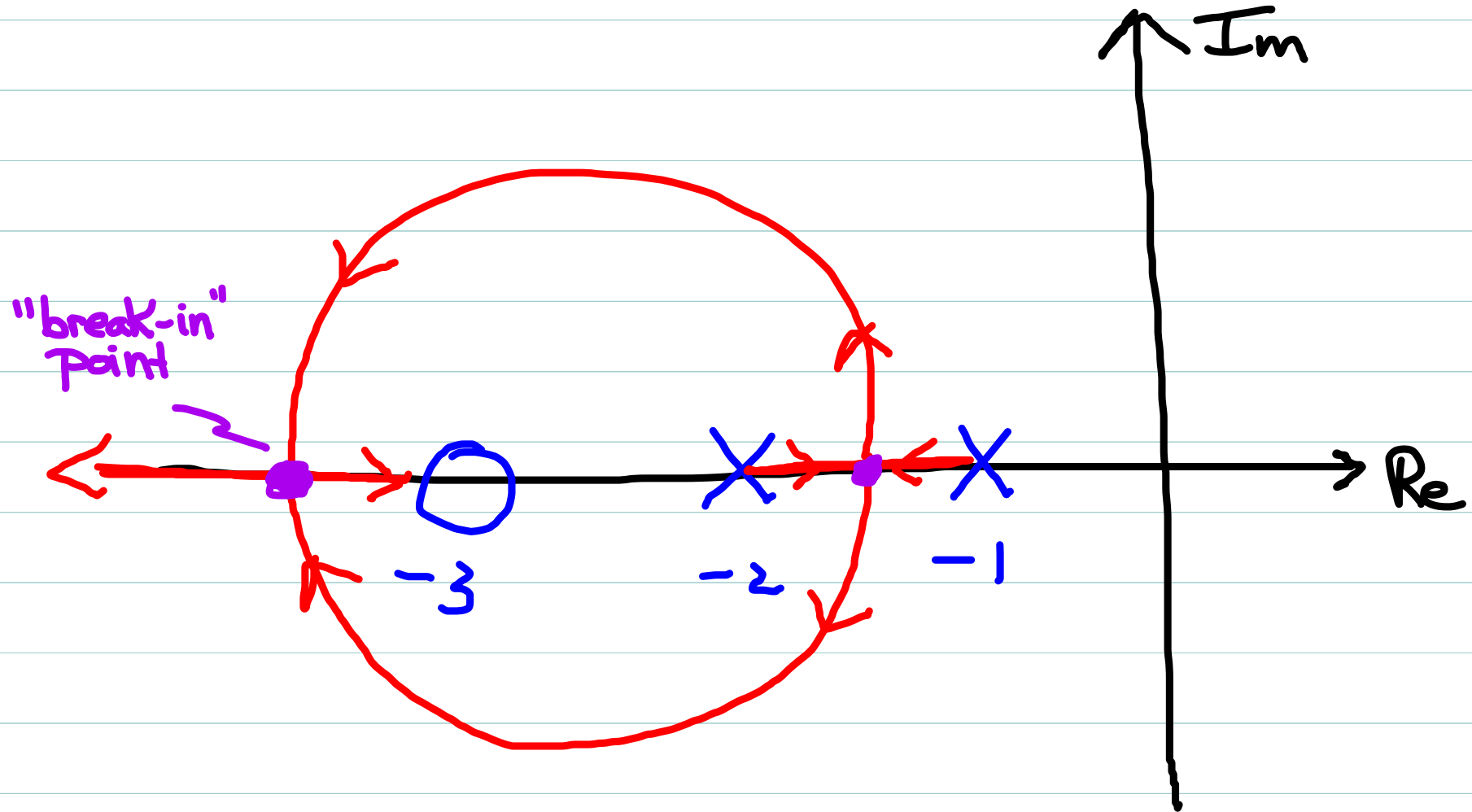
$\Rightarrow$  One branch ends at  $-3$  (OL zero). One branch goes to  $\infty$  along asymptote  $\alpha = 180^\circ$  (negative real Axis)

$\Rightarrow$  Segments of branches lie on real Axis:

$\Rightarrow$  Between  $-2$  and  $-1$

$\Rightarrow$  left of  $-3$

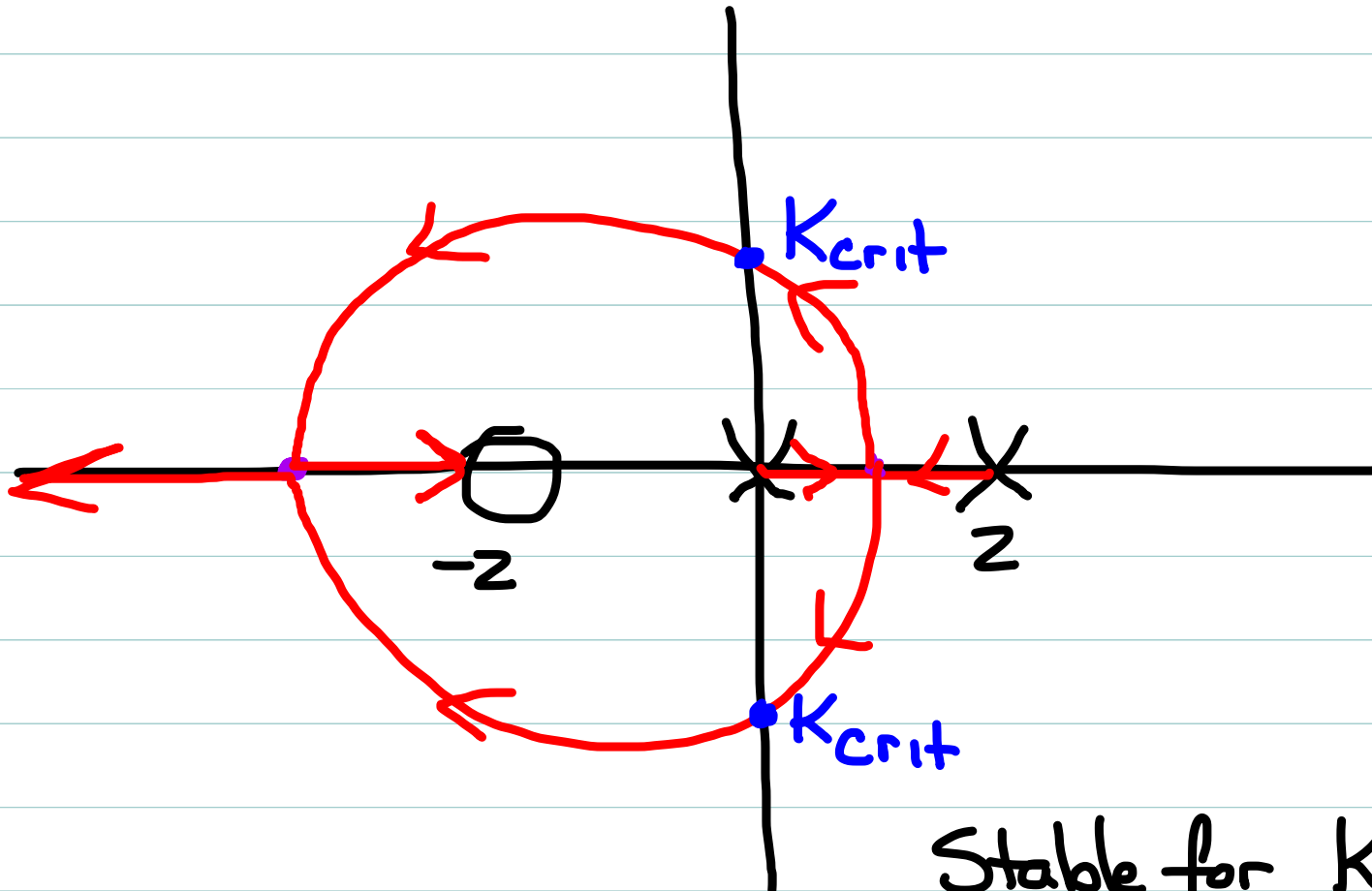
## Example #6, cont



## Example #7

$$L(s) = \frac{K(s+2)}{s(s-2)}$$

Similar analysis to above

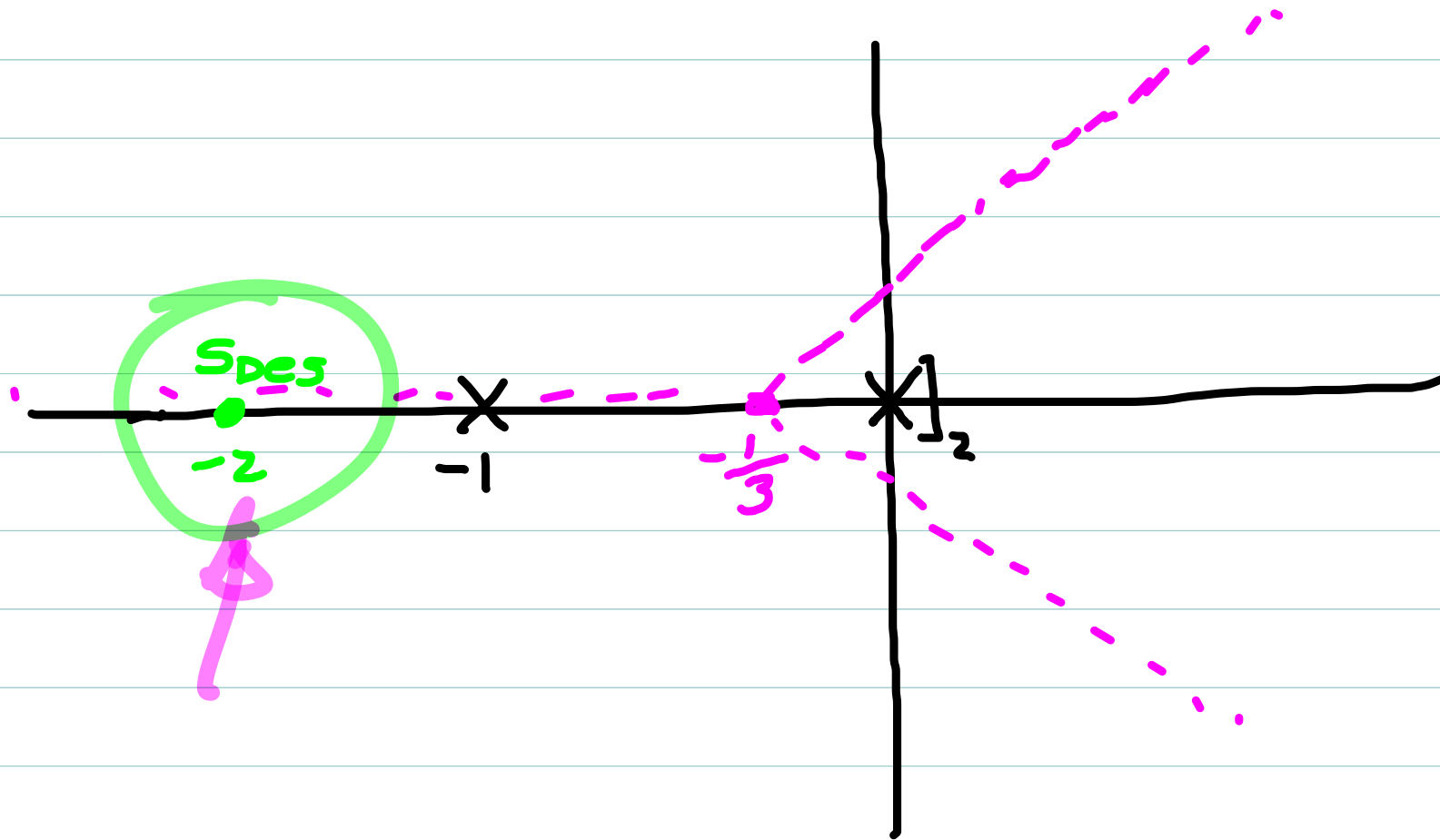


Stable for  $K > K_{crit}$

## Example #8

This is where we originally started our investigation

$$\text{with } H(s) = K, \quad L(s) = \frac{K}{s^2(s+1)}$$



## Example #8

This is where we originally started our investigation

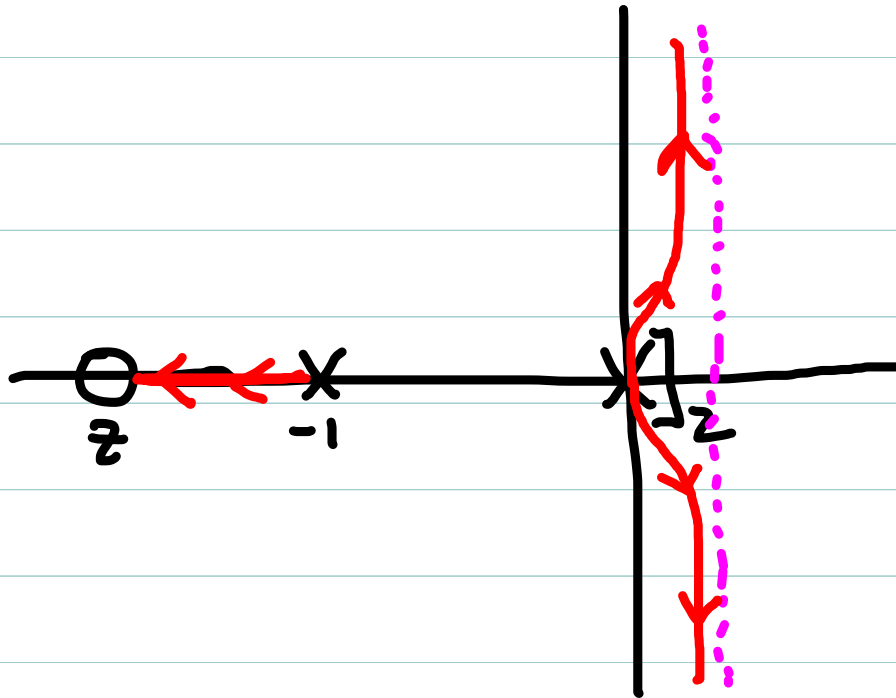
$$L(s) = \frac{K}{s^2(s+1)}$$



We can get the desired pole at -2, but will inevitably have poles of  $T(s)$  in RHP

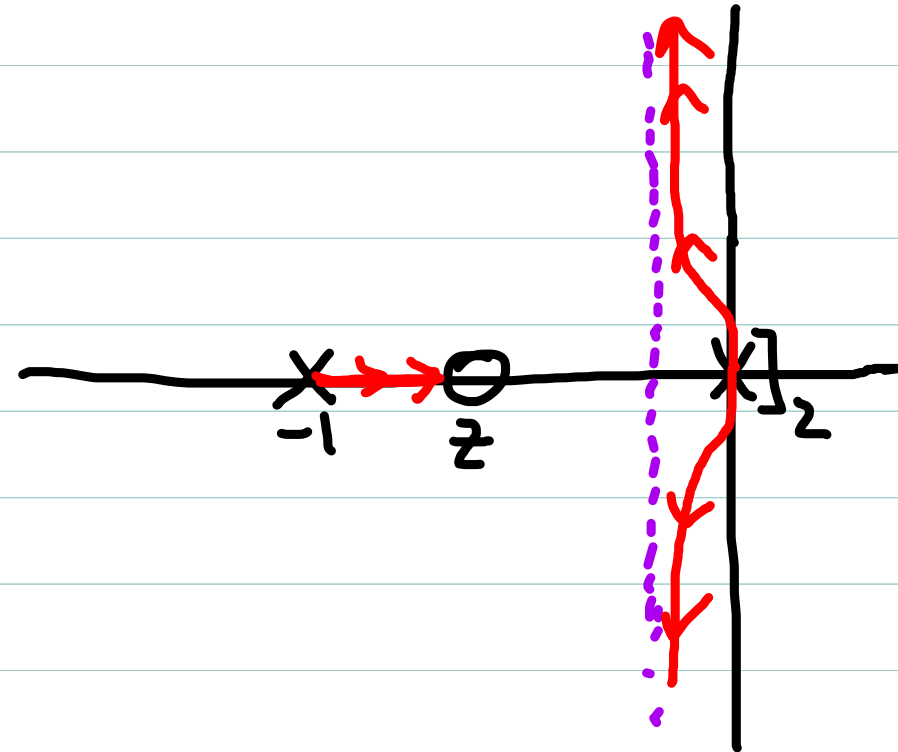


With instead  $H(s) = K(s - z)$



$$z < -1 \Rightarrow \alpha_z = \pm 90^\circ$$

$$\sigma_a = -\frac{1}{2}(1 + z) > \phi$$



$$0 < z < -1$$

$$\Rightarrow \sigma_a < \phi$$

So, with  $H(s) = K(s-z)$  we can stabilize the system as long as  $|z| < 1$  (which would agree with a Nyquist/phase margin analysis)

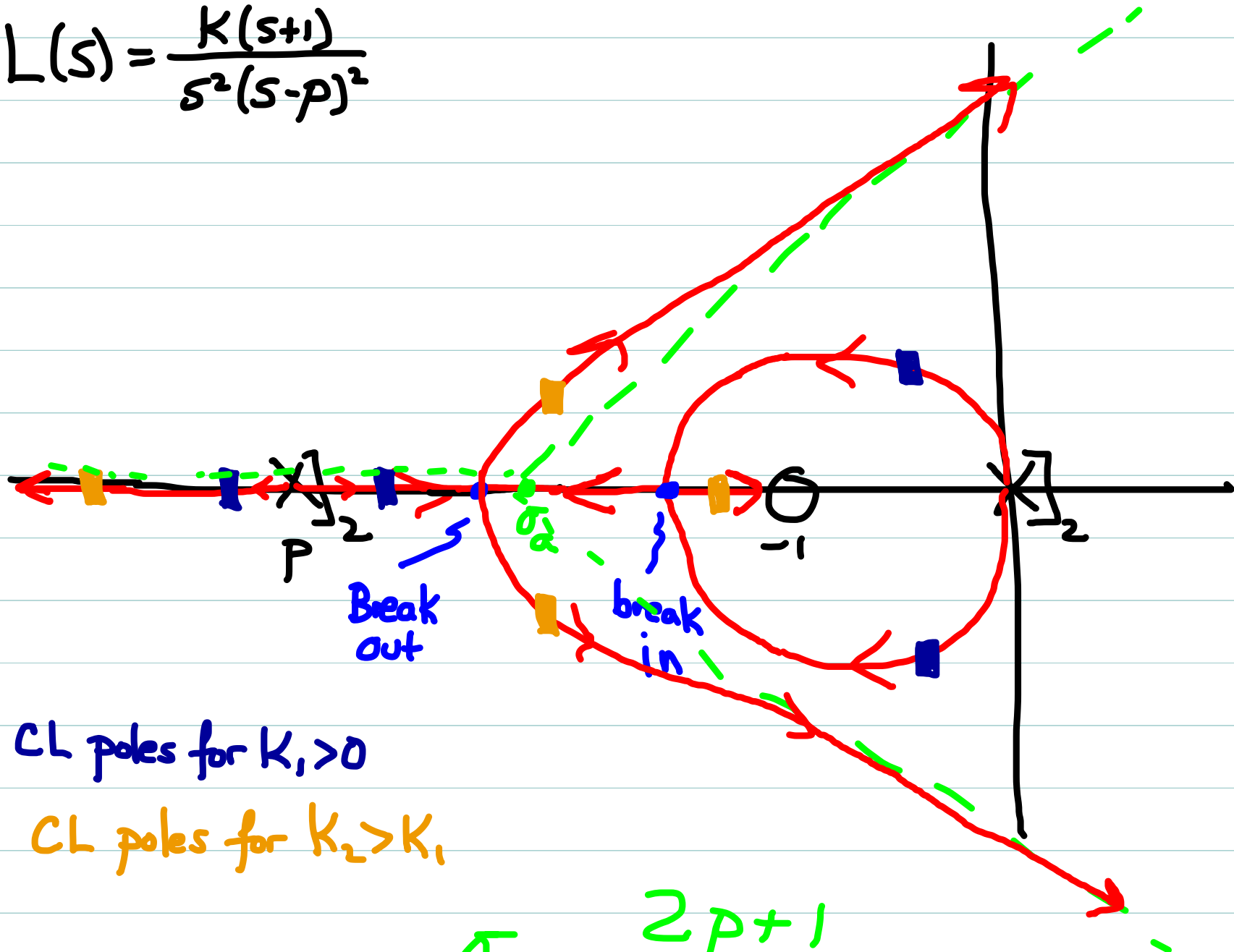
But we would have to accept a real pole  $> -1$ , and moreover this pole would not be dominant

An implementable compensator which could allow a real dominant CL pole near  $-2$  would be

$$H(s) = K \left[ \frac{(s+1)^2}{(s-p)^2} \right]$$

which has an interesting locus (next page)

$$L(s) = \frac{K(s+1)}{s^2(s-p)^2}$$



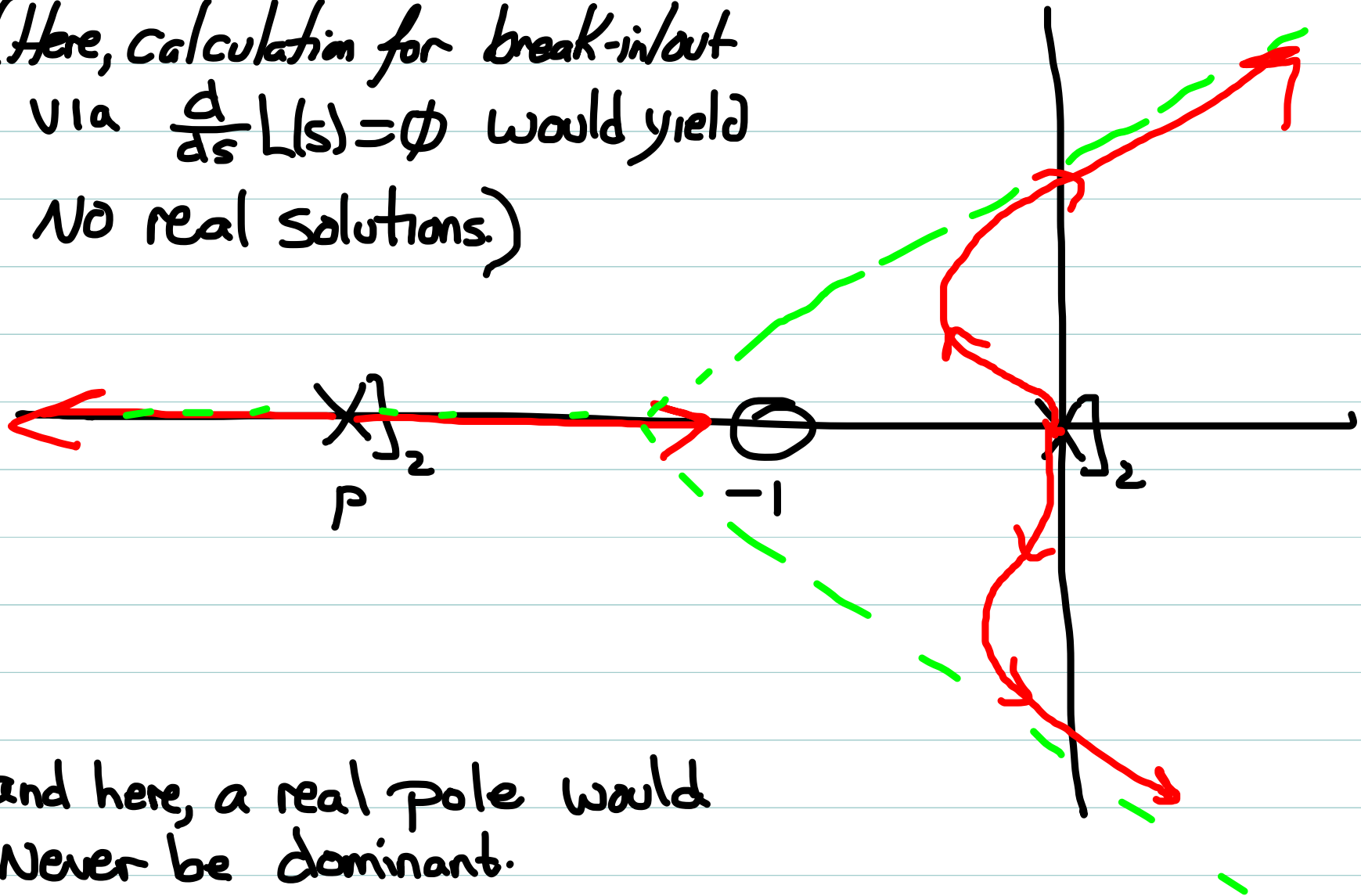
■ CL poles for  $K_1 > 0$

■ CL poles for  $K_2 > K_1$

$$\sigma_a = \frac{2p+1}{3}$$

However, depending on exact value of  $p$ , this is also possible:

(Here, calculation for break-in/out  
via  $\frac{d}{ds} L(s) = 0$  would yield  
No real solutions.)



and here, a real pole would  
Never be dominant.

## Comments on root locus method

- ⇒ Rules are not determinative; there may be many locus shapes consistent with calculations (although Matlab rlocus command will show you an exact plot).
  - ⇒ Cannot adapt method to account for effects of time delay
  - ⇒ Can adapt method only for very simple kinds of robustness analysis.
  - ⇒ Bode/Nyquist methods preferred in professional practice.
- 
- ⇒ But root locus does provide useful additional insights which are not available using freq. methods
  - ⇒ Familiarity with both gives "best of both worlds"