=> 1^{st} and 2^{nd} order step responses are "building blocks" by which we can understand response of more complex systems

=> each real pole introduces a new decaying exponential into transient response.

=> each complex pole pair introduces a decaying oscillation into the transient

=> An arbitrary number of poles of different types will typically require numerical simulation to quantify y_p, t_r, t_p, t_s

=> <u>However</u> in some cases we can still accurately predict these features.

Suppose:

$$G(s) = \frac{K}{(s-p_1)(s^2+2\zeta\omega_n s+\omega_n^2)} \quad \text{with } \zeta<1$$

$$= \frac{K}{(s-p_1)(s-p_2)(s-\bar{p}_2)}$$

for a unit step input $u(t) = \mathbb{1}(t)$ we know

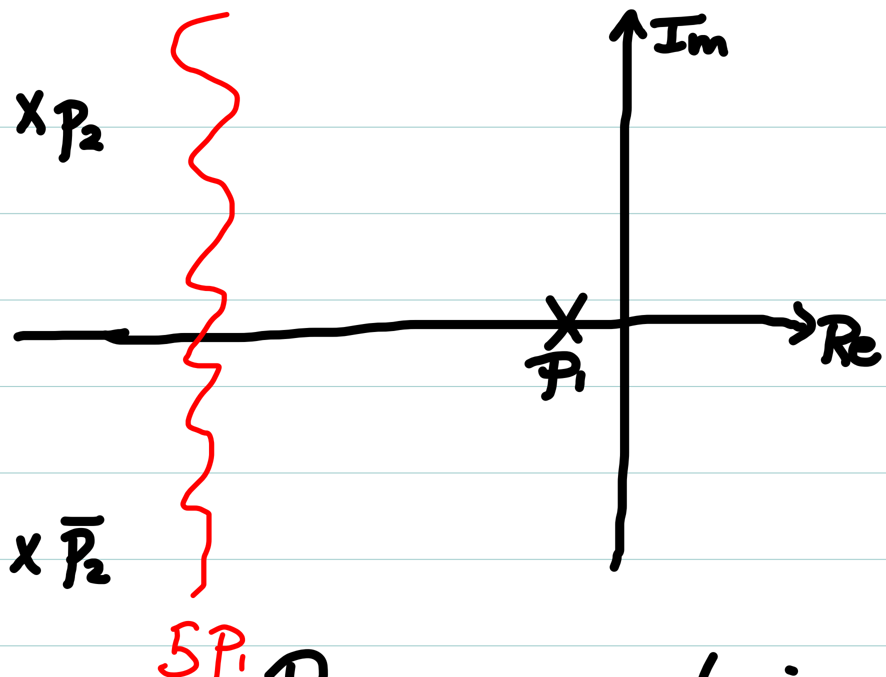
$$y_{ss} = G(0) = \frac{K}{-\omega_n^2 p_1}$$

But what can we say about y_p, t_p, t_c, t_s?

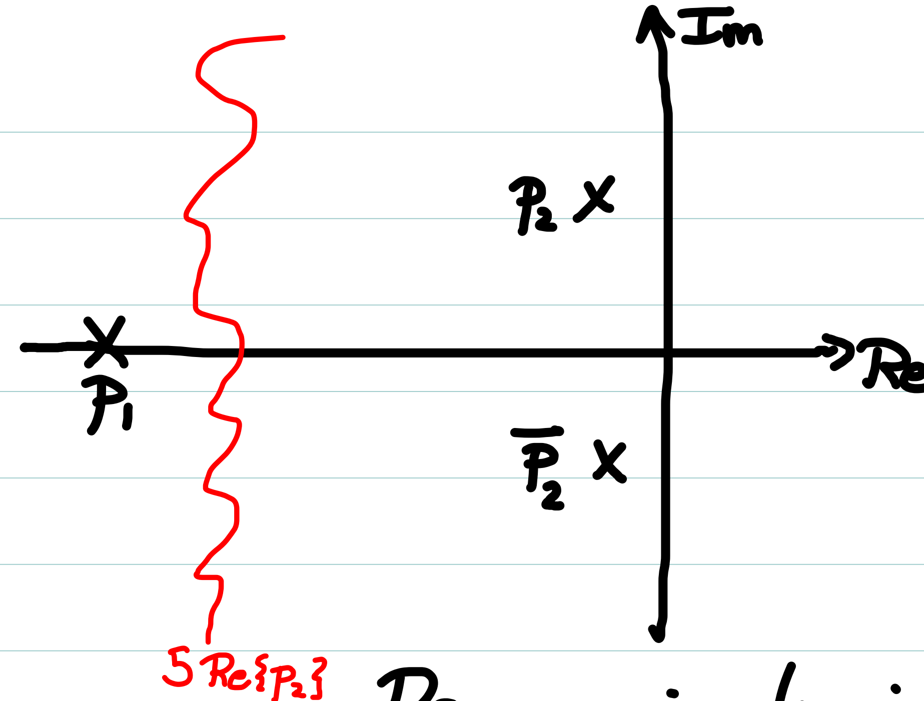
In general, not much unless either

$$|p_1| > 5\,|Re\{p_2\}| \text{ or } |Re\{p_2\}| > 5\,|p_1|$$

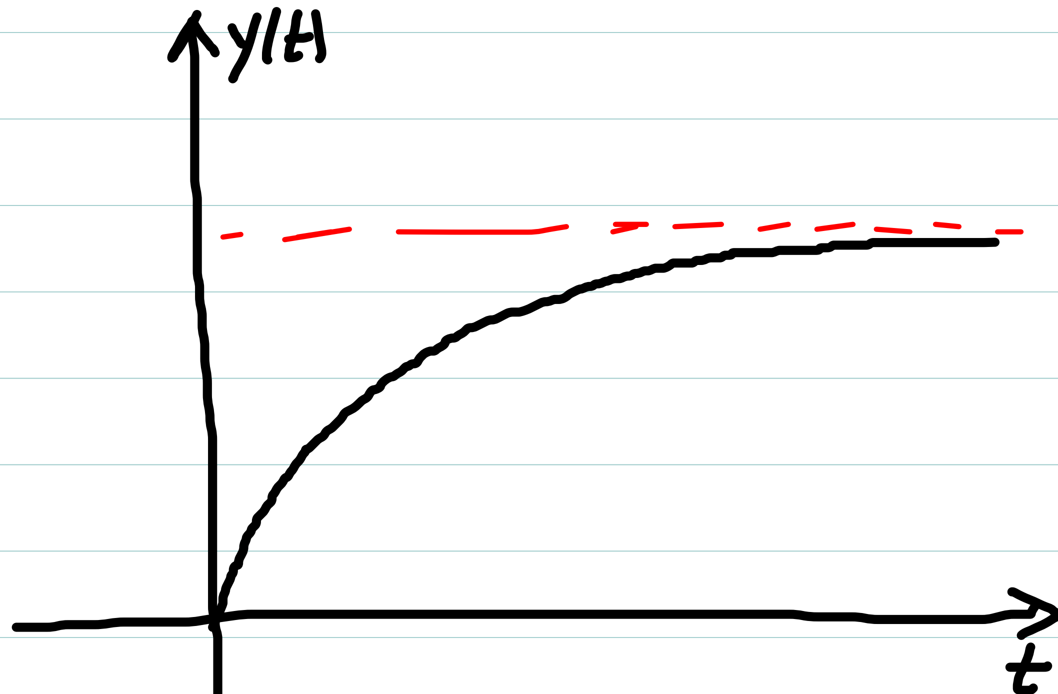
i.e. if one of the modes is <u>dominant</u>.

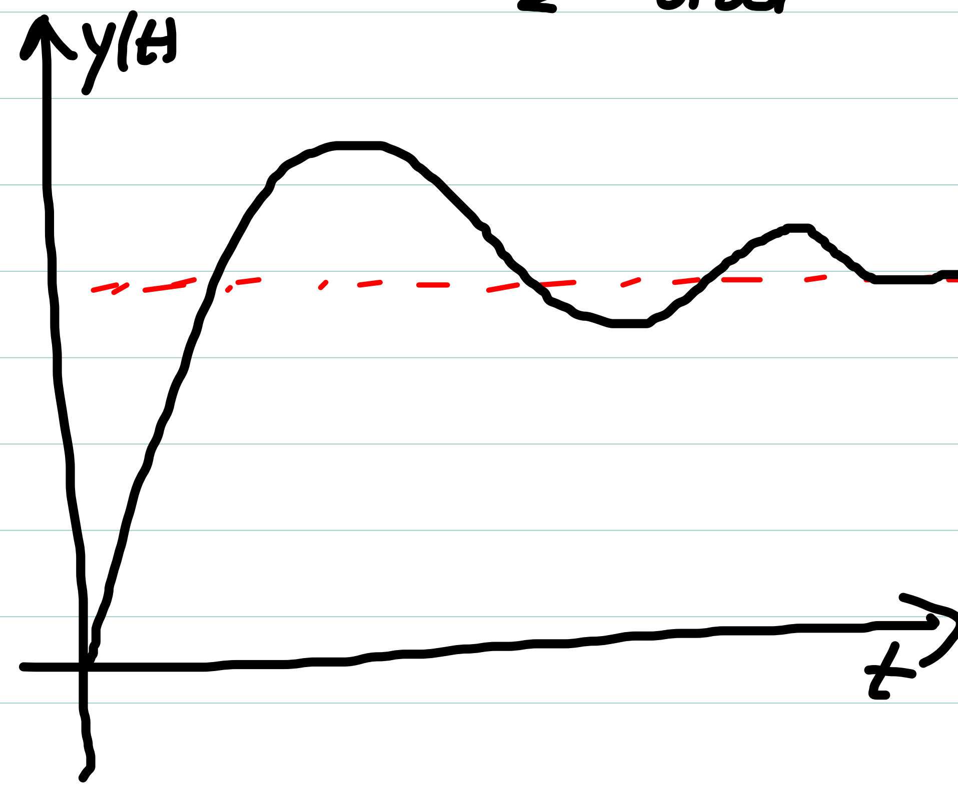


Response is dominantly 1^{st} order



Response is dominant
2^{nd} order

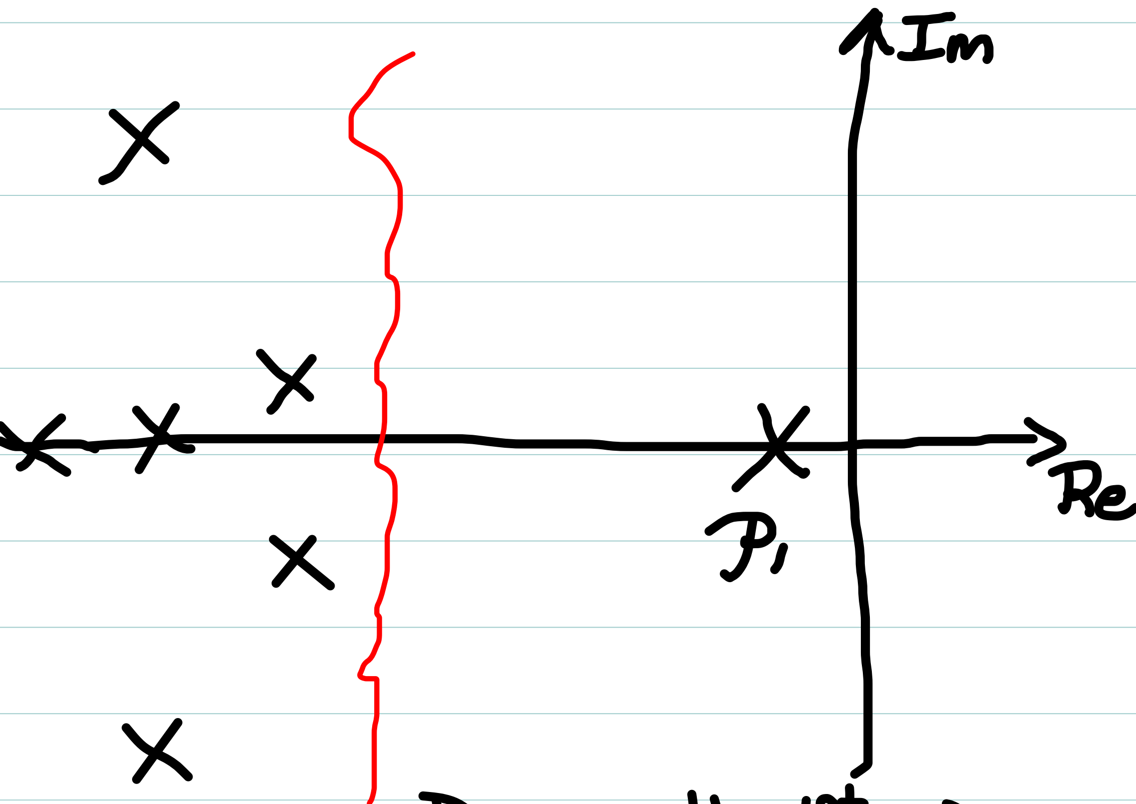




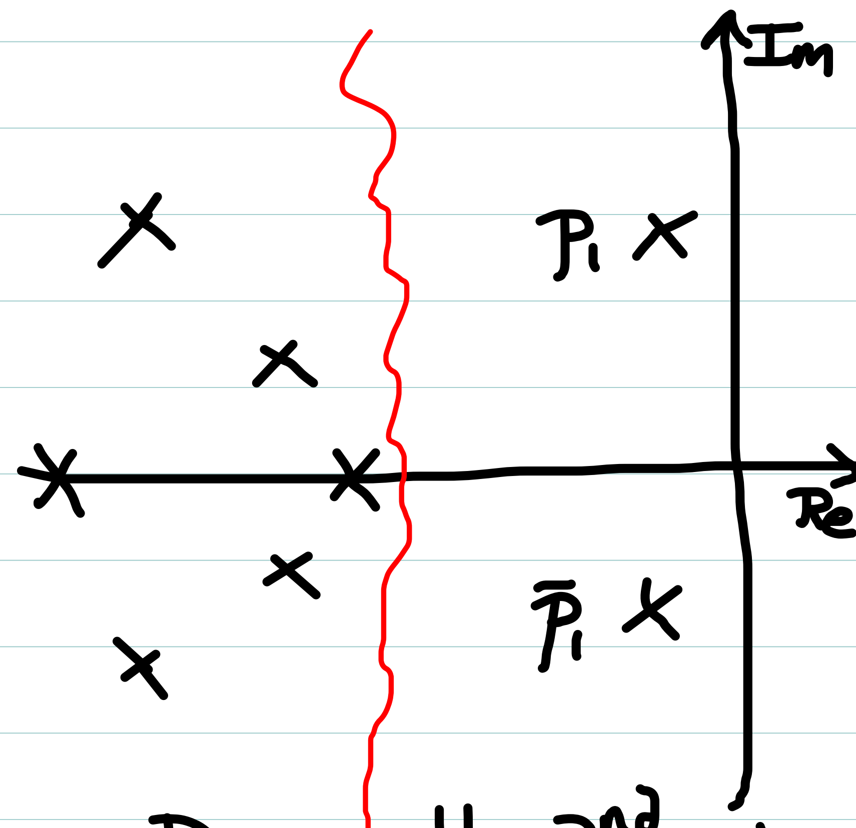
Dominant modes revisited

When a single mode is dominant, we can approximate the features of the response using just that mode

An arbitrarily complex system can be well approximated in this fashion.



Dominantly 1st order



Dominantly 2nd order

Effect of Zeros

Step response of

$$G(s) = \frac{\beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0}$$

Zero at $z_1 = -\beta_0/\beta_1$

3 important effects:

1. "Input absorbing" property
2. Transient suppression
3. Transient amplification

Both? Yes!

Depending on system

(I.) Input absorbtion

For unit step response of stable system

$$y_{ss}(t) = G(0)$$

Suppose $z_1 = -\beta_0/\beta_1 = 0 \implies \beta_0 = 0$

zero at origin

$$G(s) = \frac{\beta_1 s}{s^2 + \alpha_1 s + \alpha_0}$$

Then $y_{ss}(t) = G(0) = 0 \Leftarrow$ steady-state is zero

response contains only transient terms

In fact, $y(t)$ is the impulse response of

$$G_1(s) = \frac{\beta_1}{s^2 + \alpha_1 s + \alpha_0}$$

Effect of Zeros

Step response of

$$G(s) = \frac{\beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0}$$

Zero at $z_1 = -\beta_0/\beta_1$

3 important effects:

1. "Input absorbing" property
2. Transient suppression
3. Transient amplification

Both? Yes!

Depending on system

(2.) Transient Suppression

Suppose $s^2+\alpha_1 s+\alpha_0 = (s-p_1)(s-p_2)$ p_1, p_2 real

So

$$G(s) = \frac{\beta_1(s-z_1)}{(s-p_1)(s-p_2)}$$

Suppose $z_1 \approx p_1$, i.e. $|z_1-p_1| = \varepsilon << 1$

We know $y(t) = G(\phi) + A_1 e^{p_1 t} + A_2 e^{p_2 t}$

where $A_1 = \left[(s-p_1)Y(s)\right]_{s=p_1} = \dfrac{\beta_1 \overbrace{(p_1-z_1)}^{\sim << 1}}{p_1(p_1-p_2)}$ is small

so, for sufficiently small ε, the $e^{p_1 t}$ term in transient is negligible, and response is equivalent to a 1st order system with single pole p_2

Pole-zero Cancellation

Algebraically, if $z_1 \approx p_1$

$$G(s) = \frac{\beta_1 \cancel{(s-z_1)}}{\cancel{(s-p_1)}(s-p_2)} \approx \frac{\beta_1}{(s-p_2)}$$

Usually, if

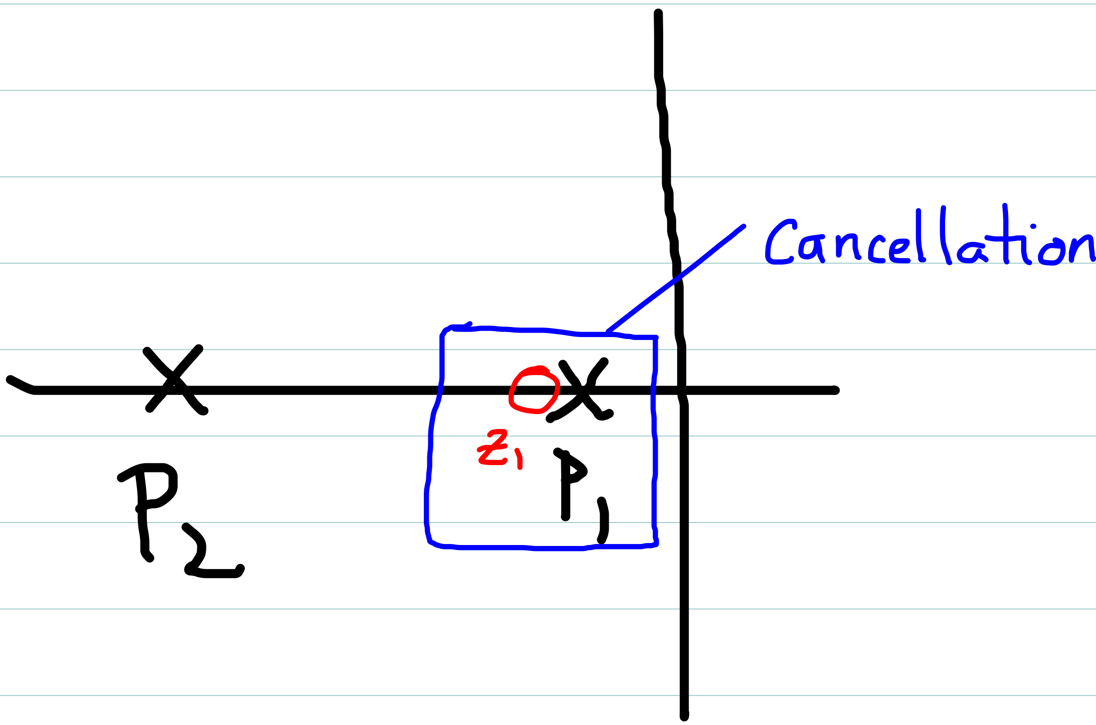
$$0.9 \le \left|\frac{z_1}{p_1}\right| \le 1.1$$

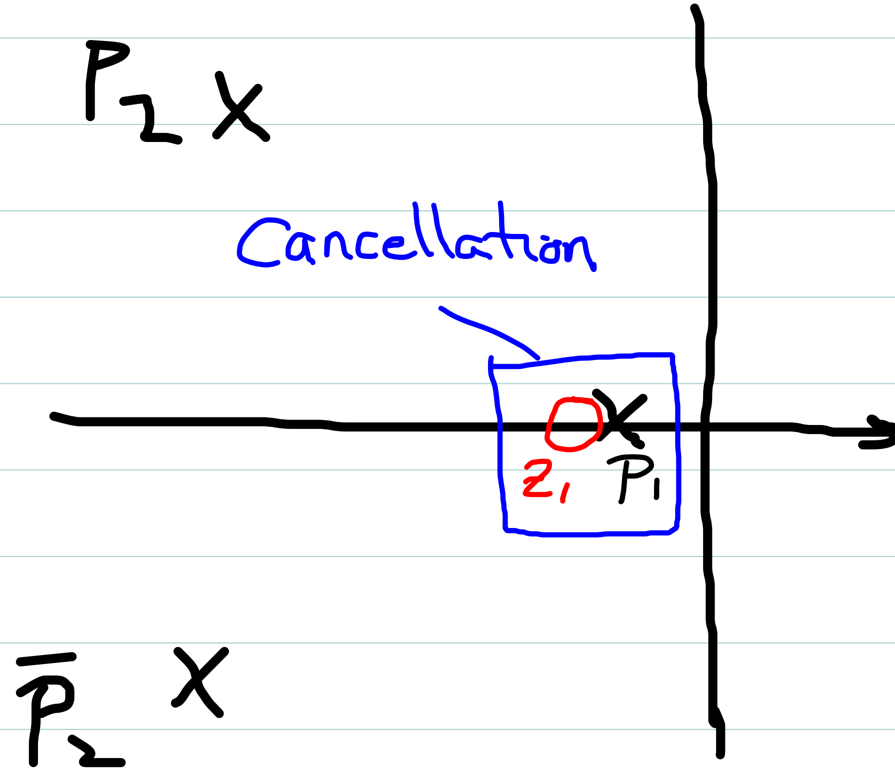
i.e. zero location within 10% of pole location

this is a good approximation

Cancellation and Dominance

Pole-zero cancellations can change dominance calculation





"fast" pole becomes dominant

2nd order poles become dominant

Cancellation is never exact!

$\Rightarrow$ z_1, p_1 come from different coefs. in diff'l eq'n.

$\Rightarrow$ These coefs come from physical properties of system whose values are not known precisely.

$\Rightarrow$ Cancellation should always be considered approx.

$\Rightarrow$ If p_1 is stable, it is a good approximation to cancel it

$$A_1 e^{p_1 t} \propto \varepsilon e^{p_1 t}$$

This term starts small, and gets smaller as t increases

But

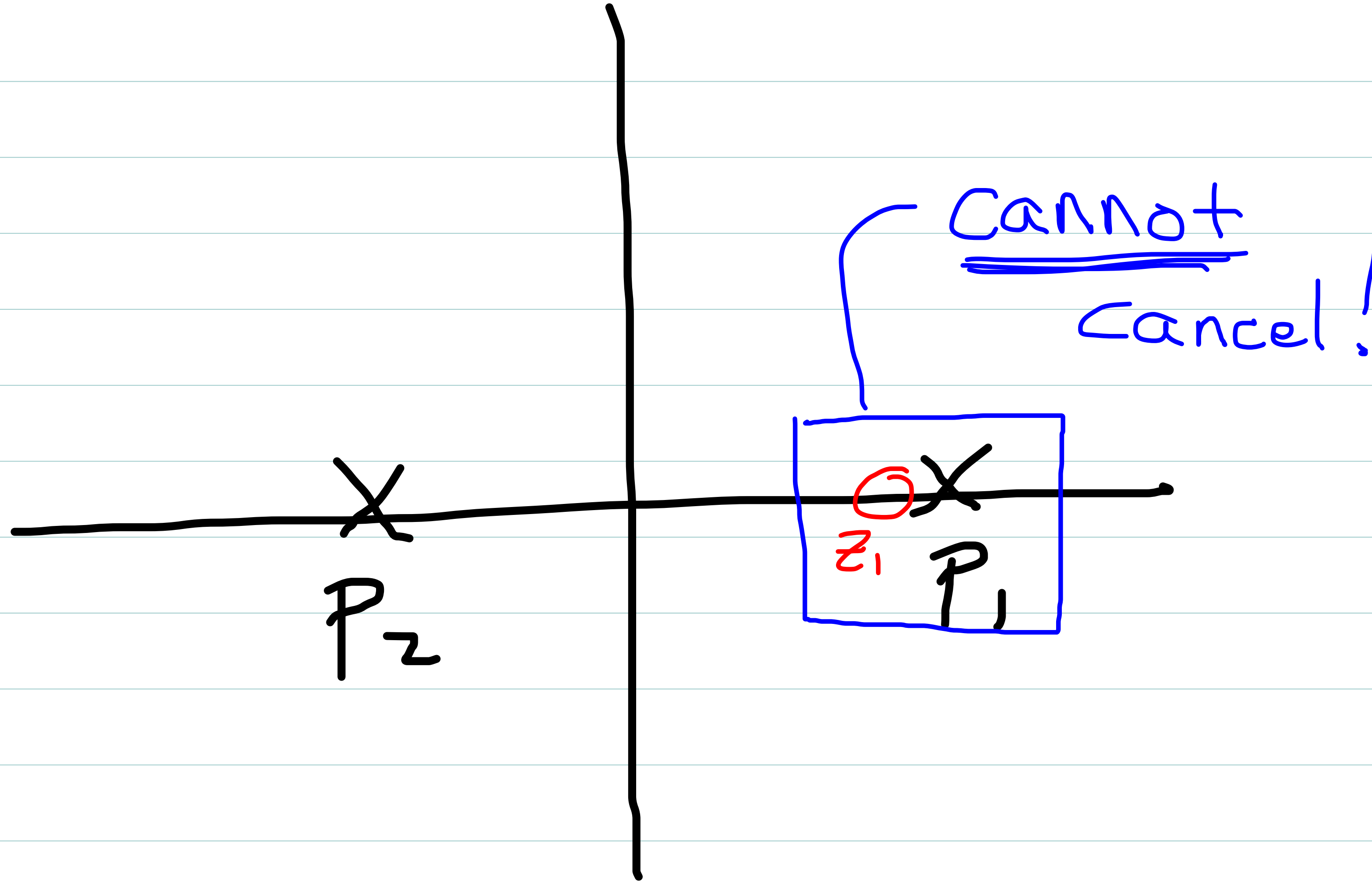
Suppose p_1 Not stable: $p_1 > 0$

then $A_1 e^{p_1 t} \propto \varepsilon e^{p_1 t}$

may start small, but increases w/o bound as t increases

Term will diverge to ∞, regardless how small ε is!

$$\boxed{\text{Pole-zero cancellation can } \underline{\text{Never}} \text{ be performed in RHP}}$$


Cannot
Cancel!
z1
P1
P2

Moreover...

Generally, if ICs on $y(t)$ are not all zero

$$Y(s) = G(s)U(s) + \frac{C(s)}{r(s)}$$

Nonzero

Will contribute terms to $y(t)$ which contain unstable mode <u>even if</u> this mode "cancels" in $G(s)$

<u>Moral</u>: Can <u>Never</u> "cancel" an unstable mode

!!!

Effects of zeros on step response

$$G(s) = \frac{\beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0}, \quad \text{zero at } z_1 = \frac{-\beta_0}{\beta_1}$$

① Input absorbtion (if $\beta_0 = \emptyset \Rightarrow z_1 = \emptyset$)

② Transient suppression via pole-zero cancellation

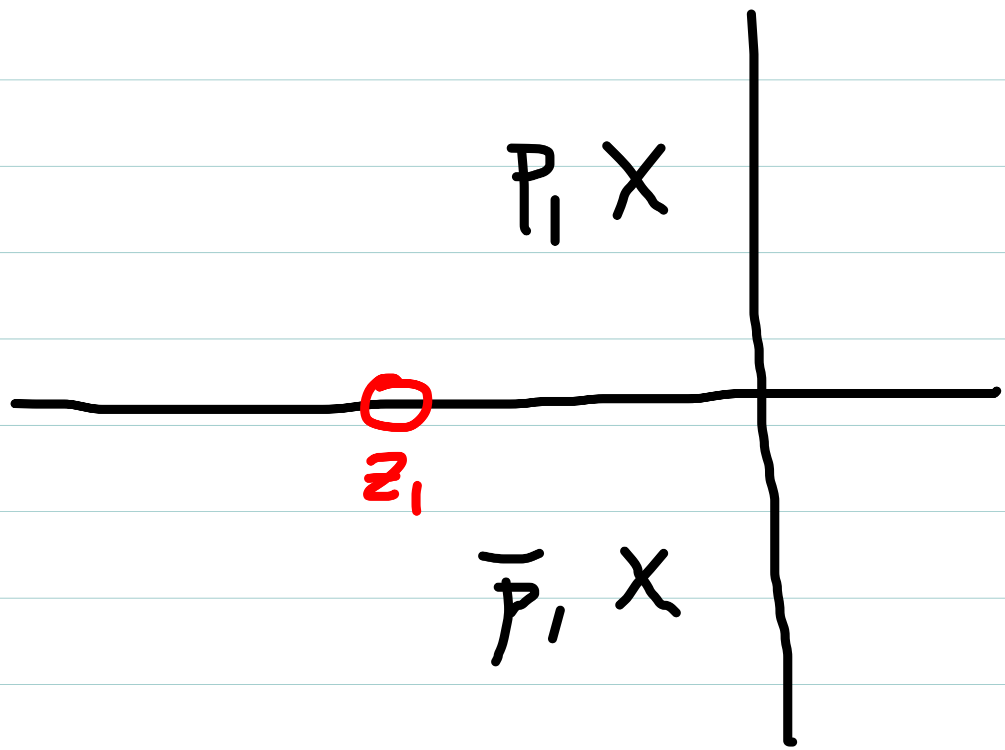
$\Rightarrow$ if $s^2 + \alpha_1 s + \alpha_0 = (s - p_1)(s - p_2)$; p_1, p_2 real
and $z_1 \approx p_1$ (or p_2)

③ Transient amplification $\Rightarrow$ examine this now.

③ Transient Amplification

Now suppose $s^2+\alpha_1 s+\alpha_0 = (s-p_1)(s-\bar{p}_1)$

$p_1 = \sigma + j\omega_d, \quad \omega_d \neq 0$



Pole-zero cancellation cannot occur here

What is the effect of the zero?

$$Y(s) = \frac{\beta_1 s + \beta_0}{s(s-p_1)(s-\bar{p}_1)} = \frac{\beta_1 s}{s(s-p_1)(s-\bar{p}_1)} + \frac{\beta_0}{s(s-p_1)(s-\bar{p}_1)}$$

$$= \left[\left(\frac{\beta_1}{\beta_0}\right) s\right]\left[\underline{\frac{\beta_0}{s(s-p_1)(s-\bar{p}_1)}}\right] + \left[\underline{\frac{\beta_0}{s(s-p_1)(s-\bar{p}_1)}}\right]$$

Let

$$Y_1(s) = \left[\frac{\beta_0}{s(s-p_1)(s-\bar{p}_1)}\right]$$

So

$$Y(s) = \left(\frac{\beta_1}{\beta_0}\right)\left[s\,Y_1(s)\right] + Y_1(s)$$

$$\Rightarrow \boxed{y(t) = \left(\frac{\beta_1}{\beta_0}\right)\dot{y}_1(t) + y_1(t)} \quad , \quad y_1(t) = \mathcal{L}^{-1}\{Y_1(s)\}$$

<u>Note:</u> $y_1(t)$ is ideal 2^{nd} order step response

$$y(t) = \left(\frac{\beta_1}{\beta_0}\right)\dot{y}_1(t) + y_1(t)$$

or equivalently:

$$\boxed{y(t) = y_1(t) - \left(\frac{1}{z_1}\right)\dot{y}_1(t)} \quad \left(z_1 = -\beta_0/\beta_1\right)$$

Where $y_1(t)$ is the "ideal" (no zero) step response

The total response $y(t)$ is the sum of the ideal response, and a fraction of the <u>derivative</u> of this response.

Suppose 1^{st} $z_1 < 0$ (LHP zero)

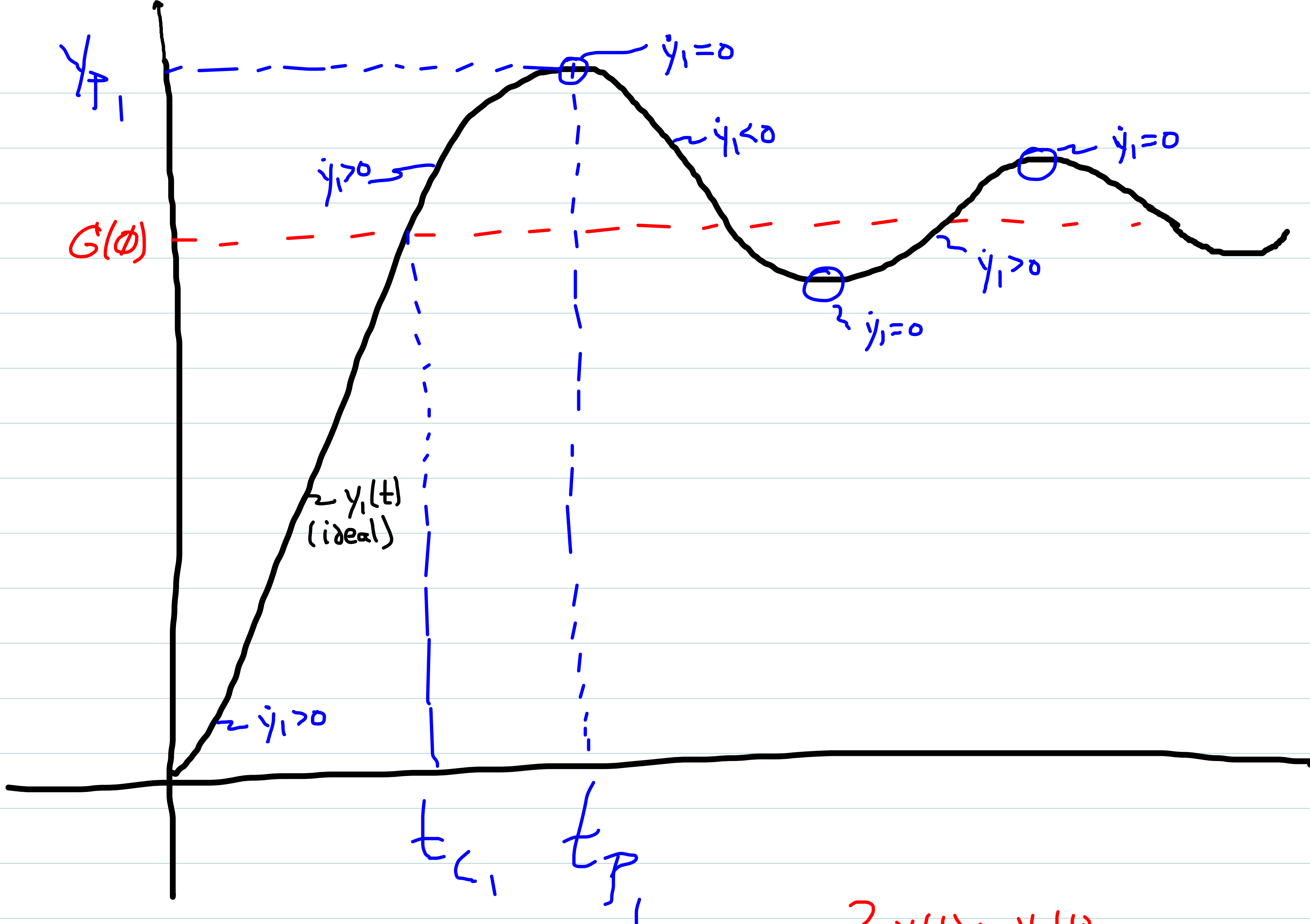
then $z_1 < 0$ and $\left(-\frac{1}{z_1}\right) > 0$ so we can write

$$y(t) = y_1(t) + \left(\frac{1}{|z_1|}\right)\dot{y}_1(t)$$

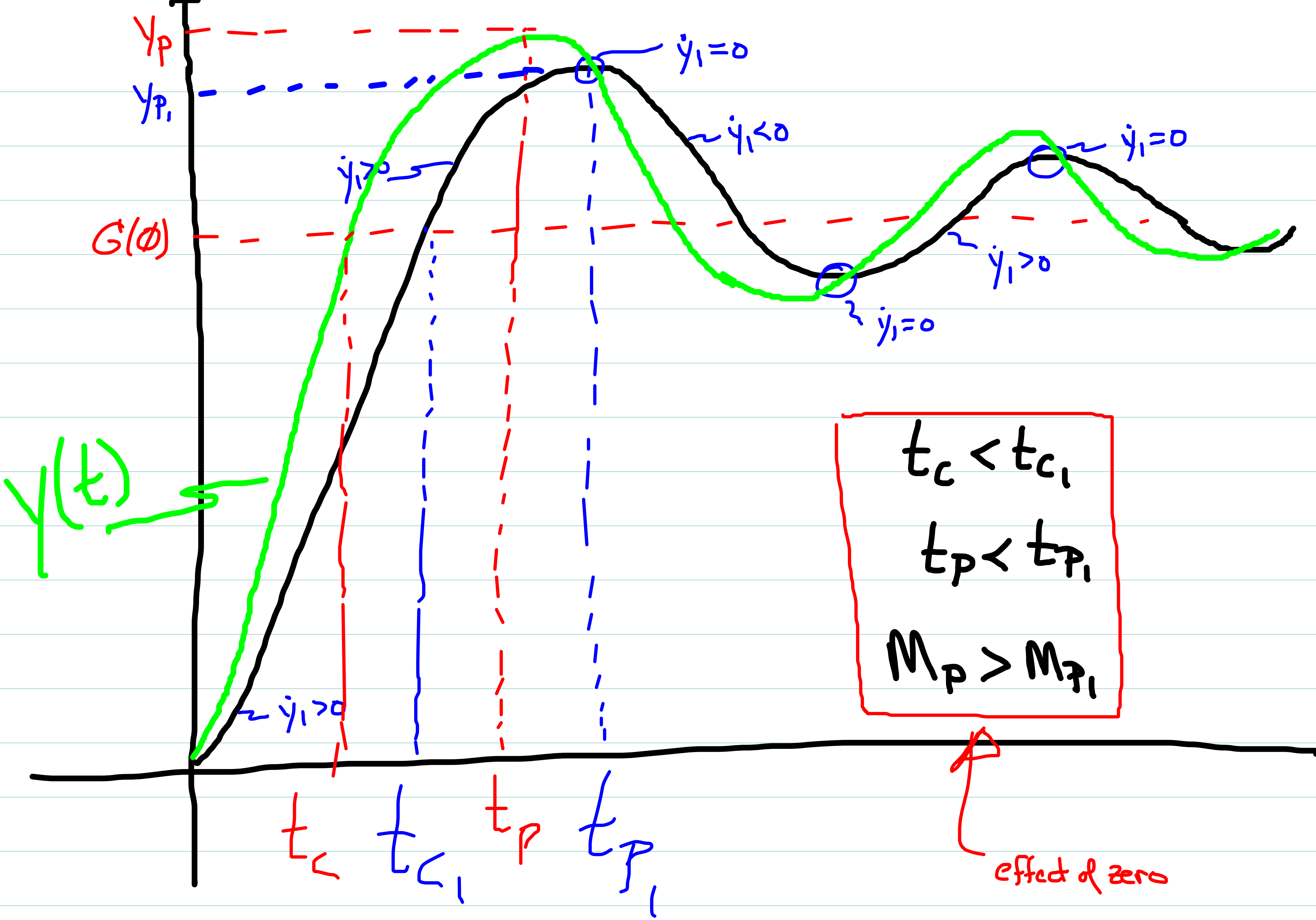
Derivative <u>adds</u> to total response. To understand effect of this, must examine behavior of $\dot{y}_1(t)$

Note that $\dot{y}_1(t) \to \phi$ As $t \to \infty$, so the steady-state of the new response will be the <u>Same</u> as the ideal response

$$y_{ss} = G(\phi)$$



<u>Note:</u> $\dot{y}_1(t) > \phi$ for all $\phi \le t < t_{p_1}$ $\Big\}$ $y(t) > y_1(t)$ in this region



y_p
y_{p_1}
$G(\emptyset)$
$y(t)$
$\dot{y}_1 > 0$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
$\dot{y}_1 < 0$
$\dot{y}_1 = 0$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
t_c
t_{c_1}
t_p
t_{p_1}
$t_c < t_{c_1}$
$t_p < t_{p_1}$
$M_p > M_{p_1}$
effect of zero