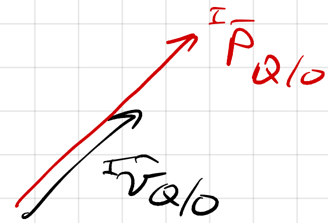
Chapter 2

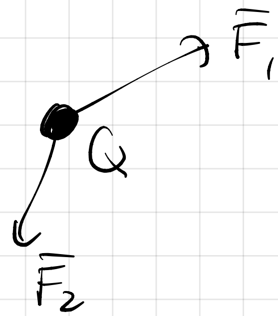
8/29/24

<u>Dfn</u> the <u>linear momentum</u> of a particle Q is

$$\boxed{{}^{I}\bar{p}_{Q/O} = m_Q\, {}^{I}\bar{v}_{Q/O}}$$



Dfn the <u>free body diagram</u> includes the forces acting on the particle



<u>Ex 2.1</u> <u>Straight-line motion with no force</u>

"which implies"

Recall $f_x = m\ddot{x} \Rightarrow \ddot{x} = \frac{f_x}{m}$

Suppose $f_x = 0 \Rightarrow \boxed{\ddot{x} = 0}$ eq. mo.

Let's solve (aka integrate) the eq. of motion for $(x, \dot{x})$

$$\ddot{x} = 0$$

$$\ddot{x} = \frac{d\dot{x}}{dt} = 0 \Rightarrow \int d\dot{x} = \int 0\,dt$$ Separation of variables

$$\dot{x}(t) = C_1$$ constant of integration

$$\dot{x} = \frac{dx}{dt} = C_1 \Rightarrow \int dx = \int C_1\, dt$$

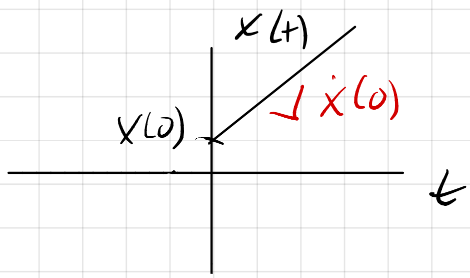
$$x(t) = C_1 t + C_2$$

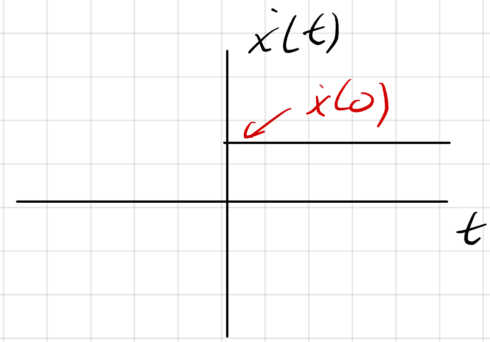
Q: How do we find C_1 and C_2?

sol'n

@ $t=0$ $\quad \left.\begin{aligned} C_1 &= \dot{x}(0) \\ C_2 &= x(0) \end{aligned}\right\}$ $\boxed{\begin{aligned} x(t) &= \dot{x}(0)t + x(0) \\ \dot{x}(t) &= \dot{x}(0) \end{aligned}}$

$y = mx + b$ line slope $\dot{x}(0)$
y-intercept $x(0)$





Ex 2.2 <u>Straight-line motion with constant force</u>

$$f_x = m\ddot{x} \Rightarrow \ddot{x} = \frac{f_x}{m} \qquad \text{"that is"}$$

Suppose f_x is constant, i.e., $f_x(t) = f$

$$\boxed{\ddot{x} = \frac{f}{m}}$$

Find $x(t)$, $\dot{x}(t)$:

$$\ddot{x} = \frac{d\dot{x}}{dt} \Rightarrow \int d\dot{x} = \int \frac{f}{m}\, dt$$

$$\dot{x}(t) = \frac{f}{m} t + c_1$$

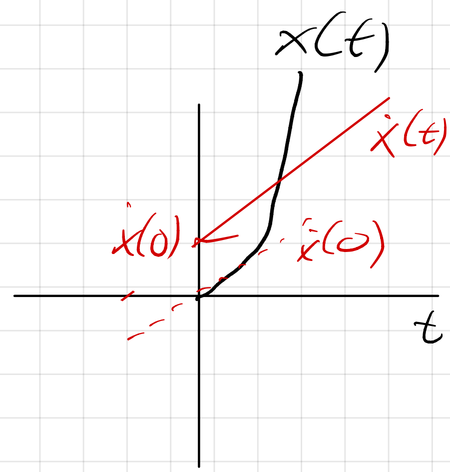
$$\dot{x} = \frac{dx}{dt} \Rightarrow \int dx = \int \left(\frac{f}{m} t + c_1\right) dt$$

$$x(t) = \frac{1}{2}\frac{f}{m} t^2 + c_1 t + c_2$$

@ $t = 0 \Rightarrow c_1 = \dot{x}(0)$

$$c_2 = x(0)$$

$$\boxed{\begin{aligned} x(t) &= \frac{1}{2}\frac{f}{m} t^2 + \dot{x}(0)t + x(0) \\ \dot{x}(t) &= \frac{f}{m} t + \dot{x}(0) \end{aligned}}$$



Ex 2.3 Straight-line motion with a position-dependent force

$$f_x = f_x(x) \,, \quad \text{eg.} \quad f_x = -\frac{k}{m}(x-x_0)$$

for example

$$\ddot{x} = \frac{f_x}{m} = \frac{f(x)}{m} = a(x) \leftarrow \boxed{\ddot{x} = a(x)}$$

$$\ddot{x} = \frac{d\dot{x}}{dt} \Rightarrow \int d\dot{x} = \int a(x)\,dt$$

* key idea: multiply both sides by dx

$$\dot{x} = \frac{dx}{dt}$$

$$\ddot{x}\,dx = a(x)\,dx$$

$$\int \frac{d\dot{x}}{\cancel{dt}}\,\dot{x}\,\cancel{dt} = \int a(x)\,dx$$

$$\boxed{\frac{1}{2}\dot{x}^2 + C = \int a(x)\,dx}$$

$$\int \dot{x}\,d\dot{x} = \int v\,dv = \frac{1}{2}v^2 + C$$

or

$$\int_{\dot{x}(t_1)}^{\dot{x}(t_2)} d\dot{x}\,\dot{x} = \int_{x(t_1)}^{x(t_2)} a(x)\,dx$$

$$\boxed{\frac{1}{2}\dot{x}^2(t_2) = \frac{1}{2}\dot{x}^2(t_1) + \int_{x(t_1)}^{x(t_2)} a(x)\,dx}$$

Anatomy of N2L

Newton's 2nd Law

$$\boxed{\vec{F}_p = m_p\, {}^{I}\vec{a}_{p/o}}$$

force is a vector

acceleration is a vector derivative

* only applies to a particle with constant mass

must be the inertial frame

Linear momentum form of N2L for particle Q

$$\bar{F}_Q = m_Q \, {}^I\bar{a}_{Q/O}$$

we have $\quad {}^I\bar{p}_{Q/O} = m_Q \, {}^I\bar{v}_{Q/O}$

$$\frac{{}^I d}{dt}\left({}^I\bar{p}_{Q/O}\right) = \frac{{}^I d}{dt}\left(m_Q \, {}^I\bar{v}_{Q/O}\right)$$

$$= m_Q \frac{{}^I d}{dt}\left({}^I v_{Q/O}\right)$$

N2L $\longrightarrow$ $= m_Q \, {}^I\bar{a}_{Q/O}$

$$\boxed{\bar{F}_Q = \frac{{}^I d}{dt}\left({}^I\bar{p}_{Q/O}\right)} \quad \text{N2L}$$

Suppose $\bar{F}_Q = 0 \Rightarrow \frac{{}^I d}{dt}\left({}^I\bar{p}_{Q/O}\right) = 0$

$${}^I\bar{p}_{Q/O} = \text{constant}$$

aka "it is conserved"