ENAE 404 - 0101: Homework 00

Numerical 2BP

Due on February 4th, 2025 at 11:59 PM

Dr. Barbee, 09:30

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Problem 1:

Plot the Earth, Didymos, and DART orbits in 3D on the same plot.

Solution

Part A

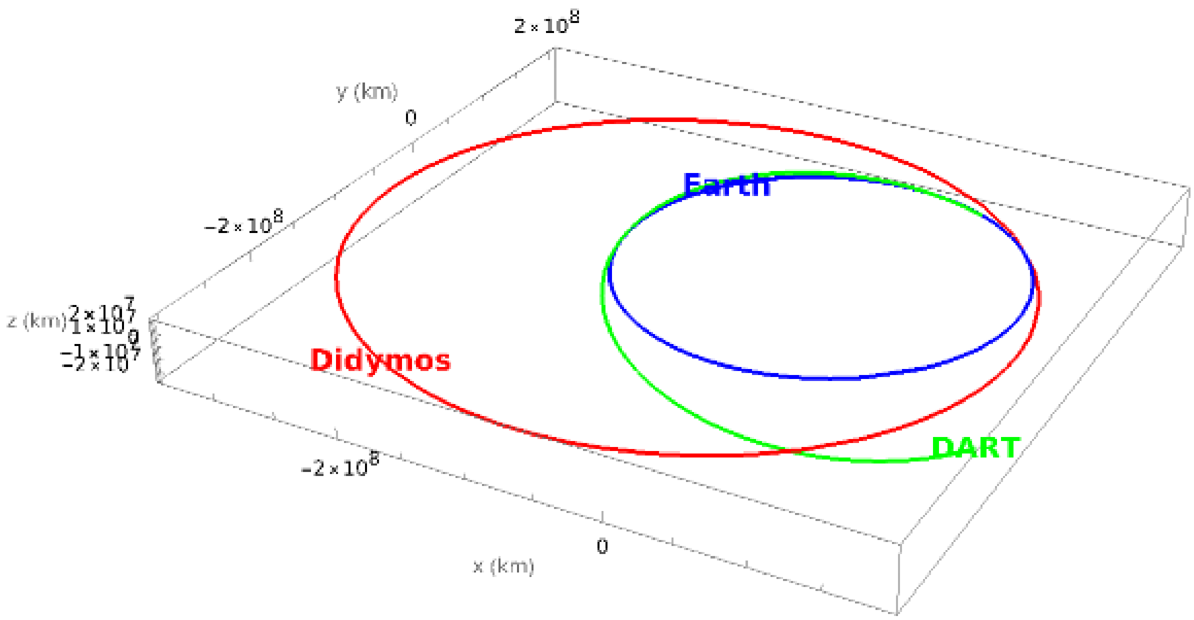

Figure 1: Earth, Didymos, and DART Orbits about the Sun

Part B

```
(* Define the Sun's gravitational parameter (km^3/s^2) *)
muSun = 1.32712*10^11;

(* Solve the two\[Hyphen]body ODE in vector form:
   r'(t) = v(t)
   v'(t) = -muSun * r(t)/Norm[r(t)]^3
   where r(t) is a 3\[Hyphen]vector {x,y,z}. *)

(* ----- Earth ----- *)
initEarth = {
  (* initial position: *) {6.825*10^7, 1.30864*10^8, 1.81329*10^4},
  (* initial velocity: *) {-26.7639, 13.8981, -9.22784*10^-4}
};
tmaxEarth = 7.0*10^7;  (* time span in seconds *)

solEarth = NDSolve[
  {
    rE'[t] == vE[t],
    vE'[t] == -muSun*rE[t]/Norm[rE[t]]^3,
    rE[0] == initEarth[[1]],
    vE[0] == initEarth[[2]]
  },
  {rE, vE},
```

Problem 1 continued on next page...

```
  {t, 0, tmaxEarth}
];

(* ----- Didymos ----- *)
initDidymos = {
  (* initial position: *) {-2.39573*10^8, -2.35661*10^8, 9.54384*10^6},
  (* initial velocity: *) {12.4732, -9.74427, -0.87661}
};
tmaxDidymos = 7.0*10^7;

solDidymos = NDSolve[
  {
    rD'[t] == vD[t],
    vD'[t] == -muSun*rD[t]/Norm[rD[t]]^3,
    rD[0] == initDidymos[[1]],
    vD[0] == initDidymos[[2]]
  },
  {rD, vD},
  {t, 0, tmaxDidymos}
];

(* ----- DART ----- *)
initDART = {
  (* initial position: *) {6.82409*10^7, 1.30854*10^8, 1.52197*10^4},
  (* initial velocity: *) {-30.6997, 8.11796, 3.95772}
};
tmaxDART = 26*10^6;

solDART = NDSolve[
  {
    rA'[t] == vA[t],
    vA'[t] == -muSun*rA[t]/Norm[rA[t]]^3,
    rA[0] == initDART[[1]],
    vA[0] == initDART[[2]]
  },
  {rA, vA},
  {t, 0, tmaxDART}
];

(* Create 3D parametric plots for each trajectory *)
trajEarth = ParametricPlot3D[
  Evaluate[rE[t] /. solEarth],
  {t, 0, tmaxEarth},
  PlotStyle -> {Blue, Thick},
  Mesh -> None
];

trajDidymos = ParametricPlot3D[
  Evaluate[rD[t] /. solDidymos],
  {t, 0, tmaxDidymos},
  PlotStyle -> {Red, Thick},
  Mesh -> None
];

trajDART = ParametricPlot3D[
  Evaluate[rA[t] /. solDART],
  {t, 0, tmaxDART},
  PlotStyle -> {Green, Thick},
  Mesh -> None
];

(* Optional: Add labels at the endpoints of each trajectory *)
```

Problem 1 continued on next page. . .

```
earthLabel = Graphics3D[{
    Blue,
    Text[Style["Earth", Bold, 14],
      Evaluate[rE[tmaxEarth] /. solEarth]]
}];
didymosLabel = Graphics3D[{
    Red,
    Text[Style["Didymos", Bold, 14],
      Evaluate[rD[tmaxDidymos] /. solDidymos]]
}];
dartLabel = Graphics3D[{
    Green,
    Text[Style["DART", Bold, 14],
      Evaluate[rA[tmaxDART] /. solDART]]
}];

(* Combine the trajectories and labels into one 3D plot *)
Show[
  trajEarth, trajDidymos, trajDART,
  earthLabel, didymosLabel, dartLabel,
  Axes -> True,
  AxesLabel -> {"x (km)", "y (km)", "z (km)"},
  Boxed -> True,
  ImageSize -> Large,
  PlotRange -> All,
  ViewPoint -> {1.3, -2.4, 1.5}
]
```

Problem 2:

Plot the position, velocity and acceleration of the Didymos Orbit magnitudes as a function of time.

Solution

Part A

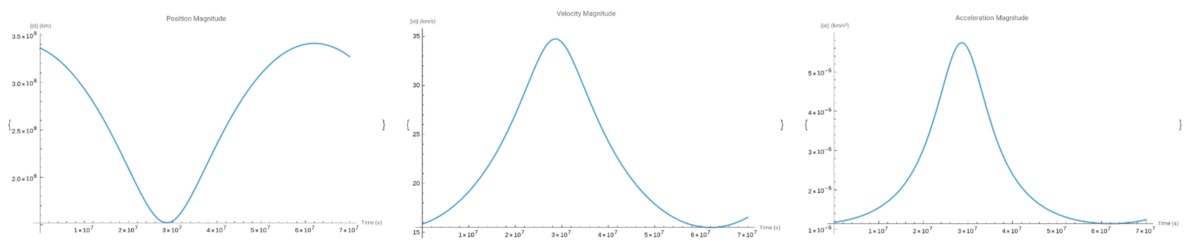

Figure 2: Didymos Orbit magnitudes vs. time

Part B

```
(* --- Define functions for the magnitudes --- *)
  (* Position magnitude as a function of time *)
  posMag[t_] := Norm[rD[t] /. solDidymos[[1]]];
(* Velocity magnitude as a function of time *)
velMag[t_] := Norm[vD[t] /. solDidymos[[1]]];
(* Acceleration is given by a = -muSun * r/|r|^3, so its magnitude is: \
*)
accMag[t_] :=
  Norm[-muSun*(rD[t] /. solDidymos[[1]])/
     Norm[rD[t] /. solDidymos[[1]]]^3];

(* --- Create subplots --- *)
plotPos = Plot[posMag[t], {t, 0, tmaxDidymos},
   PlotRange -> All,
   AxesLabel -> {"Time (s)", "||r|| (km)"},
   PlotLabel -> "Position Magnitude",
   ImageSize -> Large];
plotVel = Plot[velMag[t], {t, 0, tmaxDidymos},
   PlotRange -> All,
   AxesLabel -> {"Time (s)", "||v|| (km/s)"},
   PlotLabel -> "Velocity Magnitude",
   ImageSize -> Large];
plotAcc = Plot[accMag[t], {t, 0, tmaxDidymos},
   PlotRange -> All,
   AxesLabel -> {"Time (s)", "||a|| (km/s^2)"},
   PlotLabel -> "Acceleration Magnitude",
   ImageSize -> Large];

(* --- Arrange the plots in a grid --- *)
GraphicsRow[{
  {plotPos},
  {plotVel},
  {plotAcc}
  }]
```

Problem 3:

Explain why the magnitude plots of Problem 2 make sense given the equation of motion.

Solution

The position, velocity, and acceleration magnitude plots vary in a way that is consistent with the equations of motion.

- $|\vec{r}|$ oscillates between the perihelion and aphelion.
- $|\vec{v}|$ is at its maximum when $|\vec{r}|$ is at its minimum (and vice-versa).
- $|\vec{a}| = \frac{\mu}{|\vec{r}|^2}$ is at its maximum at the perihelion and its minimum at the aphelion.

Problem 4:

Give the final state (position and velocity vectors) of the Didymos Orbit after 70,000,000 seconds.

Solution

Part A

$$\vec{r} = \begin{bmatrix} -1.952\,68 \times 10^{8} \\ -2.6247 \times 10^{8} \\ 6.564\,72 \times 10^{6} \end{bmatrix}, \vec{v} = \begin{bmatrix} 15.0139 \\ -6.827\,68 \\ -0.970\,622 \end{bmatrix}$$

Part B

```
finalPosition = rD[tmaxDidymos] /. solDidymos[[1]]
finalVelocity = vD[tmaxDidymos] /. solDidymos[[1]]
```