Lecture 5

Wednesday, September 11, 2024 12:54 PM

Transient Circuit Analysis

Want to consider how circuits respond to a sudden change; e.g. toggling a switch.

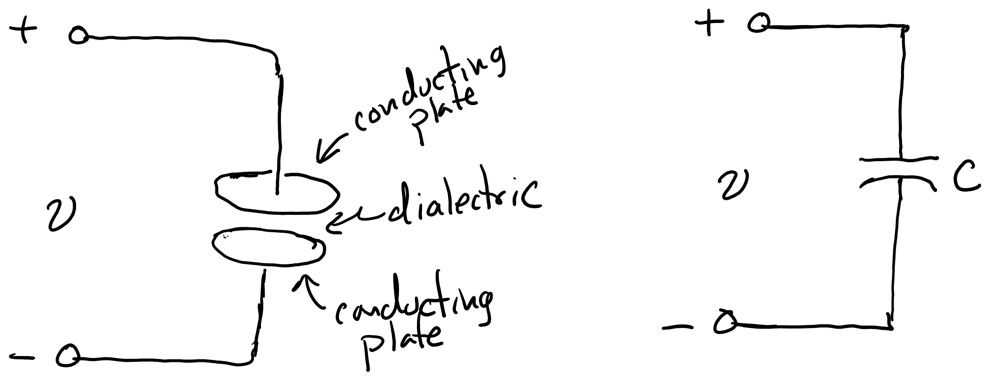
Circuit response is non-trivial when circuit elements capable of storing energy are present.

Two common examples of circuit elements capable of storing energy are:

- Capacitor
- Inductor

Capacitors

A capacitor is a circuit element that consists of two conducting surfaces separated by a dielectric material (insulator).



The charge on the capacitor is proportional to the voltage potential between the two plates

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$$q = C v$$

where C is the "capacitance" and has the units

$$\text{Farad} = \frac{\text{Coulomb}}{\text{Volt}}$$

Typical capacitors have a capacitance $\sim \mu F$.

Recall: $i = \dfrac{dq}{dt}$

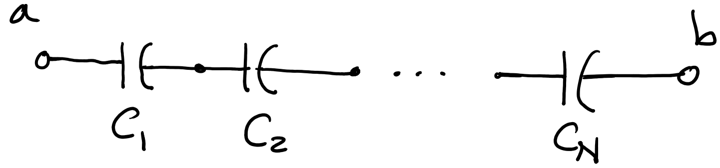
$\therefore$

$$i_c = \frac{d}{dt}(C v_c) \Rightarrow \boxed{i_c = C \frac{dv_c}{dt}}$$

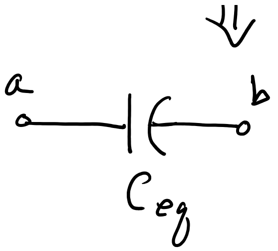
Remarks:

- In a DC circuit, there is no current flow to/from the capacitor. The capacitor acts like an open switch.
- Current does not flow through the capacitor. Current flows from one plate to the other via the connecting circuit.

Capacitors in series: Can show using KVL,

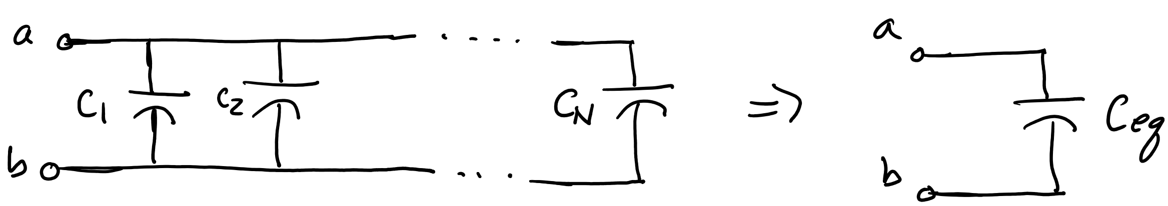


$\Downarrow$



$$C_{eq} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \right]^{-1}$$

Capacitors in parallel: Can show using KCL



$$C_{eq} = C_1 + C_2 + \cdots + C_N$$

* Opposite of how resistors combine!

Energy storage:

Recall $p(t) = v(t)\, i(t)$

Energy in capacitor = work done on capacitor = integral of power

$$e(t) = w(t) = \int_0^t p(x)\, dx = \int_0^t v(x) \left[C \frac{dv}{dx} \right] dx$$

$$= \frac{1}{2} C v^2(t) \qquad \text{units: Joules}$$

Inductors

An inductor is a circuit element consisting of a conducting wire wound around a core. The core may be nonmagnetic or ferromagnetic (iron).

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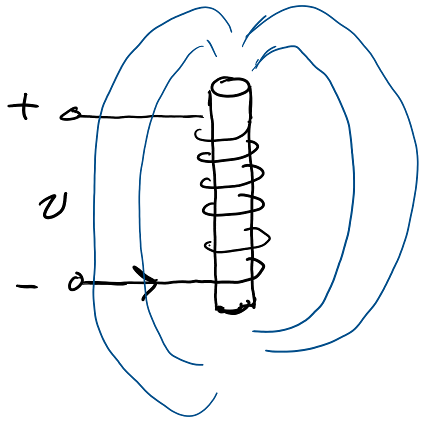
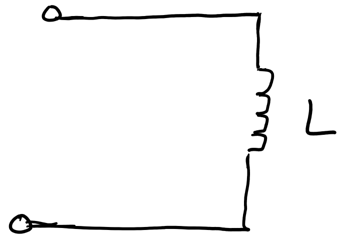




As current flows through the wire, a magnetic field forms. If core is nonmagnetic, the magnetic field will extend beyond the inductor. If a ferromagnetic core is used, the magnetic field will be contained in the core.

Joseph Henry discovered,

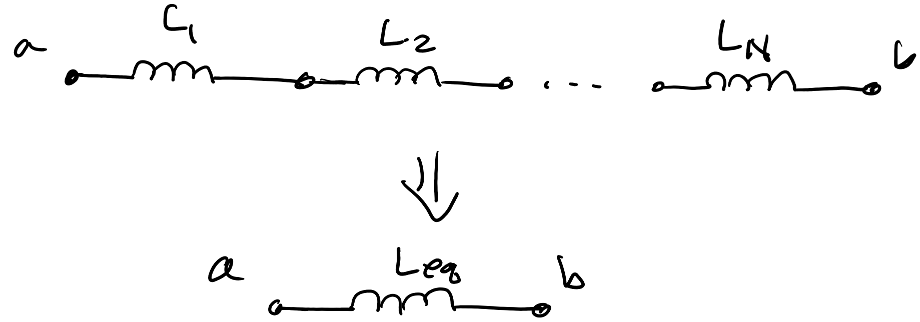
$$v(t) = L\,\frac{di}{dt}$$

where L is the "inductance" and has the unit "Henry"

$$\text{Henry} = \frac{\text{Volt}\cdot\text{second}}{\text{Amp}}$$

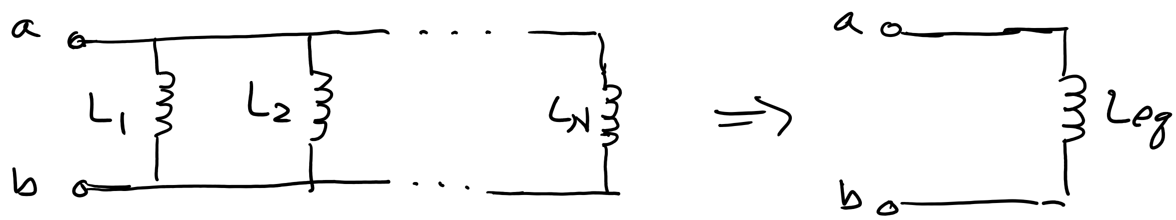
Typical inductors range from ~μH to ~10H.

Inductors in series: use KVL,



$$L_{eq} = L_1 + L_2 + \cdots + L_N$$

Inductors in parallel: Use KCL,



$$L_{eq} = \left[\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right]^{-1}$$

Combine like resisters.

Energy storage:

Apply $p(t) = v(t)\, i(t)$

$$e(t) = w(t) = \int_0^t \left[L \frac{di}{dx}\right] i(x)\, dx$$

$$= L \int_0^t i(x) \frac{di}{dx}\, dx$$

$$= \frac{1}{2} L i^2(t)$$