# 1

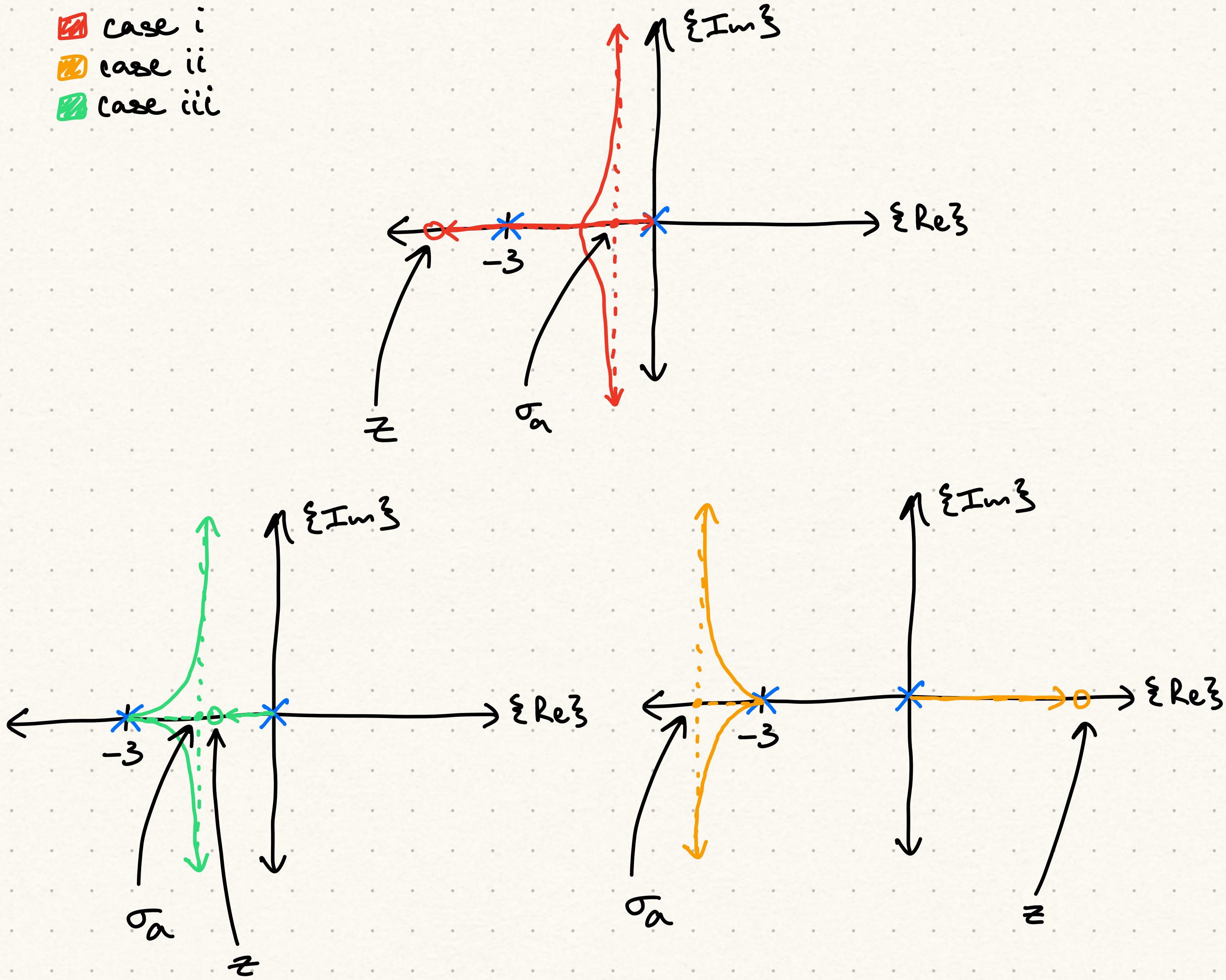
$$G(s) = \frac{6}{(s+3)^2}, \quad H(s) = K_p + \frac{K_i}{s}$$
$$= K_p\left(\frac{s + \frac{K_i}{K_p}}{s}\right)$$

a) i: $(-\infty, -3)$
ii: $(-3, 0)$
iii: $(0, +\infty)$

$$L(s) = K\left(\frac{s+z}{(s+3)^2 s}\right), \quad K = 6K_p, \quad z = \frac{K_i}{K_p}$$
$$D(s) + KN(s) = 0$$
$$s^3 + 6s^2 + 9s + Ks + Kz = 0$$

$$n = 3,\ m = 1 \Rightarrow \sigma_a = \frac{-3-3+0-\left(-\frac{K_i}{K_p}\right)}{2} = -3 + \frac{K_i}{2K_p}$$
$$\alpha_l = \pm 90^\circ$$

OL poles
case i
case ii
case iii



b) case iii is unstable, as it mandates a singular RHP zero and its LHP branch is unstable

c) Want: $\alpha < 0$

$$\alpha = \frac{-6+z}{2} \Rightarrow z < 6$$

$z > 0$ to avoid case iii

↳ $0 < z < 6$

↳ $\boxed{\text{zero must lie between } (-6, 0)}$

Need: $\measuredangle L(s) \neq -180^\circ$

Let: $\measuredangle L(s) = 180^\circ$, $\operatorname{atan}\left(\frac{w}{z}\right) - \frac{\pi}{2} - 2\operatorname{atan}\left(\frac{w}{3}\right) = -\pi$

$$-\frac{\pi}{2} = \operatorname{atan}\left(\frac{w}{z}\right) - 2\operatorname{atan}\left(\frac{w}{3}\right)$$

$$\Rightarrow \frac{w}{z} = \frac{9 - w^2}{-6w} \Rightarrow w^2 = \underbrace{-\frac{9z}{6-z} > 0}_{\text{want this to be false}}$$

↳ $0 < z < 6$

↳ zero must lie between $(-6, 0)$

d) $\zeta = \frac{1}{\sqrt{2}}$ , cancel a pole: $z = 3$

$\omega_n = \frac{z}{2\zeta}$ , $K = \omega_n^2 / 6$

$K_p = K \rightarrow \boxed{K_p = 0.75}$

$K_i = K_p z \rightarrow \boxed{K_i = 2.25}$

## 2

$$G(s) = \frac{5(s-1)}{s-6}$$

a) $H(s) = \frac{K}{s-p}$, with $p > 0$

$$L(s) = \frac{5K(s-1)}{(s-6)(s-p)} \Rightarrow 1 + L(s) = 0$$

$$(s-p)(s-6) + 5K(s-1) = 0 \Rightarrow s^2 + (-p-6)s + 6p + 5Ks - 5K = 0$$

$\hookrightarrow$ $5K - (p+6) > 0$ & $6p - 5K > 0$

$$\frac{p+6}{5} < K < \frac{6}{5}p \Rightarrow \boxed{p > \frac{6}{5}}$$

b) Want: 2 real poles @ $-2$

CL: $s^2 + (5K - (p+6))s + (6p - 5K)$

$T_{des}$: $(s+2)^2 = s^2 + 4s + 4$

$$\left[\begin{array}{ccc|c} 1, & 5K-(p+6), & 6p-5K & 0 \\ 1, & 4, & 4 & 0 \end{array}\right]$$

$$p = \frac{5K+4}{6}$$

$$5K - \left(\frac{5K+4}{6} + 6\right) = 4$$

$$5K = 10 + \frac{5K+4}{6}$$

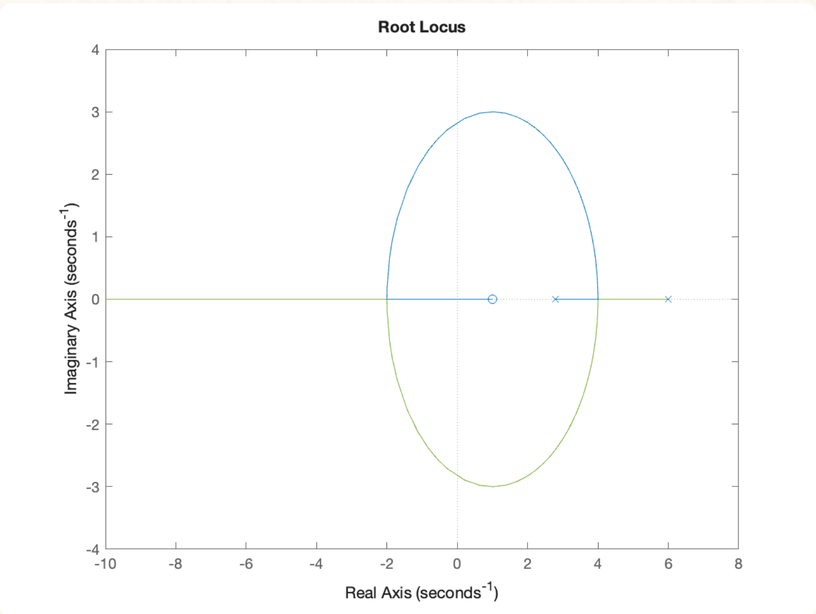
$$30K = 60 + 5K + 4$$

$$25K = 64$$

$$K = \left(\frac{8}{5}\right)^2 = \frac{64}{25}$$

$$p = \frac{\frac{64}{5} + 4}{6} = \frac{84}{30} = \frac{14}{5}$$

$$\boxed{H(s) = \frac{64}{25}\left(\frac{1}{s - \frac{14}{5}}\right)}$$

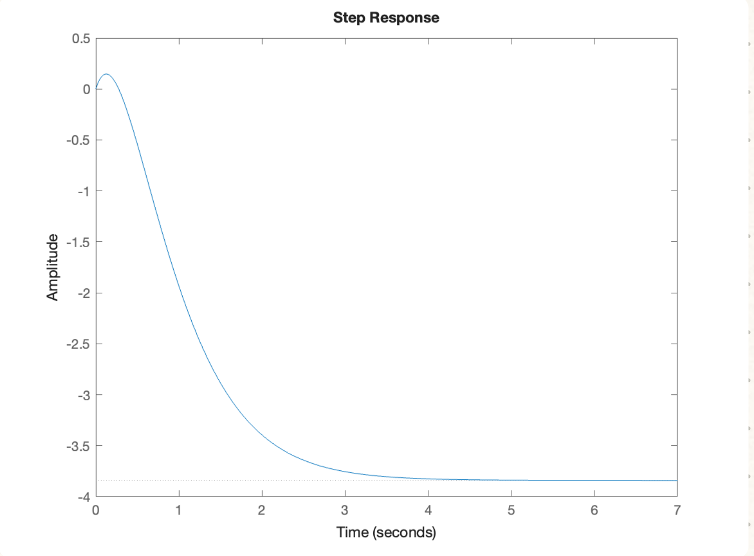


c) $y_d(t) = \mathbb{1}(t)$, $R(s) = \dfrac{H(s)}{1+L(s)}$, $U(s) = R(s)\,Y_d(s)$

$$Y_d(s) = \frac{1}{s} \Rightarrow U(s) = \frac{R(s)}{s} = \frac{H(s)}{s(1+L(s))} = \frac{2.56(s-6)}{s(s+2)^2}$$

$$\boxed{u(t) = \frac{96}{25}\left(e^{-2t} + \frac{8}{3}te^{-2t} - 1\right)}$$

$u(t)$ is bounded, as all poles of $R(s)$ are in LHP




```
R_info = struct with fields:
        RiseTime: 1.6515
   TransientTime: 3.0421
    SettlingTime: 3.0640
     SettlingMin: -3.8400
     SettlingMax: -3.4571
       Overshoot: 0
      Undershoot: 3.8196
            Peak: 3.8400
        PeakTime: 7.8058
```

d)

```
H_zoh = struct with fields:
    Ad: 1.1185
    Bd: 0.0847
    Cd: 1.2800
    Dd: 0
H_tustin = struct with fields:
    Ad: 1.1186
    Bd: 0.3639
    Cd: 0.3158
    Dd: 0.0542
```

$\underline{4}$ $G(s) = \frac{2}{s^2(s^2+3)}$

a) $H(s) = K$ ?

$L(s) = \frac{2K}{s^2(s^2+3)}$ , $L'(s) = [(s-p)L(s)]$

$n - m = 4 \Rightarrow \alpha_\ell = \pm 45°, \pm 135°$ & $\sigma_a = 0$

$\measuredangle L(s) = (1+2\ell)180° = \measuredangle L'(s) - \measuredangle(s-p)$

$\delta = \measuredangle L'(p) - (1+2\ell)180°$

$\measuredangle L'(s)\Big|_{\varepsilon\text{-circle}} = \measuredangle L'(p)$

for pole at $s = +\sqrt{3}j$:

$L'(s) = \frac{2K}{s^2(s+\sqrt{3}j)} \Rightarrow \measuredangle L'(p) = -180° - 90°$

$\delta = -90°$

by symmetry, for $s = -\sqrt{3}j$: $\delta = 90°$

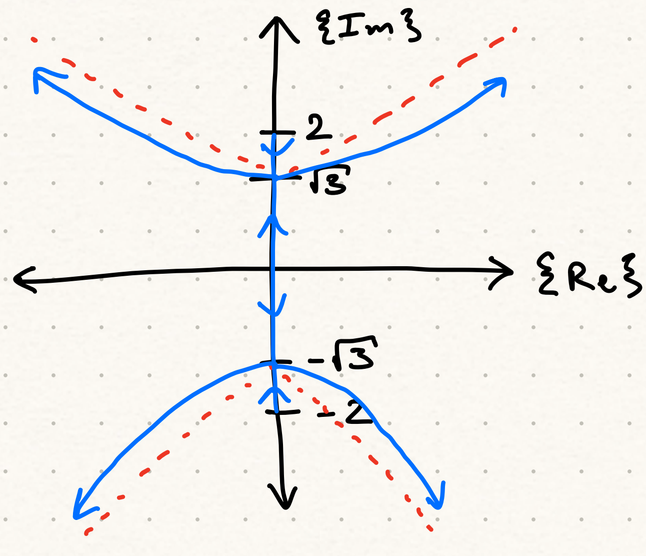
for pole at $s = 0$ (repeated):

$L'(s) = \frac{2K}{s^2+3} \Rightarrow \measuredangle L'(p) = -180°$

$\delta = \pm 180°$

$\hookrightarrow \delta = \pm 90°$

$\boxed{\text{Will be unstable at all poles for sufficiently large } K}$



b) Want: CL @ $-1 \pm j$ & all else CL @ $-4$

$$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

find: $zpk(H(s))$
& $zpk(T(s))$

Char Polys

CL: $(s^3 + a_2 s^2 + a_1 s + a_0)(s^2(s^2+3)) + (b_3 s^3 + b_2 s^2 + b_1 s + b_0)(2)$

$T_{des}$: $(s-(-1+j))(s-(-1-j))(s+4)^5$

$\hookrightarrow$ $a_2 = 22$ $\quad b_3 = 1141.5$
$a_1 = 199$ $\quad b_2 = 1031$
$a_0 = 934$ $\quad b_1 = 2304$
$b_0 = 1024$

```
H =

  1141.5 (s+0.4939) (s^2 + 0.4093s + 1.816)
  -----------------------------------------
      (s+11.86) (s^2 + 10.14s + 78.74)

Continuous-time zero/pole/gain model.
Model Properties

T =

  2283 (s+0.4939) (s^2 + 0.4093s + 1.816)
  ---------------------------------------
          (s+4)^5 (s^2 + 2s + 2)

Continuous-time zero/pole/gain model.
Model Properties
poles_T = 7x1 complex
   -4.0075 + 0.0000i
   -4.0023 + 0.0071i
   -4.0023 - 0.0071i
   -3.9939 + 0.0044i
   -3.9939 - 0.0044i
   -1.0000 + 1.0000i
   -1.0000 - 1.0000i

zeros_T = 3x1 complex
   -0.2046 + 1.3321i
   -0.2046 - 1.3321i
   -0.4939 + 0.0000i
```

c) $$H(s) = \frac{b_3 s^3 + b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

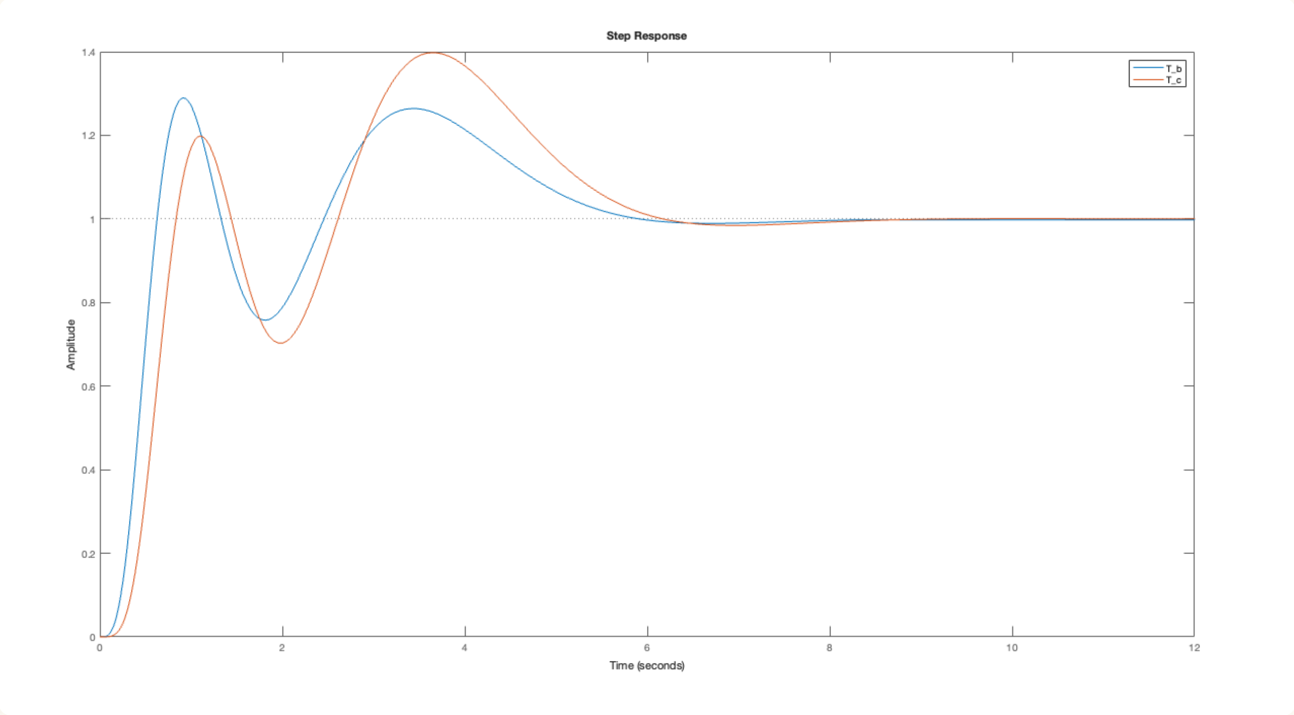
Char Polys

CL: $(s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0)(s^2(s^2+3)) + (b_3 s^3 + b_2 s^2 + b_1 s + b_0)(2)$

$T_{des}$: $(s-(-1+j))(s-(-1-j))(s+4)^6$

```
H =

     5597 (s+0.4116) (s^2 + 0.125s + 1.778)
  ---------------------------------------------
  (s^2 + 20.06s + 115.9) (s^2 + 5.939s + 51.91)

Continuous-time zero/pole/gain model.
Model Properties

T =

  11194 (s+0.4116) (s^2 + 0.125s + 1.778)
  ---------------------------------------
          (s+4)^6 (s^2 + 2s + 2)

Continuous-time zero/pole/gain model.
Model Properties
poles_T = 8x1 complex
   -4.0247 + 0.0000i
   -4.0123 + 0.0214i
   -4.0123 - 0.0214i
   -3.9877 + 0.0213i
   -3.9877 - 0.0213i
   -3.9753 + 0.0000i
   -1.0000 + 1.0000i
   -1.0000 - 1.0000i

zeros_T = 3x1 complex
   -0.0625 + 1.3320i
   -0.0625 - 1.3320i
   -0.4116 + 0.0000i
```

d) Neither controllers are good, as they both have about (or over) 5.5s settling times, and significant overshoot. This is due to the introduction of extra zeroes and their compensatory poles "spoiling" our intended behavior of the designed poles




e) Otherwise, the benefit of C is that it has an extra pole, ensuring it has noise stability, which B does not