ENAE 311H: Homework 02

Due on September 27, 2024 at 05:00 PM

Dr. Brehm, Section 0101

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Problem 1:

Given:

$$\begin{aligned} h &= 8 \times 10^4\,\text{ft} \\ M_{\infty,\text{SR-71}} &= 3.6 \\ T_{\text{alt}=h} &= 221\,\text{K} \\ \rho_{\text{alt}=h} &= 0.044\,\frac{\text{kg}}{\text{m}^3} \\ V_{\infty,\text{model}} &= 790.5\,\frac{\text{m}}{\text{s}} \\ T_{\infty,\text{model}} &= 120\,\text{K} \\ \rho_{\infty,\text{model}} &= 0.2\,\frac{\text{kg}}{\text{m}^3} \end{aligned}$$

Assume:

$$\begin{aligned} a &= \sqrt{\gamma R T} \\ \gamma &= 1.4 \\ R &= 287\,\frac{\text{J}}{\text{kg}\,\text{K}} \\ \mu &\propto T^{\frac{2}{3}} \end{aligned}$$

Find:

Are the two flows dynamically similar?

Solution

$$\begin{aligned} V &= M \times a \\ M_\infty &= \frac{V_\infty}{a_\infty} \\ Re &= \frac{\rho V L}{\mu} \end{aligned}$$

$$\begin{aligned} a_{\infty,\text{SR-71}} &= a_{\text{alt}=h} \\ a_{\text{alt}=h} &= \sqrt{1.4 \times 287 \times 221} \\ a_{\infty,\text{model}} &= \sqrt{1.4 \times 287 \times 120} \end{aligned}$$

$$\begin{aligned} a_{\infty,\text{SR-71}} &= 297.99\,\frac{\text{m}}{\text{s}} \\ a_{\infty,\text{model}} &= 219.58\,\frac{\text{m}}{\text{s}} \end{aligned}$$

$$\begin{aligned} V_{\infty,\text{SR-71}} &= 1072.76\,\frac{\text{m}}{\text{s}} \\ V_{\infty,\text{model}} &= 790.5\,\frac{\text{m}}{\text{s}} \end{aligned}$$

Problem 1 continued on next page. . .

$$\begin{aligned} M_{\infty,\text{SR-71}} &= 3.6 \\ M_{\infty,\text{model}} &= 3.60 \\ M_{\infty,\text{SR-71}} = M_{\infty,\text{model}} &\quad \square \end{aligned}$$

$$\begin{aligned} Re_{\text{SR-71}} &= 0.86 \\ Re_{\text{model}} &= 0.65 \\ \mu &= 1.50 \neq 1 \\ Re_{\text{SR-71}} \neq Re_{\text{model}} &\quad \square \end{aligned}$$

Since the Mach numbers in the SR-71 and the wind tunnel are equal, **the flows are Mach-similar**. Since the Reynolds numbers in the SR-71 and the wind tunnel are not equal, **the flows are not dynamically similar**.

Problem 2:

Given:

$$
\begin{aligned}
V_{\infty,\text{Terrier}} &= 400\,\frac{\text{m}}{\text{s}} \\
h &= 40\,000\,\text{ft} \\
T_{\text{alt}=h} &= 217\,\text{K} \\
\rho_{\text{alt}=h} &= 0.30\,\frac{\text{kg}}{\text{m}^3} \\
\text{scale factor: } s &= 5 \\
p_{0,\text{model}} &= 75\,\text{kPa}
\end{aligned}
$$

Assume:

$$
\begin{aligned}
a &\propto \sqrt{T} \\
\mu &\propto T^{\frac{2}{3}}
\end{aligned}
$$

Find:

$$
\begin{gathered}
V_{\infty,\text{model}}, T_{\infty,\text{model}}, \rho_{\infty,\text{model}}, \\
\text{Such that the flows are dynamically similar}
\end{gathered}
$$

Solution

$$
\begin{aligned}
Re = \frac{s\rho_T V_{\infty,T}}{\mu_T} &= \frac{\rho_m V_{\infty,m}}{\mu_m} \\
M = \frac{V_{\infty,T}}{a_T} &= \frac{V_{\infty,m}}{a_m} \\
\implies \frac{V_{\infty,T}}{V_{\infty,m}} = \frac{a_T}{a_m} &= \frac{T_T^{\frac{1}{2}}}{T_m^{\frac{1}{2}}} \\
\frac{\mu_T}{\mu_m} &= \frac{T_T^{\frac{2}{3}}}{T_m^{\frac{2}{3}}} \\
5\,\frac{\rho_T}{\rho_m} &= \frac{T_T^{\frac{2}{3}+\frac{1}{2}}}{T_m^{\frac{2}{3}+\frac{1}{2}}} \\
125\,\frac{\rho_T^3}{\rho_m^3} &= \sqrt{\frac{T_T}{T_m}} \\
\rho &= \frac{p}{RT} \\
T_m^{\frac{7}{2}} &= T_T^{\frac{1}{2}}\left(\frac{p_m}{5R\rho_T}\right)^3 \\
T_m &= 179.7\,\text{K} \quad \square \\
\rho_m &= 1.45\,\frac{\text{kg}}{\text{m}^3} \quad \square \\
V_{\infty,m} &= V_{\infty,T}\left(\frac{T_m}{T_T}\right)^{\frac{1}{2}}
\end{aligned}
$$

Problem 2 continued on next page. . .

$$V_{\infty,m} = 364.1\,\frac{\mathrm{m}}{\mathrm{s}} \quad \square$$