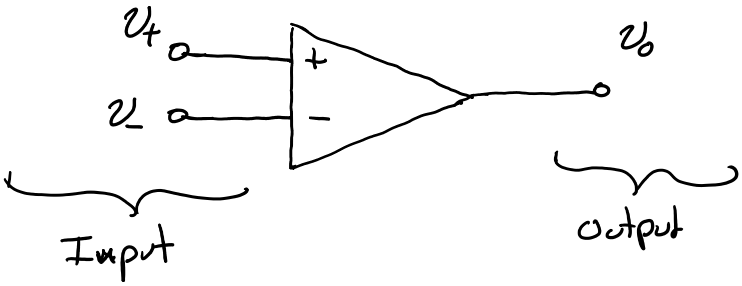
Lecture 10 and 11

Monday, September 30, 2024 12:57 PM

Operational Amplifiers

- Multipurpose component used to construct other analog circuit components.
- It can be used to create an amplifier, but it is not limited to this. May be used to create:
  - Addition, substraction, multiplication
  - Compute integrals
  - Signal inversion

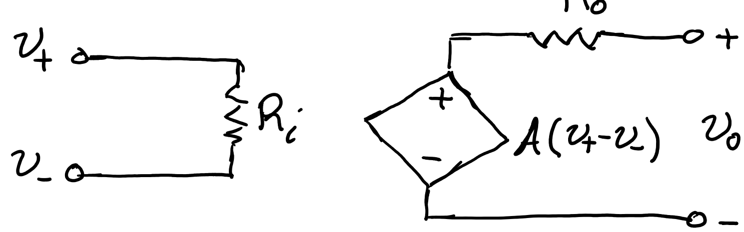
Input-Output behavior:



v_-: Inverting input

v_+: Non-inverting input

Equivalent circuit



A: Gain (typically very large)

R_i: Typically very large to limit current draw

to limit current
draw

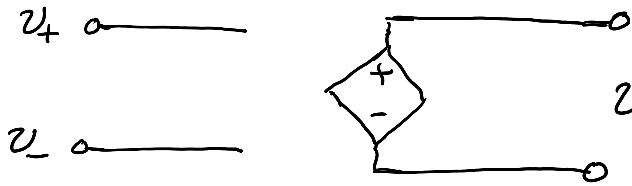
R_0: Typically as small as possible

Ideal Op Amp:

$A \to \infty$

$R_i \to \infty$

$R_o = 0$

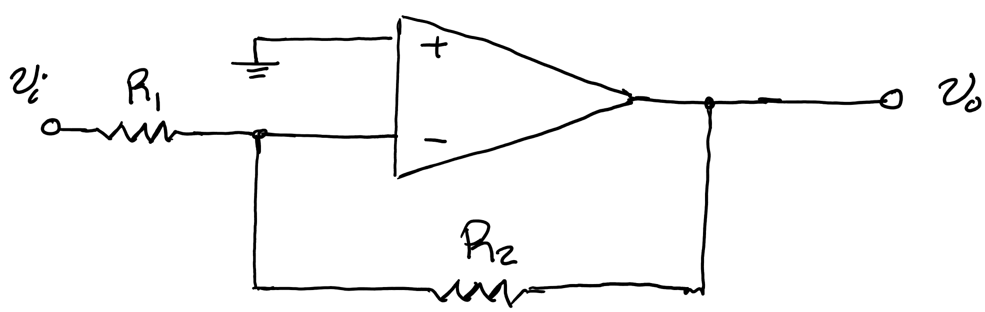


$v_o = A(v_+ - v_-)$

How can such a device be useful?

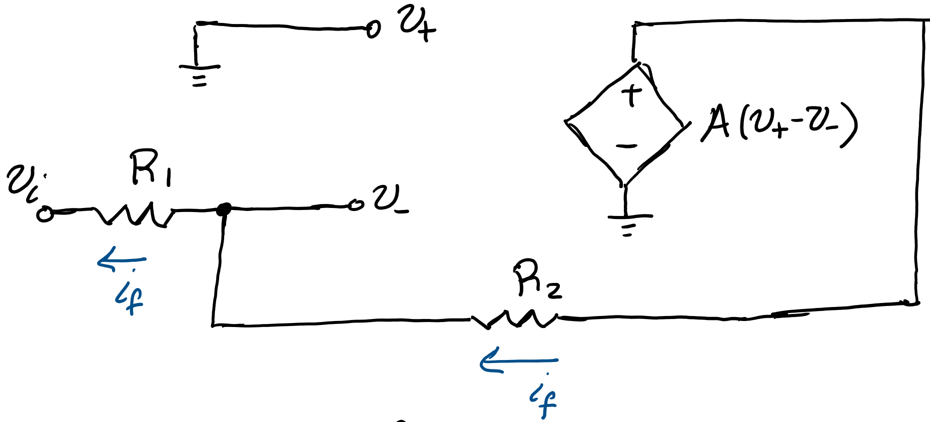
- It's never used by itself.
- Used in conjuction with other circuit elements to create a desired input-output behavior.

<u>Example</u>: Inverting amplifier



$v_o = \lim_{A \to \infty} A(v_+ - v_-)$

Equivalent Circuit (power source omitted)



$$v_o = A(v_+ - v_-)$$
$$= -A v_-$$

Let's try to find

$$A_F := \frac{v_o}{v_i} \qquad \text{Voltage gain of circuit}$$

$$v_o - v_i = (R_1 + R_2) i_f \implies i_f = \frac{v_o - v_i}{R_1 + R_2}$$

Also have

$$v_- - v_i = R_1 i_f \implies v_- = v_i + R_1 i_f$$

Substitute

$$v_- = v_i + R_1 \left[\frac{v_o - v_i}{R_1 + R_2} \right]$$

Since v_+ is grounded

$$v_o = A(\cancelto{0}{v_+} - v_-) = -A v_-$$

Substitute

$$v_o = -A \left[v_i + R_1 \left(\frac{v_o - v_i}{R_1 + R_2} \right) \right]$$

Solve for v_o / v_i,

$$\frac{v_o}{v_i} = -A \left[1 + R_1 \left(\frac{v_o/v_i - 1}{R_1 + R_2} \right) \right]$$

$$\implies A_F = \frac{v_o}{v_i} = \frac{\frac{R_1}{R_1+R_2} - 1}{1 \qquad R_1}$$

$$\Rightarrow A_F = \frac{\;}{v_i} = \frac{\frac{\;}{R_1+R_2} - 1}{\frac{1}{A} + \frac{R_1}{R_1+R_2}}$$

goes to 0 as $A \to \infty$

Let $A \to \infty$

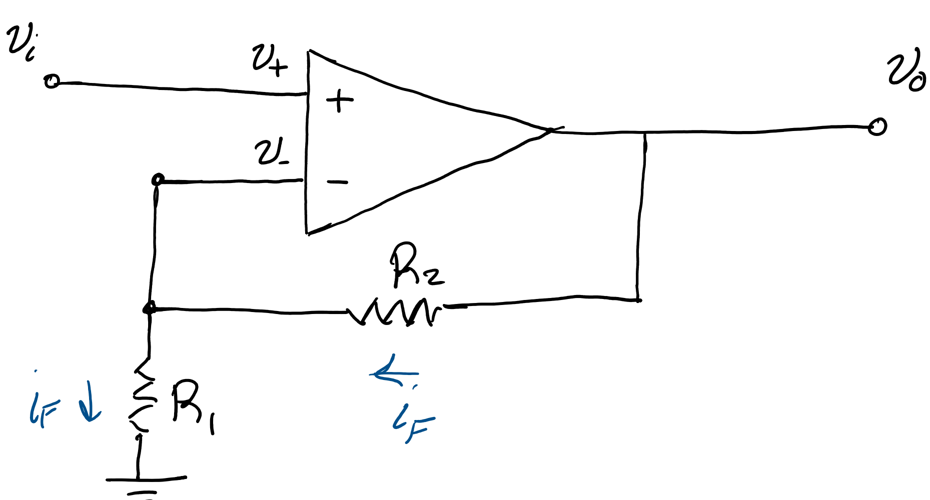
$$A_F = -\frac{R_2}{R_1}$$

Remarks:

- Gain A_F is equal to ratio of two resistances with a sign flip.
- If R_1 and R_2 have similar construction, then R_2/R_1 will be insensitive of temperature.

* - Effect of negative feedback loop is to drive v_- to v_+ (in this case $v_+ = 0$).

Example: Non-inverting amplifier



$$v_o = \lim_{A\to\infty} A(v_+ - v_-)$$

What is the effective gain for this circuit?

What is the effective gain for this circuit?

$$A_F = \frac{v_o}{v_i} \quad ?$$

Assuming ideal Op Amp:

$v_- = v_+$ due to negative feedback loop

No current flow between v_+ and v_-

$$v_o - 0 = (R_1 + R_2)\, i_F$$

$$v_- - 0 = R_1\, i_F$$

$v_- = v_+ = v_i$ because of negative feedback loop

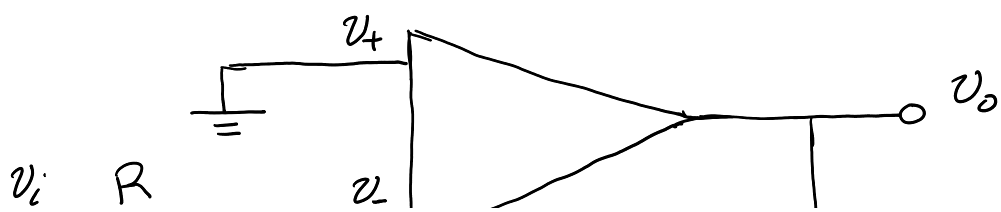
$$\therefore \quad A_F = \frac{v_o}{v_i} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

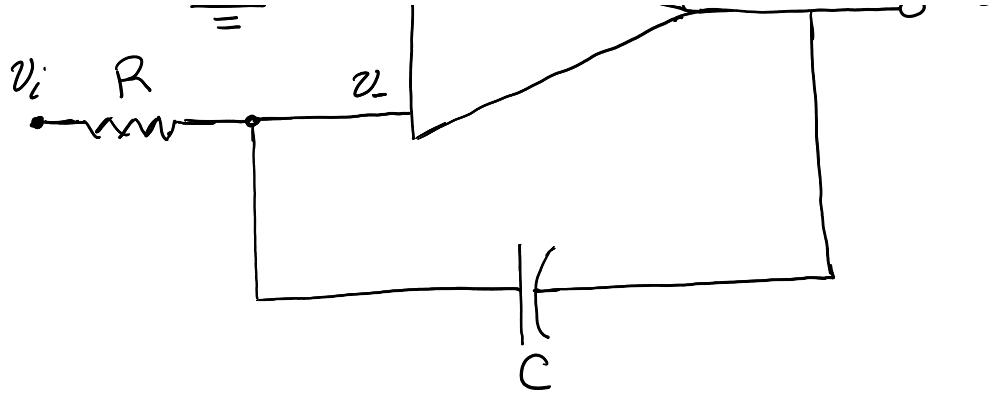
$$\boxed{A_F = 1 + \frac{R_2}{R_1}} \qquad A_F \geq 1$$

Remarks:

- Gain depends on R_2/R_1
- $A_F \geq 1$

Example: An integrating circuit





$$i_F = C\frac{dv_o}{dt}$$

$$i_i = \frac{v_i}{R}$$

Apply KCL

$$i_F + i_i = 0$$

$$\Rightarrow \quad C\frac{dv_o}{dt} + \frac{v_i}{R} = 0$$

Solve for v_o by integrating

$$C\int_0^t \frac{dv_o}{d\tau}\,d\tau + \int_0^t \frac{1}{R} v_i(\tau)\,d\tau = 0$$

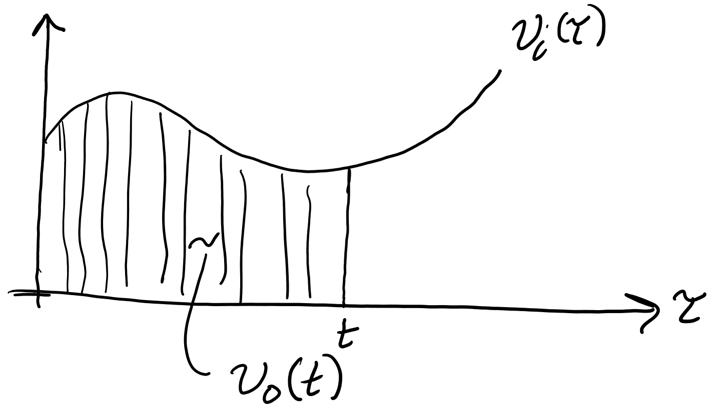
dummy variable
of integration

$$\Rightarrow \quad C\left[v_o(t) - v_o(0)\right] + \frac{1}{R}\int_0^t v_i(\tau)\,d\tau = 0$$

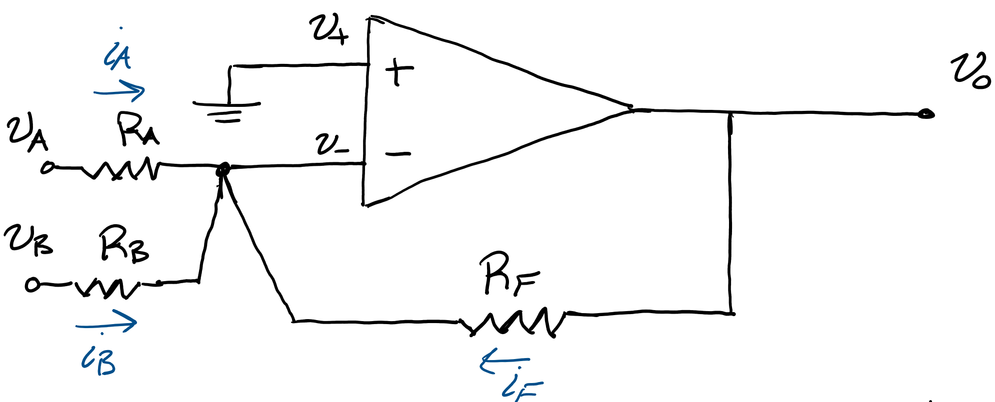
$$\Rightarrow \quad v_o(t) = v_o(0) - \frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$

If $v_o(0) = 0$,

$$\boxed{v_o(t) = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau}$$



Example: Summing Circuit



Produce an output voltage signal that is a weighted sum of input voltage signals v_A and v_B

Analysis:

Assume ideal Op Amp: v_- is held at v_+ due to negative feedback loop.

$$i_A = \frac{v_A - 0}{R_A} = \frac{v_A}{R_A}$$

$$i_B = \frac{v_B - 0}{R_B} = \frac{v_B}{R_B}$$

$$i_F = \frac{v_o - 0}{R_F} = \frac{v_o}{R_F}$$

$$i_F = \frac{v_o - 0}{R_F} = \frac{v_o}{R_F}$$

By KCL,

$$i_A + i_B + i_F = 0$$

$$\Rightarrow \quad \frac{v_A}{R_A} + \frac{v_B}{R_B} + \frac{v_o}{R_F} = 0$$

$$\Rightarrow \quad v_o = -R_F\left[\frac{v_A}{R_A} + \frac{v_B}{R_B}\right]$$

$$\Rightarrow \quad \boxed{v_o = -\left[\frac{R_F}{R_A}v_A + \frac{R_F}{R_B}v_B\right]}$$