Lecture 3

Trajectory Eqn: $r = \dfrac{p}{1+e\cos\nu}$

p = semi-latus rectum = h^2/μ

e = eccentricity = "how un-circular is the conic section"

$e=0$ = circular orbit

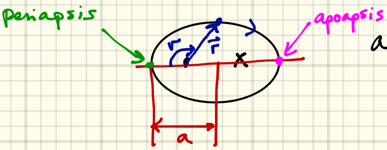
$0<e<1$ = ellipse

$e=1$: parabola

$e>1$: hyperbola

ν = true anomaly = the angle from periapsis to the s/c's location:

periapsis = the point on the orbit that is closest to the central body.



a = semi-major axis

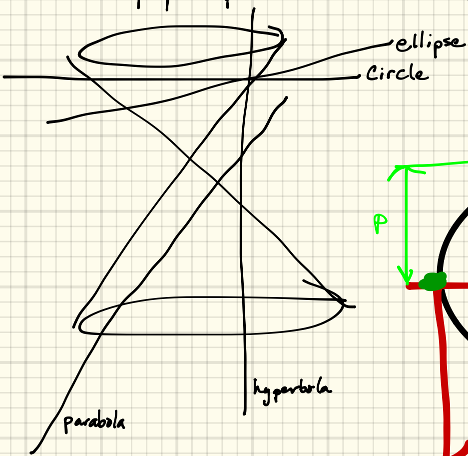
perigee = point closest to Earth

perijove = " " Jupiter

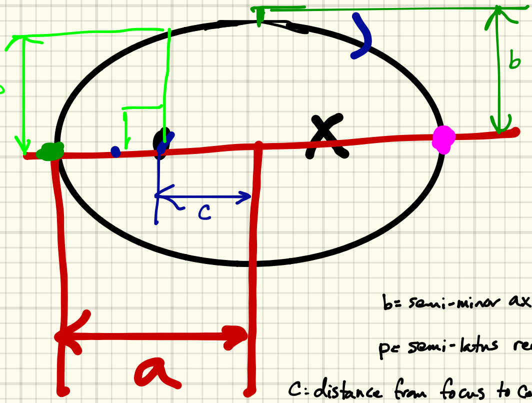
perihelion = " Sun

periwinkle = close to purple

apoapsis = point on the orbit that is farthest from the focus



Degenerate Conics: line, point



b = semi-minor axis

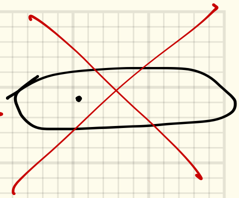
p = semi-latus rectum

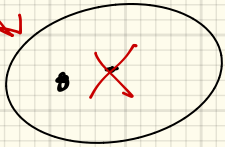
c: distance from focus to center

$e = \dfrac{c}{a}$

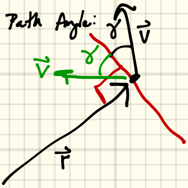
Characteristics of a good ellipse:

1. periapsis should be the point closest to the central body
2. The planet should be at a focus, not the center
3. The flight path angle at periapsis & apoapsis is zero.





Flight Path Angle: γ



— : $\gamma < 0$, $\dot{r} < 0$

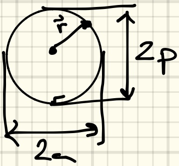
— : $\gamma > 0$, $\dot{r} > 0$

$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r}$, we now have an eqn for r.

For each location on a given orbit (a, e, etc), there is only 1 possible velocity magnitude. (v)

- $\mathcal{E} < 0$ for circles & ellipses
- $\mathcal{E} = 0$ for parabola
- $\mathcal{E} > 0$ for a hyperbolic orbit

Circle:



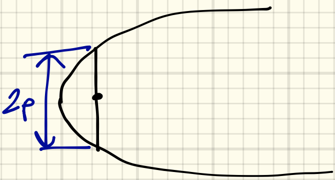
$a = r$

$e = 0$

periapsis & apoapsis are undefined.

velocity is constant

Parabola:



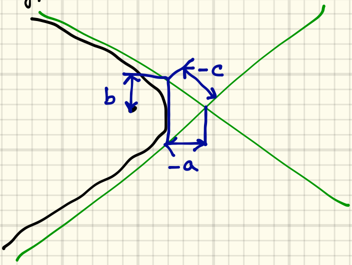
$a = \infty$

$c = \infty$

$e = 1$

Parabola has a velocity of 0 @ $r = \infty$

Hyperbola:



$e > 1$

$a < 0$

$V @ r = \infty > 0$

For all conics (except parabola):

$$p = a(1-e^2)$$

Radius of periapsis:

$$r_p = \frac{p}{1+e\cos\nu} = \frac{p}{1+e\cos(0)} = \frac{p}{1+e} = \frac{a(1-e^2)}{1+e}$$

$$\Rightarrow \boxed{r_p = a(1-e)}$$

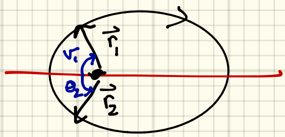
Radius of Apoapsis (same procedure):

$$\nu_a = 180°$$

$$r_a = \frac{p}{1-e} \Rightarrow \boxed{r_a = a(1+e)}$$

If $0 < \nu < 180°$, $\gamma > 0$

elseif $180 < \nu < 360°$, $\gamma < 0$



$\theta_2 = \nu_1$

$\gamma_1 = -\gamma_2$

Another expression for energy:

$$\vec{h} = \vec{r} \times \vec{v}$$

What is the expression for $|\vec{h}|$ at periapsis: $\gamma @ \nu = 0$ is 0.

$$h = rv\cos\gamma = r_p v_p$$

$$\boxed{h = \sqrt{\mu p}}$$

$$\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = \frac{v_p^2}{2} - \frac{\mu}{r_p}$$

$$r_p = a(1-e)$$

$$h^2 = r_p^2 v_p^2 \Rightarrow v_p^2 = h^2 / r_p^2$$

$$\mathcal{E} = \frac{h^2}{2r_p^2} - \frac{\mu}{a(1-e)}$$

$$\mathcal{E} = \frac{\mu a(1-e^2)}{2a^2(1-e)^2} - \frac{\mu}{a(1-e)} = \frac{\mu}{2a}\left[\frac{-e^2+2e-1}{e^2-2e+1}\right]$$

$$\Rightarrow \boxed{\mathcal{E} = -\frac{\mu}{2a}}$$

$$\boxed{\mathcal{E} = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}}$$