Lecture 15 - Lambert's Prob

TOF-related Problems:

1. Calculate the TOF between 2 points on a known orbit

2. Kepler's Pred Problem: Given $\vec{r}_1$ & $\vec{v}_1$ find $\vec{r}_2, \vec{v}_2$ @ some later time

3. Given 2 known positions ($\vec{r}_1, \vec{r}_2$) and a time of flight, find the orbit that links these positions. Lambert's Problem = Gauss' Problem

How should you check to see if your Kepler's Pred Problem code is working?

Use your 2BP integrator: IC's $\vec{r}_1, \vec{v}_1$ & integrate for the desired TOF.

Kepler's Pred Problem can be less computationally expensive than the numerical integrator

Lambert's Problem: Given $\vec{r}_1$ & $\vec{r}_2$ & TOF → what is the orbit that fits?

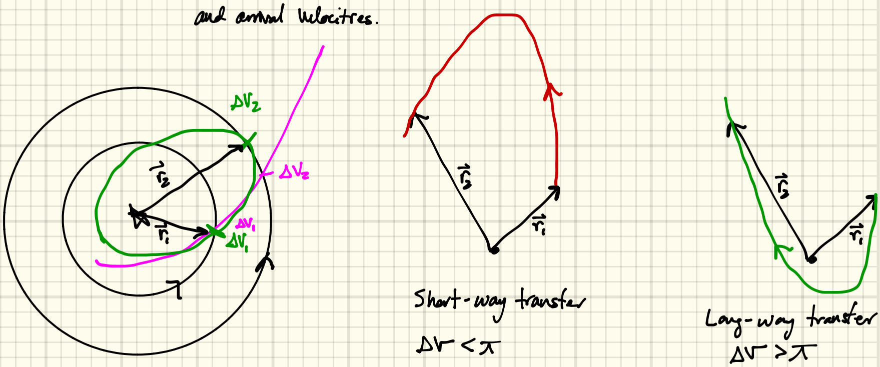
If we know $\vec{r}_1, \vec{v}_1 \Rightarrow$ OE

Motivated by desire to predict the orbit of Ceres

Modern Uses:

1. Orbit Determination: Given observations of a satellite, figure out what orbit it's on.

2. Trajectory Design: Given departure & arrival positions & a TOF, calculate the departure and arrival velocities.



Short-way transfer
$\Delta\nu < \pi$

Long-way transfer
$\Delta\nu > \pi$

Just given $\vec{r}_1$ & $\vec{r}_2$, there are an infinite number of possible transfers.

Given $\vec{r}_1, \vec{r}_2$ & TOF, there are 2 possible transfers (short-way & long-way).

Lambert's Problem: Bi-section Method
Pseudo-Code Algorithm:

Input: $\vec{r}_0, \vec{r}_f$, TOF, DM $\rightarrow$ DM = direction of motion

DM = 1 for short-way
DM = -1 for long way

Output: $\vec{v}_0, \vec{v}_f$

$$\cos\Delta\nu = \frac{\vec{r}_0 \cdot \vec{r}_f}{r_0 r_f}$$

$$A = DM\sqrt{r_0 r_f (1+\cos\Delta\nu)}$$

If $\Delta\nu = 0$, $A = 0 \Rightarrow$ Error

Else:

$\psi = 0$

$C_2 = \frac{1}{2}$, $C_3 = \frac{1}{6}$ $\quad \begin{bmatrix} C_2 = C \\ C_3 = S \end{bmatrix}$

$\psi_{up} = 4\pi^2$

$\psi_{low} = -4\pi$

while $|TOF - \Delta t| > 1\times10^{-6}$

$$y = r_0 + r_f + \frac{A(\psi C_3 - 1)}{\sqrt{C_2}}$$

If $A>0$ & $y<0$
increase ψ_{low}
end

$$\chi = \sqrt{\frac{y}{C_2}}$$

$$\Delta t = \frac{\chi^3 C_3 + A\sqrt{y}}{\sqrt{\mu}}$$

If $\Delta t < TOF$
$\psi_{low} = \psi$

else $\psi_{up} = \psi$
end

$$\psi_{new} = \frac{\psi_{up} + \psi_{low}}{2}$$

If $\psi > 1 \times 10^{-6}$

$$c_2 = \frac{1 - \cos\sqrt{\psi}}{\psi}$$

$$c_3 = \frac{\sqrt{\psi} - \sin\sqrt{\psi}}{\sqrt{\psi^3}}$$

else if $\psi < -1 \times 10^{-6}$

$$c_2 = \frac{1 - \cosh\sqrt{-\psi}}{\psi}$$

$$c_3 = \frac{\sinh\sqrt{-\psi} - \sqrt{-\psi}}{\sqrt{(-\psi)^3}}$$

else $c_2 = 1/2$, $c_3 = 1/6$

end

end (while)

Now Δt is within some tolerance of TOF

Calculate: $f = 1 - \frac{y}{r_0}$

$$g = A\sqrt{\frac{y}{\mu}}$$

$$\dot{g} = 1 - \frac{y}{r_f}$$

Output: $\vec{v}_0 = \frac{\vec{r}_f - f\vec{r}_0}{g}$

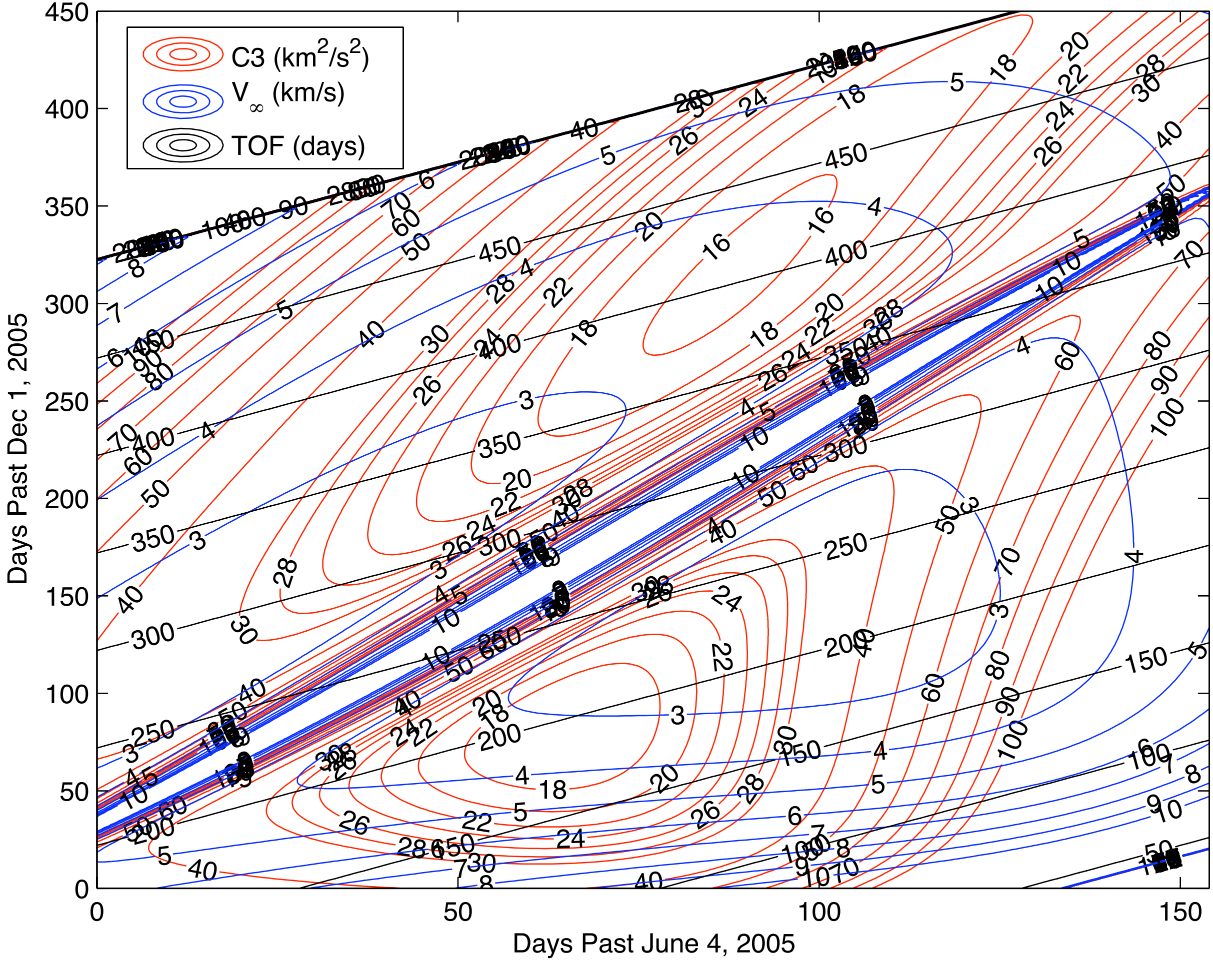
$$\vec{v}_f = \frac{\dot{g}\vec{r}_f - \vec{r}_0}{g}$$

How to check if Lambert's Prob code is working:

Given $\vec{r}_0$ & $\vec{r}_f$, TOF

From Lambert: $\vec{v}_0, \vec{v}_f$

Plug $\vec{r}_0, \vec{v}_0$ into your 2BP integrator → should get $\vec{r}_f, \vec{v}_f$


C3 (km²/s²)
V∞ (km/s)
TOF (days)
Days Past Dec 1, 2005
Days Past June 4, 2005

For Porkchop plot:

a series of launch dates $\Rightarrow \vec{r}_0$ (position of Earth at the launch date)

a series of arrival dates $\Rightarrow \vec{r}_f$ (position of Mars at the arrival date)

TOF = arrival date - launch (departure) date

$\Rightarrow$ Calculate $\vec{v}_0$ & $\vec{v}_f$ for each transfer using the Lambert solver

We also know the velocities of Earth & Mars

$$\vec{v}_{\infty E} = \vec{v}_0 - \vec{v}_E \qquad C3 = |\vec{v}_{\infty E}|^2$$

$$\vec{v}_{\infty M} = \vec{v}_f - \vec{v}_M \qquad v_{\infty} = |\vec{v}_{\infty M}|$$