ENAE311H Homework 5

Due: **Monday**, November 18th - **online** by 11:59pm

1. (a) Plot p_0/p, T_0/T, a_0/a, and a/a^* from Mach 0 to Mach 10.

   (b) Do the same for argon, which has a similar sound speed and molecular weight to air, but a ratio of specific heats of $\gamma = 5/3$, and for carbon dioxide ($\gamma = 1.28$)

   (c) What is the effect of changing the specific heat ratio of the gas?

   (d) When do compressibility effects become important for each gas?

2. In a thermally perfect, but not calorically perfect gas, the specific heats can no longer be considered constant. In this question we seek to quantify how this would affect the value of the calculated stagnation temperature for 300 K air flowing at 1500 m/s (about Mach 4.3).

   (a) Begin by showing that the energy equation in a one-dimensional inviscid adiabatic flow can be written in the differential form:
   $$c_p dT + u du = 0. \tag{1}$$

   (b) Now incorporate the effect of temperature variation by approximating the specific heat at constant pressure using the following power law in temperature:
   $$c_p = 996.96 + 0.0357T + 2 \times 10^{-4} T^2 - 8 \times 10^{-8} T^3, \tag{2}$$
   where T is in K and c_p is in J/kgK. Calculate the total temperature of air flowing at the above conditions with his expression for c_p (note that you will have to solve numerically or graphically).

   (c) How does your result in (b) differ from that if you were to assume a constant value of c_p (i.e., that appropriate for 300 K)?

   (d) In what kind of situations would be important to take into account the temperature variation of c_p?

3. A spherical model (5-cm diameter) is being tested in a hypersonic wind tunnel, where the free-stream conditions are V_∞=1200 m/s, T_∞=73 K and ρ_∞=0.05 kg/m^3.

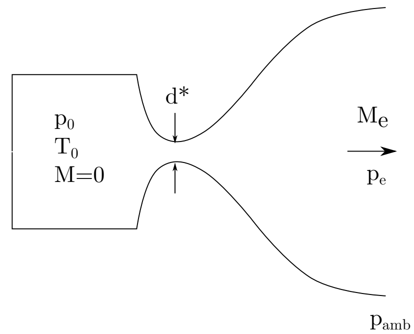
   (a) Calculate the pressure and temperature at the stagnation point of the model, and also the drag force acting on it (the drag coefficient for a sphere at high Mach numbers, based on the cross-sectional area, is 0.9). Compare the pressure at the stagnation point to the total pressure in the free-stream, and also calculate the entropy change through the shock along the stagnation streamline.

   (b) Calculate the pressure, temperature, density, sound speed and velocity at the throat of the wind tunnel, and also the ratio of areas of the nozzle exit to the throat.

4. (a) A rocket is operating at 50 km altitude, where the pressure is 76 Pa. It can be shown that rockets operate most efficiently when the exit static pressure is equal to the ambient pressure. The throat diameter of the rocket nozzle is d^*=2 cm and the exit diameter is d_e=1 m. Calculate the reservoir pressure, p_0, that will ensure optimal operation at this altitude. Assume γ=1.4.

   (b) If the ambient pressure is larger than the flow static pressure at the nozzle exit (in which case we say the nozzle is "overexpanded"; if the static pressure is larger than the ambient pressure, we say the nozzle is "underexpanded"), a shock (or shock system) will form at the nozzle exit to bring the flow pressure back up to ambient conditions. Determine the altitude at which a normal shock will form exactly at the nozzle exit, assuming the reservoir conditions are as you calculated in part (a). For atmospheric pressure, use:
   $$p = p_{ref} \exp(-z/H), \tag{3}$$
   where p_{ref} is the pressure at sea level (101 kPa) and H is the scale height (assume H=8 km).

(c) The combustion products in a rocket engine are typically more complex molecules than those found in air, and so $\gamma < 1.4$. Assuming again that the reservoir pressure is what you calculated in part (a) and the nozzle geometry is unchanged, will the nozzle at 50 km altitude be underexpanded or overexpanded with a more accurate value of γ (say, 1.2)?



5. A normal shock is propagating through a quiescent atmosphere (300 K, 101 kPa) at a Mach number M_s.

(a) Find an expression for the Mach number, M_2', behind the shock in a frame of reference fixed to the quiescent upstream flow in terms of only M_s and γ. Show that as $M_s \to \infty$, this post-shock Mach number tends to the value $\sqrt{\frac{2}{\gamma(\gamma-1)}}$. You may wish to start from the expression $M_2'=(V_s - u_2)/a_2$ rather than $M_2'=(V_s/a_2) - M_2$, as was given in class.

(b) Now consider a shock Mach number of $M_s=5$. A stationary blunt body is placed in the flow field behind the shock. Calculate the pressure and temperature at the stagnation point of this body. How would you expect this stagnation temperature to scale with M_s?