

Lecture 11: Review of Thermodynamics

ENAE311H Aerodynamics I

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Thermodynamic systems and state variables

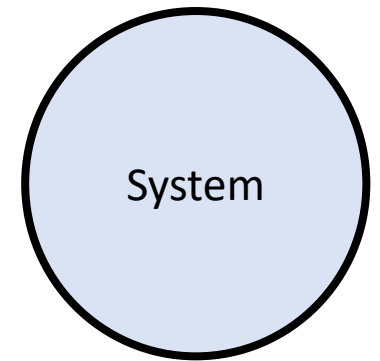
- A thermodynamic *system* is a quantity of matter separated from its surroundings by an enclosure (real or imagined).
- Classical thermodynamics deals with systems in equilibrium, i.e., exhibiting no spatial or temporal gradients (leave a system for long enough with surroundings at constant conditions, and it will reach such a state).
- Variables that depend only on the current state of the system (and not the process by which it got there) are called *variables of state*, e.g., $p, v = 1/\rho, T$.
- For a *simple* system, any two state variables are sufficient to fully specify the state of the system (and thus the value of any other state variable), so can write, for example:

$$p = p(v, T).$$

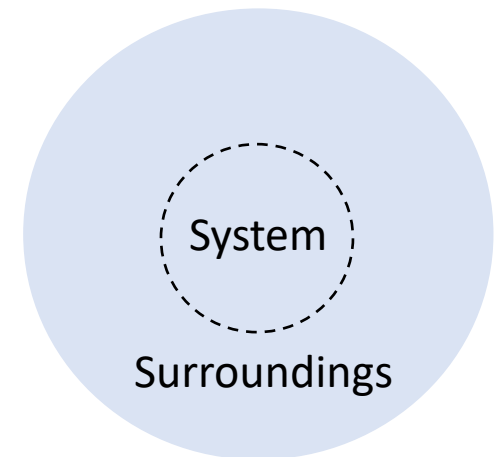
This is the “thermal equation of state”.

- For a *thermally perfect* gas the thermal equation of state takes the simple form

$$pv = RT, \quad \text{or} \quad p = \rho RT. \quad (\text{For air, } R=287 \text{ J/kg K})$$



Surroundings



Surroundings

Internal energy and specific heats

- The specific internal energy we denote by e – this in general includes translational, rotational, vibrational and electronic-excitation components.
- As e is a variable of state, we can write

$$e = e(T, v)$$

which for a thermally perfect gas becomes simply

$$e = e(T).$$

- We define the specific heat, $c = \delta q / dT$, i.e., the amount of heat required to raise the temperature by one unit. We will see shortly that the specific heat at constant volume relates differentials of T and e :

$$de = c_v dT.$$

- If the specific heats are constant (reasonable up to ~ 800 K), the gas is *calorically perfect* and

$$e = c_v T.$$

Enthalpy and specific heats

- We define an additional state variable, the enthalpy h , as

$$h = e + pv.$$

In this course we will assume a calorically perfect gas, and so we can write

$$h - e = (c_p - c_v)T.$$

- It is convenient to write

$$h = h(T, p),$$

which for a thermally perfect gas becomes

$$h = h(T).$$

- We then have

$$dh = c_p dT,$$

and for a calorically perfect gas

$$h = c_p T.$$

and thus

$$c_p - c_v = \frac{pv}{T} = R$$

We now define $\gamma = c_p/c_v$, and dividing above equation by either c_p or c_v , we find

$$\begin{aligned} c_p &= \frac{\gamma R}{\gamma - 1} \\ c_v &= \frac{R}{\gamma - 1}. \end{aligned}$$

For a calorically perfect (or just “perfect”) gas, γ is constant (for air, $\gamma=1.4$).

The first law of thermodynamics

Let δq be the heat flowing into a system from the surroundings and δw be the work done *by* the system (say, in expanding against a pressure force). Then the first law states

$$de = \delta q - \delta w.$$

Note that neither δq nor δw is a state variable as they depend on the process (not just the end state). We identify three particularly important types of processes:

1. **Adiabatic**: no heat is added to or taken away from the system ($\delta q = 0$)
2. **Reversible**: no dissipative phenomena (e.g., effects of viscosity, heat conduction, diffusion) take place in system
3. **Isentropic**: both adiabatic and reversible

The first law of thermodynamics

For a reversible process, $\delta w = p dv$, and the first law becomes

$$de = \delta q - p dv.$$

Since $e = e(T, v)$, however, we can write

$$de = \left. \frac{\partial e}{\partial T} \right|_v dT + \left. \frac{\partial e}{\partial v} \right|_T dv.$$

If the system undergoes a process at constant v , from the first law above

$$\left. \frac{\partial e}{\partial T} \right|_v = \left. \frac{\delta q}{dT} \right|_v = c_v.$$

For a thermally perfect gas, $e = e(T)$, and this becomes $de = c_v dT$, as before.

Similarly, using $dh = de + v dp + p dv = \delta q + v dp$ and $h = h(T, p)$, we recover $dh = c_p dT$.

The second law of thermodynamics

We define another variable of state, the entropy s , such that

$$ds = \left(\frac{\delta q}{T} \right)_{rev},$$

where *rev* indicates that the heat is being added reversibly.

For an irreversible (i.e., real) process

$$ds = \frac{\delta q}{T} + ds_{irrev},$$

where $ds_{irrev} \geq 0$ is the internal entropy production due to dissipative phenomena.

Thus, we have the second law of thermodynamics, which may be expressed in either of two ways:

$$\begin{aligned} ds &\geq \frac{\delta q}{T} \quad (\text{general process}) \\ ds &\geq 0 \quad (\text{adiabatic process}). \end{aligned}$$