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stumpff_C2(z::Float64) = z > 0 ? (1 - cos(sqrt(z)))/z : z < 0 ? (cosh(sqrt(-z)) - 1)/(-z) : 1/2
stumpff_C3(z::Float64) = z > 0 ? (sqrt(z) - sin(sqrt(z))) / (z*sqrt(z)) : z < 0 ? (sinh(sqrt(-z)) - sqrt(-z)) / ((-z)*sqrt(-z)) : 1/6

function solve_lambert(
    r1::Vector{Float64},
    r2::Vector{Float64},
    TOF::Float64;
    μ::Float64 = μ_Earth,
    long_way::Bool = false
)
    # Magnitudes
    r1_norm = norm(r1)
    r2_norm = norm(r2)
    # Transfer angle Δθ
    cos_dθ = dot(r1, r2) / (r1_norm * r2_norm)
    Δθ = acos(clamp(cos_dθ, -1.0, 1.0))
    if long_way
        Δθ = Δθ < π ? 2π - Δθ : Δθ
    else
        Δθ = Δθ > π ? 2π - Δθ : Δθ
    end
    # A-parameter
    A = sin(Δθ) * sqrt(r1_norm * r2_norm / (1 - cos(Δθ)))
    if iszero(A)
        error("Cannot compute Lambert solution: A = 0")
    end

    # Time-of-flight function F(z) = 0
    function F(z)
        C2 = stumpff_C2(z)
        C3 = stumpff_C3(z)
        y = r1_norm + r2_norm + A * (z*C3 - 1) / sqrt(C2)
        if y < 0
            return Inf
        end
        return ( (y/C2)^(3/2) * C3 + A*sqrt(y) ) / sqrt(μ) - TOF
    end

    # Solve for z via Newton-Raphson with finite-difference derivative
    z = 0.0
    for _ in 1:200
        Fz = F(z)
        if abs(Fz) < 1e-8
            break
        end
        δ = 1e-6
        dF = (F(z + δ) - F(z - δ)) / (2δ)
        z -= Fz / dF
    end

    # Compute y, f, g, g
    C2 = stumpff_C2(z)
    C3 = stumpff_C3(z)
    y = r1_norm + r2_norm + A * (z*C3 - 1) / sqrt(C2)

    f = 1 - y/r1_norm
    g = A * sqrt(y/μ)

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gdot = 1 - y/r2_norm

# Velocity vectors
v1 = (r2 .- f*r1) ./ g
v2 = (gdot*r2 .- r1) ./ g

# Compute eccentricity and periapsis radius from (r1, v1)
h_vec = cross(r1, v1)
e_vec = (1/μ) * ((norm(v1)^2 - μ/r1_norm)*r1 .- dot(r1,v1)*v1)
e      = norm(e_vec)
# Semi-major axis from energy
energy = norm(v1)^2/2 - μ/r1_norm
a      = -μ / (2*energy)
rp     = a * (1 - e)

return v1, v2, e, rp
end

export solve_lambert

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