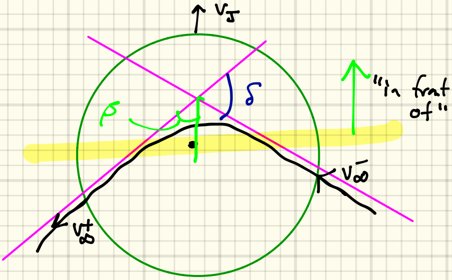
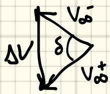
13/14: Flybys

Flybys: Consider a flyby of Jupiter.



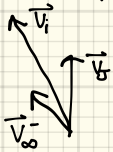
$V_\infty^- = V_\infty^+$ (from Jupiter's perspective)

δ = turn angle



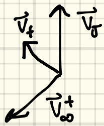
Heliocentric/inertial frame:

Initial s/c velocity:



$\vec{V}_i = \vec{V}_\infty^- + \vec{V}_J$

Final s/c velocity:



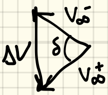
$\vec{V}_f = \vec{V}_J + \vec{V}_\infty^+$

$\Rightarrow$ This flyby decreased the s/c velocity magnitude ($v_f < v_i$) & changed its direction

"Leading edge" flyby: where periapsis of the hyperbola is "in front of" the planet

: decrease heliocentric velocity

Can use law of cosines to calculate the ΔV of the flyby

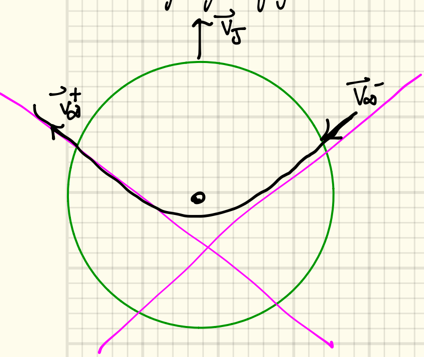


$V_\infty^- = V_\infty^+ = V_\infty$

$\Delta V^2 = v_1^2 + v_2^2 - 2 v_1 v_2 \cos\theta$

$\Delta V^2 = 2 v_\infty^2 (1 - \cos\delta)$

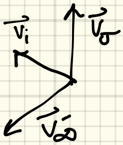
Trailing Edge Flyby:



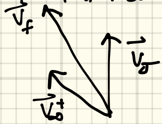
Trailing Edge: periapsis of hyperbola is behind planet

$|\vec{V}_\infty^-| = |\vec{V}_\infty^+|$

Initial Heliocentric:



Final Heliocentric:



Trailing edge flyby increases the heliocentric velocity.

Example: Earth Flyby: $e = 1.3$
$r_p = 40{,}000$ km

a. Calculate v_∞

b. Calculate β.

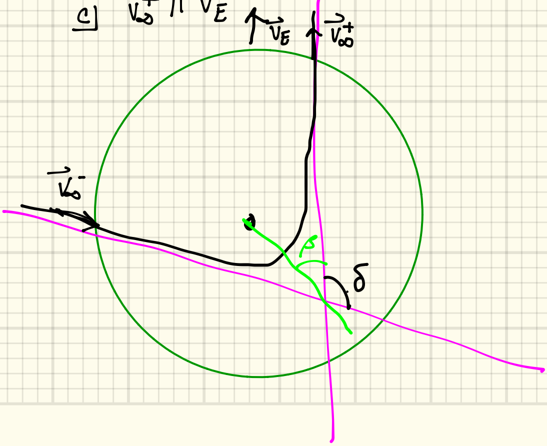
c. Sketch the flyby that will maximize the s/c's final, inertial velocity.

d. Calculate the ΔV.

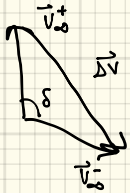
a) $e = 1 + \frac{r_p v_\infty^2}{\mu} \Rightarrow V_\infty = 1.729$ km/s

b) $\cos\beta = \frac{1}{e} \Rightarrow \beta = 39.7°$

c) $\vec{V}_\infty^+ \parallel \vec{V}_E$



d.



law of Cosines, $|\vec{V}_\infty^-| = |\vec{V}_\infty^+|$

$\Delta V^2 = 2 V_\infty^2 (1 - \cos\delta)$

$\delta = 180 - 2\beta = 100.57°$

$\Delta V = 2.66$ km/s

Need a force to change velocity:

$$\vec{F} = m\vec{a} = m_{s/c} \frac{d\vec{v}}{dt}$$

$$\Delta V_{sc} = \left| \int d\vec{v} \right| = \frac{1}{m_{sc}} \left| \int \vec{F}_{s/c} \, dt \right|$$

Note: $\vec{F}_{sc} = -\vec{F}_p$ (force on planet)

What is the ΔV of the planet?

$$\frac{1}{m_p} \left| -\int \vec{F}_{sc} \, dt \right| = \Delta V_p$$

From above: $\left| \int \vec{F}_{sc} \, dt \right| = m_{sc} \Delta V_{sc}$

$$\Delta V_p = \frac{m_{sc} \Delta V_{sc}}{m_p}$$

$$m_p \gg m_{sc} \Rightarrow \Delta V_p \simeq 0 \quad \square$$