Homework Assignment 8: Quaternions and Torque Free Motion

ENAE404: Space Flight Dynamics

Submit on Gradescope. For problems solved in Matlab, attach your code.

1. (40 pts) The orientation of a spacecraft is given in terms of the 3-2-1 Euler Angles (i.e., yaw-pitch-roll) as (30, 40, 10) degrees.
   (a) (8 pts) Give the rotation matrix from the inertial to the body-fixed frame.
   (b) (12 pts) Calculate the principal rotation axis $\hat{e}$ and the principal rotation angle ϕ.
   (c) (8 pts) Calculate the quarternions that describe this orientation of the spacecraft.
   (d) (12 pts) Assume that ${}^{B}\omega = (0.1, 0.2, 0)$ radians/sec at the current time. What is the current quarternion rate of change (i.e., $\dot{\vec{\beta}}$)?

2. (55 pts) Write a code to numerically integrate the angular velocity vector for a torque-free rigid body with principal body-fixed axes. The body has the following body-fixed inertia matrix (units of kg m^2):

$$[I] = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{bmatrix} \tag{1}$$

   (a) (6 pts) Given $\omega = [10, 0, 30]$ deg/sec in the body-fixed frame, calculate the angular momentum magnitude and kinetic energy of the body.
   (b) (6 pts plot + 10pts code) Given the above initial angular velocity, propagate the angular velocity for 100 seconds. On a single plot, plot the angular velocity components as a function of time.
   (c) (10 pts) Plot the kinetic energy deviation and angular momentum magnitude deviation as a function of time. Discuss why you believe that your code is working.
   (d) (11 pts) Create the polhode plot for this system. The angular momentum sphere should be a single color and the kinetic energy ellipsoid should be a single color. On top of the polhode, plot (as points) the time-varying body-fixed angular momentum. Use 'axis equal' and orient the plot so that you can see the angular momentum points.
   (e) (7 pts) Given $\omega = [1, 15, 0]$ deg/sec in the body-fixed frame, propagate the angular velocity for 100 seconds. On a single plot, plot the angular velocity components as a function of time.
   (f) (15 pts) Create the polhode plot for this system. The angular momentum sphere should be a single color and the kinetic energy ellipsoid should be a single color. On top of the polhode, plot (as points) the time-varying body-fixed angular momentum. Discuss why plots in e and f look significantly different from the plots in part b and d. Compare the energy between the system in part a and the current system.