

Midterm Review

10/15/24



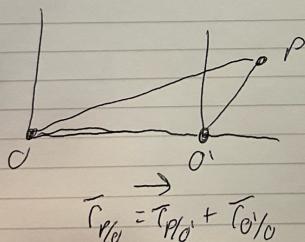
$$\overset{I}{F}_P = m \ddot{\overset{I}{v}}_{P/0}$$

$$\frac{d}{dt} (\overset{I}{\dot{r}}_{P/0}) = \overset{I}{F}_P$$

$$\frac{d}{dt} (\overset{I}{\dot{r}}_{hP/0}) = \overset{I}{m}_{P/0}$$

1. How do you know which form of NCL to use?

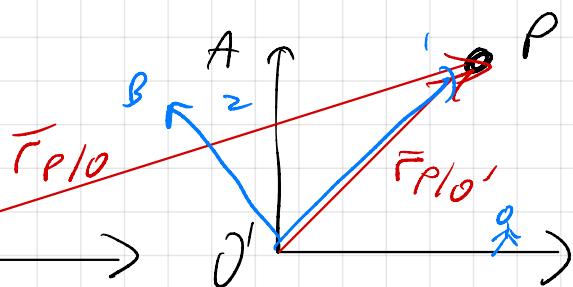
In a translation reference frame
Can you go over how to find
 $\overset{I}{\dot{r}}_{0/0}$ and $\overset{I}{\dot{v}}_{0/0}$ and $\overset{I}{\ddot{v}}_{0/0}$



$$\overset{I}{\dot{r}}_{P/0} = \overset{I}{\dot{r}}_{P/0'} + \overset{I}{\dot{r}}_{0/0}$$

Ch 3

I A



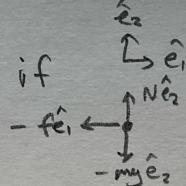
→ translating $\overset{I}{v}_0$

$$\overset{I}{\dot{r}}_{P/0} = \overset{I}{\dot{r}}_{0/0} + \overset{I}{\dot{r}}_{P/0'}$$

$$\overset{I}{\dot{v}}_{P/0} = \overset{I}{v}_{0/0} + \overset{I}{v}_{P/0'}$$

$$\overset{I}{\ddot{a}}_{P/0} = \overset{I}{\ddot{a}}_{0/0} + \overset{I}{\ddot{a}}_{P/0'}$$

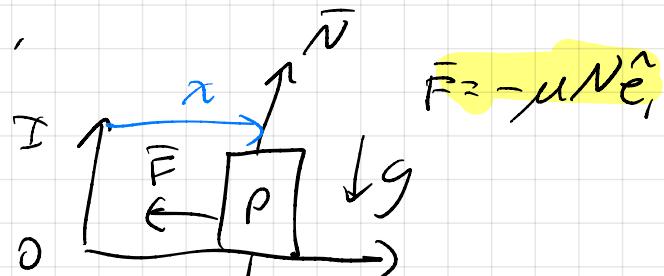
For friction; if



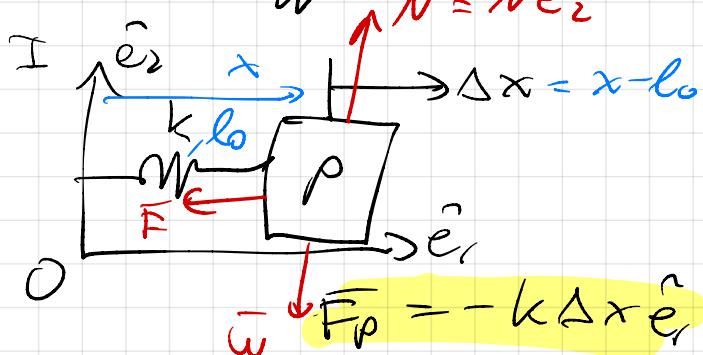
$$f = -k \Delta x$$

or

$$f = k \Delta x$$



$$\bar{F} = -\mu N \hat{e}_1$$



$$\bar{F}_P = -k \Delta x \hat{e}_2$$

* Friction opposes the motion

$$\bar{F} = -\mu N \text{sign}(\dot{x}) \hat{e}_t$$

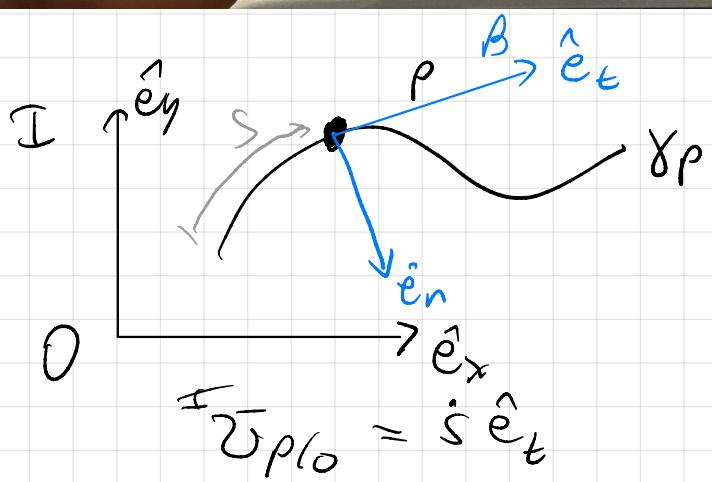
$$\bar{F}_p = m_p \bar{a}_{p/0}$$

$$\bar{F} + \bar{N} + \bar{W} = m_p \bar{a}_{p/0}$$

$$\text{Sign}(\dot{x}) = \begin{cases} +1, & \dot{x} > 0 \\ 0, & \dot{x} = 0 \\ -1, & \dot{x} < 0 \end{cases}$$

• How do you write something in path coords, I, if just a polar frame w/ diff origin? Or would you just write in cartesian w/ a vector going to it?

• Difference - b/w int and ext forces



$$I = (0, \hat{e}_x, \hat{e}_y, \hat{e}_z)$$

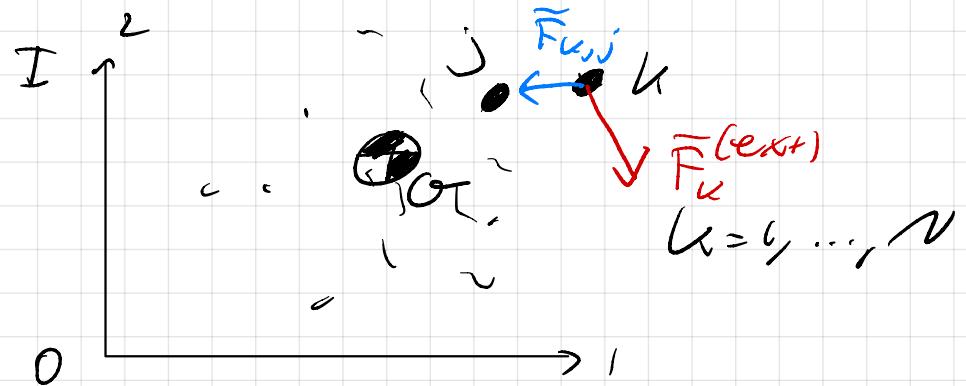
$$B = (P, \hat{e}_t, \hat{e}_n, \hat{e}_b)$$

$$\hat{e}_t = \frac{\bar{v}_{p/0}}{\|\bar{v}_{p/0}\|} = \frac{\bar{v}_p}{\|\bar{v}_{p/0}\|}$$

\hat{e}_n inward,

$$\hat{e}_t \times \hat{e}_n = \hat{e}_z$$

Recall (6.1)



$$\begin{aligned}\bar{F}_k &= \bar{F}_k^{(ext+)} + \bar{F}_{kN}^{(int+)} \\ &= \bar{F}_k^{(ext+)} + \sum_{j=1}^N \bar{F}_{kj}\end{aligned}$$

$\bar{F}_{kN} = 0$

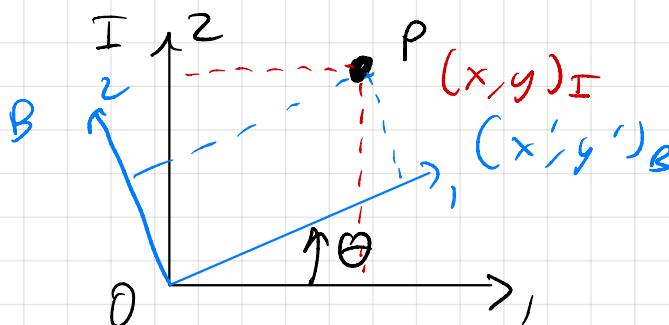
If we are in Frame B, for example, and take a derivative would the notation be like

$$B \frac{d}{dt} (\vec{r}_{P/B}) = B \vec{v}_{P/B} \quad \text{Yes}$$

Similar to problem 3.8

what does it mean to express a position vector in a frame, then express its p in another frame?

Ex: Express $\vec{r}_{P/B}$ in Frame B with components of frame A? Problem 3.8



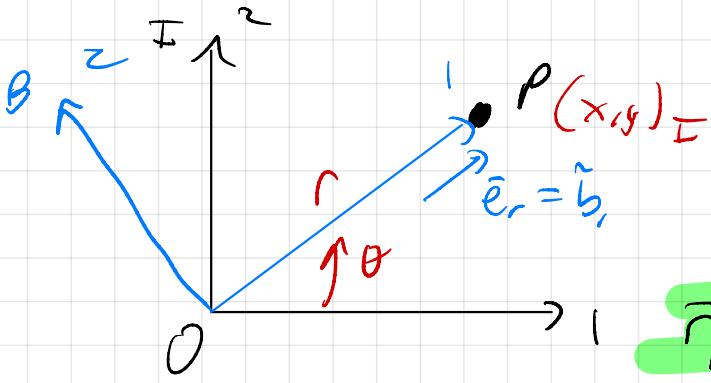
$$\begin{aligned}\bar{r}_{P/B} &= x \hat{e}_1 + y \hat{e}_2 \\ &= x' \hat{b}_1 + y' \hat{b}_2\end{aligned}$$

$$= x'((\theta \hat{e}_1 + s\theta \hat{e}_2) + y'(-s\theta \hat{e}_1 + c\theta \hat{e}_2)) = \dots$$

	\hat{b}_1	\hat{b}_2	$\frac{\hat{b}_1}{\hat{e}_1}$	$\frac{\hat{b}_2}{\hat{e}_1}$
\hat{e}_1	(θ)	$-s\theta$	$\hat{e}_1 \cdot \hat{b}_1$	$\hat{e}_1 \cdot \hat{b}_2$
\hat{e}_2	$s\theta$	$c\theta$	$\hat{e}_2 \cdot \hat{b}_1$	$\hat{e}_2 \cdot \hat{b}_2$

$$\hat{b}_1 = c\theta \hat{e}_1 + s\theta \hat{e}_2$$

$$\hat{b}_2 = -s\theta \hat{e}_1 + c\theta \hat{e}_2$$

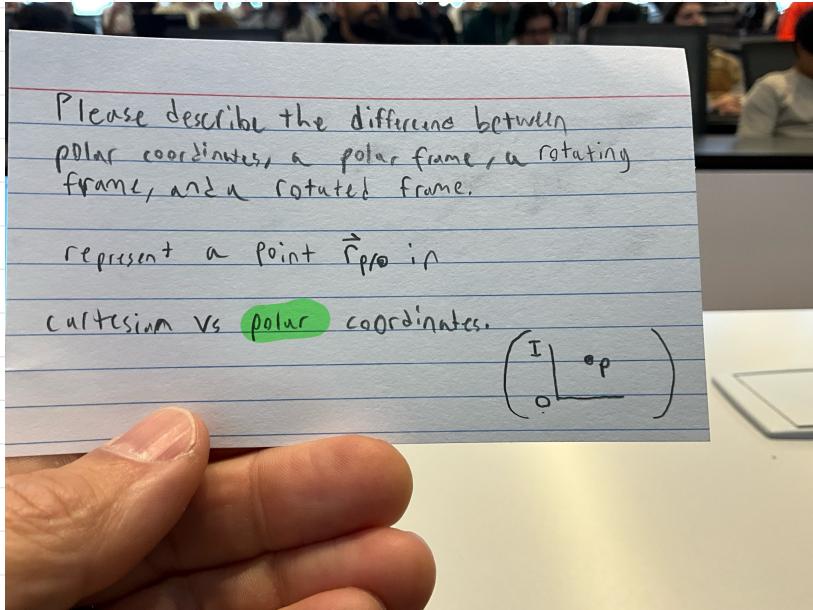


$$\mathcal{B} = (0, \hat{b}_1, \hat{b}_2, \hat{b}_3) \text{ polar}$$

$$= (0, \hat{e}_r, \hat{e}_\theta, \hat{e}_z) \text{ polar}$$

$$\vec{r}_{P/I} = r \hat{b}_1 \quad \boxed{\frac{d}{dt}}$$

$$\vec{v}_{P/I} = \dot{r} \hat{b}_1 + r \dot{\theta} \hat{b}_2$$



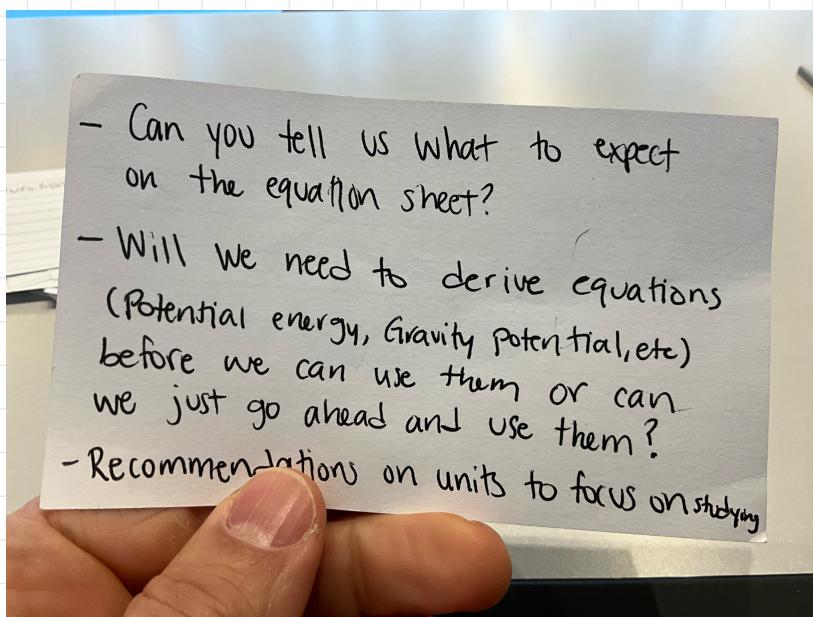
$$\vec{r}_{P/I} = x \hat{e}_1 + y \hat{e}_2$$

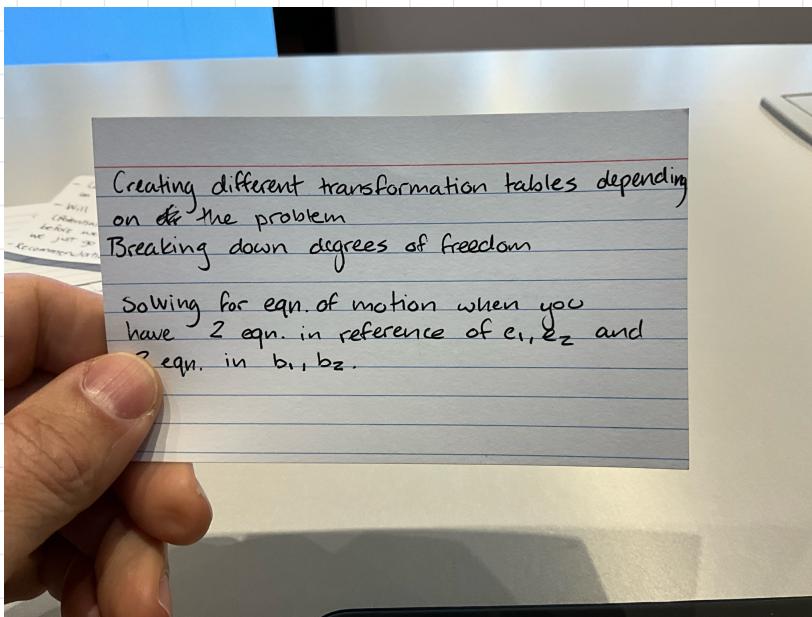
$$= r \cos \theta \hat{e}_1 + r \sin \theta \hat{e}_2$$

$$\frac{d}{dt}(\hat{b}_1) = \vec{\omega} \times \hat{b}_1,$$

$\vec{\omega}$ rotated

\hat{b}_1 rotating

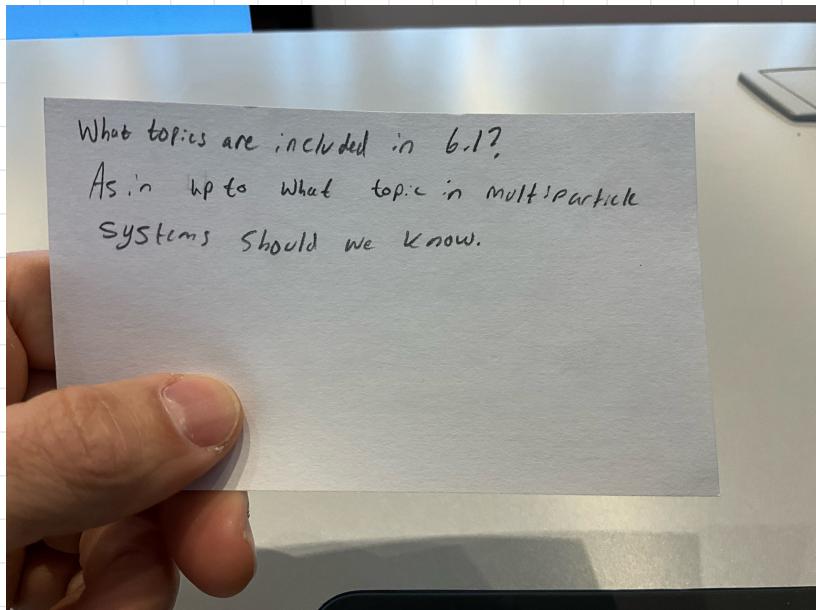




$$M = 3N - K$$

↓ ↗
 # particles # constraints

$$\begin{aligned} \hat{\mathbf{F}}_p &= mg \hat{\mathbf{e}}_1 - T \hat{\mathbf{b}}_1 = m_p \overset{I}{\hat{\mathbf{a}}}_p \cdot \hat{\mathbf{e}}_1 \\ \hat{\mathbf{e}}_1 : \quad mg - T \hat{\mathbf{b}}_1 \cdot \hat{\mathbf{e}}_1 &= m_a \overset{I}{\hat{\mathbf{c}}}_p \cdot \hat{\mathbf{e}}_1 \end{aligned}$$



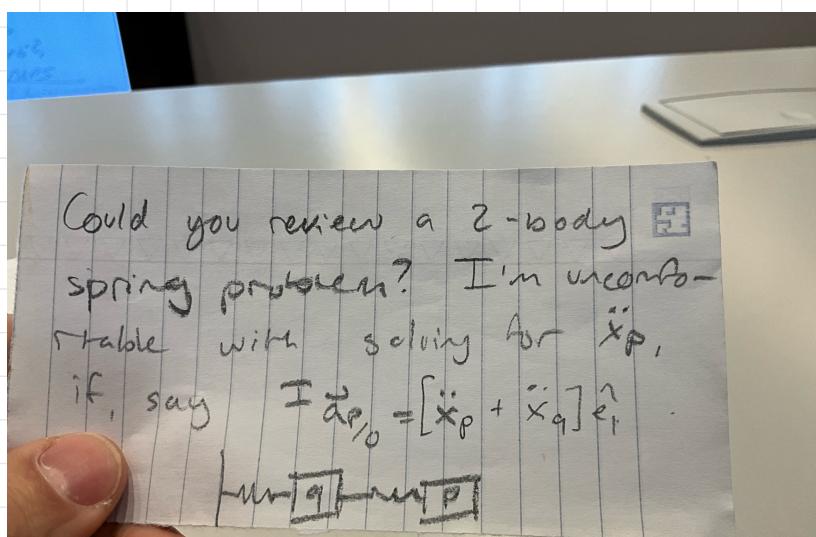
MPS

6.1 Linear momentum

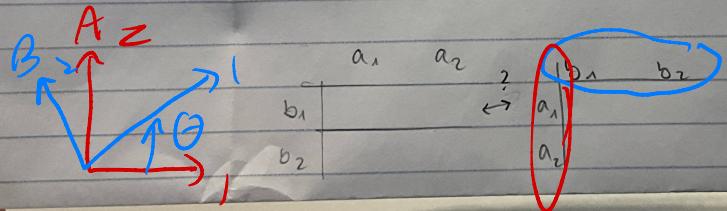
6.2 Impacts

6.3 Variable Mass

Angular Momentum, Energy



How do we know which variables go on which axis of transformation table?



How much "framing" is needed for work-energy problems?
a lot.

$$W_p = \int \vec{F}_p \cdot \vec{v}_{p/\alpha} dt$$

$\delta_p = 0$ normal forces