Muddy Review

12/5/24

For I_G calcs, $\bar{r}_{G/O}$ calcs, etc.

A bit confused on notation. Does

$\int_{\mathcal{B}} dx\,dy\,dz\,(x^2+y^2+z^2) = \int_{\mathcal{B}} (x^2+y^2+z^2)\,dx\,dy\,dz$? Yes

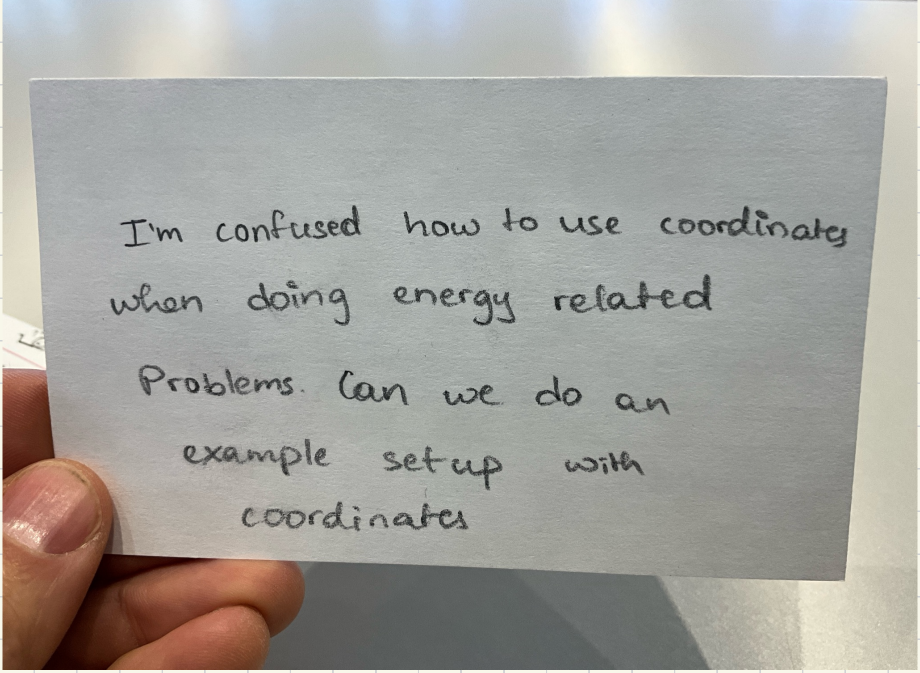
similarly, $\int_{\mathcal{B}} r\,dr\,d\theta\,dz\,(r+z) = \int_{\mathcal{B}} (r+z)\,r\,dr\,d\theta\,dz$? Yes

placement of $\bar{r}_{dm/O}$ in integral is confusing

Muddy Points

MP #1: Will we be asked to solve a problem w/ a collection of R.B.s or particles?

MP #2: Why do we represent certain things as particles v. R.B.'s. (I.e. why can we represent a plane/satellite/person/etc as a particle?)



$$E_{P/O} = T_{P/O} + U_{P/O}$$

$$(x, y, z)_I$$

$$T_{P/O} = \frac{1}{2} m_P \| {}^{I}\bar{v}_{P/O} \|^2$$

$$= \frac{1}{2} m_P \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right)$$

$$U_{P/O}^{(\bar{w})} = mgz$$

$$E_O = T_{G/O} + T_G + U_O$$

$$T_{G/O} = \frac{1}{2} m_P \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right)$$

$$(\omega_1, \omega_2, \omega_3)_C$$

$$T_G = \frac{1}{2} \left[{}^{\mathcal{I}}\bar{\omega}^{c}\right]_c^T \left[\mathbb{I}_G\right]_c \left[{}^{\mathcal{I}}\bar{\omega}^{c}\right]_c$$

$$= \frac{1}{2} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}_c^T \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}_c$$

$$= \frac{1}{2} \left(I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2 \right)$$

$$U_O^{(\bar{w})} = mgz$$

WE#3 particle $E_{P/O}(t_2) = E_{P/O}(t_1) + W_P^{(nc)}$

RB $E_O(t_2) = E_O(t_1) + W^{(nc)}$

$$= W_{G/O}^{(nc)} + W_G^{(nc)}$$

$$W_P^{(\bar{F}_P)} = \int_{\gamma_P} \bar{F}_P \cdot {}^{\mathcal{I}}d\bar{r}_{P/O}$$

$$= \int_{\gamma_P} \underbrace{\bar{F}_P \cdot {}^{\mathcal{I}}\bar{v}_{P/O}}_{\stackrel{?}{=} 0} \, dt$$

$$W_{G/O}^{(\bar{F}_G)} = \int_{\gamma_G} \bar{F}_G \cdot {}^{\mathcal{I}}\bar{v}_{G/O} \, dt$$

$$W_G^{(\bar{M}_G)} = \int \bar{M}_G \cdot {}^{\mathcal{I}}\bar{\omega}^{c} \, dt$$

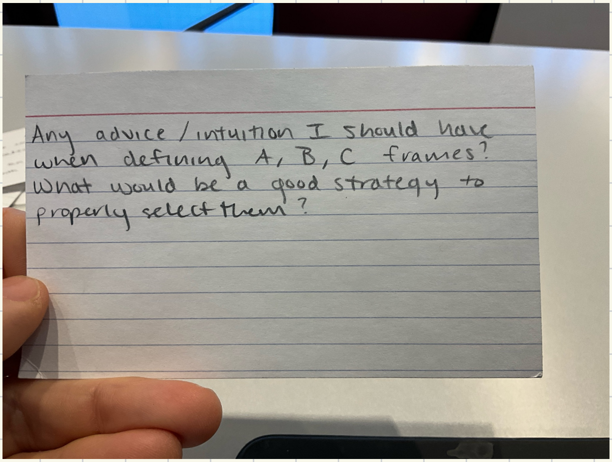
MPS

$$W^{(nc)} = W^{(nc,ext)} + W^{(nc,int)}$$

$$W^{(nc,int)} = \sum_{k=1}^{N} W_k^{(nc,int)}$$

$$= \sum_{k=1}^{N} \sum_{j=1}^{N} \bar{F}_{k,j} \cdot d\bar{r}^{I}_{k/O}$$

MPS $E_0(t_2) = E_0(t_1) + W^{(nc,ext)} + W^{(nc,int)}$



3D RB

I inertial

A } intermediate

B

C body frame

3 simple rotation

3D Rotation Coordinate

301 → Euler Angles

~~Quaternions~~

~~Exponential Coords~~

$$\left(1-2-3 \qquad \psi, \theta, \phi\right)_I^C$$

$$\left(3-2-3 \qquad \psi, \theta, \phi\right)_I^C$$

12 options

$$ {}^I\bar{\omega}^A = \dot{\psi}\,\hat{a}_1 \qquad {}^I R^A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix} $$

$$ {}^A\bar{\omega}^B = \dot{\theta}\,\hat{b}_2 \qquad {}^A R^B = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix} $$

← 2 axis rotation

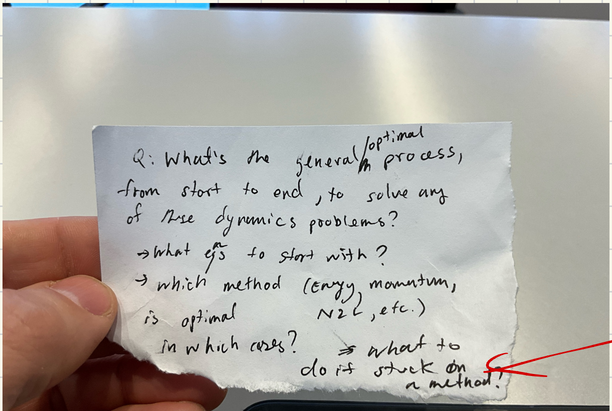
$$ {}^B\bar{\omega}^C = \dot{\phi}\,\hat{c}_3 \qquad {}^B R^C = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

$$ {}^I\bar{\omega}^C = {}^I\bar{\omega}^A + {}^A\bar{\omega}^B + {}^B\bar{\omega}^C = \dot{\psi}\,\hat{a}_1 + \dot{\theta}\,\hat{b}_2 + \dot{\phi}\,\hat{c}_3 $$

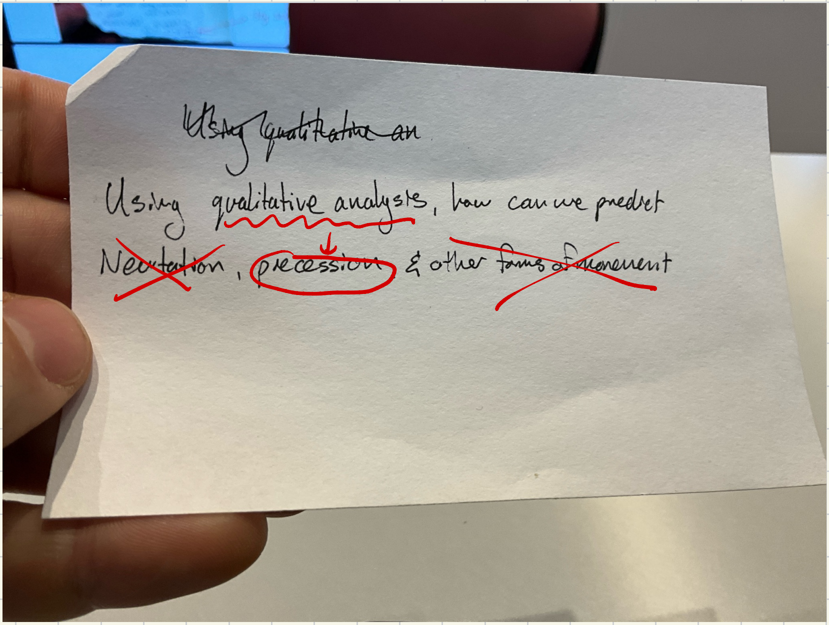
$$ {}^C R^A = \left({}^C R^B\right)\left({}^B R^A\right) $$

$$ \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}_C = \left[{}^I\bar{\omega}^C\right]_C = {}^C R^A \begin{bmatrix} \dot{\psi} \\ 0 \\ 0 \end{bmatrix}_A + {}^C R^B \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix}_B + \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix}_C $$

$$ {}^I R^C = \left({}^I R^A\right)\left({}^A R^B\right)\left({}^B R^C\right) $$



try something else.



$$\bar{M}_G = \frac{{}^{I}d}{dt}\left({}^{I}\bar{h}_G\right)$$

intermediate frame sharing axis of spin

$$= \underbrace{\frac{{}^{B}d}{dt}\left({}^{I}\bar{h}_G\right)}_{\approx 0} + {}^{I}\bar{\omega}^{B} \times {}^{I}\bar{h}_G$$

rapidly spinning

$$\bar{M}_G \approx \underline{{}^{I}\bar{\omega}^{B}} \times {}^{I}\bar{h}_G$$

unknown

$$ {}^{I}\bar{\omega}^{C} = \overbrace{\underbrace{{}^{I}\bar{\omega}^{A} + {}^{A}\bar{\omega}^{B}}_{\text{(small)}}}^{\text{unknown}} + \underbrace{{}^{B}\bar{\omega}^{C}}_{\text{spin (large)}}$$

Can you give us any reference on what the curve could be like?

storical mean curve?
ast years curve?
Is it a flat curve or weighted?

Why do we need both $\omega_1, \omega_2, \omega_3$ & $\dot{\theta}, \dot{\psi}, \dot{\phi}$ for the rotational kinematics of a rigid body?
They're
What makes them different? Frame? Use case?

Aerospace Conventions

$$\left[{}^{I}\underline{v}_{o/o}\right]_C = \begin{bmatrix} u \\ v \\ w \end{bmatrix}_C$$

$(x, y, z)_I$

$$\left[{}^{I}\underline{v}_{o/o}\right]_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}_I = {}^{I}R^{C} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\left[{}^{I}\underline{\omega}^{C}\right]_C = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\left[\mathbb{I}_{o}\right]_C = \begin{bmatrix} I_1 & 0 & -I_{13} \\ 0 & I_2 & 0 \\ -I_{13} & 0 & I_3 \end{bmatrix}$$

Fwd-Rt-Dwn body frame