Lecture 4: Dynamical Similarity and Flow Regimes

ENAE311H Aerodynamics I

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Flow similarity

Consider the flow fields generated by two different bodies. We define the two flows as *dynamically similar* if:

1. The streamline patterns (i.e., paths in space taken by the flow) are geometrically similar
2. The distributions of $\frac{V}{V_\infty}$, $\frac{p}{p_\infty}$, $\frac{T}{T_\infty}$, etc. are identical throughout if plotted in common nondimensional coordinates.
3. The force and moment coefficients are the same.

To ensure dynamical similarity, it is required that:

1. The bodies (and any other solid boundaries) are geometrically similar
2. All relevant nondimensional similarity parameters (e.g., Re, M) are the same for the two flows.

Continuum versus rarefied flows

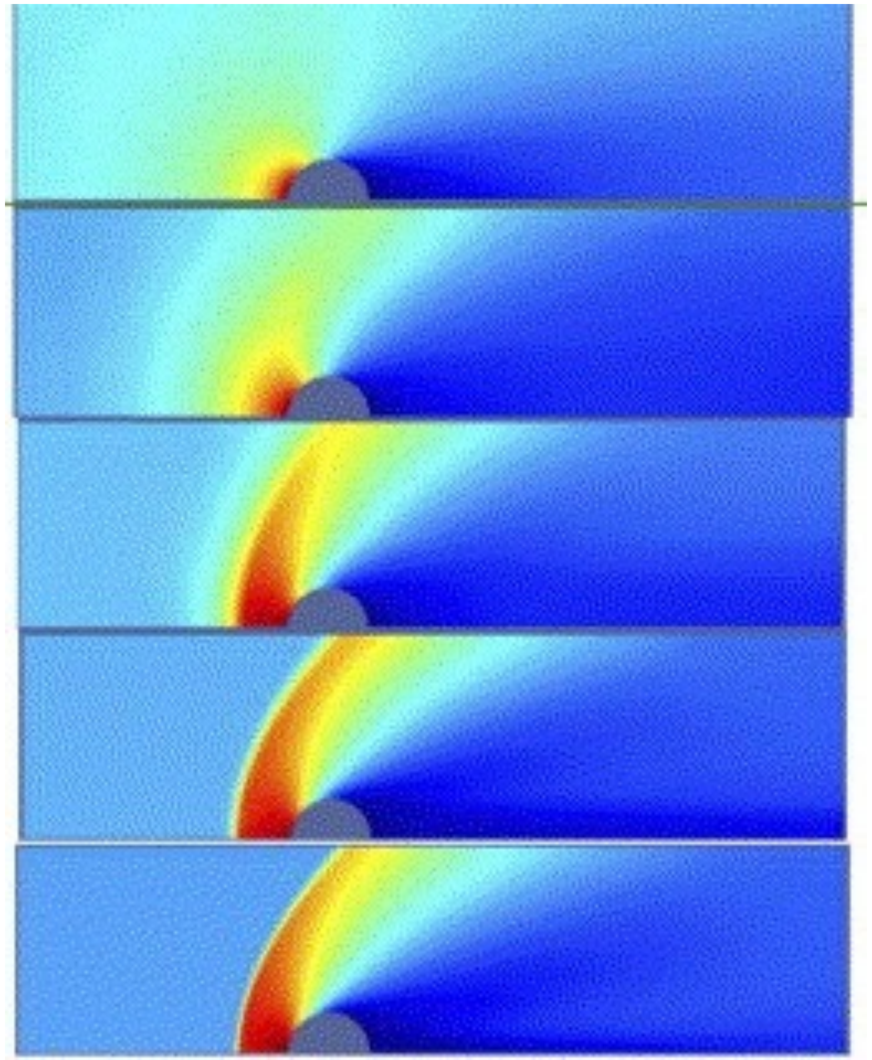
We have already discussed the continuum approximation, which holds for the flows we will be interested in in this course. If we imagine, however, either shrinking the body in question or decreasing the density of the gas so that the body dimension is comparable to the mean free path (mean distance travelled by molecules between collisions), certain non-continuum effects become important:

- No-slip boundary conditions no longer hold
- Shocks become diffuse
- In free-molecular regime, molecules impact surfaces without interacting with one another.

The parameter that governs the degree of rarefaction of a flow is the Knudsen number, Kn:

$$Kn = \frac{\lambda}{d} = \frac{mean\ free\ path}{body\ dimension}$$

$Kn = \lambda/R$



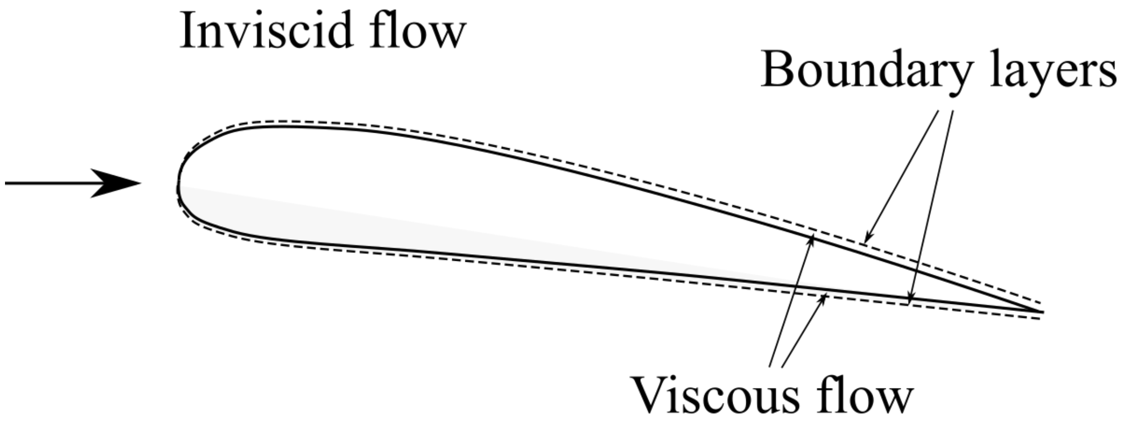
From Kolobov et al., JCP, 2007

Inviscid versus viscous flows

- Viscosity is brought about by molecular transport of momentum → all flows to some extent viscous

- The degree of influence of flow viscosity depends on the Reynolds number:
  - At low Re, the entire flowfield is essentially viscous
  - As $Re \to \infty$, the flowfield becomes effectively inviscid
  - For large but finite Re, the viscous effects are typically confined to a “boundary layer” close to the surface (pressure forces determined by external inviscid flow; viscous forces determined by viscous boundary-layer flow).

- Certain flowfields (e.g., separated flows)are dominated by viscous effects, even at large Re.



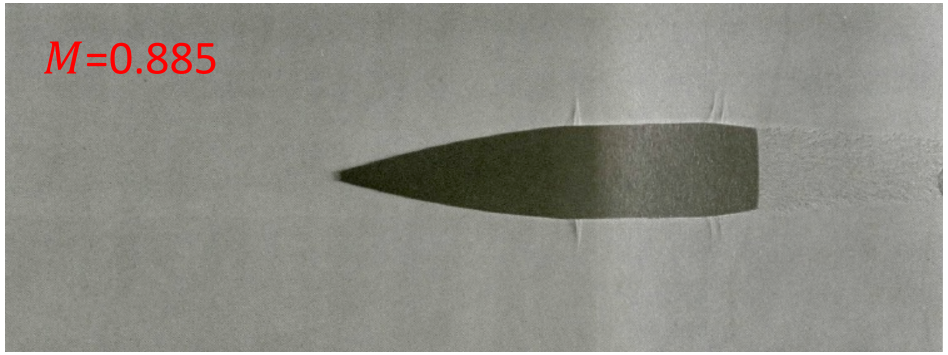
Compressible versus incompressible flow

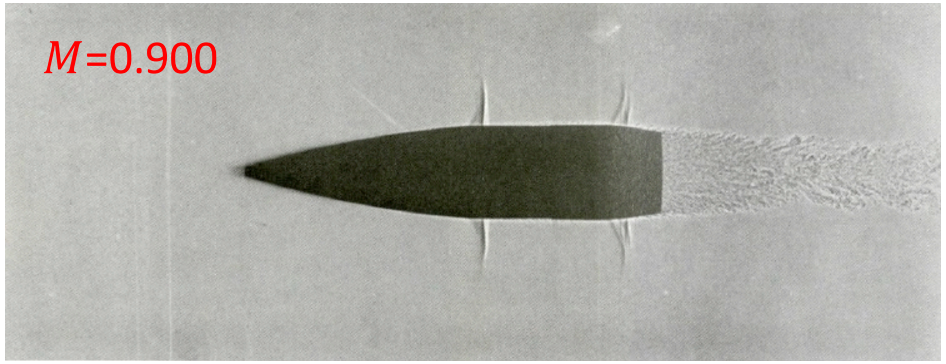
- All fluids are compressible to some extent, in that the density will change, if ever so slightly.
- Liquids are effectively incompressible, except at rather extreme conditions.
- The flow of gases can, to a reasonable approximation, be treated as incompressible for Mach numbers up to approximately 0.3.

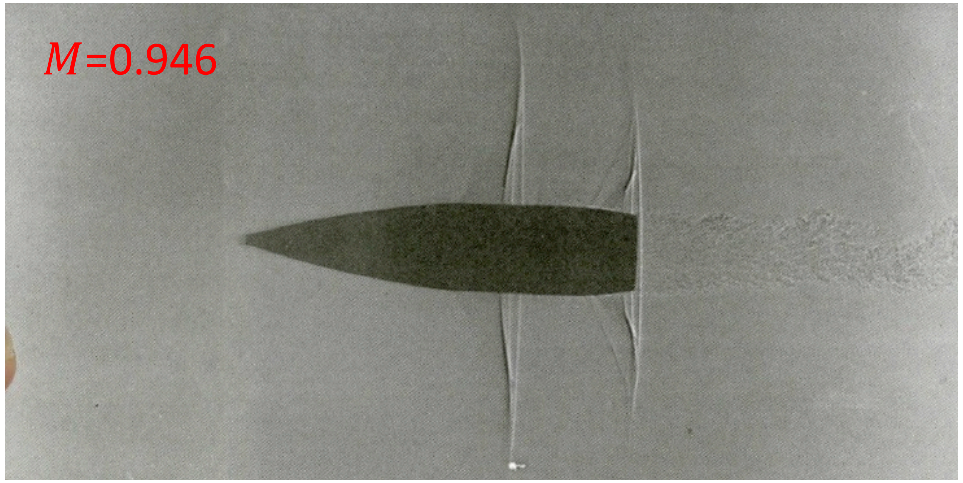
Mach-number regimes

- Subsonic flow ($M < 1$)
  - Mach number is <1 everywhere; information can propagate everywhere within the fluid domain and streamlines smoothly varying

- Transonic flow ($0.8 \lesssim M \lesssim 1.2$)
  - Contains mixed regions of subsonic and supersonic flow; characterized by weak shocks at very steep angles







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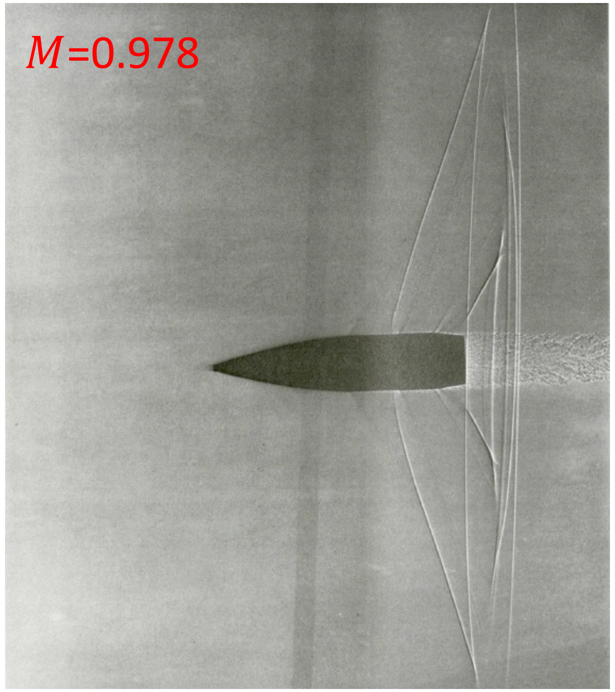

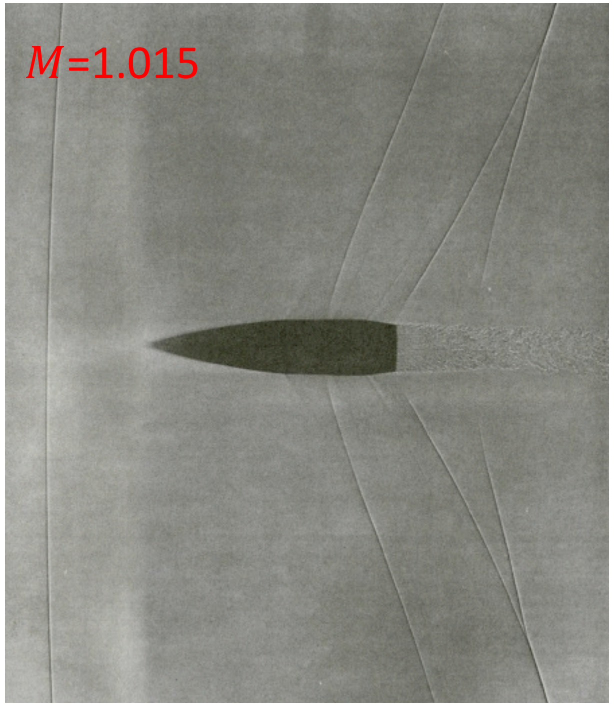

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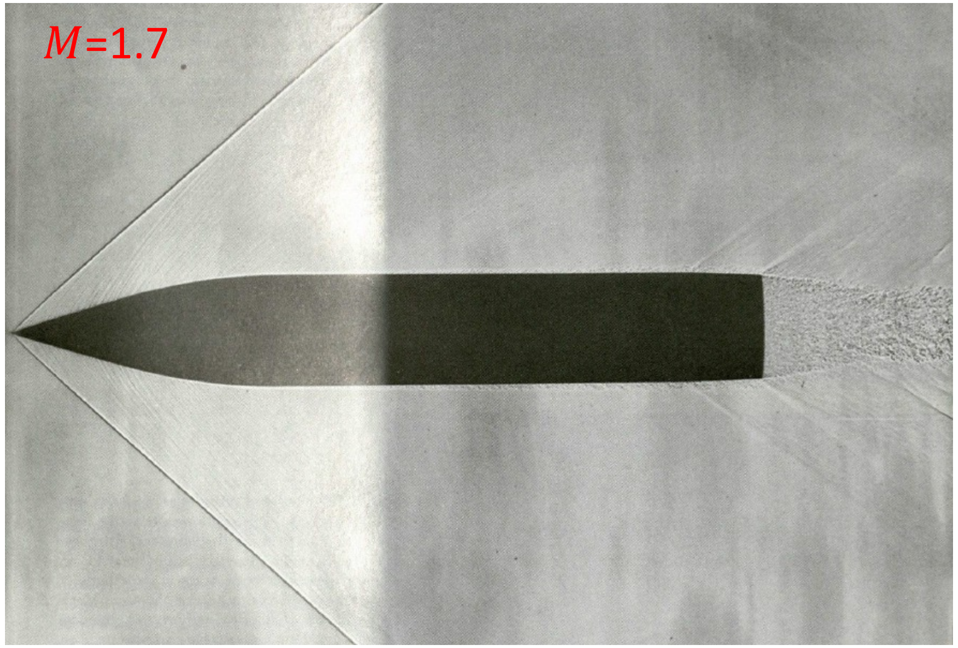

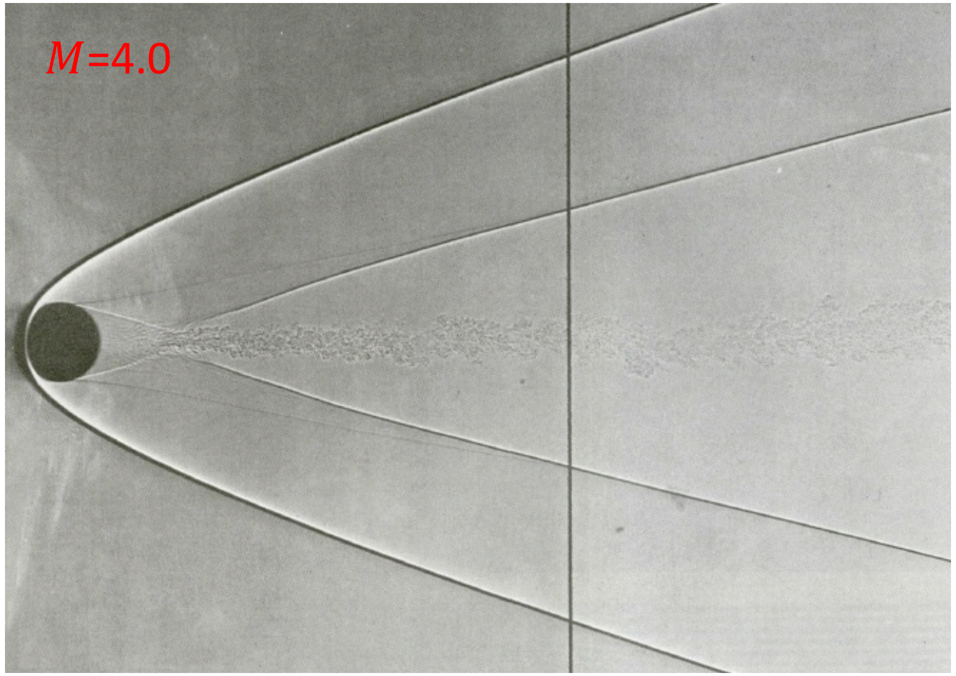

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- Supersonic flow ($M > 1$)
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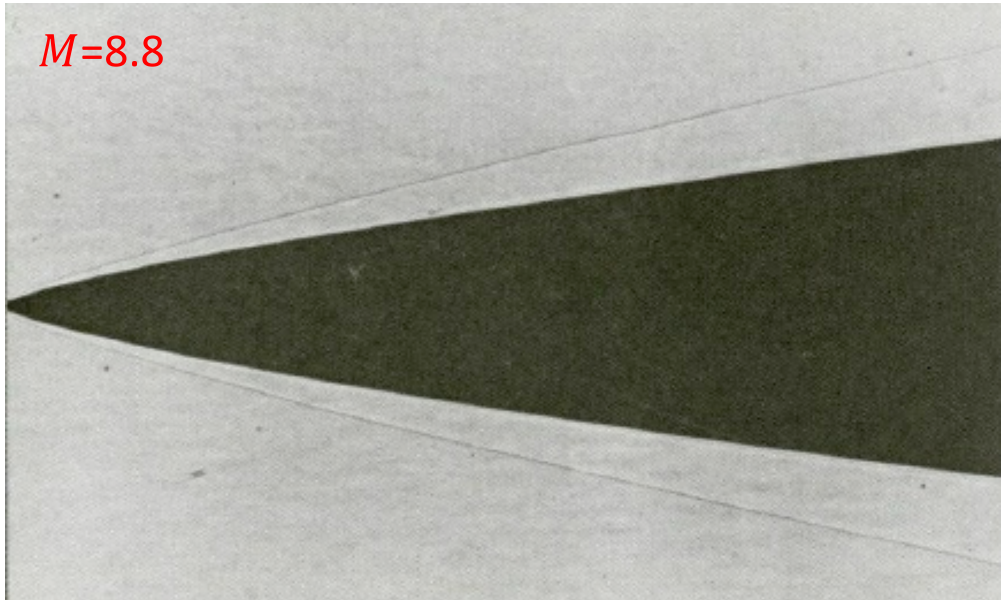
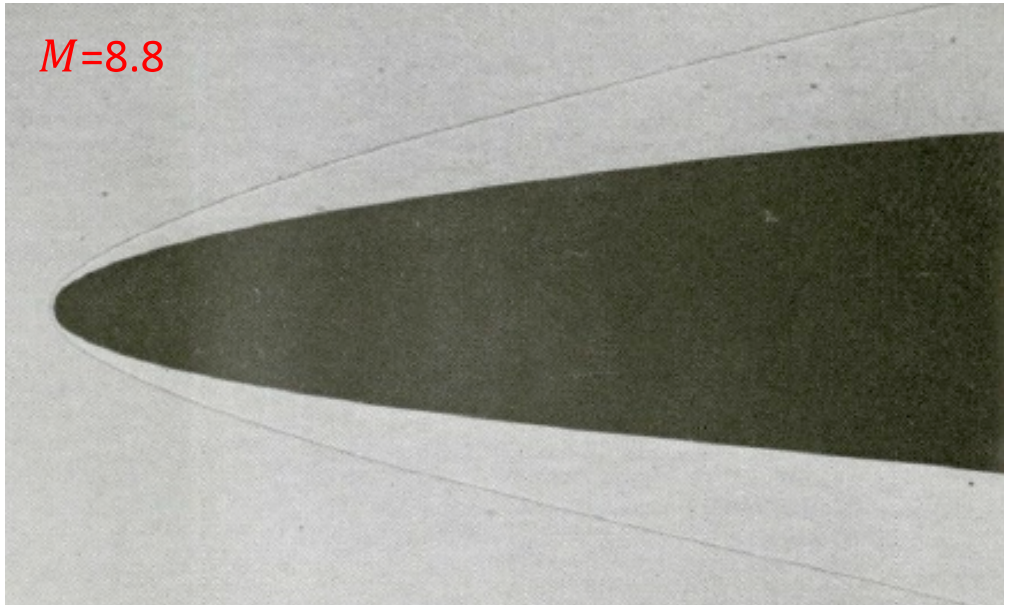
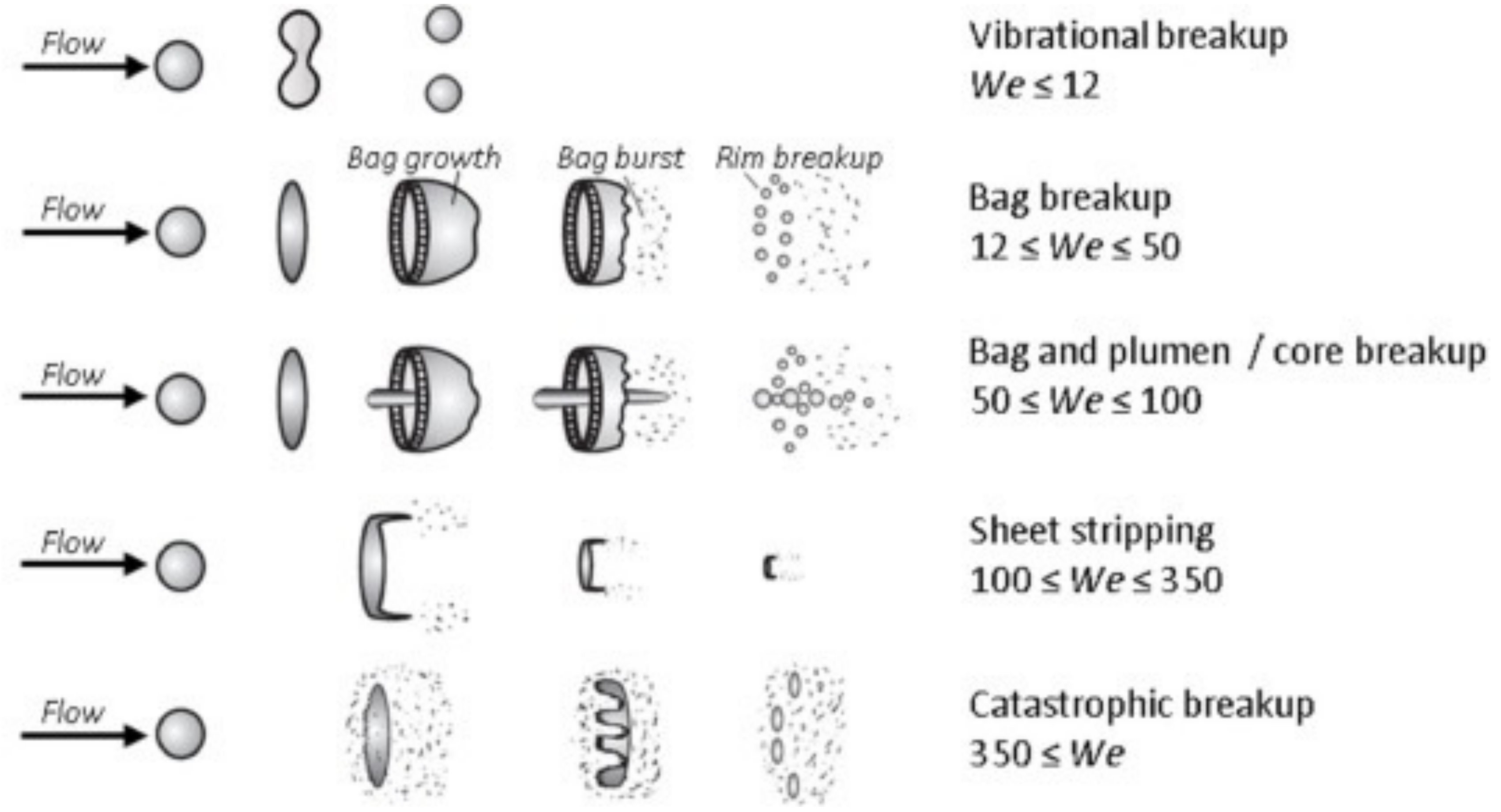
- Hypersonic flow ($M \gg 1$)
  - Characterized by thin shock layers for slender bodies and strongly curved shocks for blunt bodies
  - At very high Mach numbers, real-gas effects such as molecular dissociation become important





Droplet breakup



$$\mathrm{We} = \frac{\text{Drag Force}}{\text{Cohesion Force}} = \left(\frac{8}{C_{\mathrm{D}}}\right) \frac{\left(\frac{\rho\, v^2}{2}\, C_{\mathrm{D}}\, \pi\, \frac{l^2}{4}\right)}{(\pi\, l\, \sigma)} = \frac{\rho\, v^2\, l}{\sigma}$$