ENAE 301: Homework 01

Due on September 06, 2024 at 01:00 PM

Dr. Paley, Section 0103

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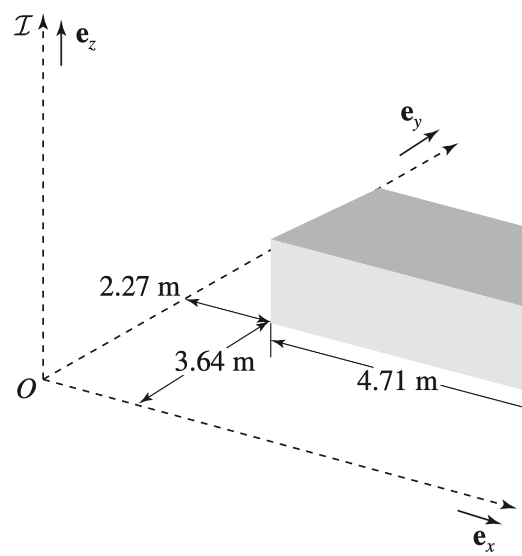
Problem 1

1.1: What are the Cartesian coordinates of point P in frame I, as shown in the following figure?

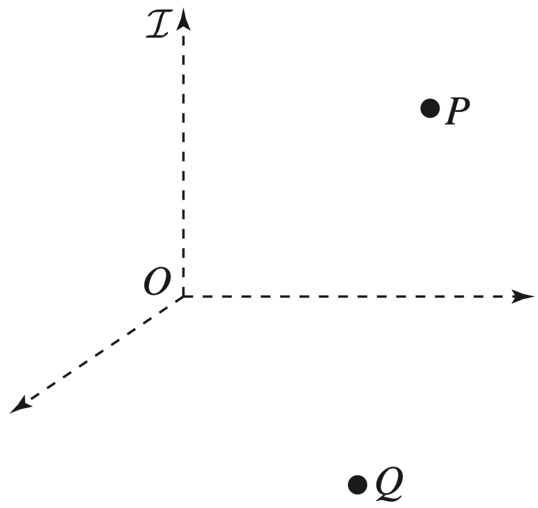


Solution

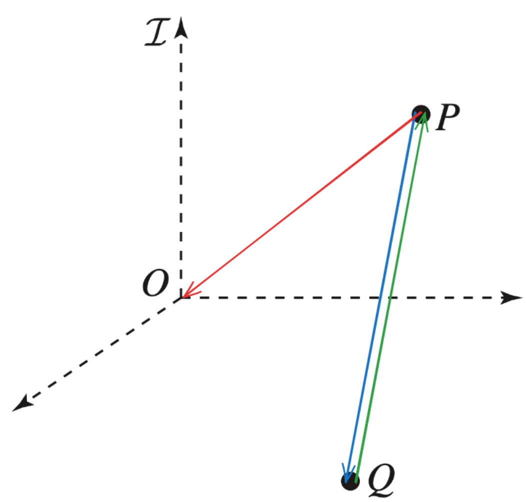
The cartesian coordinates of point P in frame I are $(6.98, 3.64, 1.92)$ m.

Problem 2

1.2: Sketch and label the vectors $\vec{r}_{\frac{P}{O}}$, $\vec{r}_{\frac{P}{Q}}$, $\vec{r}_{\frac{Q}{P}}$ in the following figure:



Solution



Problem 3

1.3: Match each of the following definitions to the appropriate term below:

1. A perspective for observations regarding the motion of a system
2. A mathematical quantity with both magnitude and direction
3. Second-order differential equations whose solution is the trajectory of a point
4. A set of scalars used to locate a point relative to another point

- Vector
- Reference frame
- Coordinate system
- Equations of motion

Solution

Part A

Reference Frame: A perspective for observations regarding the motion of a system.

Part B

Vector: A mathematical quantity with both magnitude and direction.

Part C

Equations of motion: Second-order differential equations whose solution is the trajectory of a point.

Part D

Coordinate system: A set of scalars used to locate a point relative to another point.

Problem 4

2.3: Consider the straight-line motion of a particle of mass m_P acted on only by air resistance, $F_P = -b\dot{x}^2$. Find analytically an expression for the velocity of the particle as a function of time if it starts with initial velocity v_0 at time t_0.

Solution

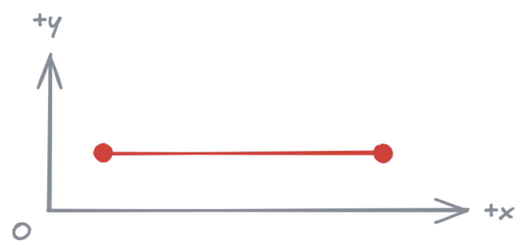
$$\begin{aligned}
m_P \frac{d\dot{x}}{dt} &= F_P \\
m_P \frac{d\dot{x}}{dt} &= -b\dot{x}^2 \\
\frac{d\dot{x}}{\dot{x}^2} &= -\frac{b}{m_P} dt \\
\left[-\frac{1}{\dot{x}}\right]_{v_0}^{\dot{x}} &= -\frac{b}{m_P}\left(t - t_0\right) \\
\dot{x} = \left[v_0^{-1} + \frac{b}{m_P}\left(t - t_0\right)\right] \quad \square.
\end{aligned}$$

Problem 5

2.6: Sketch a planar model of a weightlifter's barbell using two point masses and a rigid massless rod. How many degrees of freedom are there in your model?

Solution

In my model, there are 3 degrees of freedom.



Problem 6

2.7: In Example 2.5 we found that the three-link robot arm in Figure 2.6a has three degrees of freedom. We described them by three angles in Figure 2.6b. Suppose, instead, you desired to describe the system by the six Cartesian coordinates for the end of each link, $(x_1, y_1)_I$, $(x_2, y_2)_I$, $(x_3, y_3)_I$. How many constraint equations would be necessary and what are they?

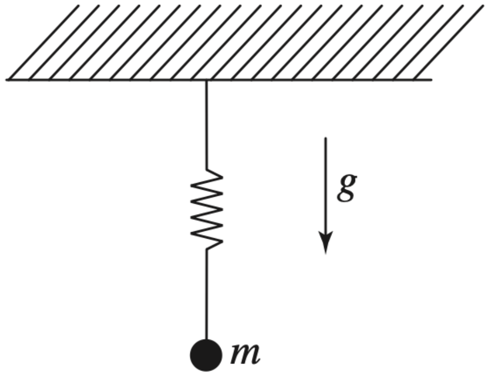
Solution

For this description of the system, we would need two constraint equations:

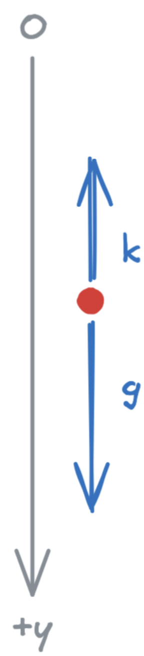
$$r_a = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$r_b = \sqrt{(x_2 - x_3)^2 + (y_2 - y_3)^2}.$$

Problem 7

2.12: Find the equations of motion for a mass m suspended vertically from a spring as shown in the following figure, assuming that the mass is constrained to move only vertically and that it is subject to the force of gravity. Draw a free-body diagram, choose a coordinate system, and use Newton's second law to find the equation of motion.



Solution



$$\ddot{y} = \frac{g - ky}{m}.$$

Problem 8

2.17: Consider the equation of motion of a driven simple harmonic oscillator $\ddot{x} = a - \frac{k}{m}x$, where a is a constant.

1. Integrate the equation to find the solution analytically.
2. Solve the equation of motion using MATLAB `ODE45`, and plot $x(t)$ and $\dot{x}(t)$. Let $a = 1$, $m = 0.5$, and $k = 3$. Be sure to label your axes (with units!) and add a legend to the plot.

Solution

$$\begin{aligned}
\ddot{x} &= a - \frac{k}{m}x \\
d\dot{x} + \frac{k}{m}x &= adt \\
\int \frac{k}{m}xd\dot{x} &= \int adt \\
\frac{k}{2m}x^2\dot{x} &= at + C_1 \\
\frac{k}{2m}x^2dx &= (at + C_1)\,dt \\
\int \frac{k}{2m}x^2dx &= \int at + C_1dt \\
\frac{k}{6m}x^3 &= \frac{a}{2}t^2 + C_1t + C_2 \\
x = \sqrt[3]{\left(\frac{a}{2}t^2 + C_1t + C_2\right)\frac{6m}{k}} & \quad \square.
\end{aligned}$$

Code:

```
x0 = [0; 0];
tspan = [0 10];
[t, x] = ode45(@harmonic_oscillator, tspan, x0);

figure;
subplot(2,1,1);
plot(t, x(:,1), 'b', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Displacement x(t) (m)');
title('Displacement vs. Time');
legend('x(t)');

subplot(2,1,2);
plot(t, x(:,2), 'r', 'LineWidth', 2);
xlabel('Time (s)');
ylabel('Velocity \dot{x}(t) (m/s)');
title('Velocity vs. Time');
legend('\dot{x}(t)');

grid on;

function dxdt = harmonic_oscillator(t, x)
    a = 1;
    m = 0.5;
    k = 3;
```

```
            dxdt = zeros(2,1);
            dxdt(1) = x(2);
            dxdt(2) = a - (k/m)*x(1);
        end

```

Plot: