ENAE 311H: Homework 02

Due on September 27, 2024 at 05:00 PM

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Problem 1

Mach number: $M = 3.6$, Altitude temperature: $T = 221\,\mathrm{K}$.

$$a = \sqrt{1.4 \times 287 \times 221}\,\mathrm{m/s}$$

$$V = M \times a$$

Free-stream temperature: $T_\infty = 120\,\mathrm{K}$, Free-stream velocity: $V_\infty = 790.5\,\mathrm{m/s}$.

$$a_\infty = \sqrt{1.4 \times 287 \times 120}\,\mathrm{m/s}$$

$$M_\infty = \frac{V_\infty}{a_\infty}$$

$$Re = \frac{\rho V L}{\mu}$$

$$Re_{\text{SR-71}} = Re_{\text{model}}$$

Speed of sound for the SR-71: $297.99\,\mathrm{m/s}$
Velocity of the SR-71: $1072.76\,\mathrm{m/s}$
Mach number in the wind tunnel: 3.60
Speed of sound in the wind tunnel: $219.58\,\mathrm{m/s}$
The Mach number in both the SR-71 flight and the wind tunnel experiment is essentially the same ($M = 3.6$), meaning that **the flows are Mach-similar**.
Reynolds number for the SR-71: 0.86 Reynolds number for the wind tunnel: 0.65 Viscosity ratio (SR-71 to wind tunnel): 1.50
Since the Reynolds numbers are not equal, the flows are **not dynamically similar**.

Problem 2

Step 1: Calculate speed of sound in flight:
$\mathrm{a}_{\text{flight}} = \sqrt{\gamma R T_{\text{flight}}}$
$a_{\text{flight}} = \sqrt{1.4 \times 287 \times 217} = 295.28\,\mathrm{m/s}$

Step 2: Calculate Mach number in flight:
$\mathrm{M}_{\text{flight}} = \frac{V_{\text{flight}}}{a_{\text{flight}}} = \frac{400}{295.28} = 1.3546$

Step 3: Calculate speed of sound in wind tunnel based on guessed temperature $\mathrm{T}_{\text{wind}} = 300\,\mathrm{K}$:
$a_{\text{wind}} = \sqrt{\gamma R T_{\text{wind}}}$
$a_{\text{wind}} = \sqrt{1.4 \times 287 \times 300} = 347.19\,\mathrm{m/s}$

Step 4: Calculate required velocity for Mach similarity in wind tunnel:
$\mathrm{V}_{\text{wind}} = M_{\text{flight}} \times a_{\text{wind}} = 1.3546 \times 347.19 = 470.32\,\mathrm{m/s}$

Step 5: Calculate density in wind tunnel using ideal gas law:
$\rho_{\text{wind}} = \frac{P_{\text{wind}}}{R T_{\text{wind}}} = \frac{75000}{287 \times 300} = 0.871\,\mathrm{kg/m^3}$

Step 6: Calculate Reynolds numbers:
$\mu \propto T^{2/3}$
$\mu_{\text{ratio}} = \frac{T_{\text{flight}}^{2/3}}{T_{\text{wind}}^{2/3}} = \frac{217^{2/3}}{300^{2/3}} = 0.8058$

Reynolds number for flight:
$\mathrm{Re}_{\text{flight}} = \frac{\rho_{\text{flight}} V_{\text{flight}} L_{\text{flight}}}{\mu_{\text{flight}}} = \frac{0.30 \times 400 \times 1}{217^{2/3}} = 3.323$

Reynolds number for wind tunnel (with L = 1/5):
$\mathrm{Re}_{\text{wind}} = \frac{\rho_{\text{wind}} V_{\text{wind}} L_{\text{wind}}}{\mu_{\text{wind}}} = \frac{0.871 \times 470.32 \times \frac{1}{5}}{300^{2/3}} = 1.828$

Step 7: Solve for temperature to match Reynolds number similarity:
$\mathrm{Re}_{\text{flight}} = Re_{\text{wind}}$
$\Rightarrow \frac{\rho_{\text{flight}} V_{\text{flight}}}{T_{\text{flight}}^{2/3}} = \frac{\rho_{\text{wind}} V_{\text{wind}}}{T_{\text{wind}}^{2/3}}$
$T_{\text{wind}} = 179.77\,\mathrm{K}$

Final calculations:
$\mathrm{T}_{\text{wind}} = 179.77\,\mathrm{K}$
$V_{\text{wind final}} = 1.3546 \times 268.83 = 364.07\,\mathrm{m/s}$
$\rho_{\text{wind final}} = \frac{75000}{287 \times 179.77} = 1.454\,\mathrm{kg/m^3}$