ENAE 311H: Homework 01

Due on September 10, 2024 at 05:00 PM

Dr. Brehm, Section 0101

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Problem 1

Consider an infinitely thin flat plate with a $1\,\text{m}$ chord at an angle of attack of $15°$ to an oncoming flow. The pressure distributions on the upper and lower surfaces are given by $p_u = 2\text{e}4\,(x-1) + 2.7\text{e}4$ and $p_l = 1\text{e}4\,(x-1) + 1.1\text{e}5$, where x is the distance from the leading edge along the chord; the shear stress distributions are $\tau_u = 144x^{-0.3}$ and $\tau_l = 360x^{-0.3}$. Here, the units of p and τ are $\frac{\text{N}}{\text{m}^2}$. Calculate the normal and axial forces, the lift and drag, moments about the leading edge and quarter chord, all per unit span, as well as the center of pressure.

Solution
Part A
Normal Force is calculated here by integrating the pressure difference between the upper and lower faces over the chord length.

$$N = \int_0^1 p_l(x) - p_u(x)\mathrm{d}x = 88\,\text{kN} \quad \square$$

Part B
Axial Force is calculated here by integrating the shear stress on the upper and lower faces over the chord length.

$$A = \int_0^1 \tau_l(x) - \tau_u(x)\mathrm{d}x = 720\,\text{N} \quad \square$$

Part C
Lift is computed here using the Normal and Axial forces and the angle of attack α.

$$L = N\cos\alpha - A\sin\alpha = 81.815\,12\,\text{kN} \quad \square$$

Part D
Drag is computed here using the Normal and Axial forces and the angle of attack α.

$$D = A\cos\alpha + N\sin\alpha = 23.471\,54\,\text{kN} \quad \square$$

Part E
Moment about the leading edge is calculated here by integrating the pressure difference in the same way as we calculated the Normal Force, but also taking into account the position along the chord.

$$M_{LE} = \int_0^1 x\,(p_l(x) - p_u(x))\,\mathrm{d}x = 43.166\,67\,\text{kN m} \quad \square$$

Part F
Moment at quarter-chord is calculated here by integrating the pressure difference in the same way as we calculated the Moment about the leading edge, but now at the quarter-chord, and not the leading edge.

$$M_{\frac{c}{4}} = \int_0^1 (x - 0.25)\,(p_l(x) - p_u(x))\,\mathrm{d}x = 21.166\,67\,\text{kN m} \quad \square$$

Part G
Center of Pressure is calculated here by dividing the Moment about the leading edge by the Normal Force.

$$x_{COP} = \frac{M_{LE}}{N} = 0.49\,\text{m} \quad \square$$

Problem 2

A series of experiments is performed on a two-dimensional airfoil in which the lift, drag, and moment coefficients (the latter about the quarter chord) are measured over a range of angles of attack from $0°$ to $10°$. The lift coefficient curve is found to be well approximated by the equation:
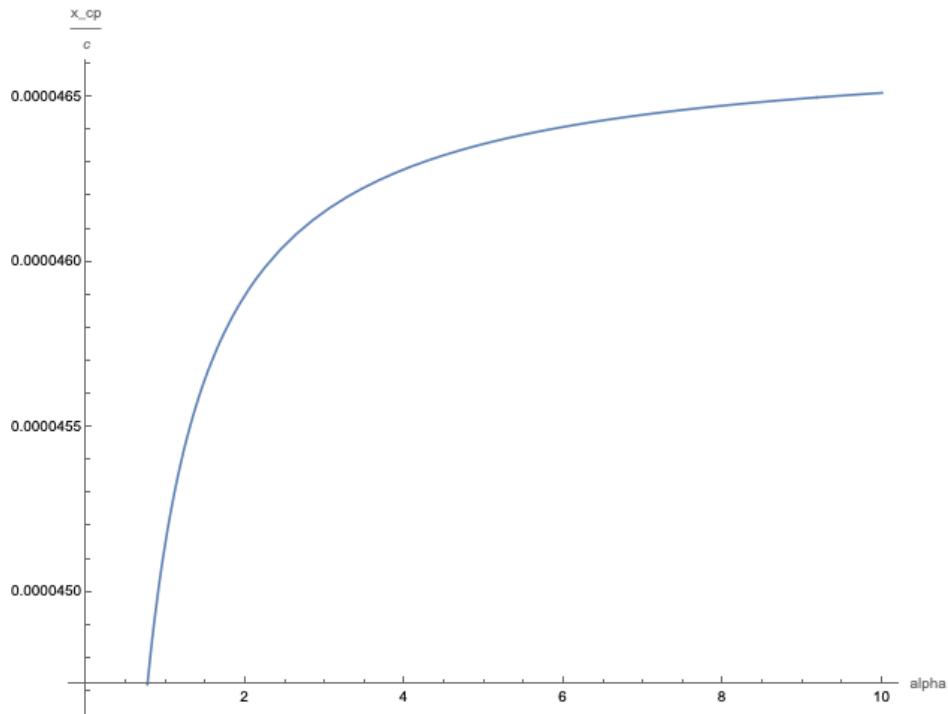
$$c_l = 0.2 + 6\alpha,$$

where α is the angle of attack in radians. The drag is found to be well approximated by

$$c_d = 0.006 + 0.3\alpha^2$$

while $c_{m,\frac{c}{4}}$ increases linearly from -0.04 for $\alpha = 0°$ to -0.03 for $\alpha = 10°$. Make a plot of $\frac{x_{cp}}{c}$ as a function of α for this airfoil.
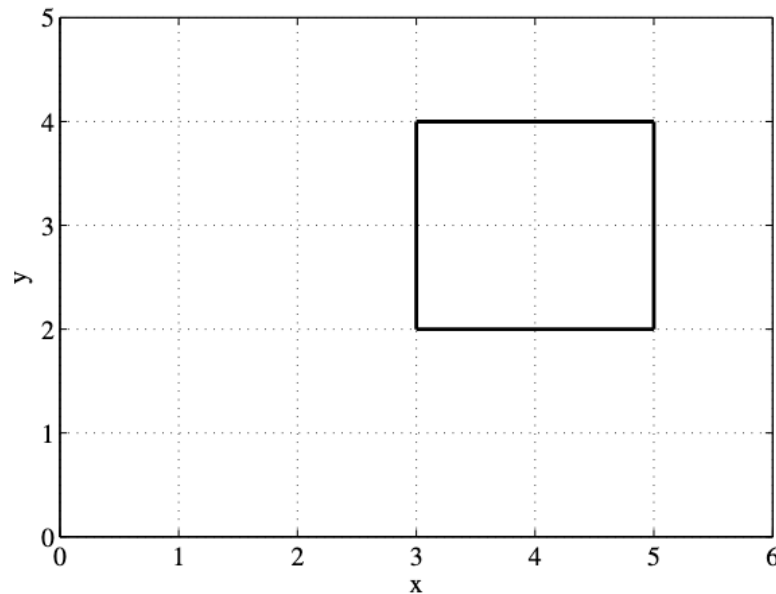
Solution

Problem 3

Given the pressure distribution $p(x, y) = x^2 + 2y^2 + 50$ acting on the body shown below, calculate the following:

1. The pressure force per unit depth on each face of the body.

2. The magnitude and direction of the net force per unit depth on the body.

3. The magnitude and direction of the net moment per unit depth on the body acting about the origin of the coordinate system.

4. The location of the object's center of pressure with respect to the origin.

Solution
Part A
Face 1 coordinates: $x = 3, y = 2 : 4$.
Face 2 coordinates: $x = 3 : 5, y = 2$.
Face 3 coordinates: $x = 3 : 5, y = 4$.
Face 4 coordinates: $x = 5, y = 2 : 4$.

$$p(x, y) = x^2 + 2y^2 + 50$$

$$F_1 = \int_2^4 p(3, y)\mathrm{d}y = 155.333$$

$$F_2 = \int_3^5 p(x, 2)\mathrm{d}x = 148.667$$

$$F_3 = \int_3^5 p(x, 4)\mathrm{d}x = 196.667$$

$$F_4 = \int_2^4 p(5, y)\mathrm{d}y = 187.333 \quad \Box$$

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Part B
Net Force is given by the vector sum of the pressure force on each face.

$$\vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4}$$

$$\vec{F} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} F_1 & F_4 \\ F_2 & F_3 \end{bmatrix} = \begin{bmatrix} 342.667 \\ 345.333 \end{bmatrix}$$

$$\vec{F} = \sqrt{F_x^2 + F_y^2} = 486.493, 45.222° \quad \square$$

Part C
Net Moment requires is given by reintegrating the pressure force on each face, now factoring in the distance from the origin of the coordinate system.

$$p(x, y) = x^2 + 2y^2 + 50$$

$$M_1 = \int_2^4 p(3, y) * 3\mathrm{d}y = 466$$

$$M_2 = \int_3^5 p(x, 2) * 2\mathrm{d}x = 297.333$$

$$M_3 = \int_3^5 p(x, 4) * 4\mathrm{d}x = 786.667$$

$$M_4 = \int_2^4 p(5, y) * 5\mathrm{d}y = 936.667$$

$$\vec{M} = M_1 - M_2 + M_3 - M_4 = 18.667, \text{anti-clockwise} \quad \square$$

Part D
COP can be calculated via $\sum \frac{M_i}{F_i}$

$$x_{cp} = \frac{M_1 + M_4}{F_1 + F_4} = 4.0934$$

$$y_{cp} = \frac{M_2 + M_3}{F_2 + F_3} = 3.1390$$

$$COP = (x_{cp}, y_{cp}) = (4.0934, 3.1390) \quad \square$$

Problem 4

Consider the steady flow of viscous incompressible fluid through a smooth, square pipe. The friction between the pipe wall and the fluid will result in a drop in pressure, Δp, from one end of the pipe to the other that will depend on the length and width of the pipe, l and d, the density and coefficient of viscosity of the fluid, ρ and μ, and the flow velocity, V, i.e., $\Delta p = f(l, d, \rho, \mu, V)$. Use the Buckingham Pi theorem to show that:

$$\frac{\Delta p}{\rho V^2} = F(Re, \frac{l}{d}),$$

where Re is the Reynolds number, $Re = \frac{\rho V d}{\mu}$. If we extended the analysis to include pipe roughness (with a characteristic roughness height ϵ), how would the above equation be modified?

Solution
Part A

Pressure drop Δp depends on the following:

$$\Delta p : \left[ML^{-1}T^{-2} \right]$$
$$l : [L]$$
$$d : [L]$$
$$\rho : \left[ML^{-3} \right]$$
$$\mu : \left[ML^{-1}T^{-1} \right]$$
$$V : \left[LT^{-1} \right]$$

We will repeat ρ, V, d. Our dimensionless groups are as follows:

$$\pi_1 = \frac{\Delta p}{\rho V^2}$$
$$\pi_2 = \frac{\rho V d}{\mu} = Re$$
$$\pi_3 = \frac{l}{d}$$

We now have all of our dimensionless groups, and can relate $\pi_1 = F(\pi_2, \pi_3) \therefore$

$$\frac{\Delta p}{\rho V^2} = F(Re, \frac{l}{d}) \quad \square$$

Part B

Now including the roughness, we need another dimensionless group:

$$\pi_4 = \frac{\epsilon}{d}$$

This makes our new relationship $\pi_1 = F(\pi_2, \pi_3, \pi_4) \therefore$

$$\frac{\Delta p}{\rho V^2} = F(Re, \frac{l}{d}, \frac{\epsilon}{d}) \quad \square$$

Problem 5

Consider a long cylindrical wire that suddenly has a large electrical current passed through it, such that the wire explodes, depositing an energy per unit depth, E, into the surrounding air. This causes a cylindrically expanding shock wave to propagate outwards from the initial position of the wire. If E is large, the initial propagation rate of this shock is independent of the pressure and temperature of the gas (this is known as the strong-shock limit); the only gas property that is important then is the density, ρ.

In fact, other thermodynamic properties of the quiescent air enter into the problem through the specific heats at constant volume and pressure, c_v and c_p.

1. Using dimensional analysis, show that in the first case the shock velocity, V_s, decays with time t as

$$V_s = c\left(\frac{E}{\rho}\right)^{\frac{1}{4}} t^{-\frac{1}{2}},$$

   where c is a constant.

2. In the second case, use the Buckingham Pi theorem to show that

$$\frac{V_s^4 t^2 \rho}{E} = F(\gamma),$$

   where $\gamma = \frac{c_p}{c_v}$.

Solution
Part A
Shock Velocity here depends on the following:

$$V_s : \left[LT^{-1}\right]$$
$$t : [T]$$
$$E : \left[MLT^{-2}\right]$$
$$\rho : \left[ML^{-3}\right]$$

We want a relationship of the form:
$$V_s = cE^a \rho^b t^c$$
, where a, b, c are constants.

$$\left[M^a L^a T^{-2a}\right]\left[M^b L^{-3b}\right]\left[T^c\right] = \left[M^{a+b} L^{a-3b} T^{-2a+c}\right] = \left[LT^{-1}\right]$$

As such, we can lay out the following:

$$\left[\begin{array}{ccc|c} a & b & c & V_s \\ \hline 1 & 1 & 0 & 0 \\ 1 & -3 & 0 & 1 \\ -2 & 0 & 1 & -1 \end{array}\right]$$

We now have $a = \frac{1}{4}, b = -\frac{1}{4}, c = -\frac{1}{2}$, giving us $V_s = cE^{\frac{1}{4}}\rho^{-\frac{1}{4}}t^{-\frac{1}{2}}$ $\therefore$

$$V_s = c\left(\frac{E}{\rho}\right)^{\frac{1}{4}} t^{-\frac{1}{2}} \quad \square$$

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Part B

For the Buckingham Pi theorem, we know Shock Velocity here depends on the following:

$$V_s : \left[LT^{-1} \right]$$
$$t : [T]$$
$$E : \left[MLT^{-2} \right]$$
$$\rho : \left[ML^{-3} \right]$$
$$c_p : \left[L^2 T^{-2} K^{-1} \right]$$
$$c_v : \left[L^2 T^{-2} K^{-1} \right]$$

As such, $\gamma = \frac{c_p}{c_v}$ is dimensionless.

We will repeat V_s, t, ρ. Our dimensionless groups are as follows:

$$\pi_1 = \frac{V_s^4 t^2 \rho}{E}$$
$$\pi_2 = \gamma$$

Now, we simply have:

$$\frac{V_s^4 t^2 \rho}{E} = F(\gamma) \quad \square$$