University of Maryland at College Park
Dept. of Aerospace Engineering

ENAE 432: Aerospace Control Systems

Problem Set #8

Question 1:

We wish to design a feedback controller for the orientation of a space science satellite, whose dynamics can be written as

$$G(s) = \frac{1}{Is^2(s+2)}$$

where I is the rotational inertia, which is nominally 15.

a.) Design a lead compensator $H(s)$ so the the feedback loop will have a phase margin of $45°$ with a magnitude crossover at 0.2 rad/sec. Show the compensated open-loop frequency response, and determine the gain margin of this design.

b.) Do a Nyquist analysis for your design in a.) and formally prove that the closed-loop system will be stable. Verify by determining the exact location of the poles of $T(s)$ for your design.

c.) Use Matlab to obtain the response $y(t)$ when $y_d(t)$ is a unit step with your controller design. Quantify all relevant features of the transient response. What is the steady-state tracking error?

d.) Suppose that the actual inertia is uncertain, or changes during the operational lifetime of the satellite. Without changing the $H(s)$ in a.), determine by how much the inertia I in $G(s)$ could be different from its assumed value of 15 before the feedback loop would become unstable. Express your answer as a fraction of the nominal inertia value of 15. How does your result relate to the gain margin found in a?

e.) Determine the peak of the sensitivity magnitude diagram that results using your design in a (using the nominal inertia $I = 15$)

Question 2:

a.) Suppose that you are told to try to decrease the settling time of the controlled satellite, by increasing the magnitude crossover an order of magnitude to 2.5 rad/sec, while maintaining the same phase margin. Can this be accomplished with a lead compensator?

b.) Repeat Question #1 with this new crossover specification, using whatever modifications to the compensator structure you deem necessary.

c.) Compare the maximum required control inputs when $y_d(t)$ is a unit step using your new design, and compare with that obtained using the design in Question #1. Does the new design require larger magnitudes for $|u(t)|$? Why does this make sense physically?

Question 3:

As we will show shortly, in order to reject constant external disturbances (from wind guests, solar pressure, etc) the compensator $H(s)$ must contain at least one pole at the origin. Of course this pole will contribute $-90°$ of phase at all frequencies, so to counteract this and still be able to obtain net positive phase from $H(s)$ we will require an additional zero. Thus, let us consider redesigning the compensator from Q1 as

$$H(s) = \frac{K(s+z_1)(s+z_2)}{s(s+p_1)}$$

Repeat Question #1 using this new compensator structure, and determine the corresponding derivative free set of equations that describes how $u(t)$ is computed from $e(t)$ for your $H(s)$.

HINTS: In a situation like this, where certain parts of the compensator structure are *fixed* by design requirements, it is useful to "pretend" in the design process that the fixed terms are actually part of $G(s)$. Of course they aren't really, and must be "absorbed" back into the final definition of $H(s)$ at the end of the design process. In the present situation, this is equivalent to doing the design for K, z_1, z_2 and p_1 by initially moving the required pole at the origin for $H(s)$ into $G(s)$. Once these four parameters of $H(s)$ have been designed to satisfy the margin/crossover specifications with the modified $G(s)$, the pole added to $G(s)$ is moved back to $H(s)$ to complete the final design.

This works because the design process focuses on the properties of $L(s) = G(s)H(s)$, and from this perspective it is irrelevant whether a pole or zero in $L(s)$ comes from $G(s)$ or $H(s)$. Only in the final specification of $H(s)$, and its ultimate physical implementation as a piece of code, do we need to correctly account for the ZPK structure specific to $H(s)$.

In designing for the four compensator parameters, I'd suggest splitting the ϕ_{req} so that half is provided by a lead compensator design for z_1 and p_1, and the other half can then be obtained by properly locating z_2. Finally, the gain K can be chosen as usual to force $|L(j\omega_\gamma)| = 1$ at the desired crossover frequency.

(This suggested splitting of the positive phase contribution from the different factors in $H(s)$ is by no means optimal, and exploring other possibilities would be part of a more comprehensive design study.)