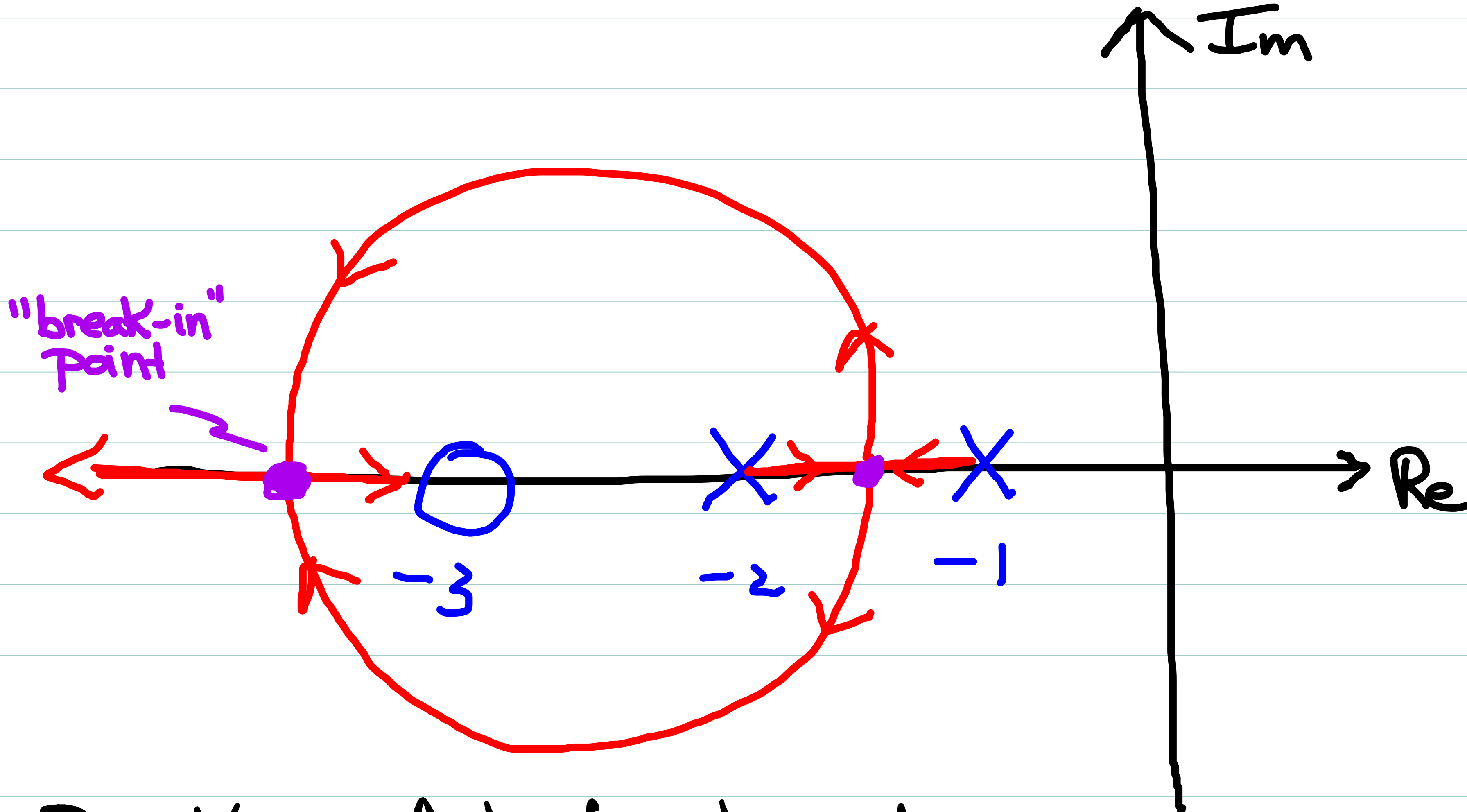
Example #6, cont



Break-in, like break-out satisfies

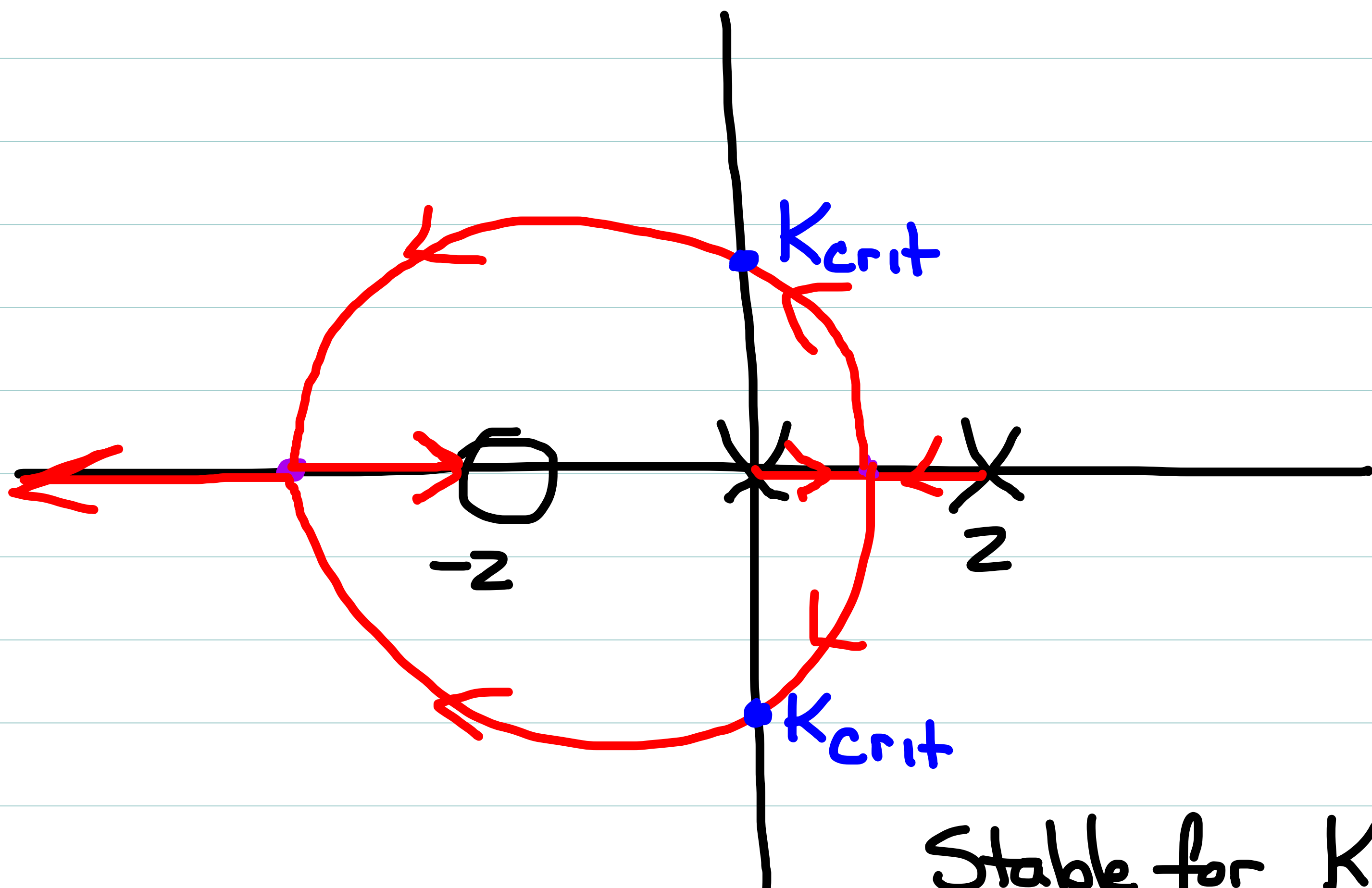
$$\frac{d}{ds} h(s) = 0$$

Example #7

$$L(s) = \frac{K(s+2)}{s(s-2)}$$

Same pole-zero pattern, shifted to the Right.

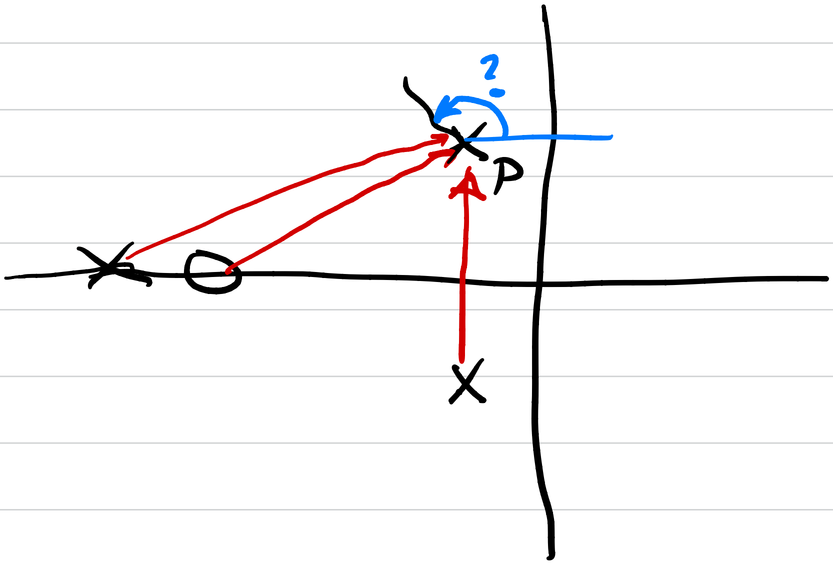
Similar analysis to above



Stable for $K > K_{crit}$

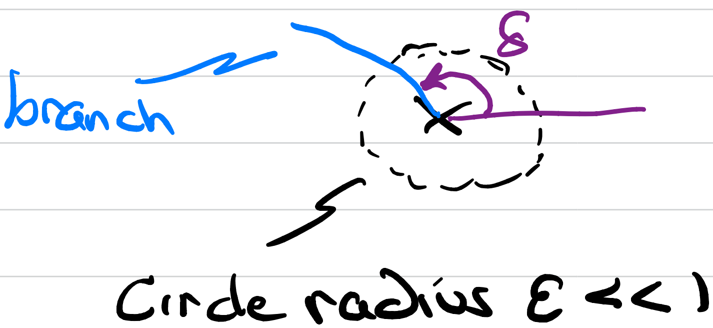
Root Locus: Add'l Rules

Angle of departure/arrival: complex pole/zero



We know branch will start at complex pole here, but what direction does this branch leave from the pole?

Consider a tiny circle around complex pole p



Branch leaving pole will puncture this tiny circle at angle δ

This is the "angle of departure"

What is departure angle δ?

Let $L'(s) = [(s-p)L(s)]$ (remove pole being examined from $L(s)$)

Then

$$\measuredangle L(s) = \measuredangle L'(s) - \measuredangle(s-p) \quad \text{for any } s$$

$$= (1+2l)180^\circ \quad (\text{if } s \text{ is a CL pole})$$

Since ε-circle is tiny compared to distance to other poles/zeros in $L'(s)$

Then

$$\measuredangle L'(s)\Big|_{\varepsilon\text{-circle}} \approx \measuredangle L'(p)$$

hence

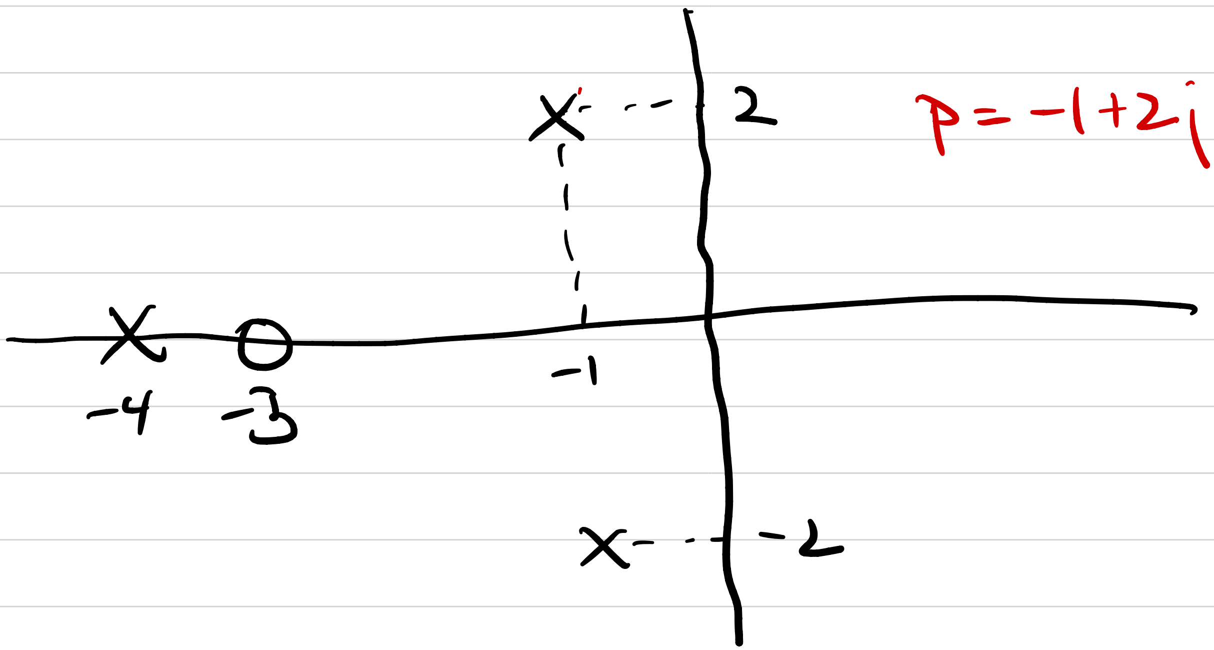
$$\measuredangle L(s)\Big|_{\varepsilon\text{-circle}} = \measuredangle L'(p) - \delta$$

$\delta = \measuredangle(s-p)$ on ε-circle.

$$\Rightarrow \quad \delta = \measuredangle L'(p) - (1+2l)180^\circ$$

choose l so $\delta \in [-180^\circ, +180^\circ]$

Example



$p = -1 + 2j$

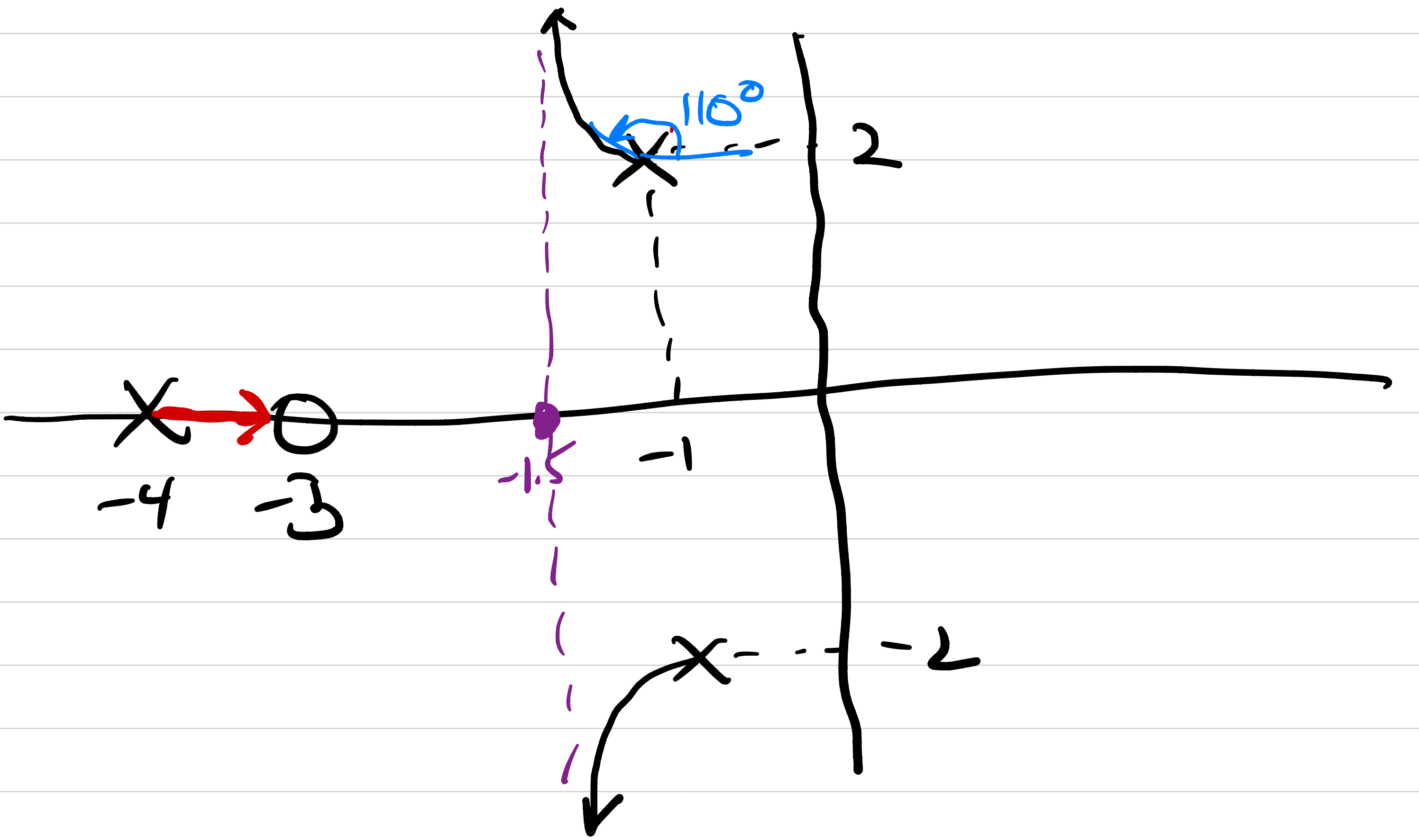
$$L'(s) = \frac{K(s+3)}{(s+4)(s-\bar{p})} \quad \text{here}$$

$$\angle L'(p) = \angle(2+2j) - \angle(3+2j) - \angle 4j$$

$$= 45^\circ - \tan^{-1}(2/3) - 90^\circ$$

$$\cong -78.7^\circ$$

$$\delta = -78.7^\circ + 180^\circ = \underline{\underline{101.3^\circ}}$$



=

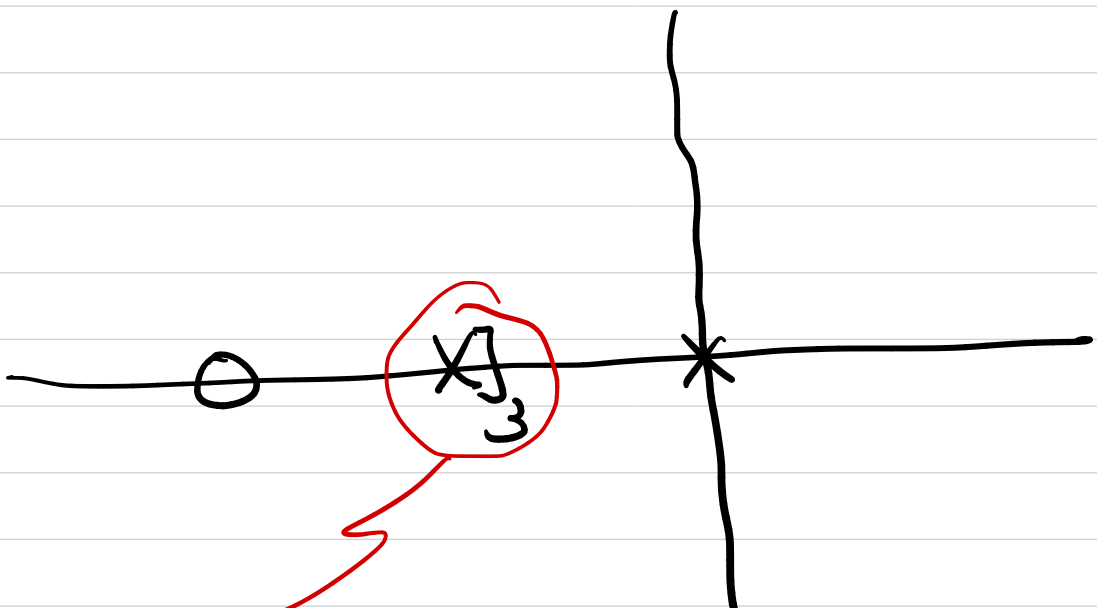
Similar calculations apply to arrival angle at complex zero.

Here $L'(s) = \left[\left(\frac{1}{s-z}\right) L(s)\right]$ (remove zero from $L(s)$)

and arrival angle δ satisfies

$$\delta = (1+2\ell)180^\circ - \measuredangle L'(z)$$

Angle of departure/arrival multiple poles



3 branches leave.
One to Right on real axis, what about other 2?

Argument is similar. Suppose p repeated q times.

Let $L'(s) = [(s-p)^q L(s)]$ (remove repeated pole p from $L(s)$)

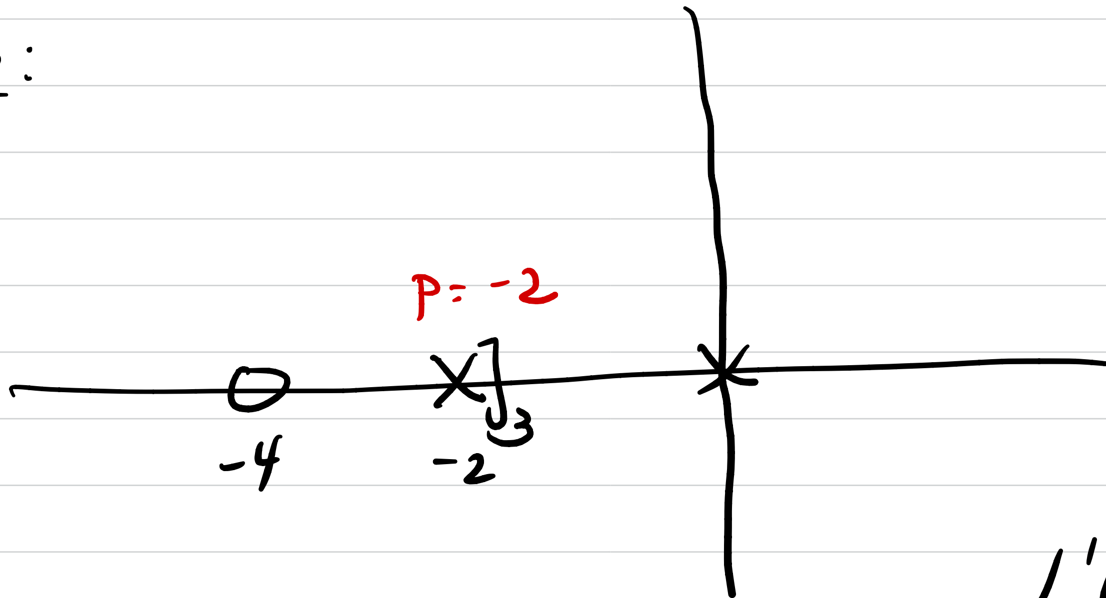
and consider tiny ε-circle around p.

Same calculation now gives

$$\delta = 1/q \left[\angle L'(p) - (1+2\ell)180^\circ \right]$$

Defines q unique angles

Example:



$$L'(s) = \frac{K(s+4)}{s} \text{ here}$$

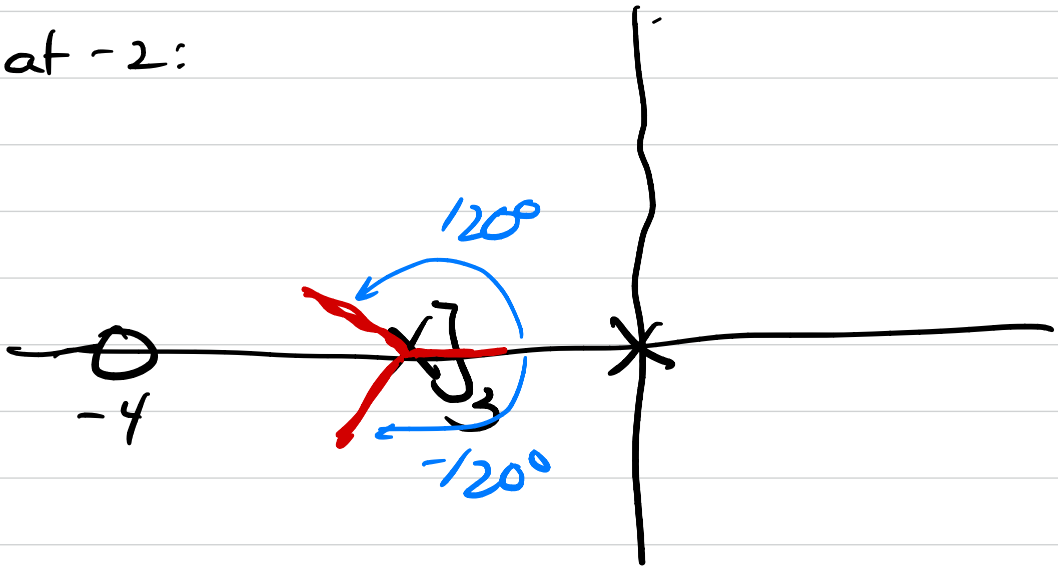
$$\measuredangle L'(p) = \measuredangle\left(\frac{2}{-2}\right) = -180^\circ$$

with $\ell = -1$, $\delta = 1/3\,[-180^\circ - (-180^\circ)] = 0^\circ$

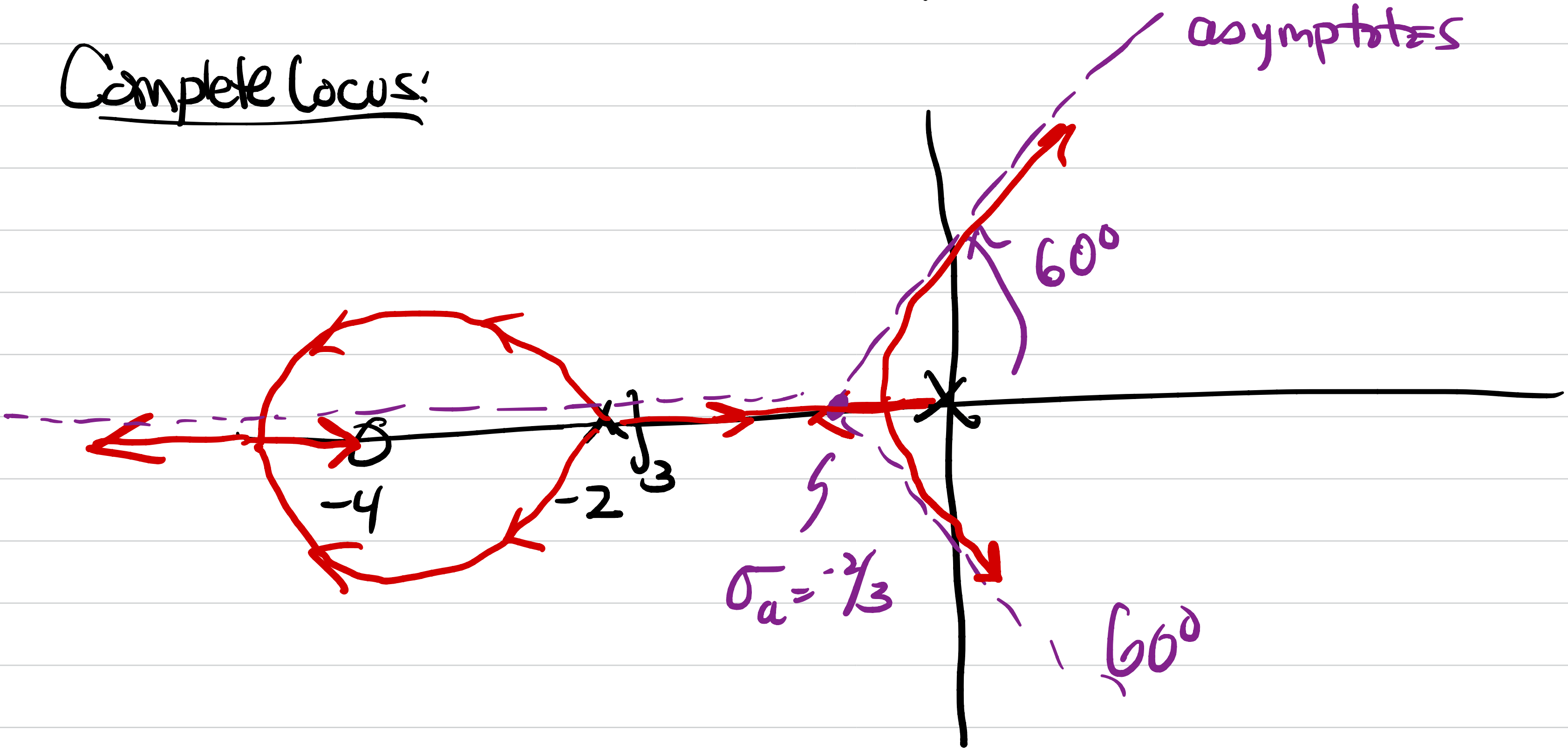
$\ell = 0$, $\delta = 1/3\,[-180^\circ - 180^\circ] = -120^\circ$

$\ell = -2$, $\delta = 1/3\,[-180^\circ - (-540^\circ)] = +120^\circ$

Branches at −2:



Complete Locus:



Again, similar considerations apply to calculating arrival angles for branches at a repeated zero:

Let $\angle'(s) = \left[\frac{1}{(s-z)^q}\angle(s)\right]$

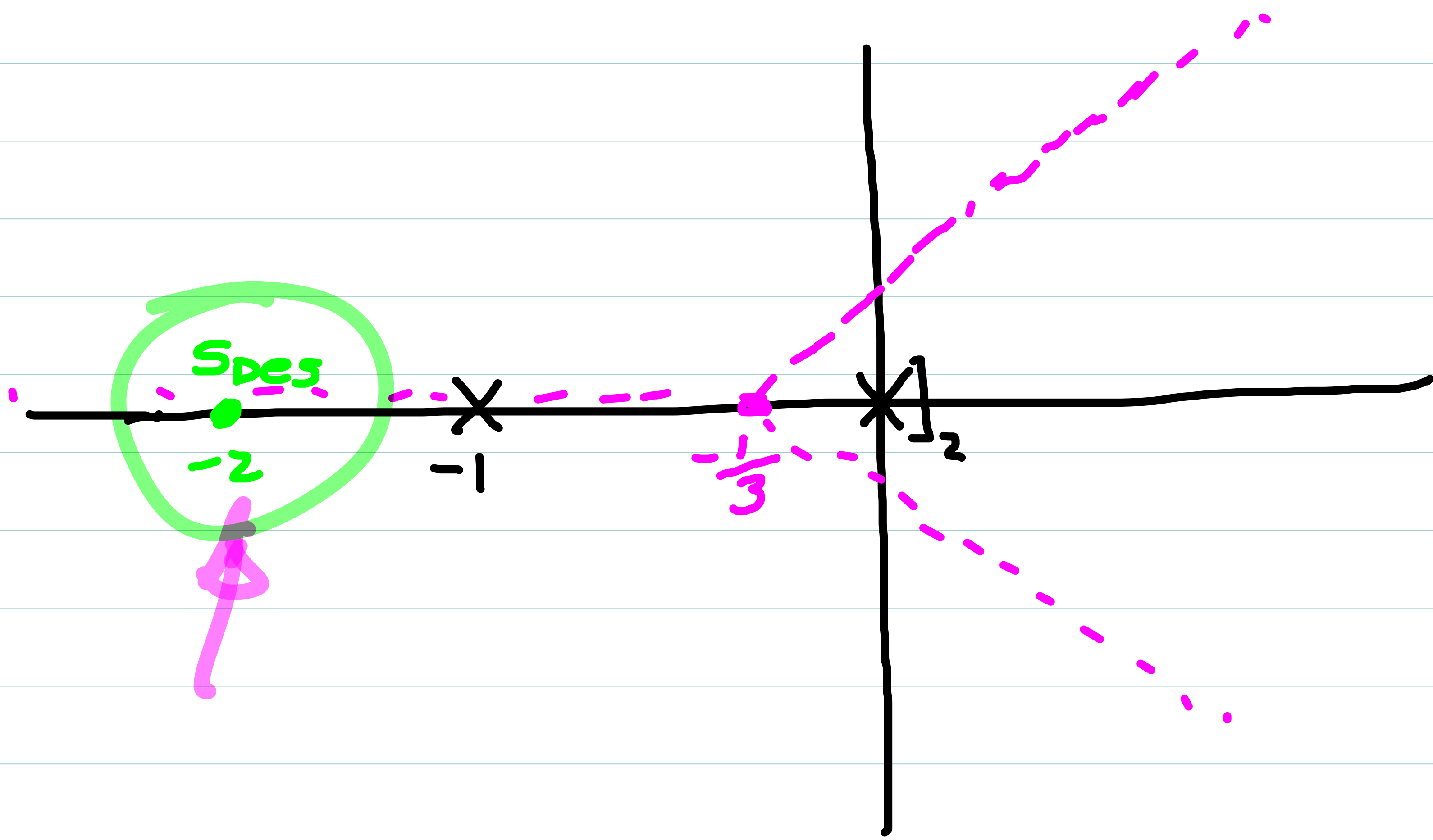
and then

$$\delta = 1/q\left[(1+2\ell)180° - \angle'(z)\right]$$ | q unique directions

Example #3

This is where we originally started our investigation

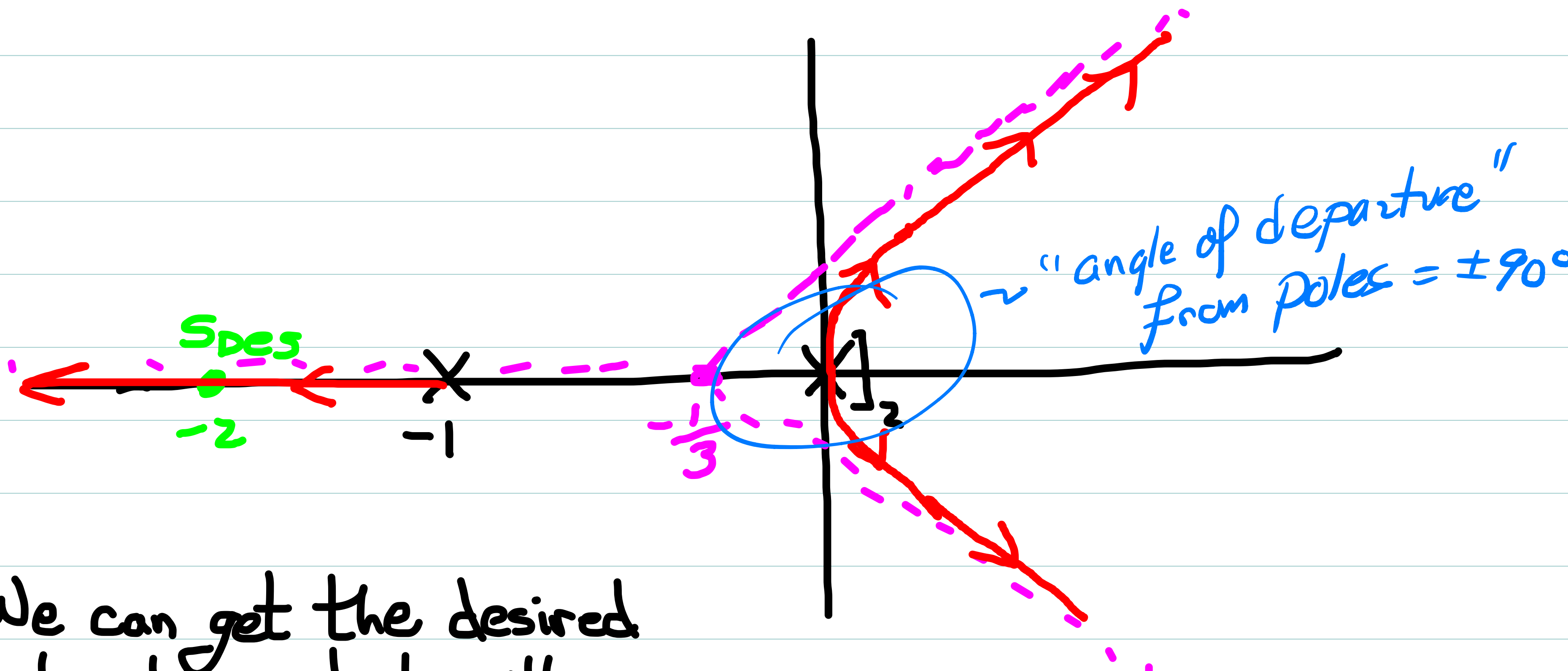
with $H(s) = K$, $L(s) = \dfrac{K}{s^2(s+1)}$



Example #3

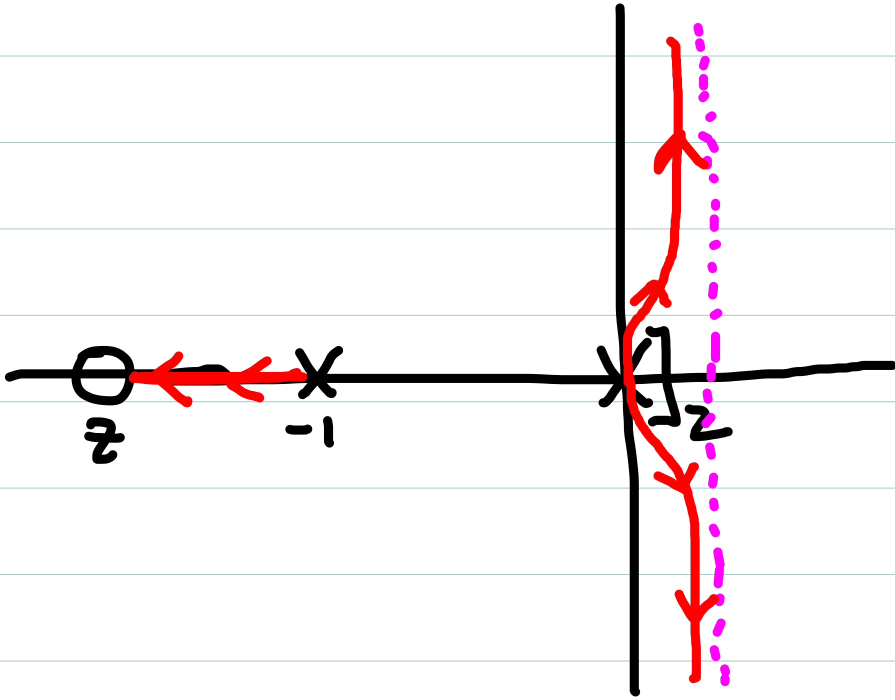
This is where we originally started our investigation

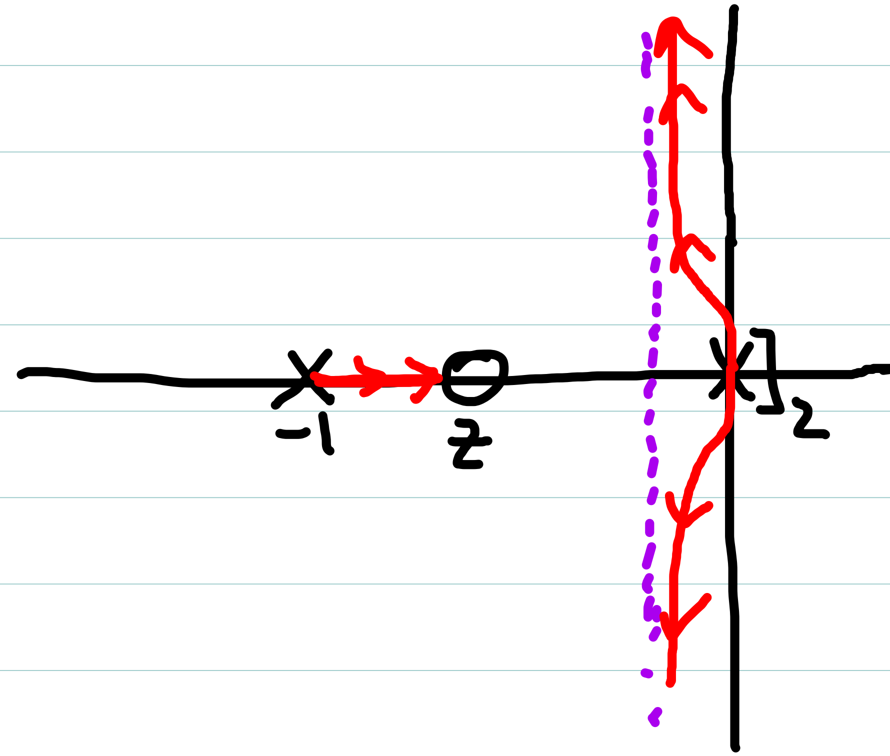
$$L(s) = \frac{K}{s^2(s+1)}$$



We can get the desired pole at -2, but will inevitably have poles of $T(s)$ in RHP

With instead $H(s) = K(s-z)$ (PD compensator)





$z < -1 \Rightarrow \alpha_\ell = \pm 90^\circ$

$$\sigma_a = \frac{-1}{2}(1+z) > 0$$

$0 < z < -1$

$\Rightarrow \sigma_a < 0$

So, with $H(s) = K(s-z)$ we can stabilize the system as long as $|z| < 1$ (which would agree with a Nyquist/phase margin analysis)

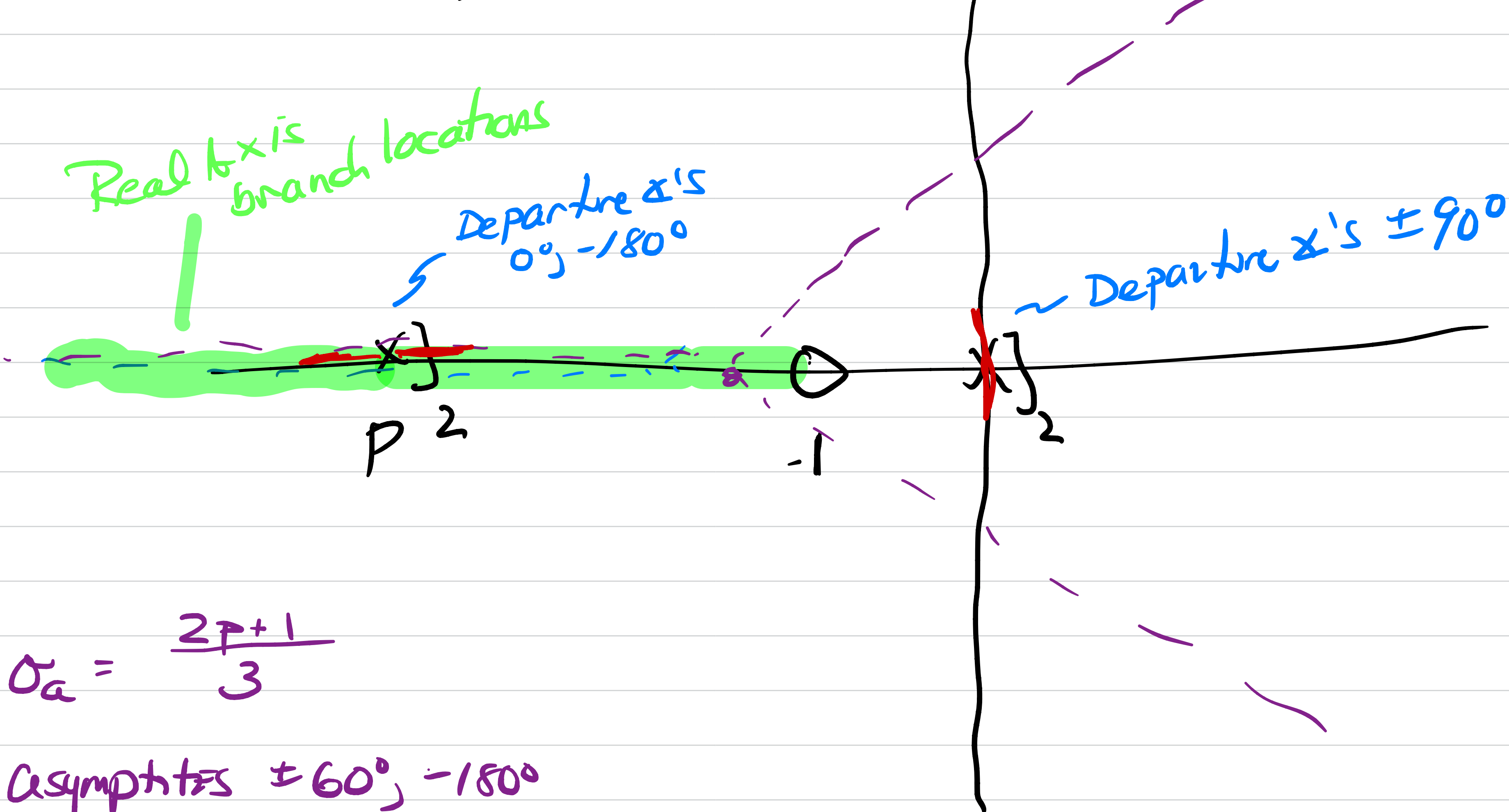
But we would have to accept a real pole > -1, and moreover this pole would not be dominant

An implementable compensator which could allow a real dominant CL pole near -2 would be

$$H(s) = K\left[\frac{(s+1)^2}{(s-p)^2}\right]$$

which has an interesting locus (next page)

$$L(s) = \frac{K(s+1)}{s^2(s-p)^2}$$

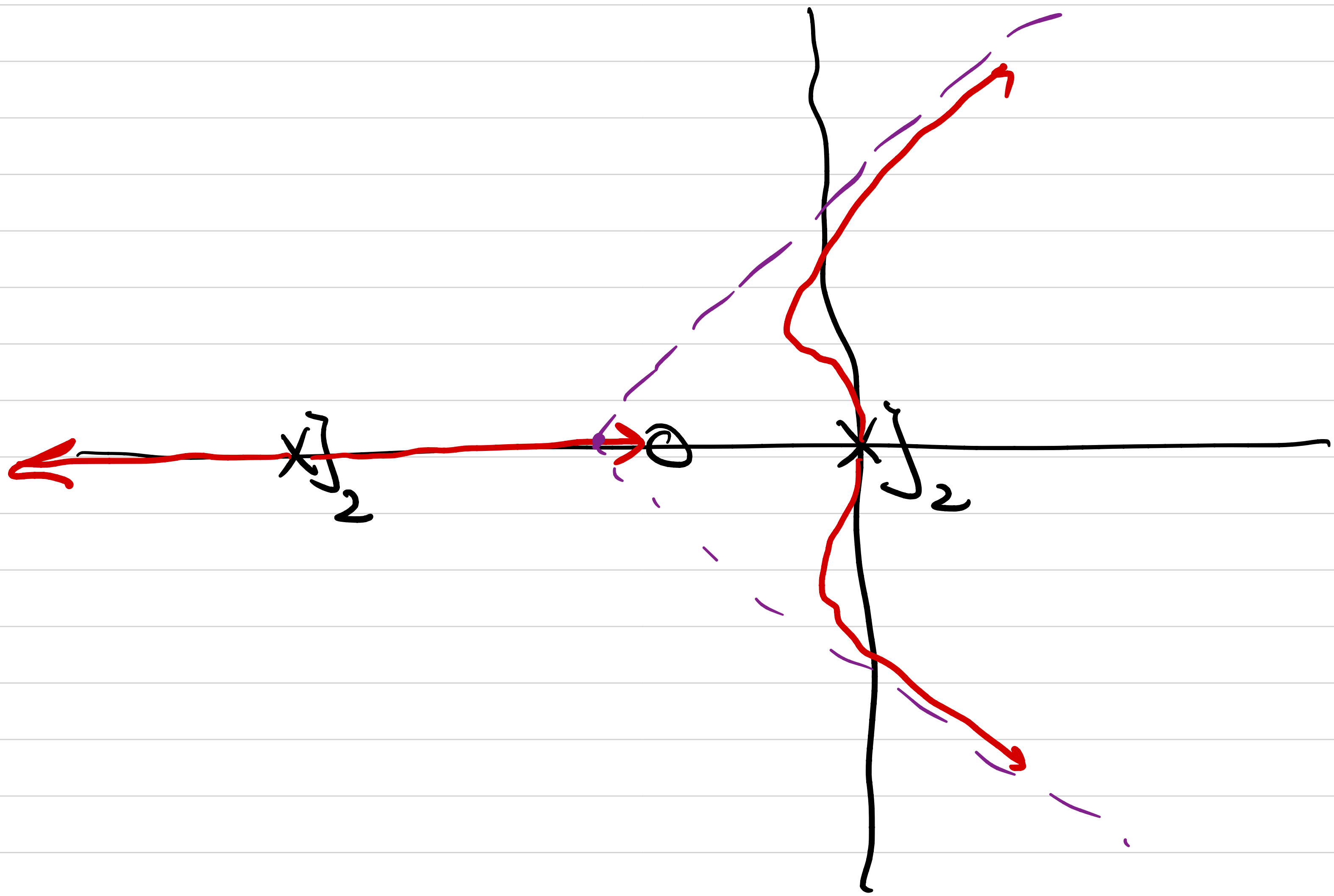


$$\sigma_a = \frac{2p+1}{3}$$

asymptotes $\pm 60°, -180°$

$(n-m=3 \text{ here})$

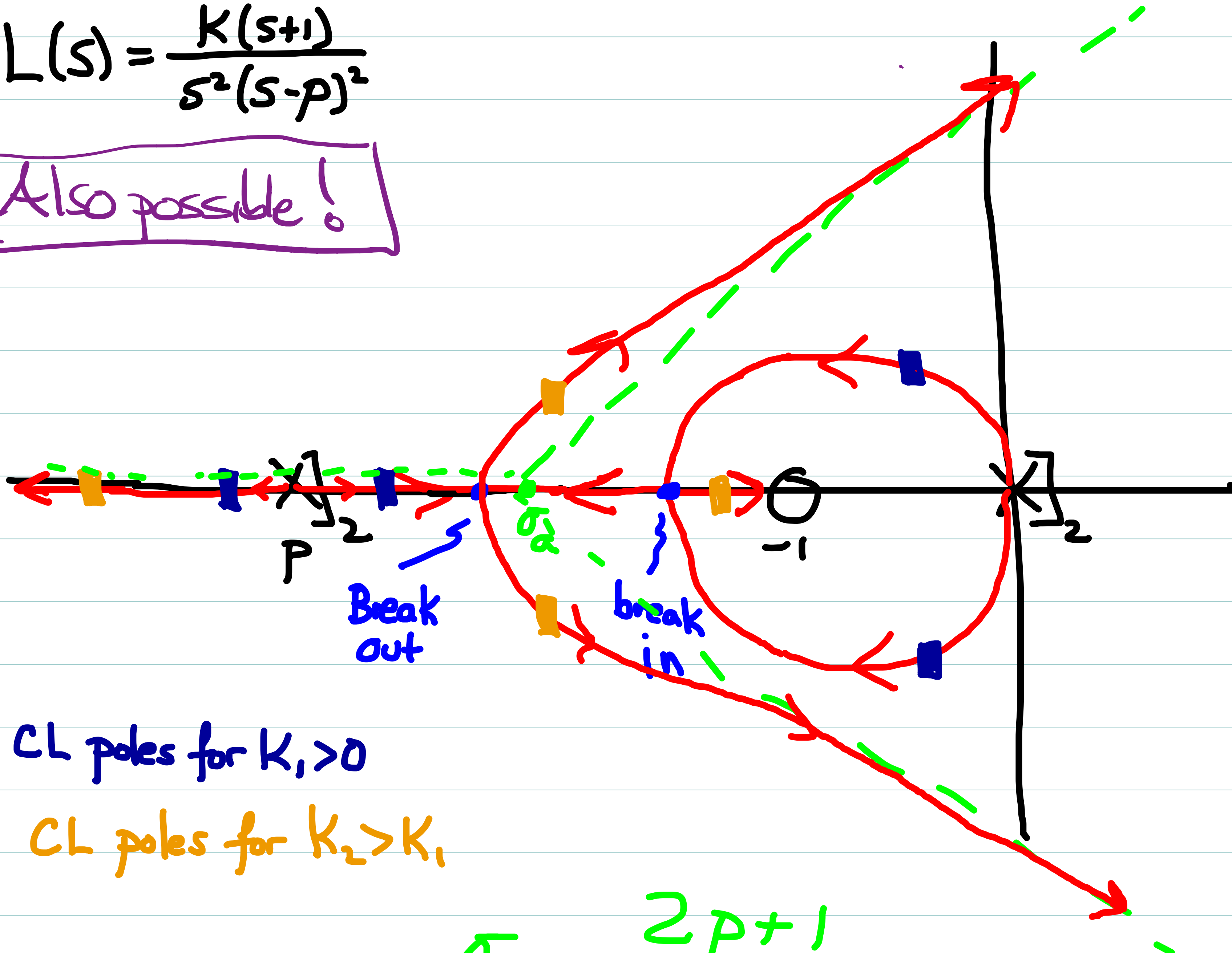
So, we could have this:



Simplest shape consistent w/rules, but not the only one!

$$L(s) = \frac{K(s+1)}{s^2(s-p)^2}$$

$\boxed{\text{Also possible!}}$



■ CL poles for $K_1 > 0$

■ CL poles for $K_2 > K_1$

$$\sigma_a = \frac{2p+1}{3}$$

- Which shape we get is highly dependent on exact location of poles in $H(s)$
- Simple rules are insufficient to identify which shape we get

  (But note the break-in calculations would show the differences, however they require factoring a 3rd order polynomial here)
- 2nd shape would get close to our requirements (dominant CL pole at -2), 1st shape would not.

Comments on root locus method

⇒ Rules are not determinative; there may be many locus shapes consistent with calculations (although Matlab rlocus command will show you an exact plot).

⇒ Cannot adapt method to account for effects of time delay

⇒ Can adapt method only for very simple kinds of robustness analysis.

⇒ Bode/Nyquist methods preferred in professional practice.

⇒ But root locus does provide useful additional insights which are not available using freq. methods

⇒ Familiarity with both gives "best of both worlds"