

### ENRE 447 Homework 2

Problems are similar to the book problems from Modarres & Groth 2023 (problem number in brackets). In some cases I have modified the problem slightly. Approved software ok, open everything. All policies of course syllabus and grading rubric apply.

1. Find the constant  $c$  so that  $f(x, y) = cxy$ ,  $0 \leq x, y \leq 1$ , and  $f(x, y) = 0$ , otherwise, is a joint pdf of  $X$  and  $Y$ . Are  $X$  and  $Y$  independent? Find  $E(X)$ ,  $E(Y)$ ,  $E(XY)$ ,  $\text{Var}(X)$ ,  $\text{Var}(Y)$ , and  $\text{Cov}(X, Y)$ .
2. [2.16] Given that  $\text{Pr}=0.006$  is the probability of an engine failure on a flight between two cities, find the probability of:
  - a. No engine failure in 1000 flights.
  - b. At least one failure in 1000 flights.
  - c. At least two failures in 1000 flights.
3. The manufacturer of a type pump states that, on average, this type of pump experiences 3.0 failures per 100,000 operational hours. At a factory with many of these pumps, they will accumulate 200,000 operational hours this year. Find the probability that there will be \_\_\_\_ pump failures at the factory this year.
  - a. 0
  - b. 2
  - c. 6
  - d. 8
  - e. Between 4 and 8
  - f. Fewer than 3.
4. If the diameter of a given kind of ball bearings are normally distributed with the mean 0.6140 inches and standard deviation 0.0025 inches, determine the percentage of ball bearings with diameters
  - a. Between 0.610 and 0.618 inches inclusive.
  - b. Greater than 0.617 inches.
  - c. Less than 0.608 inches.
  - d. Equal to 0.615 inches.
5. [3.1] Assume that  $T$ , the random variable that denotes life in hours of specified component, has a cumulative density function (cdf) of

$$F(t) = \begin{cases} 1 - \frac{100}{t} & t \geq 100 \\ 0 & t < 100 \end{cases}$$

Determine the following:

- a. PDF  $f(t)$
  - b. Reliability function  $R(t)$
  - c. MTTF (Using a practical upper limit of 1 million hrs to avoid trivial solution)
6. [3.13] A manufacturer uses the exponential distribution to model the number of cycles to for a product. The product has  $\lambda = 0.003$  failures/cycle,
    - a. What is the mean cycle to failure for this product?
    - b. If the product survives for 300 cycles, what is the probability that it will fail sometimes after 500 cycles? If operational data show that 1,000 components have

survived 300 cycles, how many of these would be expected to fail after 500 cycles?

7. [3.15] Time to failure of a relay follows a Weibull distribution with  $\alpha=10$  years,  $\beta=0.5$ . Find the following:
  - a.  $\Pr(\text{failure after 1 year})$
  - b.  $\Pr(\text{failure after 10 years})$
  - c. MTTF
8. [3.19] An electronic device has a time to failure modeled by the lognormal distribution with parameters  $\mu = 5.8$  and  $\sigma = 1.2$ .
  - a. Find the MTTF.
  - b. If this device is used in an application which requires it to be replaced when its reliability falls below 0.9, when should the device be replaced?
  - c. Find the hazard function for the device at the time calculated in (b).