Lecture 7: The Substantial Derivative and Conservation of Momentum

ENAE311H Aerodynamics I

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QUIZ

5.11 At cruise conditions, air flows into a jet engine at a steady rate of 65 lbm/s. Fuel enters the engine at a steady rate of 0.60 lbm/s. The average velocity of the exhaust gases is 1500 ft/s relative to the engine. If the engine exhaust effective cross section area is 3.5 ft^2, estimate the density of the exhaust gases in lbm/ft^3.

The substantial derivative

Consider again the differential form of the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0.$$

We can expand out the second term to give:

$$\nabla \cdot (\rho \mathbf{v}) = \mathbf{v} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{v}$$

Our original equation thus becomes

$$\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = -\rho \nabla \cdot \mathbf{v}$$

We can expand out the LHS to give

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} \equiv \frac{D\rho}{Dt}.$$

The derivative D/Dt is known as the total, substantial, or material derivative. It describes the rate of change of a property of a fluid element moving with the flow.

To see this, imagine we have a fluid element of density ρ, at position (x, y, z) at time t, i.e., $\rho(x, y, z, t)$.

At time $t + \Delta t$, it will have moved to $(x + \Delta x, y + \Delta y, z + \Delta z)$ and its density will be:

$$\begin{aligned} \rho|_{t+\Delta t} &= \rho(x + \Delta x, y + \Delta y, z + \Delta z, t + \Delta t) \\ &= \rho(x, y, z, t) + \frac{\partial \rho}{\partial x} dx + \frac{\partial \rho}{\partial y} dy + \frac{\partial \rho}{\partial z} dz + \frac{\partial \rho}{\partial t} dt \\ &\quad + H.O.T. \end{aligned}$$

The change in density is then

$$d\rho = \frac{\partial \rho}{\partial x} dx + \frac{\partial \rho}{\partial y} dy + \frac{\partial \rho}{\partial z} dz + \frac{\partial \rho}{\partial t} dt$$

and thus

$$\begin{aligned} \frac{d\rho}{dt} &= \frac{\partial \rho}{\partial x}\frac{dx}{dt} + \frac{\partial \rho}{\partial y}\frac{dy}{dt} + \frac{\partial \rho}{\partial z}\frac{dz}{dt} + \frac{\partial \rho}{\partial t} \\ &= v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} + \frac{\partial \rho}{\partial t} \\ &= \frac{D\rho}{Dt}. \end{aligned}$$

Applies to any intrinsic property of the flow!

Conservation of momentum

Let us return now to the Reynolds Transport Theorem:

$$\frac{dN_s}{dt} = \frac{\partial}{\partial t}\iiint_{CV} \eta\rho dV + \iint_{CS} \eta\rho\mathbf{v}\cdot\mathbf{dA},$$

Let us now consider the case of $N_s = \boldsymbol{P}$, i.e., the momentum of the fluid system. We have seen already that the corresponding intensive variable is the fluid velocity, i.e, $\eta = \boldsymbol{v}$.

From Newton's second law, the rate of change of momentum is equal to the sum of applied forces, i.e.,

$$\frac{d\mathbf{P_s}}{dt} = \sum \mathbf{F}$$

Applying the RTT, we thus have

$$\underbrace{\frac{\partial}{\partial t}\iiint_{CV} \rho\mathbf{v}dV}_{\text{Rate of change of momentum within CV}} + \underbrace{\iint_{CS} \mathbf{v}(\rho\mathbf{v}\cdot\mathbf{dA})}_{\text{Net momentum flux through CV boundaries}} = \underbrace{\sum \mathbf{F}}_{\text{Applied forces}}$$

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Applying the RTT, we thus have

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\mathbf{v}dV + \iint_{CS} \mathbf{v}(\rho\mathbf{v}\cdot\mathbf{dA}) = \sum \mathbf{F}$$

Note that this is a vector equation with three components. The x-component, for example, is

$$\frac{\partial}{\partial t}\iiint_{CV} \rho v_x dV + \iint_{CS} v_x\rho\mathbf{v}\cdot\mathbf{dA} = \sum F_x$$

Conservation of momentum (integral form)

The forces relevant here are of two types:

1. Surface forces (pressure and shear stress), for which we can write

$$\sum \mathbf{F_s} = -\iint_{CS} p\mathbf{dA} + \iint_{CS} \bar{\bar{\tau}} \cdot \mathbf{dA}$$

2. The gravity body force:

$$\sum \mathbf{F_b} = \iiint_{CV} \rho \mathbf{f} dV,$$

where $\boldsymbol{f} = -g\hat{\boldsymbol{j}}$ is the gravitational acceleration (assumed in the y direction).

Substituting into the momentum equation, we have

inviscid gravity negligible

$$\frac{\partial}{\partial t}\iiint_{CV} \rho \mathbf{v} dV + \iint_{CS} \mathbf{v}(\rho \mathbf{v} \cdot \mathbf{dA}) = -\iint_{CS} p\mathbf{dA} + \cancel{\iint_{CS} \bar{\bar{\tau}} \cdot \mathbf{dA}} + \cancel{\iiint_{CV} \rho \mathbf{f} dV}$$

Or, in x-direction (inviscid, no body force):

$$\frac{\partial}{\partial t}\iiint_{CV} \rho v_x dV + \iint_{CS} \rho v_x(\mathbf{v} \cdot \mathbf{dA}) = -\iint_{CS} p dA_x$$

Conservation of momentum (differential form)

We can derive a corresponding differential form by using similar arguments as in the continuity case.

Since the CV is spatially fixed, we have

$$\frac{\partial}{\partial t}\iiint_{CV} \rho\mathbf{v}dV = \iiint_{CV} \frac{\partial}{\partial t}(\rho\mathbf{v})dV.$$

From the divergence theorem:

$$\begin{aligned}\iint_{CS} \mathbf{v}(\rho\mathbf{v}\cdot\mathbf{dA}) &= \iiint_{CV} \nabla\cdot(\rho\mathbf{v}\mathbf{v})dV\\ \iint_{CS} \bar{\bar{\tau}}\cdot\mathbf{dA} &= \iiint_{CV} \nabla\cdot\bar{\bar{\tau}}dV,\end{aligned}$$

And from the gradient theorem:

$$\iint_{CS} p\mathbf{dA} = \iiint_{CV} \nabla p dV.$$

Substituting into the integral momentum equation:

$$\iiint_{CV}\left[\frac{\partial}{\partial t}(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) + \nabla p - \nabla\cdot\bar{\bar{\tau}} - \rho\mathbf{f}\right]dV = 0.$$

We argue, as before, that since the CV is arbitrary, the term in [] must be identically zero, i.e.,

$$\frac{\partial}{\partial t}(\rho\mathbf{v}) + \nabla\cdot(\rho\mathbf{v}\mathbf{v}) + \nabla p - \nabla\cdot\bar{\bar{\tau}} - \rho\mathbf{f} = 0.$$

A more useful form of this results if we expand the first two terms and use the continuity equation:

$$\boxed{\rho\frac{\partial\mathbf{v}}{\partial t} + \rho(\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla p - \nabla\cdot\bar{\bar{\tau}} - \rho\mathbf{f} = 0.}$$

Or alternatively

$$\boxed{\rho\frac{D\mathbf{v}}{Dt} + \nabla p - \nabla\cdot\bar{\bar{\tau}} - \rho\mathbf{f} = 0.}$$

Conservation of momentum (differential form)

We can derive a corresponding differential form by using similar arguments as in the continuity case.

Often we will have the case that the flow is (approximately) inviscid and the body forces are negligible, in which case these equations simplify to:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \frac{1}{\rho}\nabla p = 0$$

and

$$\frac{D\mathbf{v}}{Dt} + \frac{1}{\rho}\nabla p = 0$$

The x-component of this equation is

$$\frac{\partial v_x}{\partial t} + (\mathbf{v} \cdot \nabla)v_x + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

or

$$\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0.$$

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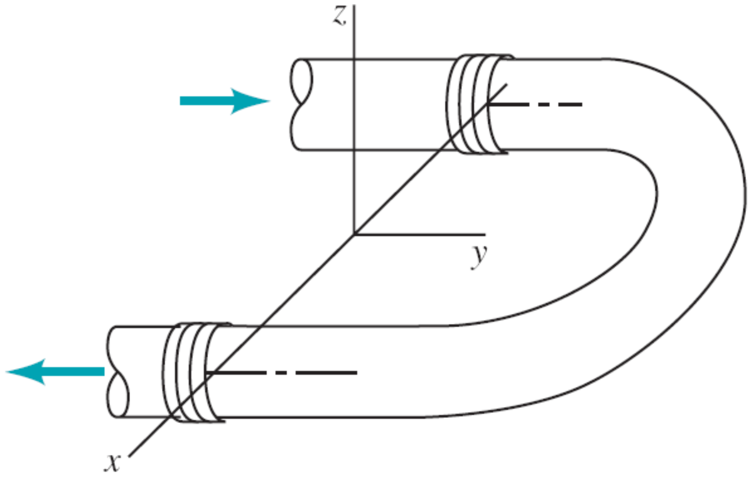
5.28 Water flows through a horizontal, 180° pipe bend as is illustrated in Fig. P5.28. The flow cross section area is constant at a value of 9000 mm^2. The flow velocity everywhere in the bend is 15 m/s. The pressures at the entrance and exit of the bend are 210 and 165 kPa, respectively. Calculate the horizontal (x and y) components of the anchoring force needed to hold the bend in place.



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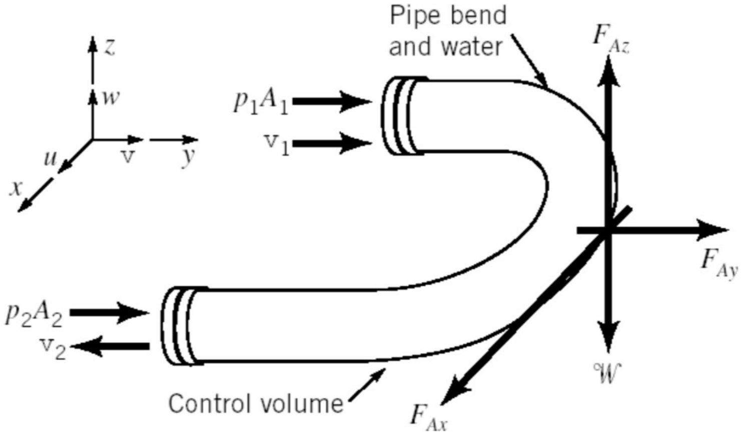


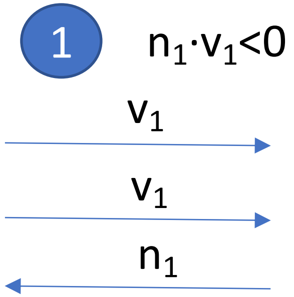
Step. 1 Select a proper CV: Inside of the valve

Step. 2 Find all forces acting on the CV (Free-body diagram)

Then, $\cancel{\dfrac{\partial}{\partial t}\int_{CV}\vec{V}\rho dV} + \int_{CS}\vec{V}\rho\vec{V}\cdot\hat{n}dA = \sum\vec{F}_{CV}$ (Steady flow)

$\vec{V}\perp\hat{n}$ at the side wall

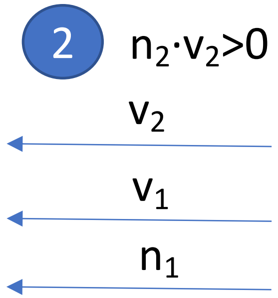
1. $x-$comp. : $\int_{CS}u\rho\vec{V}\cdot\hat{n}dA = \int_{(1)}u_1\rho\vec{V}\cdot\hat{n}dA + \cancel{\int_{\text{Side}}u\rho\vec{V}\cdot\hat{n}dA} + \int_{(2)}u_2\rho\vec{V}\cdot\hat{n}dA = F_{Ax}$



No x component of fluid velocity at sections (1) and (2), ($u_1 = u_2 = 0$)

$$\therefore \int_{CS}u\rho\vec{V}\cdot\hat{n}dA = F_{Ax} = 0$$

2. $y-$comp. : $\int_{CS}v\rho\vec{V}\cdot\hat{n}dA = v_1\int_{(1)}\rho\vec{V}\cdot\hat{n}dA + v_2\int_{(2)}\rho\vec{V}\cdot\hat{n}dA = F_{Ay} + p_1A_1 + p_2A_2$



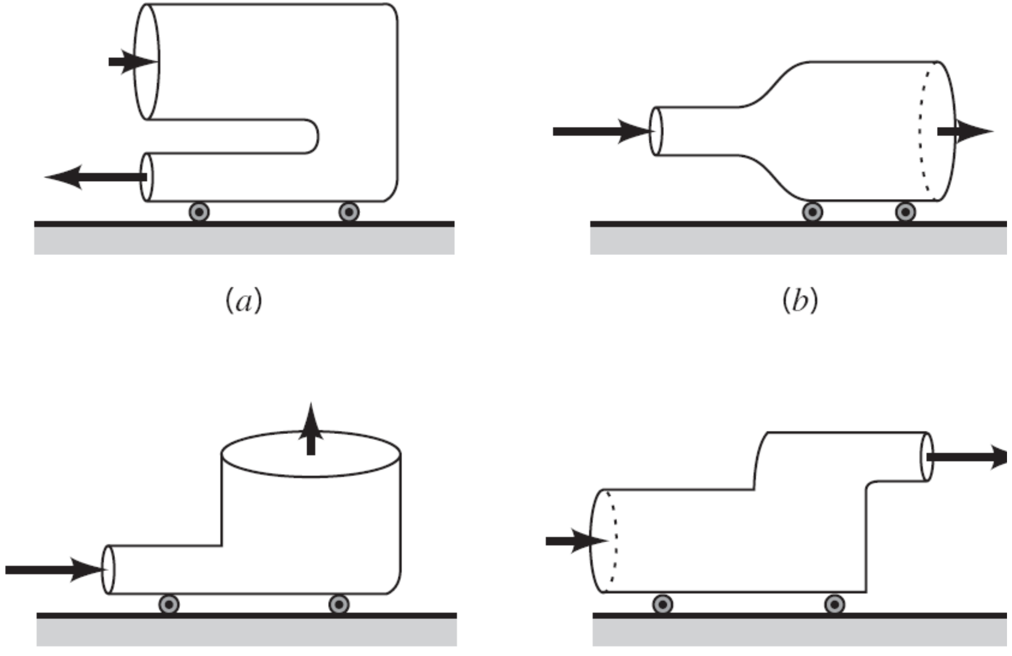
or $$(v_1)(-\dot{m}_1) + (-v_2)(\dot{m}_2) = F_{Ay} + p_1A_1 + p_2A_2$$

$$\therefore F_{Ay} = -\dot{m}(v_1 + v_2) - p_1A_1 - p_2A_2$$

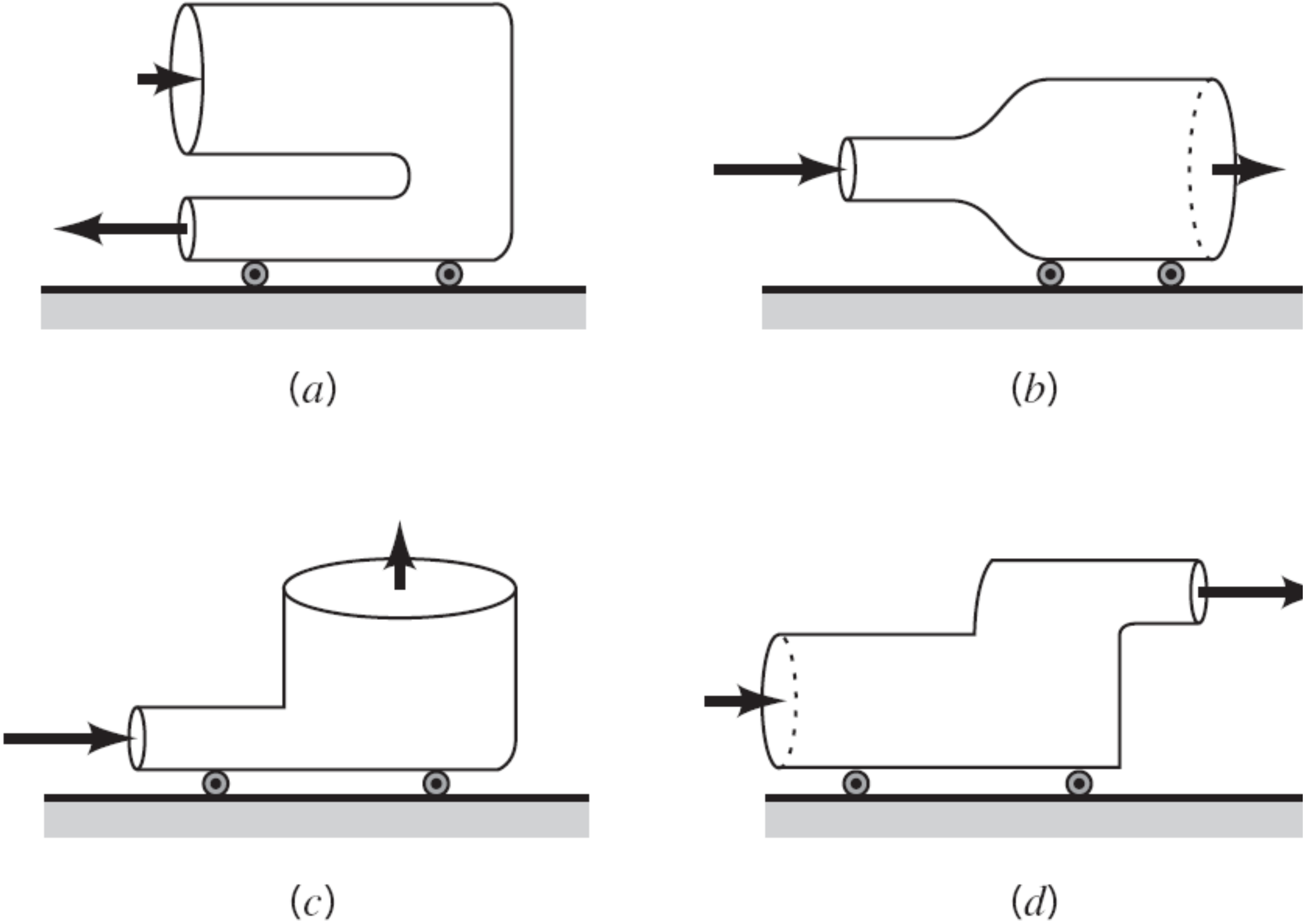
where $\dot{m} = \rho A_1 v_1 = (1.94)(0.1)(50) = 9.70$ slug/s

$p_1 = 30$ psia, $p_2 = 24$ psia, and $A_1 = A_2 = 0.1\ \text{ft}^2(144\ \text{in}^2/\text{ft}^2) = 14.4\ \text{in}^2$

5.58 The four devices shown in Fig. P5.58 rest on frictionless wheels, are restricted to move in the x direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.



(a) (b) (c) (d)


(a)
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(c)
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5.58

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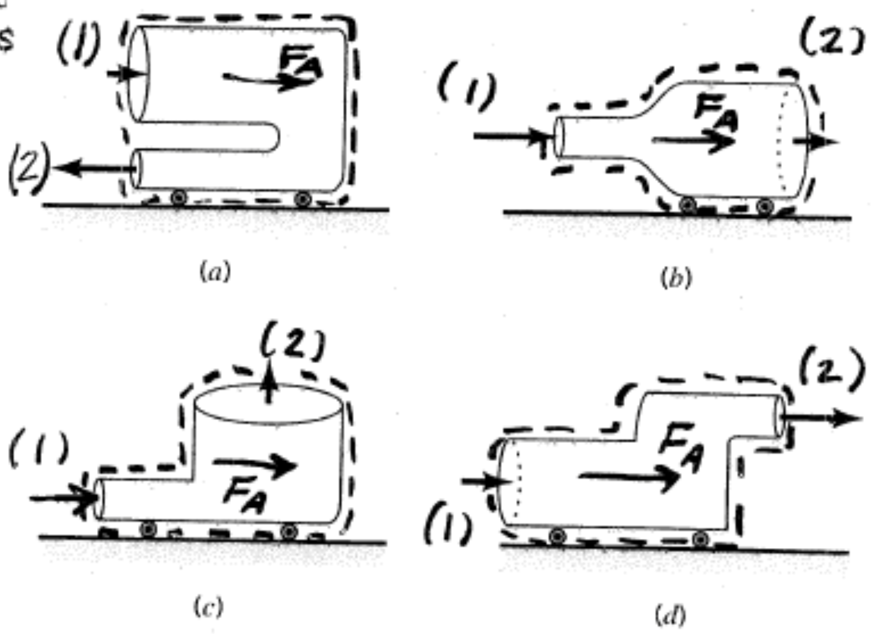


FIGURE P5.58

We apply the horizontal component of the linear momentum equation to the contents of the control volume (broken lines) and determine the sense of the anchoring force F_A. If F_A is in the direction shown in the sketches, motion will be to the left. If F_A is in a direction opposite to that shown, the motion is to the right. If $F_A = 0$, there is no horizontal motion.

For sketch (a)

$$-V_1 \rho V_1 A_1 - V_2 \rho V_2 A_2 = F_A$$

Since F_A is to the left, motion is to the right.

For sketch (b)

$$-V_1 \rho V_1 A_1 + V_2 \rho V_2 A_2 = F$$

and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and since $V_1 > V_2$, then F_A is to the left and motion is to the right.

For sketch (c) (note: flow is into cv at (1)

$$-V_1 \rho V_1 A_1 = F_A$$

and F_A is to the left so motion is to the right.

For sketch (d)

$$-V_1 \rho V_1 A + V_2 \rho V_2 A_2 = F_A$$

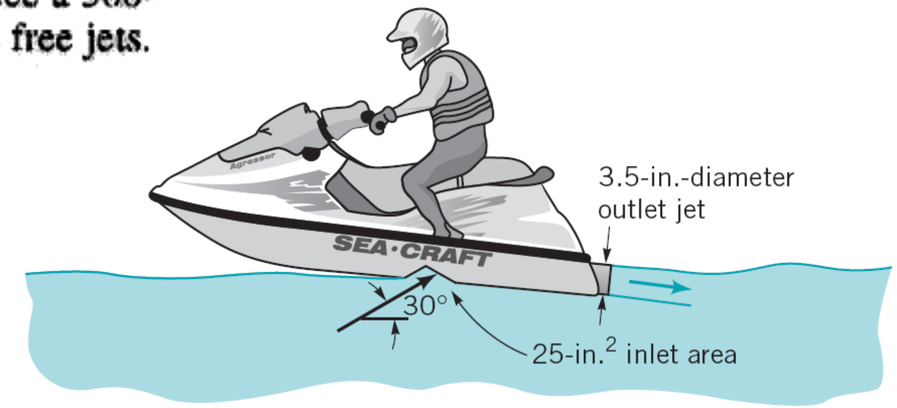
and from conservation of mass

$$\rho V_1 A_1 = \rho V_2 A_2$$

and $V_1 < V_2$

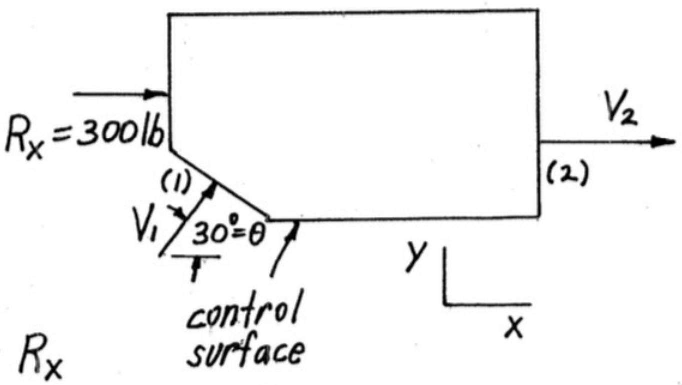
so F_A is to the right and motion is to the left.

5.36 The thrust developed to propel the jet ski shown in **Video V9.7** and Fig. P5.36 is a result of water pumped through the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300-lb thrust? Assume the inlet and outlet jets of water are free jets.



For the control volume indicated the x-component of the momentum equation

$$\int_{cs} u\rho\vec{V}\cdot\hat{n}\,dA = \sum F_x \text{ becomes}$$



(1) $$(V_1\cos 30°)\rho(-V_1)A_1 + V_2\,\rho(+V_2)A_2 = R_x$$

where we have assumed that $p=0$ on the entire control surface and that the exiting water jet is horizontal.

With $\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$ Eq. (1) becomes

$$R_x = \dot{m}(V_2 - V_1\cos\theta) = \rho V_1 A_1 (V_2 - V_1\cos 30°) \qquad (1)$$

Also, $A_1V_1 = A_2V_2$ so that

$$V_2 = \frac{A_1V_1}{A_2} = \frac{25\,in.^2}{\frac{\pi}{4}(3.5\,in.)^2}V_1 = 2.60\,V_1 \qquad (2)$$

By combining Eqs. (1) and (2):

$$R_x = \rho V_1^2 A_1 (2.60 - \cos 30°)$$

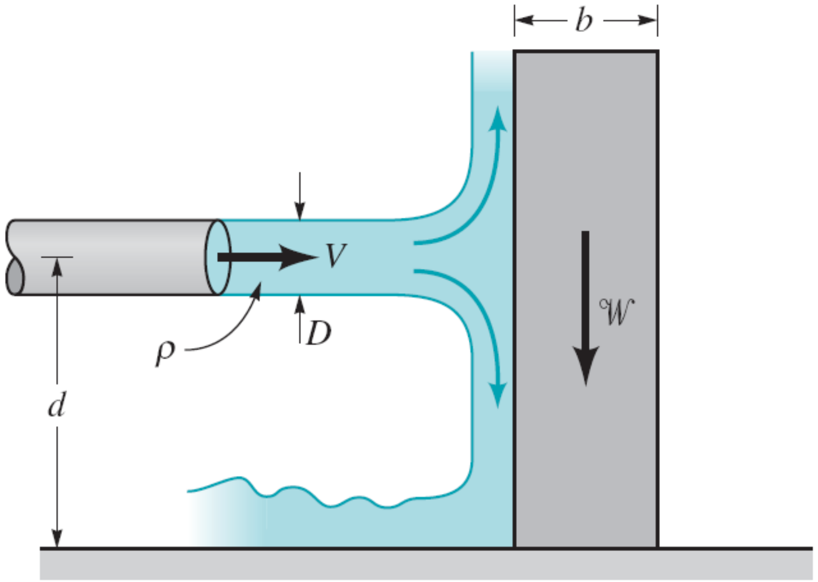
or

$$V_1 = \left[\frac{300\,lb}{\left(1.94\,\frac{slugs}{ft^3}\right)\left(\frac{25}{144}ft^2\right)(2.60-\cos 30°)}\right]^{1/2} = 22.7\,\frac{ft}{s}$$

Thus,

$$Q = A_1V_1 = \left(\frac{25}{144}ft^2\right)\left(22.7\,\frac{ft}{s}\right) = \underline{\underline{3.94\,\frac{ft^3}{s}}}$$

7.14 As shown in Fig. P7.14 and Video V5.4, a jet of liquid directed against a block can tip over the block. Assume that the velocity, V, needed to tip over the block is a function of the fluid density, ρ, the diameter of the jet, D, the weight of the block, $\mathcal{W}$, the width of the block, b, and the distance, d, between the jet and the bottom of the block. **(a)** Determine a set of dimensionless parameters for this problem. Form the dimensionless parameters by inspection. **(b)** Use the momentum equation to determine an equation for V in terms of the other variables. **(c)** Compare the results of parts **(a)** and **(b)**.



(a) $V = f(\rho, D, \omega, b, d)$

(a) $V = f(\rho, D, \mathcal{W}, b, d)$

$V \doteq LT^{-1}$ $\quad \rho \doteq FL^{-4}T^{2}$ $\quad D \doteq L$ $\quad \mathcal{W} \doteq F$ $\quad b \doteq L$ $\quad d \doteq L$

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From the pi theorem, $6-3 = 3$ pi terms required.
By inspection for Π_1 (containing V)

$$\Pi_1 = VD\sqrt{\frac{\rho}{\mathcal{W}}} \doteq (LT^{-1})(L)\left(\sqrt{\frac{FL^{-4}T^{2}}{F}}\right) \doteq F^0L^0T^0$$

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Check using MLT:

$$VD\sqrt{\frac{\rho}{\mathcal{W}}} = (LT^{-1})(L)\left(\sqrt{\frac{ML^{-3}}{MLT^{-2}}}\right) \doteq M^{0}L^{0}T^{0} \therefore \text{ok}$$

(a) $$V = f\,(\rho, D, \mathcal{W}, b, d)$$

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For π_2 let

$$\pi_2 = \frac{b}{d}$$

(a) $$V = f\,(\rho,\ D,\ \mathcal{W},\ b,\ d)$$

$$V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad D \doteq L \quad \mathcal{W} \doteq F \quad b \doteq L \quad d \doteq L$$

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$$\Pi_2 = \frac{b}{d}$$

and for Π_3

$$\Pi_3 = \frac{d}{D}$$

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For Π_2 let

$$\Pi_2 = \frac{b}{d}$$

and for Π_3

$$\Pi_3 = \frac{d}{D}$$

and both Π_2 and Π_3 are obviously dimensionless.
Thus,

$$VD\sqrt{\frac{\rho}{\mathcal{W}}} = \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$

(a) $$V = f(\rho, D, \mathcal{W}, b, d)$$

$$V \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad D \doteq L \quad \mathcal{W} \doteq F \quad b \doteq L \quad d \doteq L$$

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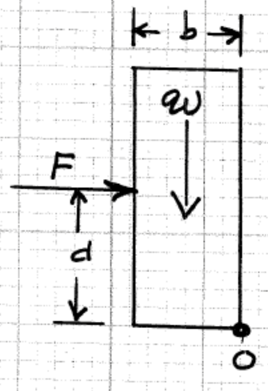


(b) For impending tipping around O

$$\Sigma M_o = 0$$

so that

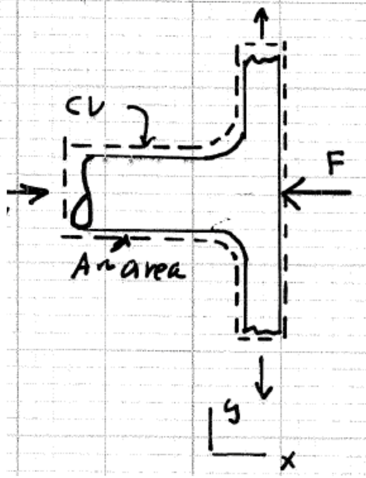
$$Fd = \mathcal{W}\left(\frac{b}{2}\right) \qquad (1)$$

(con't)

From momentum considerations using the CV shown

$\overset{+}{\rightarrow}$ $\int \rho u \vec{V} \cdot \hat{n} \, dA = \sum F_x$

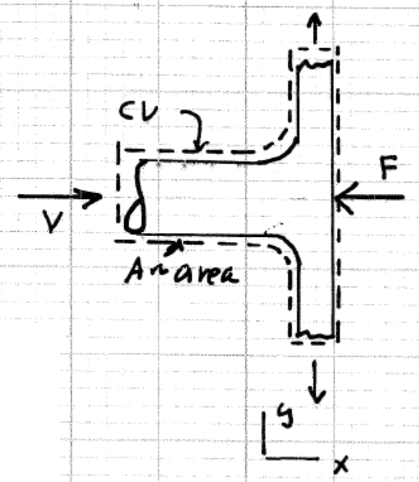
$\rho V^2 A = F$



From momentum considerations using the CV shown

$$\overset{+}{\rightarrow} \quad \int \rho u \vec{V} \cdot \hat{n} \, dA = \sum F_x$$

$$\rho V^2 A = F$$



Thus, from Eq. (1)

$$(\rho V^2 A)(d) = \mathcal{W}\left(\frac{b}{2}\right)$$

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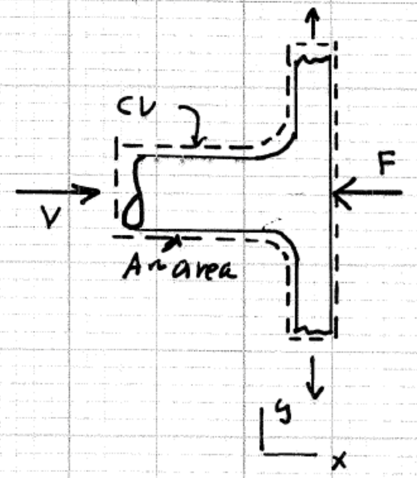

Thus, from Eq. (1)

$$(\rho V^2 A)(d) = \mathcal{W}\left(\frac{b}{2}\right)$$

so that

$$V = \sqrt{\frac{\mathcal{W}(b)}{2\rho A d}}$$

and with $A = \pi/4 \, D^2$

$$\underline{\underline{V = \sqrt{\frac{2\mathcal{W} b}{\pi \rho d D^2}}}}$$

(2)

From momentum considerations using the CV shown

$\overset{+}{\rightarrow}$ $$\int \rho u \vec{V}\cdot\hat{n}\, dA = \sum F_x$$

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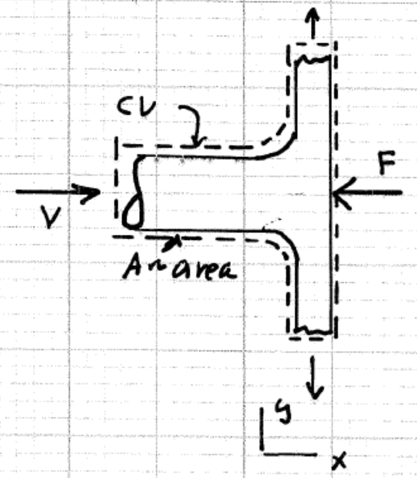

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$$\underline{\underline{V = \sqrt{\frac{2\mathcal{W}\, b}{\pi \rho\, d\, D^2}}}} \qquad (2)$$

(c) From part (a)

$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}}\ \phi\left(\frac{b}{d}, \frac{d}{D}\right)$$

Eq. (2) can be written as

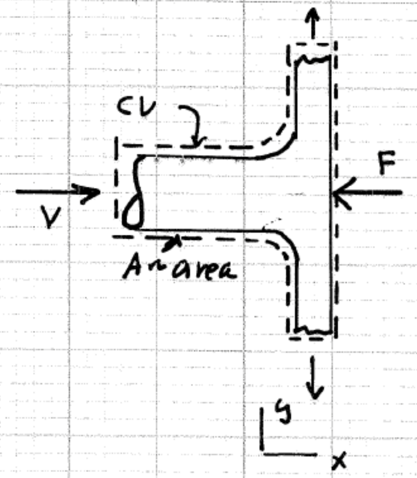
$$V = \sqrt{\frac{\mathcal{W}}{\rho D^2}}\left(\sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}\right) \qquad (3)$$

It follows by comparing Eqs. (2) and (3) that

$$\phi\left(\frac{b}{d}, \frac{d}{D}\right) = \sqrt{\left(\frac{2}{\pi}\right)\left(\frac{b}{d}\right)}$$

so that $\phi\left(\frac{b}{d}, \frac{d}{D}\right)$ is actually independent of $\frac{d}{D}$.

From momentum considerations using the CV shown



$\xrightarrow{+}$ $$\int \rho u \vec{V}\cdot\hat{n}\, dA = \sum F_x$$

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Thus, from Eq. (1)

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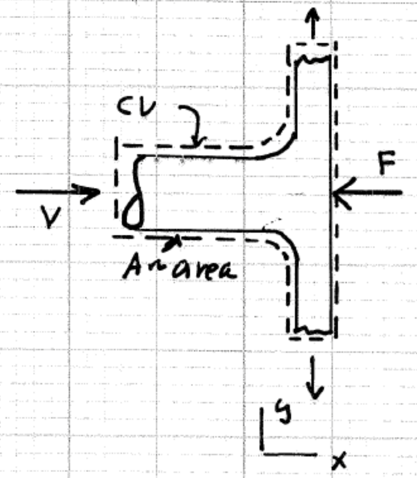

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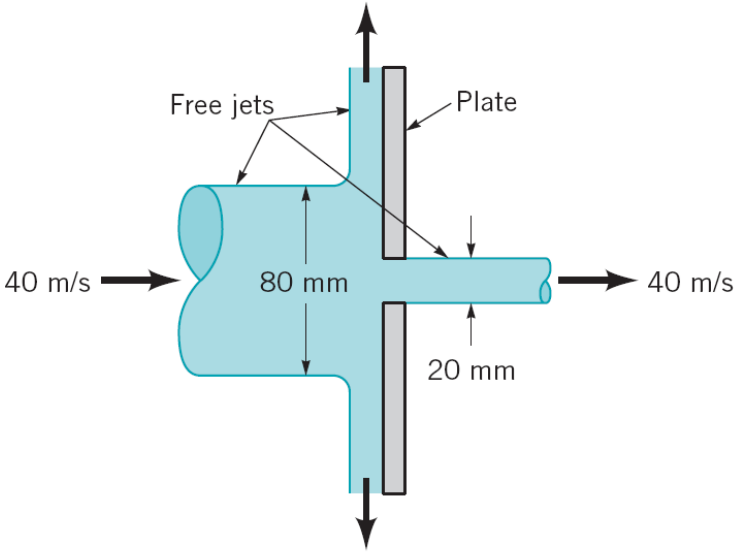
5.38 A circular plate having a diameter of 300 mm is held perpendicular to an axisymmetric horizontal jet of air having a velocity of 40 m/s and a diameter of 80 mm as shown in Fig. P5.38. A hole at the center of the plate results in a discharge jet of air having a velocity of 40 m/s and a diameter of 20 mm. Determine the horizontal component of force required to hold the plate stationary.



$$\int_{cs} -p$$

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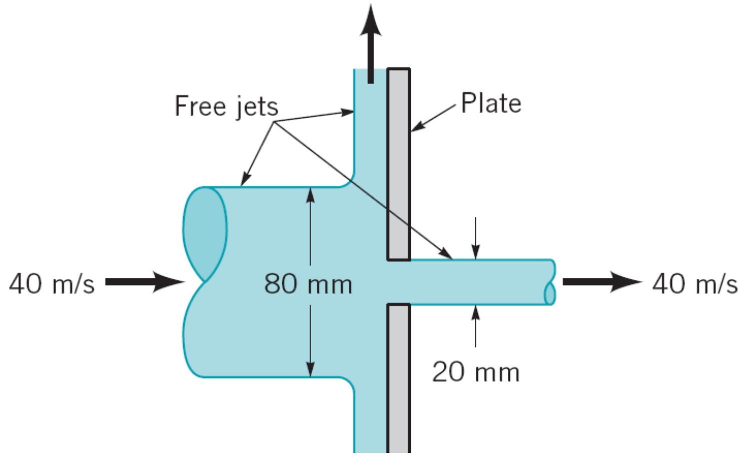

The control volume contains the plate and flowing air as indicated in the sketch above. Application of the horizontal or x direction component of the linear momentum equation yields

$$-u_1 \rho u_1 A_1 + u_2 \rho u_2 A_2 = -F_{A,x}$$

or

$$F_{A,x} = u_1^2 \rho \frac{\pi D_1^2}{4} - u_2^2 \rho \frac{\pi D_2^2}{4} = u_1^2 \rho \frac{\pi}{4}\left(D_1^2 - D_2^2\right)$$

Thus

$$F_{A,x} = \left(40 \frac{m}{s}\right)^2 \left(1.23 \frac{kg}{m^3}\right) \frac{\pi}{4} \left[\frac{(80\ mm)^2 - (20\ mm)^2}{\left(1000 \frac{mm}{m}\right)^2}\right] \left(1 \frac{N}{kg \cdot \frac{m}{s^2}}\right)$$

and

$$F_{A,x} = \underline{\underline{9.27\ N}}$$