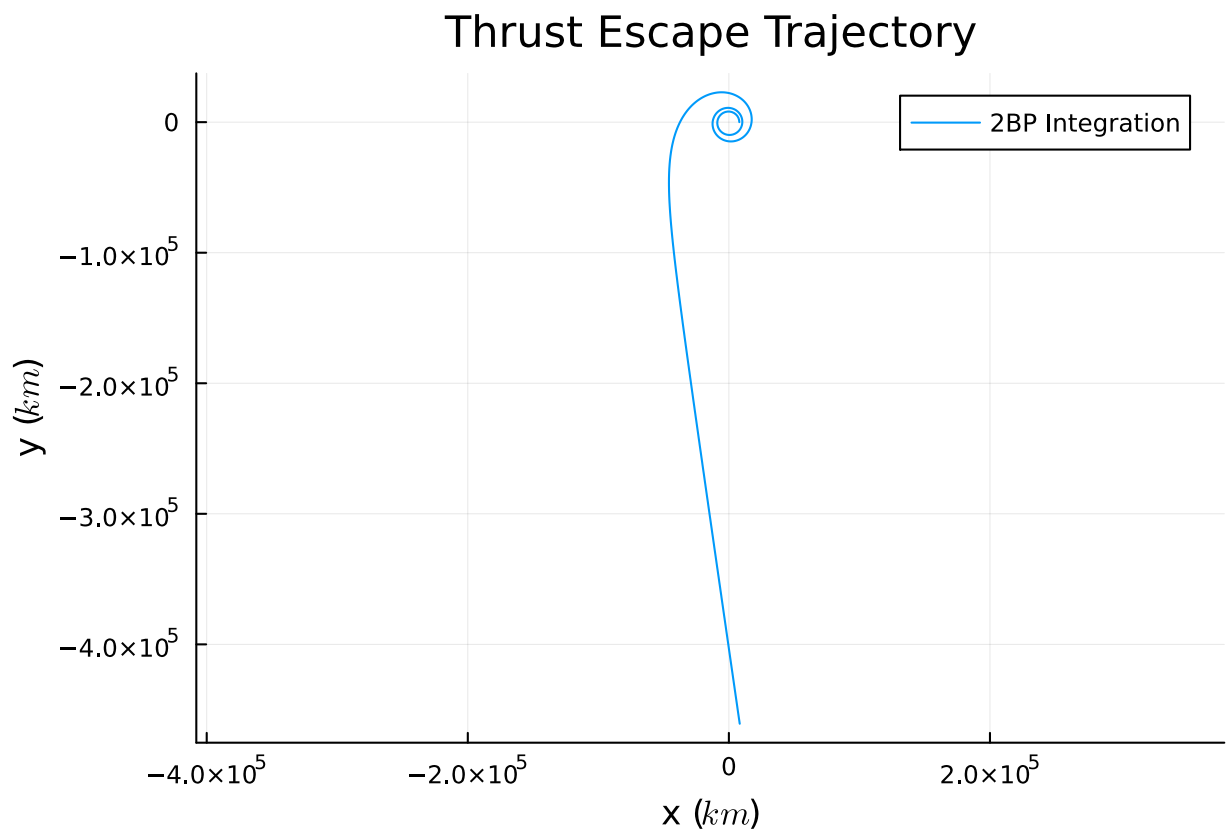
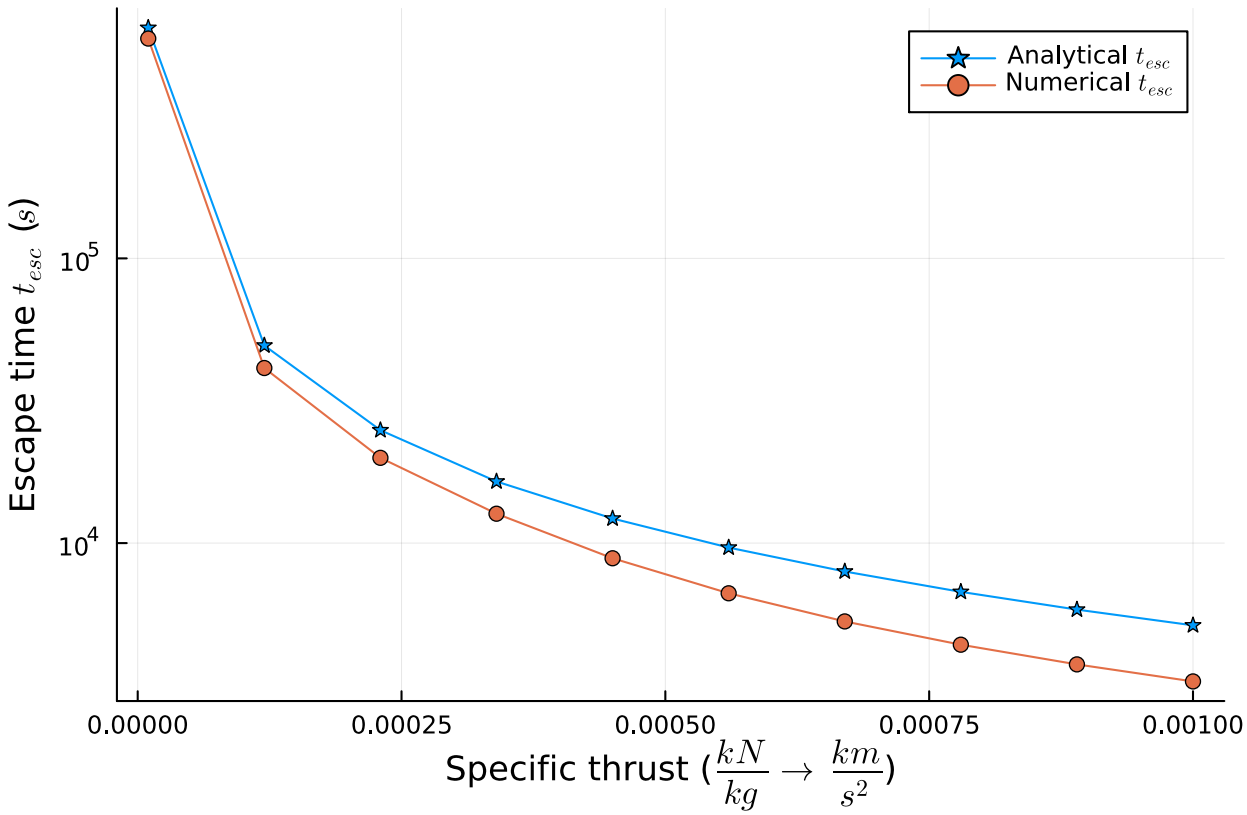
```julia
# enae404 hw07
include("../../../code/sfd.jl")
using .SpaceFlightDynamics
using LinearAlgebra
using Plots
using LaTeXStrings

# problem 01
# givens
T_s = 1e-4
r_o = 8000.0
r_1 = r_o * [1.0, 0.0, 0.0]

# part a
v_o = sqrt(μ_Earth / r_o)
v_1 = v_o * [0.0, 1.0, 0.0]
@show v_1

# part b
a_r = -1 * μ_Earth / r_o^2
a_t = T_s
a = [a_r, a_t]
@show a

# part c
t_e = v_o / a_t * (1 - (20 * a_t^2 * r_o^2 / v_o^9)^(1 / 8))
@show t_e

# part d
sv = solve_2BP_thrust(StateVectors(r_1, v_1), (0.0, 2 * t_e), μ=μ_Earth, T_spec=T_s,
int_pts=500)
v_e = sv[end].v
@show v_e

xs = [sv.r[1] for sv in sv]
ys = [sv.r[2] for sv in sv]
plt = plot(
    xs, ys, label="2BP Integration",
    title="Thrust Escape Trajectory",
    xlabel=L"x ($km$)",
    ylabel=L"y ($km$)",
    aspect_ratio=:equal,
    grid=true)
display(plt)

# part e
time_step = 2 * t_e / length(sv)
t_e_num = 0
for i ∈ eachindex(sv)
    ε = 0.5 * norm(sv[i].v)^2 - μ_Earth / norm(sv[i].r)
    if ε > 0
        global t_e_num = i * time_step
        break
    end
end
@show t_e_num

# part f
analytic_tesc(a_t) = v_o / a_t * (1 - (20 * a_t^2 * r_o^2 / v_o^9)^(1 / 8))
```

```
function numeric_tesc(a_t; int_pts=2000)
    t_e = analytic_tesc(a_t)
    t_end = 10 * t_e
    sv = solve_2BP_thrust(
        StateVectors(r_1, v_1),
        (0.0, t_end),
        μ=μ_Earth,
        T_spec=a_t,
        int_pts=int_pts
    )
    N = length(sv)
    ts = range(0, t_end, length=N)
    ε = [0.5 * norm(sv[i].v)^2 - μ_Earth / norm(sv[i].r) for i in 1:N]
    idx = findfirst(ε .>= 0)
    if idx === nothing
        return NaN
    elseif idx == 1
        return ts[1]
    else
        t1, t2 = ts[idx-1], ts[idx]
        e1, e2 = ε[idx-1], ε[idx]
        return t1 - e1 * (t2 - t1) / (e2 - e1)
    end
end

T_specs = range(1e-5, 1e-3, length=10)
t_anal = [analytic_tesc(T) for T in T_specs]
t_num = [numeric_tesc(T) for T in T_specs]

plot(
        T_specs, t_anal,
    label=L"Analytical $t_{esc}$",
    xlabel=L"Specific thrust ($\frac{kN}{kg}\to\frac{km}{s^2}$)",
    ylabel=L"Escape time $t_{esc}$ ($s$)",
    yscale=:log10,
    marker=:star5,
    legend=:topright,
    grid=true
)
plot!(
    T_specs, t_num,
    label=L"Numerical $t_{esc}$",
    marker=:circle
)

v_1 = [0.0, 7.0586865084801715, 0.0]
a = [-0.006228131903125, 0.0001]
t_e = 59814.507545356515
v_e = [1.3415898646080981, -9.296827531119956, 0.0]
t_e_num = 50483.4443682809
```


Thrust Escape Trajectory
2BP Integration
0
−1.0×10^5
−2.0×10^5
−3.0×10^5
−4.0×10^5
y (km)
−4.0×10^5
−2.0×10^5
0
2.0×10^5
x (km)


Analytical t_{esc}
Numerical t_{esc}
Escape time t_{esc} (s)
10^5
10^4
0.00000
0.00025
0.00050
0.00075
0.00100
Specific thrust ($\frac{kN}{kg} \rightarrow \frac{km}{s^2}$)