University of Maryland at College Park

DEPT. OF AEROSPACE ENGINEERING

ENAE 432: Aerospace Control Systems

Problem Set #2

Issued: 8 Feb. 2025
Due By: 14 Feb. 2025

Question 1:

Neglecting friction, the differential equation modeling an inverted pendulum is

$$ml^2\ddot{\theta}(t) = \tau(t) + mgl\theta(t)$$

where physical constants m, l, and g are all positive. The input $\tau(t)$ is an external torque that can be applied to the system. Since the pendulum is inverted, the reference $\theta = 0$ orientation is straight upwards here.

a.) Intuitively this system is unstable – the natural (homogeneous) dynamics of a pendulum do not want to remain at $\theta = 0$ (upright). Prove this is true mathematically: identify the modes and characterize their stability, and use this to characterize the stability of the system as a whole.

b.) Does the natural motion of the inverted pendulum modeled above exhibit oscillations? Why or why not?

c.) Which do you think would be easier to balance on your hand, a pencil or a broomstick? HINT: Humans have limited reaction times, hence making very fast corrective hand motions would be more difficult than slower more gradual ones. Show explicitly how the length of the pendulum affects the speed of instability.

Question 2:

The main objective of this course is *feedback control*, that is varying the input $u(t)$ in response to the measured difference between the output $y(t)$ and its desired values. This is a powerful technique to stabilize unstable systems, or to improve the performance of stable but poorly behaved systems.

Suppose we use a DC motor attached to the pivot of the pendulum in Question #1 through which we can apply torques $\tau(t)$ that drive the system according to the following rule:

$$\tau(t) = -K_P\theta(t)$$

with constant "gain" $K_P > 0$. Thus, we are varying the torque from the motor *proportional* to the deviation of the pendulum from its zero reference (upright, in this case).

a.) Can you find value(s) for K_P that will stabilize the system? Is there a range of values that will work? If so, identify this range.

b.) Above a certain value for K_P, the controlled dynamics will exhibit stable oscillations. Identify this value of K_P and determine the frequency of the resulting oscillations as a function of K_P and the other physical parameters.

c.) What is the damping ratio of the oscillations in b)? What is their settling time? Your answers should again be in terms of K_P and the physical parameters.

Question 3:

Consider the function

$$f(t) = \begin{cases} \frac{1}{T^2} & 0 \leq t < T \\ \frac{-1}{T^2} & T \leq t <= 2T \\ 0 & \text{otherwise} \end{cases}$$

where T is a constant.

a.) Find $F(s) = \mathcal{L}\{f(t)\}$ from the defining integral. NOTE: $F(s)$ will be a single, well-defined function of s; in particular, despite the form of $f(t)$, $F(s)$ is *not* a piecewise function of s (or t).

b.) Find $\lim_{T \to 0} F(s)$.

Question 4:

An F16 aircraft nominally in straight, level flight at 10,000 ft with speed 500 ft/sec has the following transfer function relating aileron deflection (in deg) to roll rate (in deg/sec)

$$G(s) = \frac{-300s^2 - 600s + 8}{13s^4 + 46s^3 + 130s^2 + 170s + 6}$$

a.) Rewrite $G(s)$ in ZPK form; use the second order form for complex poles/zeros. Is the aircraft stable? Why or why not?

b.) There are three modes in this system: an oscillatory (second-order) mode called the Dutch roll, a very slowly decaying real exponential mode called the spiral mode, and a faster decaying exponential mode called the roll mode. Identify the frequency of the oscillations associated with the Dutch roll mode, the damping ratio of the Dutch roll mode, and the settling times of all 3 modes.

c.) If the aileron is deflected by 2.5 degrees, find the steady-state roll rate.

d.) Find the constant aileron deflection needed to produce a 7 deg/sec steady-state roll rate.

e.) Approximately how long will it take for the aircraft to achieve the steady-state in d.)?

f.) Determine the actual differential equation which models the roll dynamics of the aircraft. The input $u(t)$ is the aileron angle, while the output $y(t)$ is the roll rate.

g.) This model predicts that there is an exponentially increasing input $u(t) = e^{at}$ with $a > 0$ such that the steady-state roll rate will in fact be exactly zero. Find the numerical value of a.