University of Maryland at College Park

Dept. of Aerospace Engineering

ENAE 432: Aerospace Control Systems

Problem Set #9

Issued: 12 Apr. 2025
Due By: 18 Apr. 2025

Question 1:

Motor speed controllers are very common applications, controlling propeller speeds on quadcopters, momentum wheel speeds on spacecraft, etc. A simple model for a DC motor is

$$I\dot{\omega}(t) = -b\omega(t) + K_m u(t).$$

where ω is the motor speed, I is the inertial load, and $b\omega$ term is due to drag or friction. Suppose for this problem that $I = 5$, $b = 1$ and $K_m = 3$.

a.) Show that the proportional feedback control law $u(t) = Ke(t)$, where $e(t) = \omega_d(t) - \omega(t)$ can produce a stable closed-loop system, with any desired settling time and no transient oscillation. (Nyquist should hardly be needed for this).

b.) Choose a specific value of K which will result in a 2 second settling time for the closed-loop transient response. Determine the resulting steady-state tracking error when $\omega_d(t) = 50$ (constant). Does this design exhibit perfect tracking?

c.) To ensure that the speed perfectly tracks at least constant ω_d, the PI ("proportional+integral") controller

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau$$

is commonly used. This can be alternately written as

$$\begin{aligned} u(t) &= K_p e(t) + K_i x_1(t) \\ \dot{x}_1(t) &= e(t) \end{aligned}$$

Determine the corresponding transfer function $H(s)$ and prove that this controller will indeed ensure perfect tracking for constant ω_d.

d.) Another common rule of thumb in feedback control design is "make sure the slope at magnitude crossover is at least -20 dB/dec". Having a slope which is too shallow near crossover means that the actual crossover point (and hence the actual phase margin) can be very sensitive to small (but inevitable) errors in the model for $G(s)$. For this problem, adhering to this rule constains the choice of the two gains. Derive at least one set of constraints which will ensure the desired property is satisfied.

e.) Choose values for the two gains which both respect the rule in d), and achieves the same result as the design in b) but using repeated real closed-loop poles. Will a step response of the resulting closed-loop system show overshoot? Why or why not?

Question 2:

The compensator in Question #1 is known as a "PI" (proportional+integral) compensator. A PID (PI+derivative) compensator instead has the form

$$u(t) = K_p e(t) + K_d \dot{e}(t) + K_i \int_0^t e(\tau) d\tau$$

or equivalently

$$\begin{aligned} u(t) &= K_p e(t) + K_d \dot{e}(t) + K_i x_1(t) \\ \dot{x}_1(t) &= e(t) \end{aligned}$$

This is one of the most commonly used controllers, at least for systems with relatively simple dynamics and when derivative measurements are available. Logic to implement this calculation is often directly implemented in the circuitry of standard embedded motion control chips or off-the-shelf drone autopilots. Since it is so popular, there are many *ad hoc* "tuning rules" for the three gains, so that people who really don't know anything about controls, or even the exact dynamics of their system, can still get a feedback loop up and running using these controllers.

The most common of these tuning rules are the Ziegler-Nichols (ZN) tuning rules, which can be summarized as follows:

- Set K_d and K_i to zero. Increase K_p until the controlled system begins to exhibit persistent oscillations (this can be done experimentally, if needed). Let K_u be this value of K_p, and let T_u be the observed period of the oscillations.
- Choose the values: $K_p = 3K_u/5$, $K_i = 2K_p/T_u$, and $K_d = K_p T_u/8$.

Pretty cookbook, no?

Let's try this for a system whose dynamics are assumed to be

$$G(s) = \frac{10}{s(s+4)^2}$$

Since we actually have a model of the system here, we can use Bode methods to directly identify the numerical values of K_u and T_u, and thus the numerical values of the three PID gains. To this end, note that the first step is equivalent to setting $H(s) = K$, and K_u is then the value of K for which the phase margin and gain margins of $L(s)$ are zero. As we have seen previously, in such a situation the resulting closed-loop system will then have poles on the imaginary axis at $\pm j\omega_\gamma$, and the period of the corresponding oscillations is thus $T_u = 2\pi/\omega_\gamma$.

Using these observations, determine the $L(s)$ which results from Ziegler-Nichols tuning of a PID controller for this $G(s)$. (Note that it should be possible to determine K_u and T_u *by inspection* of this system!) Quantify the resulting crossovers and margins, compute the resulting closed-loop poles, and quantify the significant features of a unit step response for $T(s)$. In your judgement, is this a particularly good feedback loop design?

Question 3:

Let's use our deeper understanding of controls to better tune the PID controller in Question #2. Specifically let's use what we have learned, rather than the *ad hoc* rules above, to tune the same compensator so the loop has a phase margin of 40° at a crossover of 4 rad/sec. Determine the equivalent PID gains (K_p, K_d, and K_i) which will achieve this target, generate a step response of the resulting $T(s)$, and compare with Question #2.

NOTE: You still have some freedom in the placement of the zeros of $H(s)$. The simplest design will assume these zeros are repeated, but this is in no way required nor necessarily optimal. Time permitting, you might explore the extent to which you can improve the metrics for the step response of $T(s)$, while maintaining the same phase margin and crossover, by varying the zero locations in $H(s)$.

Question 4:

For your design in Question #3:

a.) What is the delay margin, in seconds? What is the minimum tolerable loop update rate for the controller calculations, in Hz (this is just the inverse of the delay margin)?

b.) If the loop rate is fixed by the available electronics at 20 Hz (i.e. 20 updates/sec, or 50 msec between updates), what is the effective phase margin of your feedback loop? What is the effective gain margin? Qualitatively, how do you think these reductions would affect the step response properties of $T(s)$?

c.) If the system is subjected to a sinusoidal input disturbance, determine the range of disturbance frequencies for which the induced additional tracking steady-state tracking error will be no more than 1% of the amplitude of the disturbance. Determine also the frequency for the disturbance which would result in the largest additional tracking steady-state error. You may neglect delay effects in these calculations.

d.) Neglecting delay effects, find the minimum distance from the Nyquist plot to the -1 point for the open-loop dynamics corresponding to your design. How does this change when the delay in b) is included? HINT: Matlab can generate Bode diagrams that include a delay term in the transfer function.

e.) Suppose the model of $G(s)$ used for the design is suspected to be lacking a high frequency pole, so that $G_{true}(s) = G(s)/(as+1)$. If we continue to use the same compensator as designed in Question #3, what is the largest value for a that could be tolerated while guaranteeing the actual feedback loop will remain stable? Use the multiplicative uncertainty robustness test, and neglect the effects of delay in the calculation.