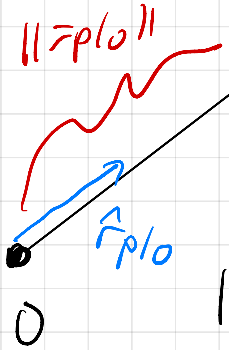
Chapter 1

8/27/24

Dynamics is the science that describes the motion of bodies aka mechanics, kinetics

"also known as"

Dfn a **vector** is a geometric entity that has both magnitude & direction in space

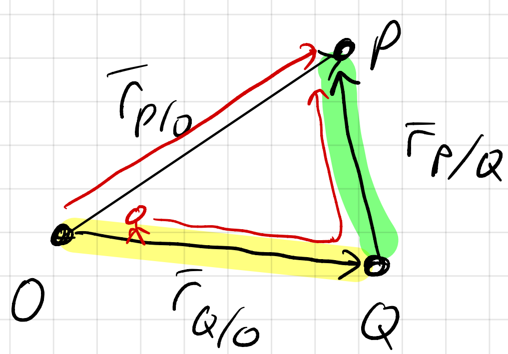


"with respect to"

$\bar{r}_{P/O}$ the position of P wrt O

$\|\bar{r}_{P/O}\|$ the norm of a vector is its magnitude aka length

$$\hat{r}_{P/O} = \frac{\bar{r}_{P/O}}{\|\bar{r}_{P/O}\|}$$ is a unit vector



vector addition property

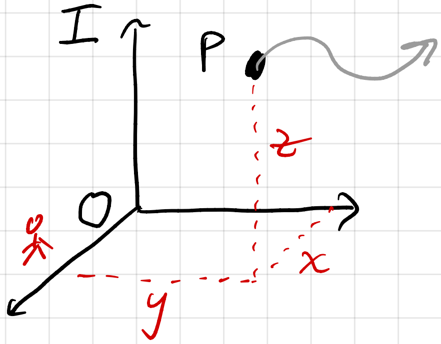
$\bar{r}_{Q/O} + \bar{r}_{P/Q} = \bar{r}_{P/O}$ "vector triad"

$\bar{r}_{P/Q} = \bar{r}_{P/O} - \bar{r}_{Q/O}$

Claim: $\bar{r}_{P/Q} = -\bar{r}_{Q/P}$

$\bar{r}_{P/Q} + \bar{r}_{Q/P} \stackrel{\checkmark}{=} 0$ zero vector

Dfn a **reference frame** is a point of view from which observations and measurements are made



$(x, y, z)_I$

a **coordinate system** is a set of scalars that locate the position of a point w/t another point in a reference frame.

position	$\bar{r}_{P/O}$	$(x, y, z)_I$	
velocity	${}^I\bar{v}_{P/O}$	$(\dot{x}, \dot{y}, \dot{z})_I$	$\dot{x} = \frac{d}{dt}(x)$
acceleration	${}^I\bar{a}_{P/O}$	$(\ddot{x}, \ddot{y}, \ddot{z})_I$	$\ddot{x} = \frac{d}{dt}(\dot{x}) = \frac{d^2}{dt^2}(x)$

mass "which implies"

Newton's 2nd Law (1D)

$$f_x = m\ddot{x} \Rightarrow \boxed{\ddot{x} = \frac{f_x}{m}}$$

$$f_y = m\ddot{y} \qquad \ddot{y} = \frac{f_y}{m}$$

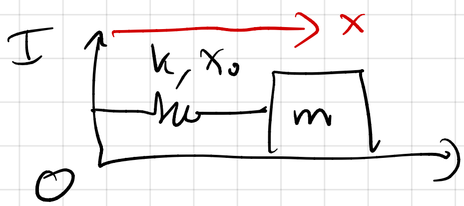
$$f_z = m\ddot{z} \qquad \ddot{z} = \frac{f_z}{m}$$

Dfn **equations of motion** are the three 2nd-order differential equations whose solution is the position and velocity of a point as a function of time.

Ex spring force aka Hooke's law

Stiffness

$$f_x = -k(x - x_o)$$



$$\boxed{\ddot{x} = -\frac{k}{m}(x - x_o)}$$

Dfn an <u>equilibrium point</u> of a dynamic system is a special solution in which the rate of change of the states are all zero

<u>Ex</u> States are $(x, \dot{x})$ aka position and velocity

$$\Rightarrow (\dot{x}^*, \ddot{x}^*) = (0, 0) \text{ @ equilibrium}$$

$$\ddot{x}^* = 0 = -\frac{k}{m}(x^* - x_0) \Rightarrow \boxed{\begin{aligned} x^* &= x_0 \\ \dot{x}^* &= 0 \end{aligned}} \text{ eq. pt.}$$