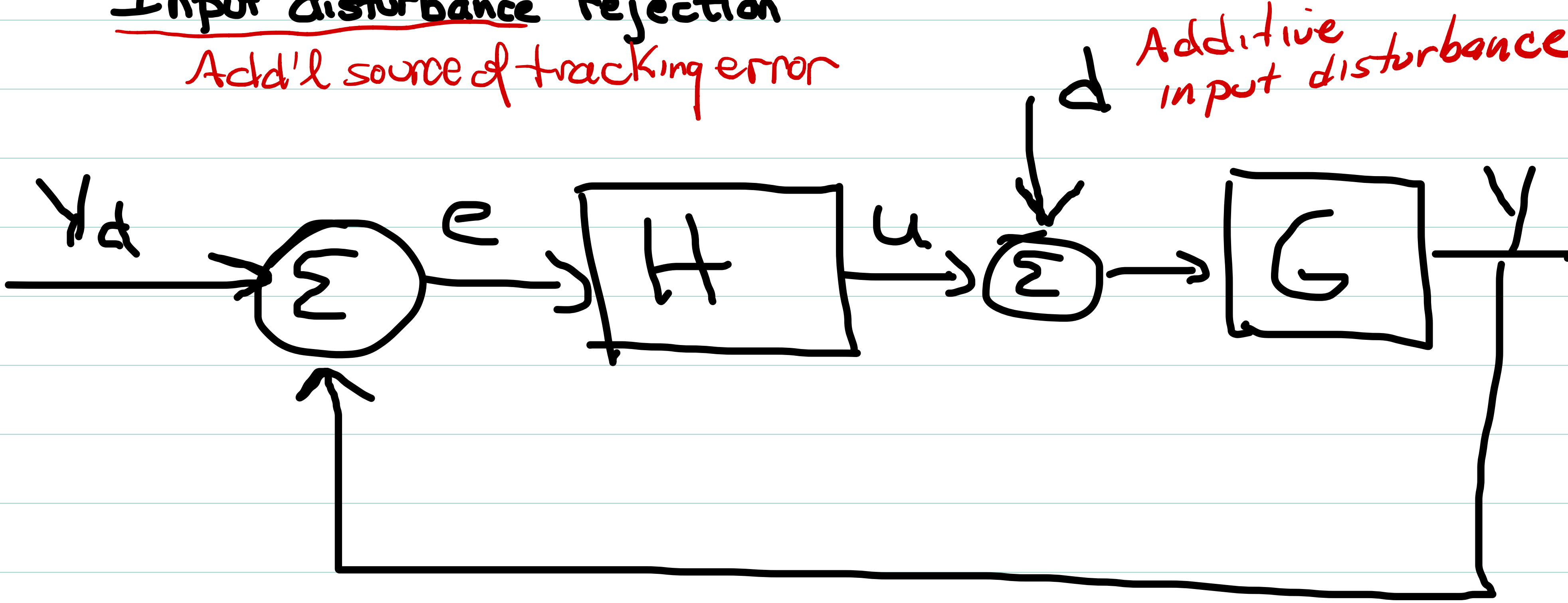
Input disturbance rejection

Add'l source of tracking error



$d(t)$ external "disturbance" input to the system: Not under our direct control, and cannot be predicted or measured during operation of the system.

What effect will this have on stability or accuracy?

"rejection": ability to maintain $e_{ss}(t)$ small even when $d(t) \neq \emptyset$.

Re-derive feedback loop equations:

$$Y(s) = G(s)\left[U(s)+D(s)\right] = G(s)\left[H(s)E(s)+D(s)\right]$$

$$= G(s)H(s)E(s) + G(s)D(s)$$

$$= G(s)H(s)\left[Y_d(s) - Y(s)\right] + G(s)D(s)$$

So $\left(1+L(s)\right)Y(s) = L(s)\,Y_d(s) + G(s)D(s)$

or $$Y(s) = \underbrace{\left[\frac{L(s)}{1+L(s)}\right]}_{T(s)} Y_d(s) + \underbrace{\underbrace{\left[\frac{G(s)}{1+L(s)}\right]}_{S_i(s)} D(s)}_{\text{Added term due to disturbance}}$$

$S_i(s)$ "input sensitivity" TF.

Note: poles of $S_i(s)$ same as $T(s) \Rightarrow S_i(s)$ is stable if $T(s)$ is.

$$\boxed{\Rightarrow \text{Disturbance cannot destabilize system!}}$$

Disturbance can, however, worsen tracking:

$$Y(s) = T(s)Y_d(s) + S_i(s)D(s)$$

$$E(s) = Y_d(s) - Y(s) = \underbrace{(1-T(s))}_{S(s)}Y_d(s) - S_i(s)D(s)$$

So:

$$E(s) = \underbrace{\boxed{S(s)Y_d(s)}}_{\text{ideal tracking error}} \underbrace{- S_i(s)D(s)}_{\text{extra error due to disturbance}}$$

Want to quantify the added errors due to disturbance

Can analyze similarly to above, but need a bit more care:

$$S_i(s) = \frac{G(s)}{1+L(s)}$$

Let $G(s) = \dfrac{N_G(s)}{D_G(s)}$, $H(s) = \dfrac{N_H(s)}{D_H(s)}$ so $L(s) = \dfrac{N_G(s)N_H(s)}{D_G(s)D_H(s)}$

Dist. rejection

Want $|S_i(j\omega)| \ll 1$ for ω in freq range of $d(t)$

i.e. if $d(t)$ has sig. freq content in $[\omega_1, \omega_2]$

want $|S_i(j\omega)| \ll 1$ for $\omega \in [\omega_1, \omega_2]$

Note $S_i(s) = \dfrac{G(s)}{1+L(s)} = \dfrac{1}{G^{-1}(s) + H(s)}$

So $|S_i(j\omega)| \ll 1 \Rightarrow$ either

- $|G(j\omega)| \ll 1$, or
- $|H(j\omega)| \gg 1$ $\Leftarrow$ Can design for this!

Note: $|G(j\omega)| \to 0$ as $\omega \to \infty$ for physical systems

$\Rightarrow |S_i(j\omega)| \to 0$ as $\omega \to \infty$

But usually $|G(j\omega)| \gtrsim 1$ for mid/low freq.

Dist rejection, cont

$|S_c(j\omega)| \to 0$ As $\omega \to \infty$

But freq. band where dist. is significant $[\omega_1, \omega_2]$ is typically at mid-low freqs where $|G(j\omega)| \approx 1$

$\Rightarrow$ Need $|H(j\omega)| \gg 1$ at these freqs!

Thus, good dist. rejection typically requires

$$\underbrace{|H(j\omega)| \gg 1}_{\text{Note: reqt on } H(s) \text{ only!}} \text{ for } \omega \in \underbrace{[\omega_1, \omega_2]}_{\text{freqs. where dist is significant}}$$

As with tracking error and $S(s)$, the IMP provides Add'l insights.

Then:

$$S_i(s) = \frac{G(s)}{1+L(s)} = \frac{N_G(s)D_H(s)}{D_G(s)D_H(s) + N_G(s)N_H(s)}$$

Again suppose $D(s) = \frac{a(s)}{b(s)}$; a, b polynomials

Then additional error:

$$S_i(s)D(s) = \left[\frac{N_G(s)D_H(s)}{D_G(s)D_H(s) + N_G(s)N_H(s)}\right]\left[\frac{a(s)}{b(s)}\right]$$

Internal model principle again!

If $N_G(s)D_H(s)$ cancels non-stable roots of $b(s)$ then in steady-state $\mathcal{L}^{-1}\{S_i D\} = \emptyset$

i.e. disturbance creates <u>NO</u> additional error!

Implications:

If $N_G(s)D_H(s)$ cancels non-stable roots of $b(s)$, then cancellation is either due to:

$\Rightarrow$ $N_G(s)$ cancelling (extremely rare)

$\Rightarrow$ $D_H(s)$ cancelling (can design for this)

So generally, external disturbances create NO add'l error if compensator contains an internal model of disturbance.

That is, if compensator $H(s)$ has same non-stable poles as the disturbance.

"perfect rejection" of dist.

i.e. if $d(t) = d_0$ (constant), no add'l tracking error if $H(s)$ has pole at origin.

Summary of error analysis

For *perfect tracking* of "type p" desired behaviors

$$y_d(t) = \left(\frac{A_p}{p!}\right) t^p$$

$L(s)$ must have $p+1$ poles at origin

For *perfect rejection* of type p disturbances $d(t)$, $H(s)$ must have $p+1$ poles at origin

in Both cases, p poles at origin (one less) will ensure finite, but nonzero errors

Note: tracking objectives can be satisfied if required poles come from plant, compensator, or a combination of both

But dist. rejection req't's can be satisfied *only* by poles in the compensator.

$\Rightarrow$ Above are special cases of IMP.

Good accuracy thus often requires $H(s)$ to have at least one pole at origin.

$\Rightarrow$ This pole adds $-90°$ of phase at all frequencies!

$\Rightarrow$ Works against our stability/performance guidelines of increasing phase margin.

$\Rightarrow$ Even adding a LHP zero doesn't help here:

$$H(s) = K\left[\frac{s - z_c}{s}\right] \qquad z_c < 0$$

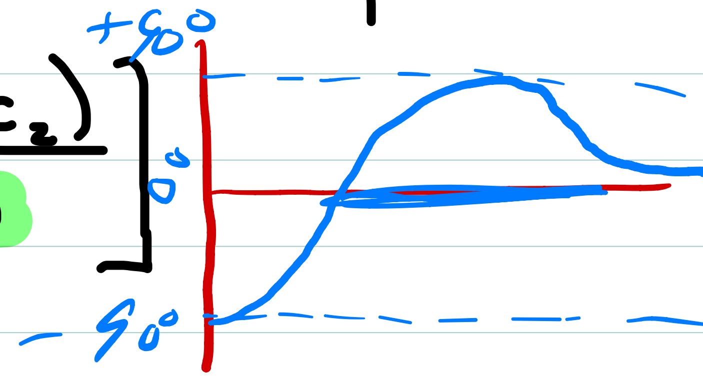
has $\measuredangle H(j\omega) < 0°$ for all $\omega \geq 0$.

$\Rightarrow$ May be acceptable if $\measuredangle G(j\omega)$ already has "adequate" positive phase, so $\measuredangle L = \measuredangle G + \measuredangle H$ can tolerate a small reduction.

More generally, we'd require extra LHP zero(s) to still provide positive phase changes to $\angle L(s)$ despite required pole at origin

Implementability then requires an additional LHP pole:

$$H(s) = K\left[\frac{(s-z_{c_1})(s-z_{c_2})}{s(s-p_c)}\right]$$



4 degrees of freedom total!

Things get even more complicated if $H(s)$ needs ≥ 2 poles at origin to achieve tracking objectives!

Remember: Tracking of $y_d(t)$ depends on IMP properties of $L(s)$

Disturbance rejection depends on IMP properties of $H(s)$

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Disturbance rejection depends on IMP properties of $H(s)$