$$y(t) = \left(\frac{\beta_1}{\beta_0}\right)\dot{y}_1(t) + y_1(t)$$

or equivalently:

$$\boxed{y(t) = y_1(t) - \left(\frac{1}{z_1}\right)\dot{y}_1(t)} \quad \left(z_1 = -\beta_0/\beta_1\right)$$

Where $y_1(t)$ is the "ideal" (no zero) step response

The total response $y(t)$ is the sum of the ideal response, and a fraction of the <u>derivative</u> of this response.

Suppose 1st $z_1 < 0$ (LHP zero)

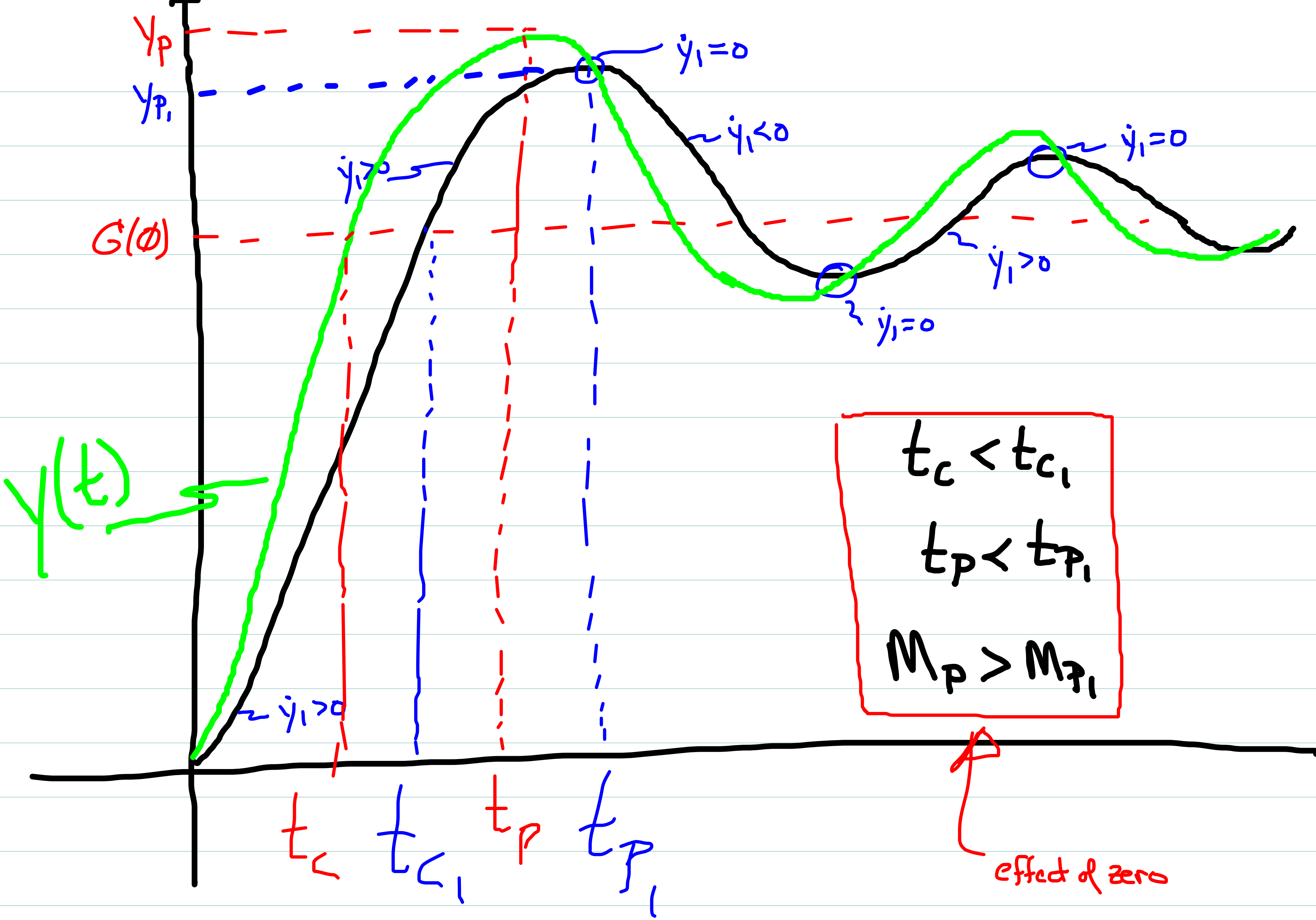
then $z_1 < 0$ and $\left(-\frac{1}{z_1}\right) > 0$ so we can write

$$y(t) = y_1(t) + \left(\frac{1}{|z_1|}\right)\dot{y}_1(t)$$

Derivative <u>adds</u> to total response. To understand effect of this, must examine behavior of $\dot{y}_1(t)$

Note that $\dot{y}_1(t) \to \emptyset$ as $t \to \infty$, so the steady-state of the new response will be the <u>same</u> as the ideal response

$$y_{ss} = G(\emptyset)$$



y_p
y_{p_1}
$G(\emptyset)$
$y(t)$
$\dot{y}_1 = 0$
$\dot{y}_1 < 0$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
$\dot{y}_1 > 0$
$t_c < t_{c_1}$
$t_p < t_{p_1}$
$M_p > M_{p_1}$
effect of zero
t_c
t_{c_1}
t_p
t_{p_1}

Summary of observations

A LHP zero changes a 2^{nd} order step response by:

$\Rightarrow$ Increasing overshoot y_p and M_p

$\Rightarrow$ decreasing t_c and t_p

In a sense, system "responds" faster (crosses y_{ss} more quickly), but price is greater overshoot.

$\Rightarrow$ Note: tricky to quantify exact changes to t_c, t_p, y_p based on z_1

$\Rightarrow$ However, note change from "ideal" response is proportional to $\frac{1}{|z_1|}$

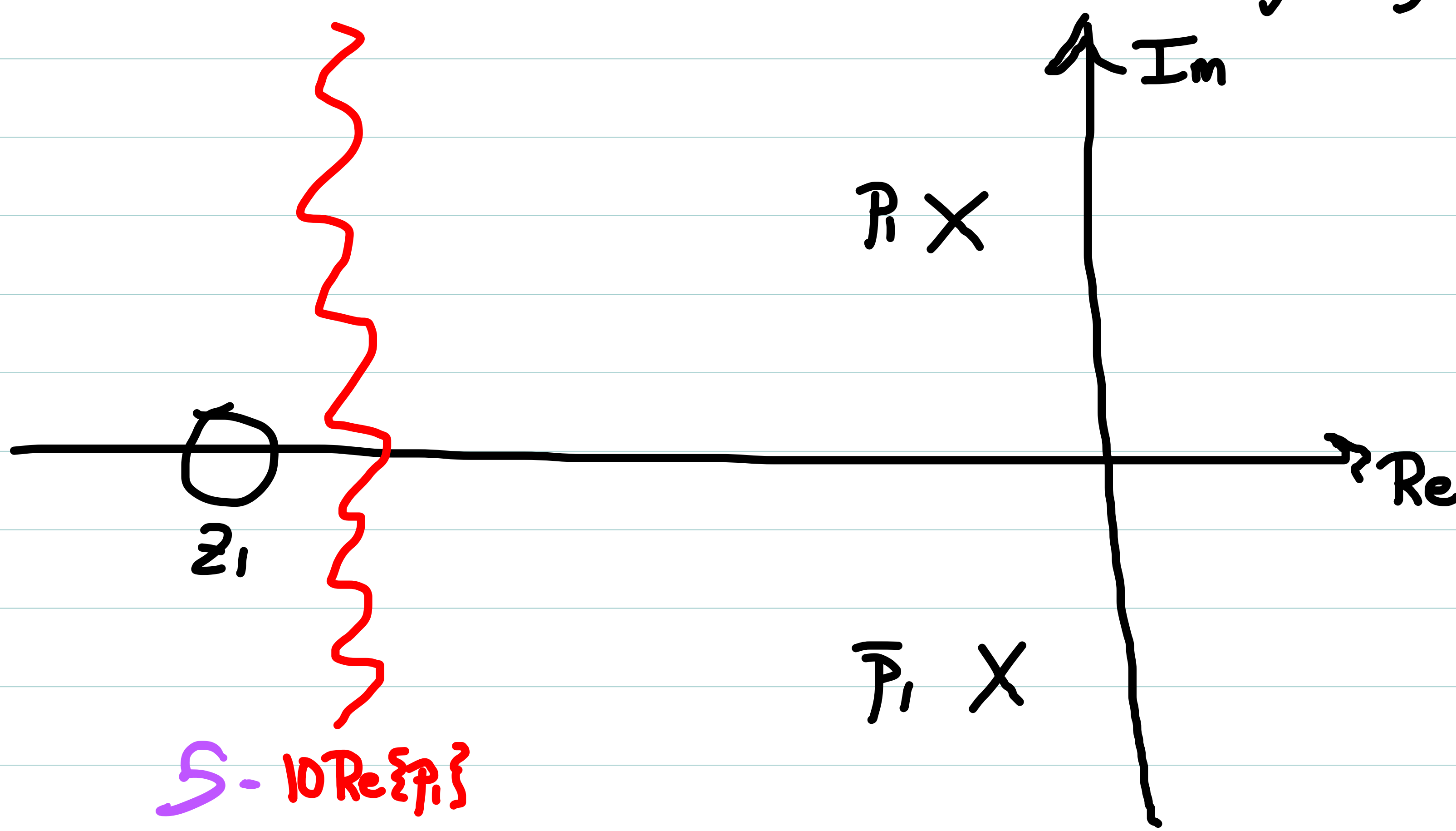
$\Rightarrow$ The further z_1 is from imag axis, the smaller the effect

Rule of Thumb

Effect of zero in this case is negligible if

$$|z_1| > \overset{5}{\cancel{10}}\,|Re\{p_1\}|$$

i.e. zero is 10 times further into LHP than complex poles.



Question

$\Rightarrow$ A zero increases (amplifies) the overshoot of a 2^{nd} order system with $\zeta < 1$ (complex poles).

$\Rightarrow$ Can it actually <u>create</u> overshoot in a system with 2 real poles ($\zeta \geq 1$)?

$\Rightarrow$

Question

$\Rightarrow$ A zero increases (amplifies) the overshoot of a 2^{nd} order system with $\zeta < 1$ (complex poles).

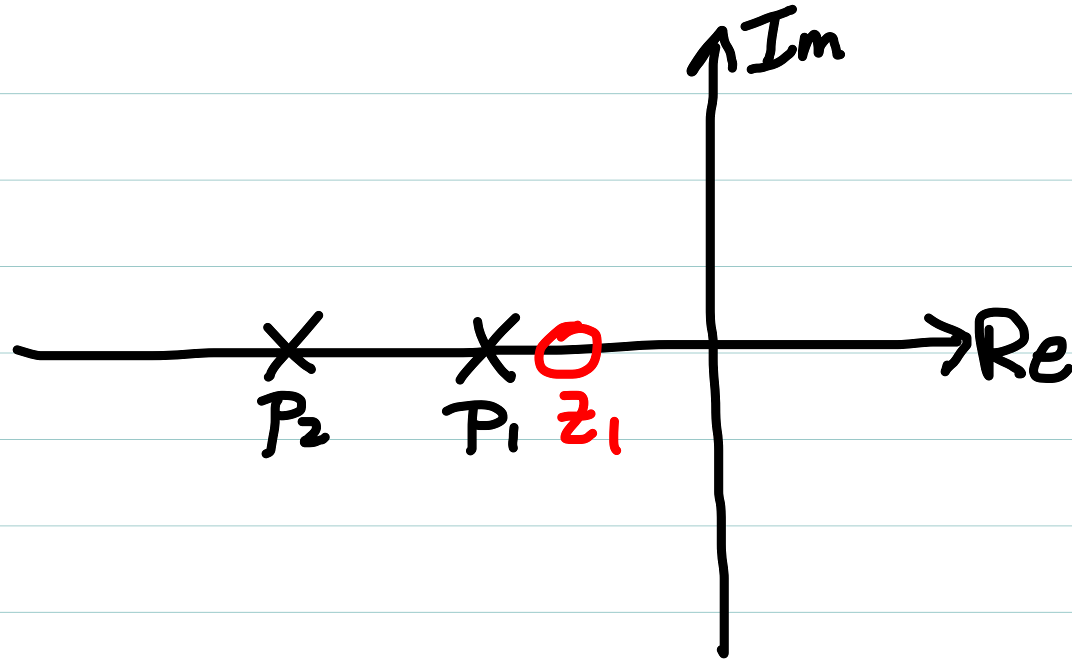
$\Rightarrow$ Can it actually create overshoot in a system with 2 real poles ($\zeta \geq 1$)?

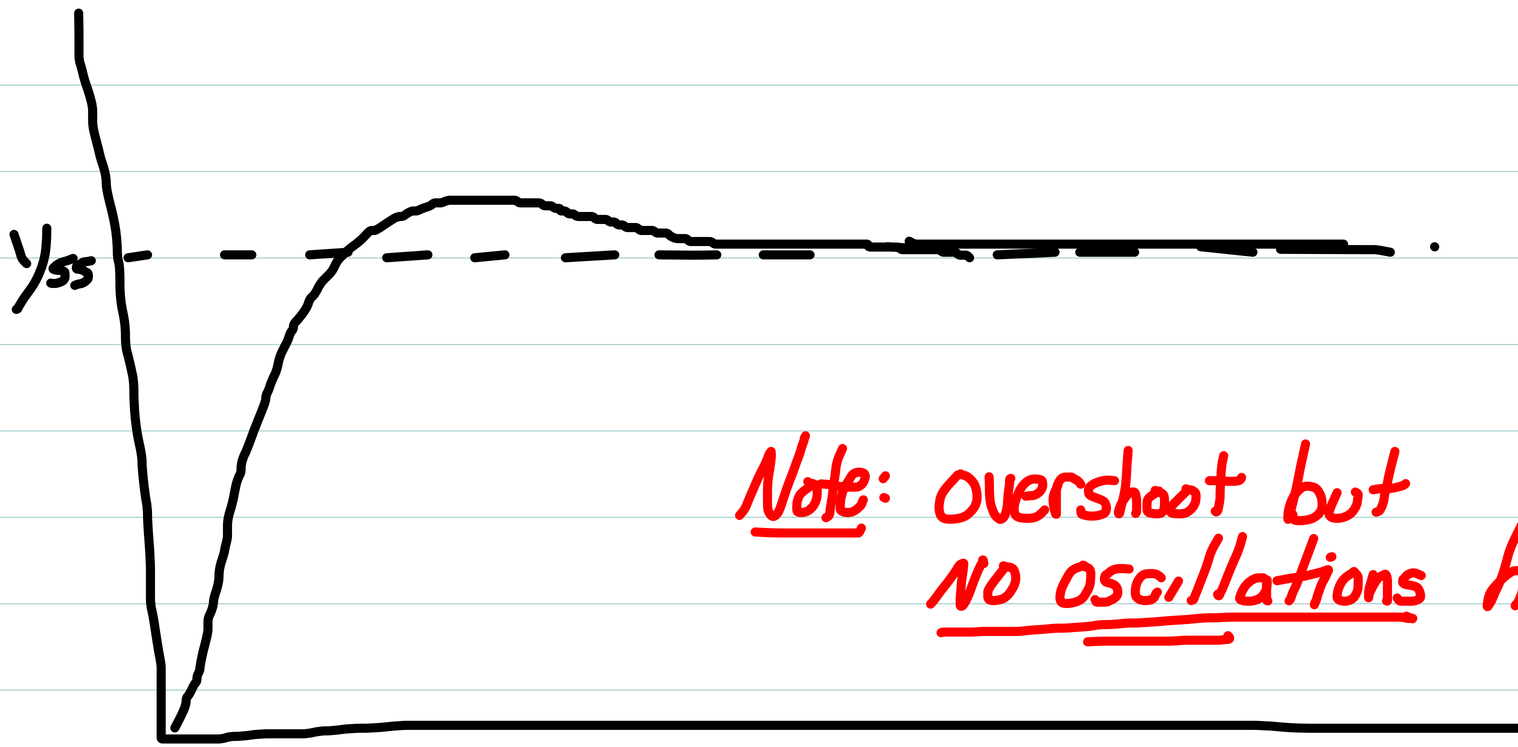
$\Rightarrow$ Yes!

$\Rightarrow$ With 2 real poles p_1 and p_2, $y_p > y_{ss}$ if

$$|z_1| < \min(|p_1|, |p_2|)$$

i.e. if zero is closer to imag axis than ~~either~~ both of the two poles.





Note: overshoot but NO oscillations here

Back to 2^{nd} order ($\zeta < 1$ case)

Suppose $z_1 > \phi$, i.e. z_1 in RHP, then

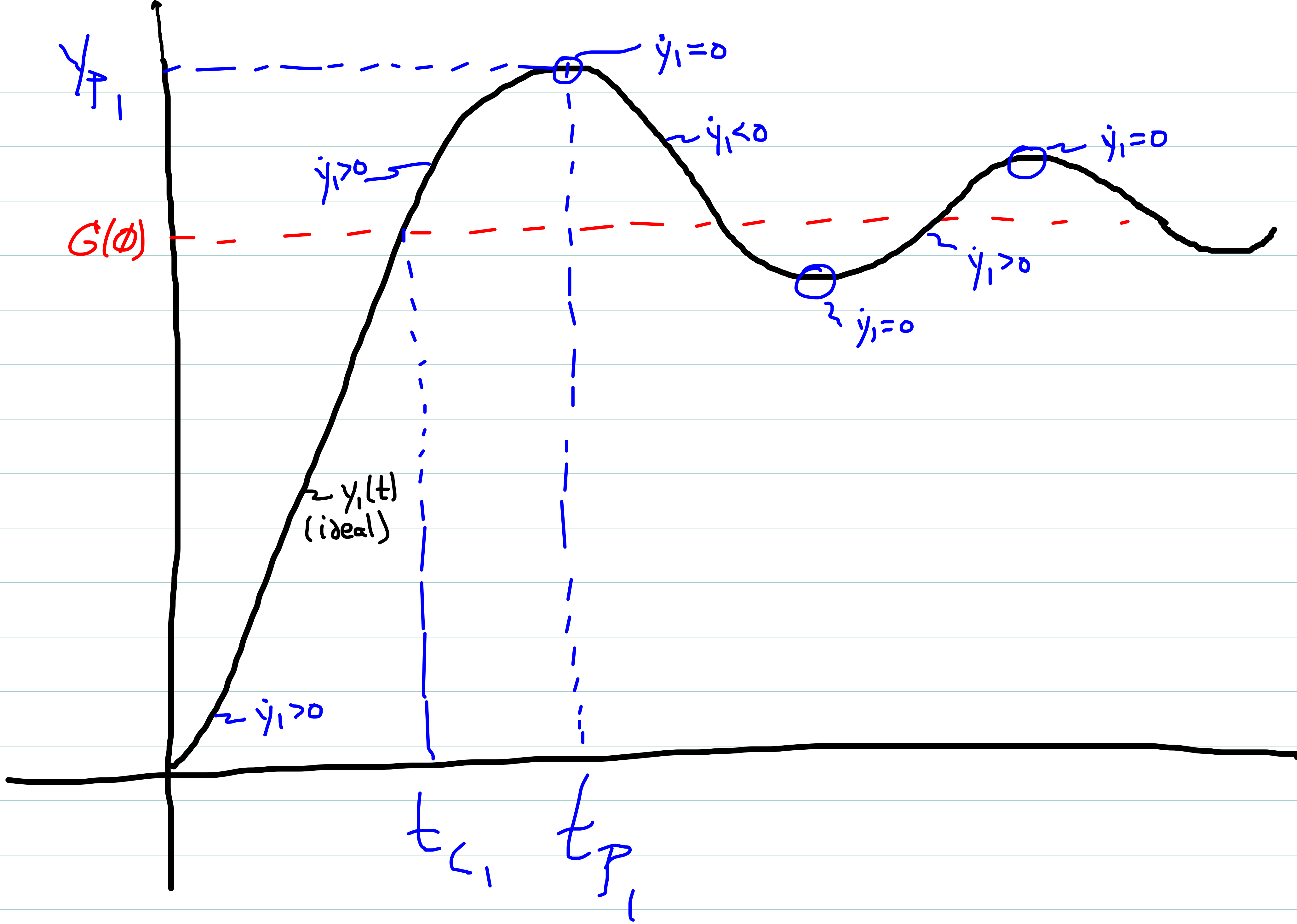
$$y(t) = y_1(t) - \left(\frac{1}{z_1}\right) \dot{y}_1(t)$$

So we are subtracting the derivative from the ideal response.

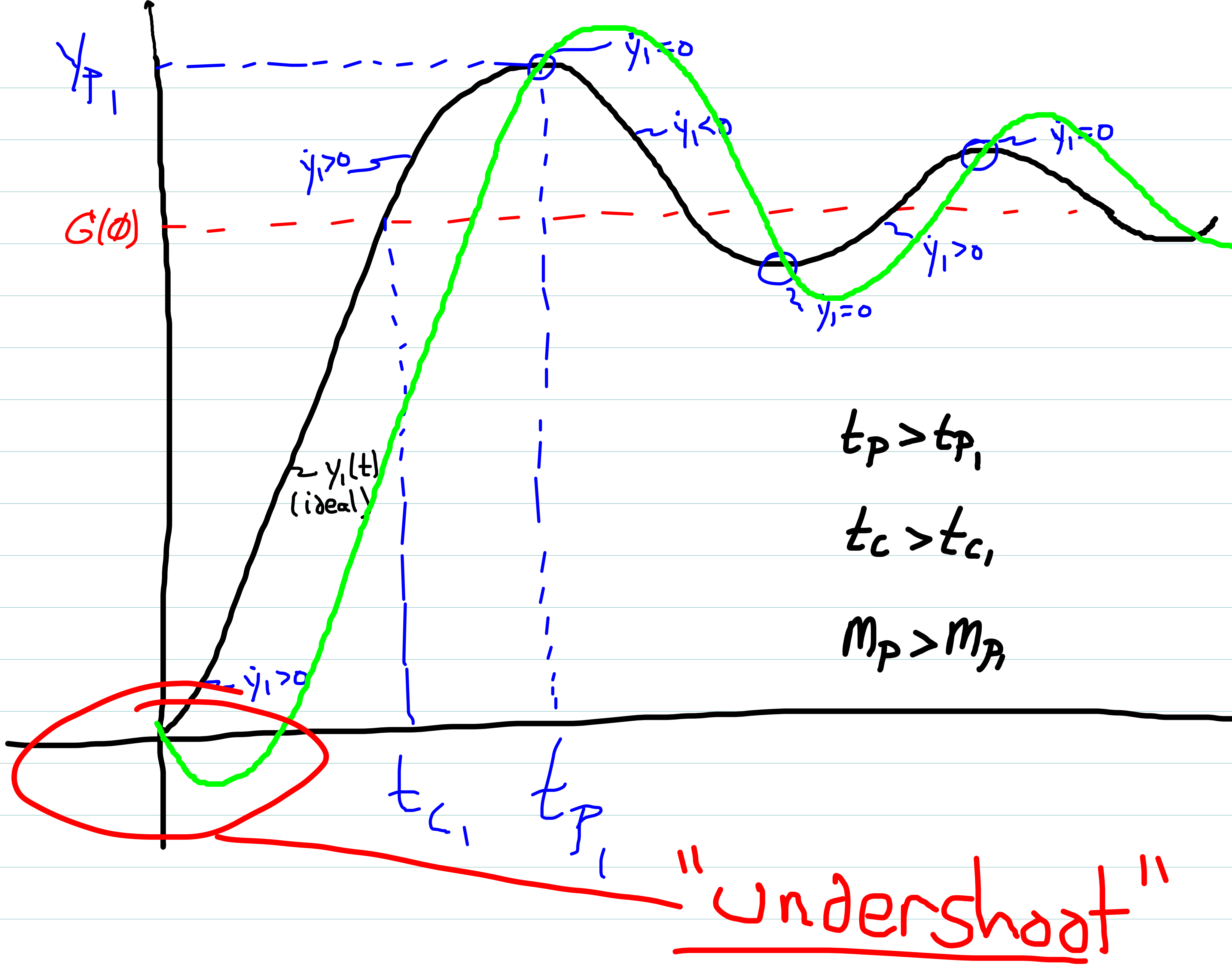
But recall $\dot{y}_1(t) \geq \phi$ for $\phi < t < t_p$,

And $y_1(t) \approx \phi$ for t close to zero

Seems to suggest that $y(t)$ may become negative for times near $t = \phi \ldots$?



y_{P_1}
$\dot{y}_1 = 0$
$\dot{y}_1 < 0$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
$G(\emptyset)$
$\dot{y}_1 > 0$
$\dot{y}_1 = 0$
$y_1(t)$ (ideal)
$\dot{y}_1 > 0$
t_{c_1}
t_{P_1}



$t_p > t_{p_1}$

$t_c > t_{c_1}$

$M_p > M_{p_1}$

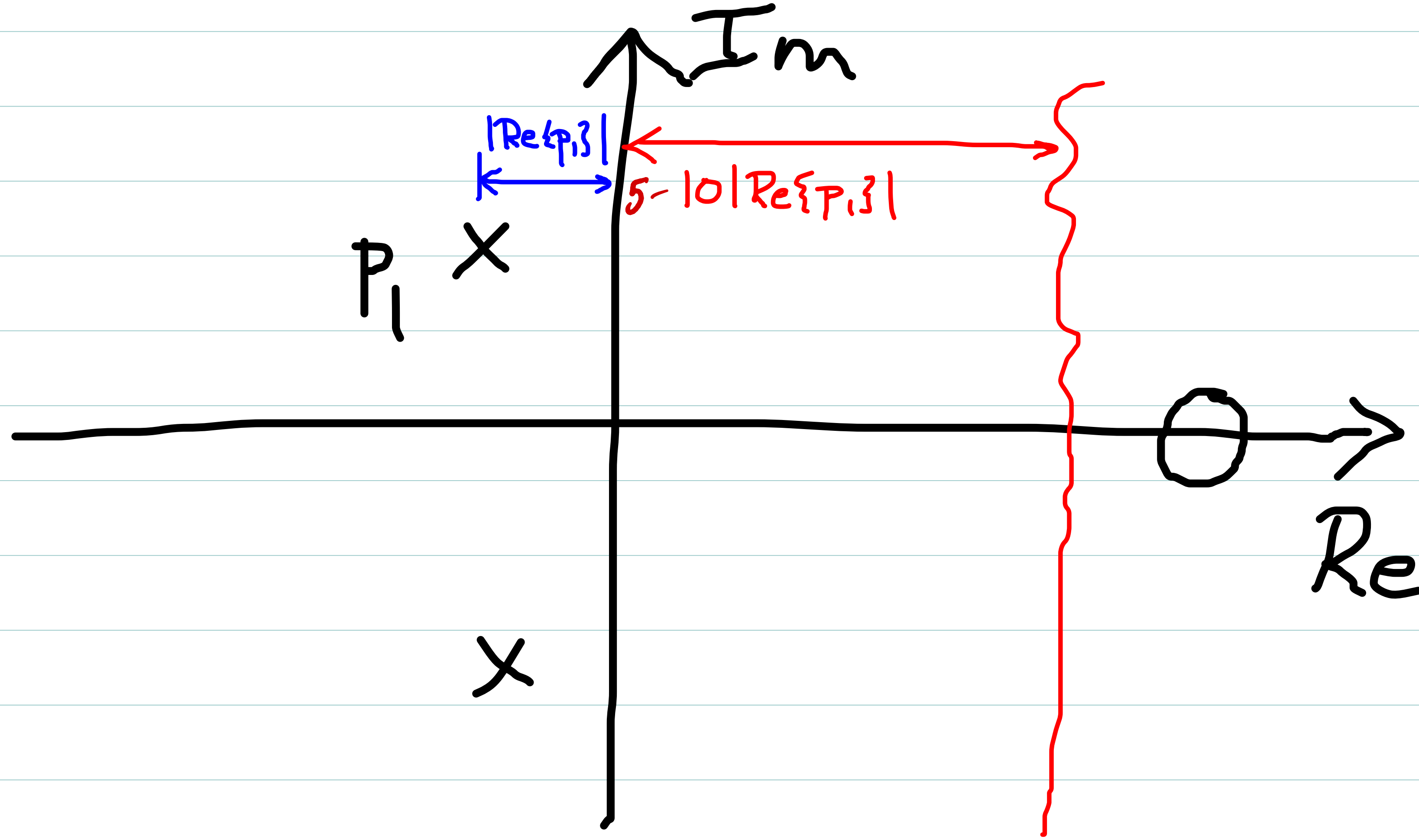
Observations (RHP zero)

- ⇒ Again, the peak response is greater
- ⇒ However, t_c and t_p have increased
- ⇒ Appearence of a new feature : "undershoot"
- ⇒ Response initially heads "in wrong direction" before ultimately returning to the same steady-state
- ⇒ Such behavior is NOT UNstable
- ⇒ It is, however, very tricky to design controllers for such systems.

Effect is still proportional to $\frac{1}{|z_1|}$

hence diminishes as z_1 moves further from Im axis

Again negligible if $|z_1| > 10|\mathrm{Re}\{p_1\}|$



Effect on settling time

How a zero, either LHP or RHP, affects t_s is difficult to predict.

$\Rightarrow$ Often, but not always, t_s is longer with zero due to increased amplitude of transient oscillations

$\Rightarrow$ No hard and fast rule here

$\Rightarrow$ Primary effect is increased overshoot and:

- reduction of t_r, t_p (LHP)
- undershoot, with increase of t_r, t_p (RHP)

Performance Specifications

$\Rightarrow$ Step inputs representative for many desired behaviors

- move to new pointing angle (spacraft)
- Move to new altitude or heading (aircraft)

$\Rightarrow$ Required performance often specified as upper limits on acceptable t_s, M_p

- System must settle quickly enough, and not overshoot too much.

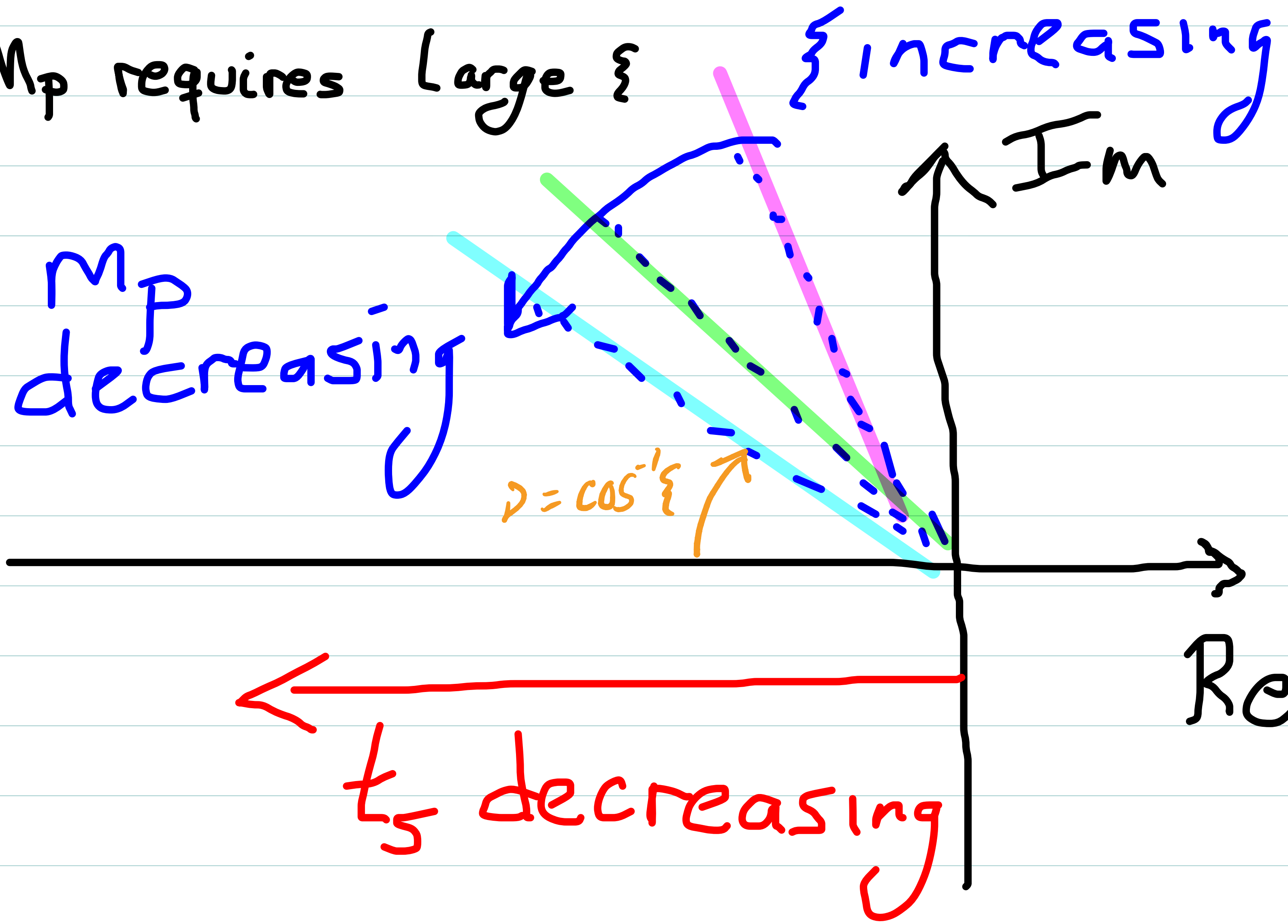
$\Rightarrow$ Recall:

- t_s inversely proportional to $|Re\{p_i\}|$
- M_p a decreasing function of ζ

$$t_s \approx \frac{4}{|Re\{p_1\}|}, \quad \boxed{M_p = \exp\left[\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right]}$$

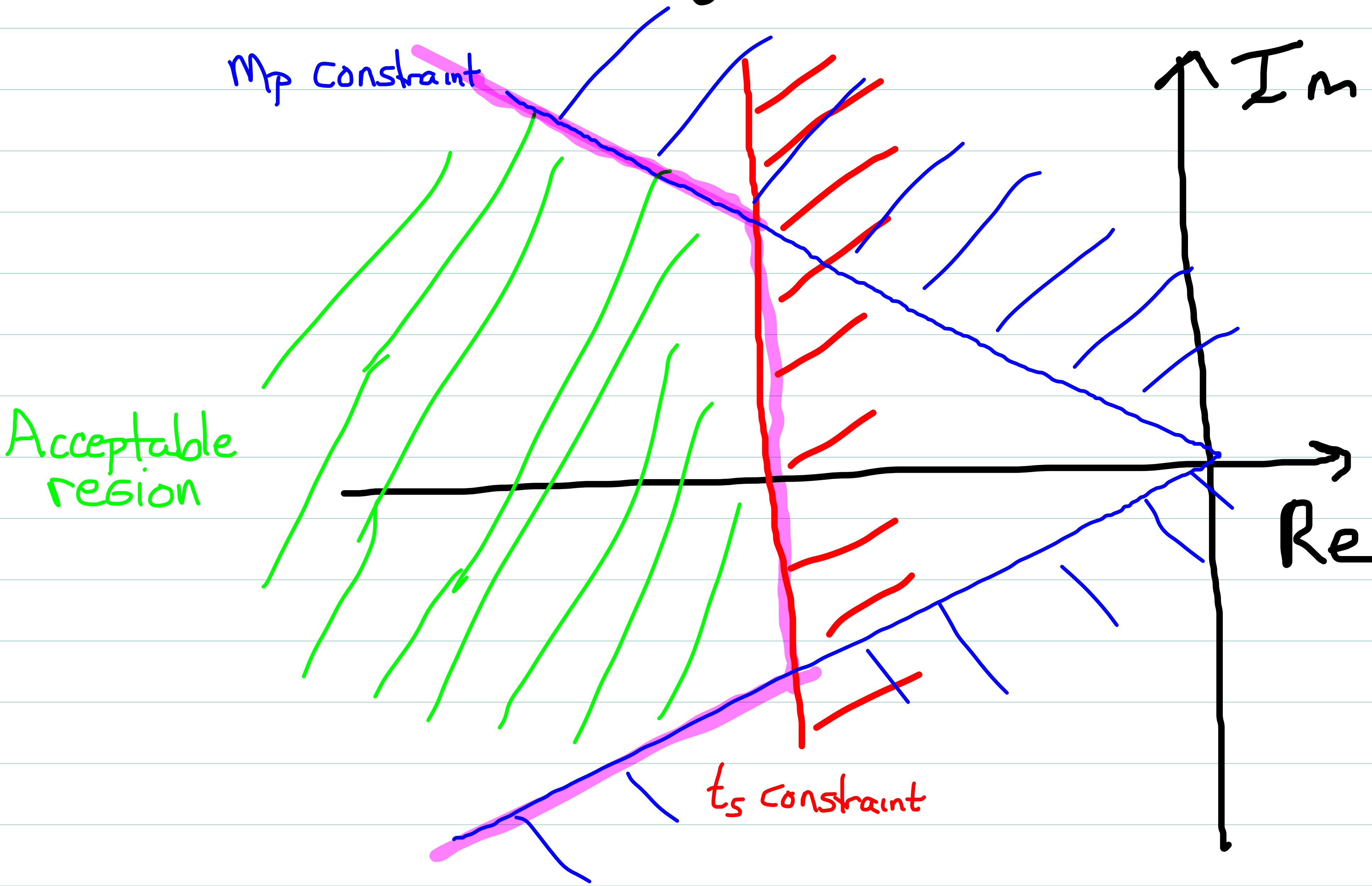
$\Rightarrow$ small t_s requires large $|Re\{p_1\}|$

$\Rightarrow$ small M_p requires large ζ



$\Rightarrow$ Upper bound on t_s gives lower bound on $|Re\{p_i\}|$

$\Rightarrow$ Upper bound on M_p gives lower bound on ζ



Desireable Pole Locations

⇒ "Good" poles satisfy all transient performance constraints (upper bounds on t_s, M_p)

⇒ "Bad" poles are unstable

⇒ "Ugly" poles are stable, but have too much overshoot or take too long to settle.

⇒ Most aerospace system have natural dynamics which are "bad" or "ugly"

⇒ Goal of control is to make these systems "good"

Feedback "moves" poles

⇒ Already seen this on previous homeworks.

⇒ But it can be tricky!

Suppose $u(t) = K(y_d(t) - y(t))$

If system is modeled with $Y(s) = G(s)U(s)$

where $$G(s) = \frac{\beta_1 s + \beta_0}{s^2 + \alpha_1 s + \alpha_0}$$

Then poles are moved to roots of

$$r_{CL}(s) = s^2 + (\alpha_1 + K\beta_1)s + (\alpha_0 + K\beta_0)$$

⇒ Tricky to predict movement of poles for all possible values of $K, \alpha_0, \alpha_1, \beta_0, \beta_1$

⇒ Even more complicated for $G(s)$ with additional poles and/or zeros

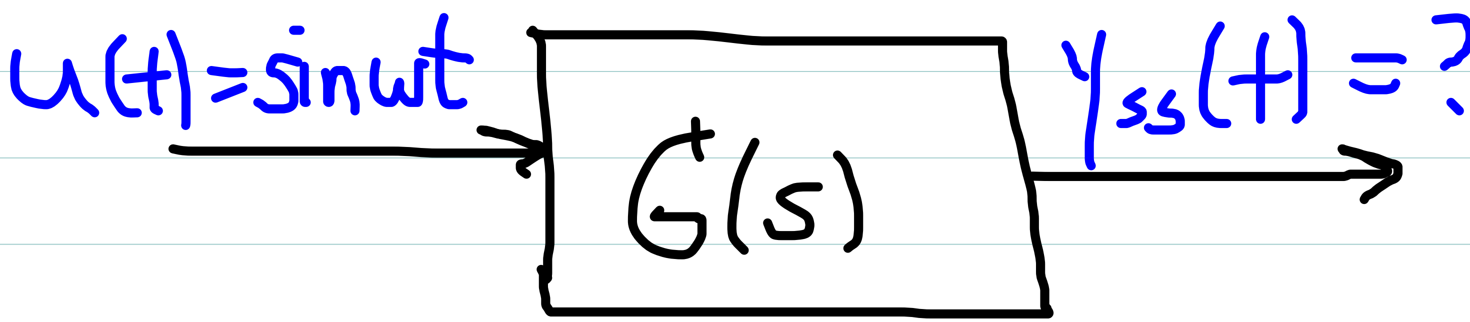
⇒ Need a more systematic tool to predict effectiveness of a control strategy.

⇒ One approach is BASED on a more careful analysis of the behavior of $G(j\omega)$.

Sinusoidal Response

Here we wish to understand the properties of the steady-state response of a $\boxed{\text{stable}}$ system when $\boxed{u(t) = \sin \omega t.}$

Note: our focus is shifting (temporarily) away from the transient response



$$\Longrightarrow y_{ss}(t) = \operatorname{Im}\{G(j\omega)e^{j\omega t}\}$$