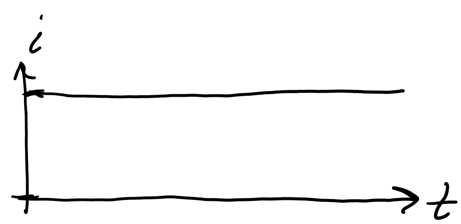
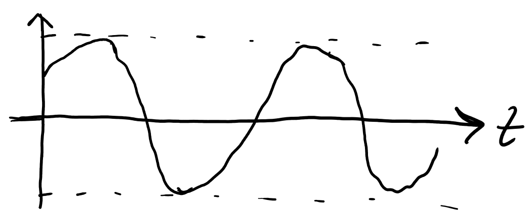
AC Circuits

DC:



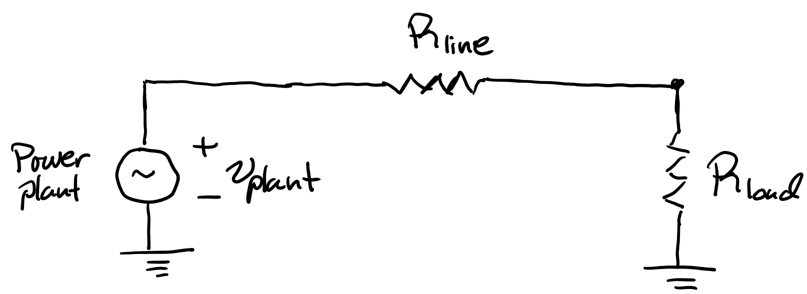
No change in current flow direction

AC:



current flow oscillates (changes direction)

Why AC?



$$P_{loss} = i^2 R_{line}$$

$$P_{in} = i\, v_{plant}$$

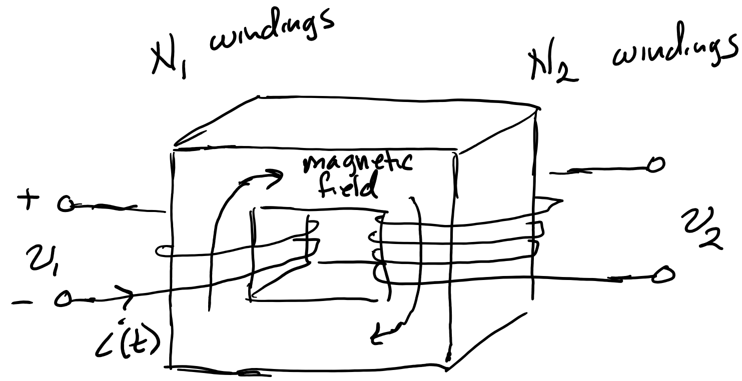
$\therefore$

$$P_{loss} = \left(\frac{P_{in}}{v_{plant}} \right)^2 R_{line}$$

$$P_{loss} \rightarrow 0 \quad \text{when} \quad v_{plant} \rightarrow \infty$$

Faraday's Law: "The induced voltage in a coil is

Faraday's Law: "The induced voltage in a coil is proportional to the <u>time rate of change</u> of flux linkage."

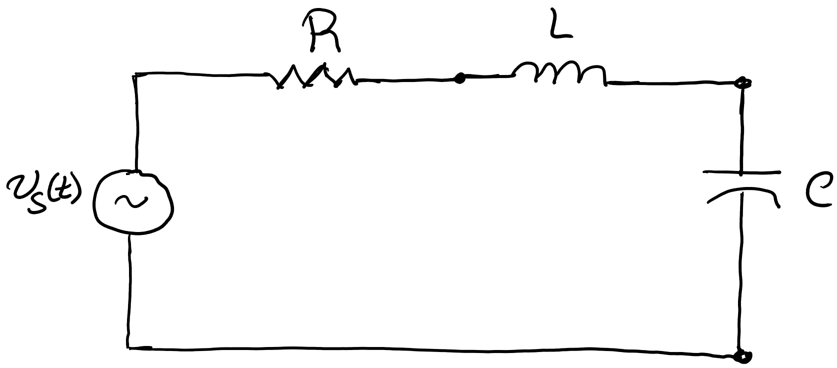


$$\frac{v_2}{v_1} = \frac{N_1}{N_2}$$

But this only works if i is time varying

$$\frac{di}{dt} \neq 0$$

Consider again an RLC circuit,



What is the current flow $i(t)$ if $v_s(t) = V_0 \sin(\omega t)$

From last lecture,

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{dv_s}{dt} = \underbrace{\omega V_0}_{F_0} \cos(\omega t)$$

$$= F_0 \cos(\omega t)$$

Recall from ODE's

The solution is

$$i(t) = \underbrace{i_H(t)}_{\text{Homogeneous solution}} + \underbrace{i_p(t)}_{\text{particular solution}}$$

If we want to know the "steady-state" response $(t \to \infty)$, we know

$$i_H(t) \to 0 \quad \text{as} \quad t \to \infty \qquad R > 0$$

So, the steady-state solution

$$i(t) = i_p(t)$$

For harmonic forcing,

$$i_p(t) = I_0 \cos(\omega t + \phi)$$

Observe,

$$F(t) := F_0 \cos(\omega t) + j F_0 \sin(\omega t)$$

$$= F_0 e^{j\omega t} \qquad ; \quad e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$$

Now,

$$I(t) = i(t) + j \varphi(t)$$

Then

$$\frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC} I = F$$

$$\Rightarrow \left[\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i\right] + j\left[\frac{d^2 \varphi}{dt^2} + \frac{R}{L}\frac{d\varphi}{dt} + \frac{1}{LC}\varphi\right]$$

$$= F_0 \cos \omega t + j F_0 \sin \omega t$$

$$\Rightarrow \quad \frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = F_0 \cos \omega t \quad !$$

$\therefore$ We can represent any physical harmonic forcing by a complex function of the form

$$A e^{j\omega t}$$

and the real component of the solution will correspond to the physical response.

Since the forcing has the form $F_0 e^{j\omega t}$,

$$\frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC} I = F_0 e^{j\omega t}$$

Know, the steady sol'n

$$I(t) = I_0 e^{j(\omega t + \phi)}$$

Note:
$j = \sqrt{-1}$
$j^2 = -1$

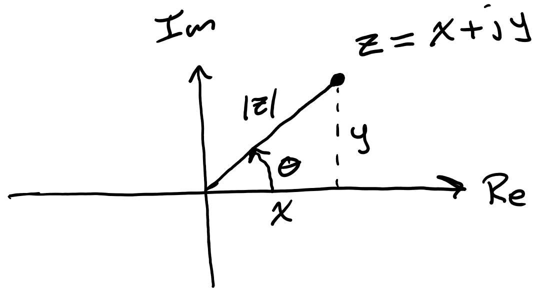
Substitute I into ODE,

$$-\omega^2 I_0 e^{j(\omega t+\phi)} + \frac{R}{L} j\omega I_0 e^{j(\omega t+\phi)} + \frac{1}{LC} I_0 e^{j(\omega t+\phi)} = F_0 e^{j\omega t}$$

$$\Rightarrow \left[-\omega^2 + j\omega\frac{R}{L} + \frac{1}{LC}\right] I_0 \cancel{e^{j\omega t}}\, e^{j\phi} = F_0 \cancel{e^{j\omega t}}$$

$$\Rightarrow I_0 e^{j\phi} = \frac{F_0}{\left(\frac{1}{LC} - \omega^2\right) + j\left(\omega\frac{R}{L}\right)}$$

Recall:



$$z = |z| e^{j\theta} = x + iy$$

$$|z|^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x}$$

$\therefore$

$$I_0 e^{j\phi} = \frac{F_0}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega\frac{R}{L}\right)^2}}$$

$$= \frac{F_0}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega\frac{R}{L}\right)^2}}\, e^{-j\theta}$$

$\therefore$

$$I_0 = \frac{F_0}{\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \left(\omega\frac{R}{L}\right)^2}}$$

$$\phi = \tan^{-1}\left(-\frac{\omega\frac{R}{L}}{\frac{1}{LC} - \omega^2}\right)$$