

Lecture 4: Complexity and Sorting

ENAE 380 Flight Software Systems
September 9, 2024



Find the number 50

1

2

3

4

5

6

7

Find the number 50

15

23

16

8

42

50

4

Find the number 50

1

2

3

4

5

6

7

Find the number 50

50

42

23

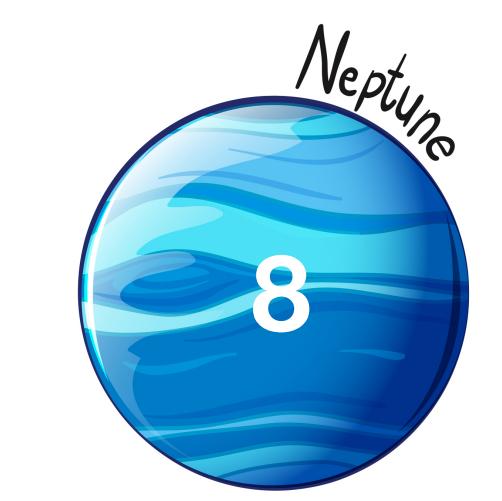
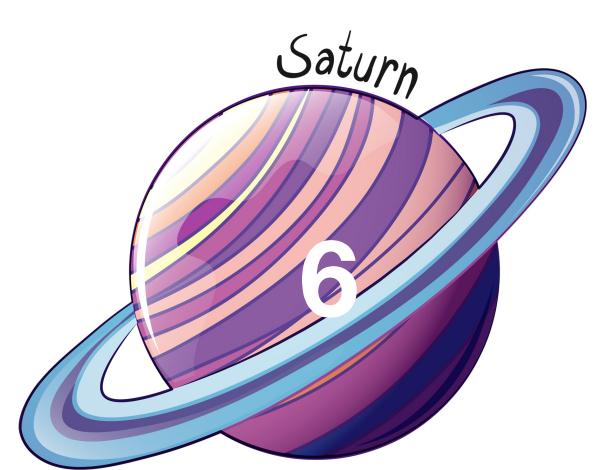
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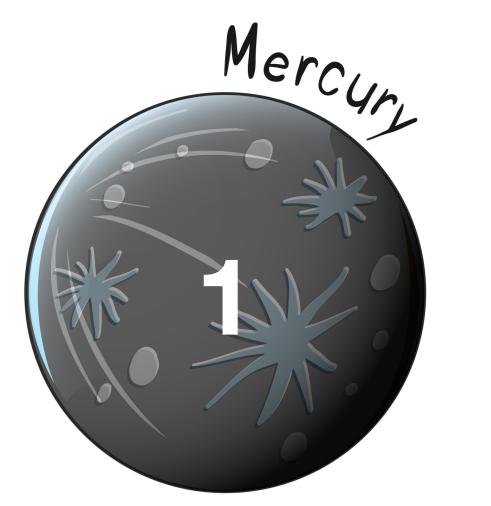
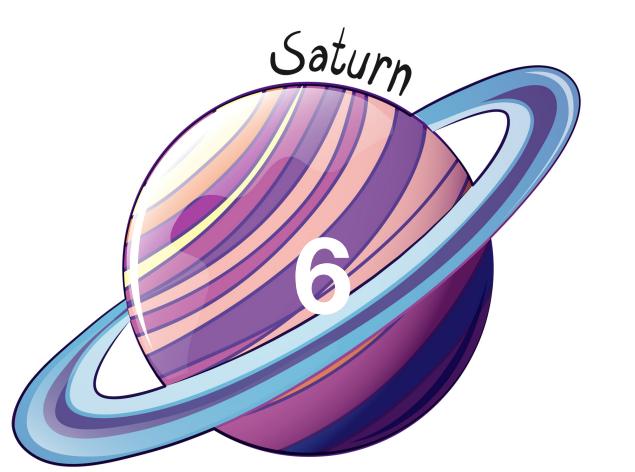
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8

4

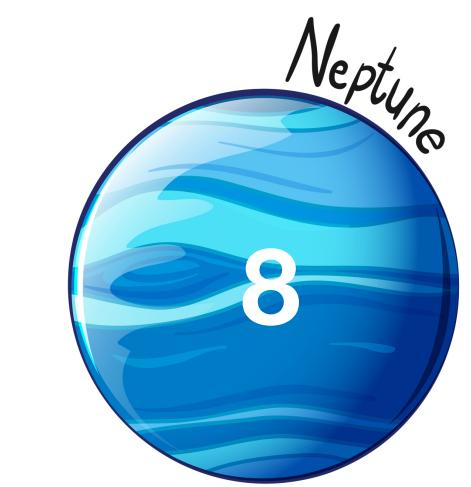
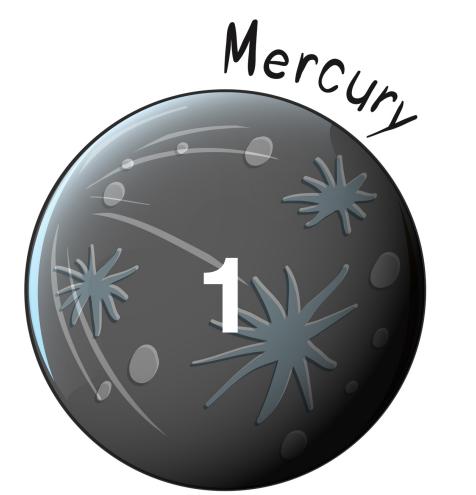
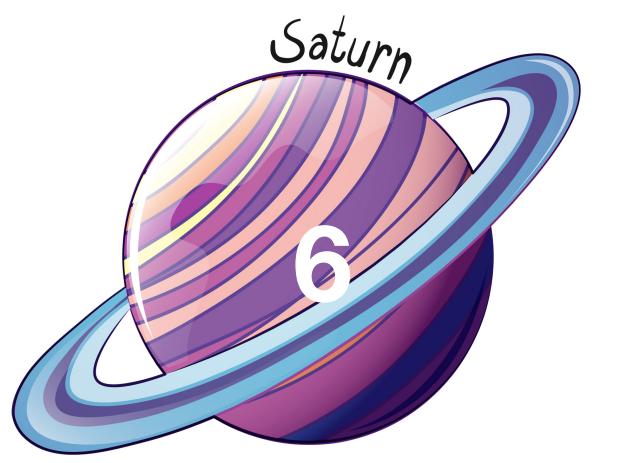
Bubble Sort

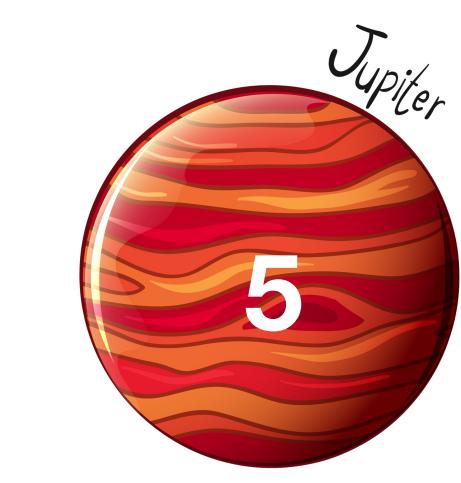
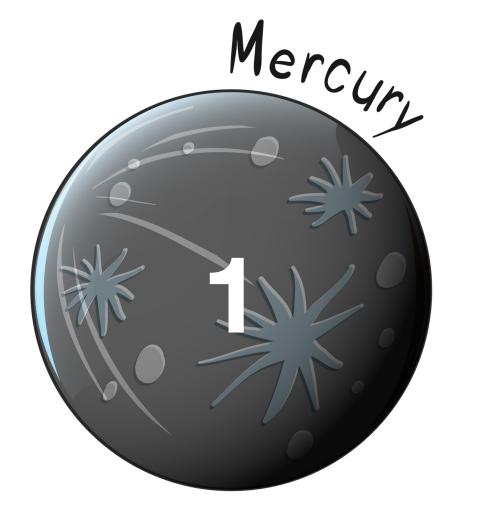


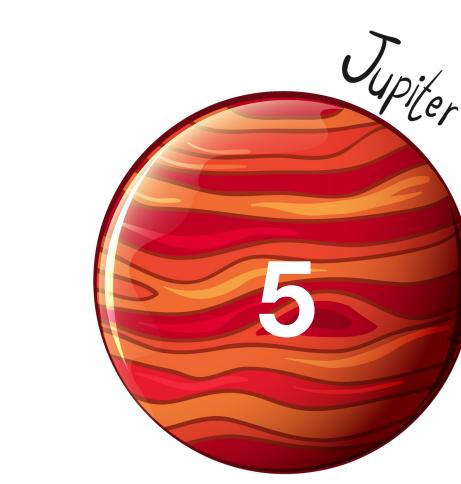
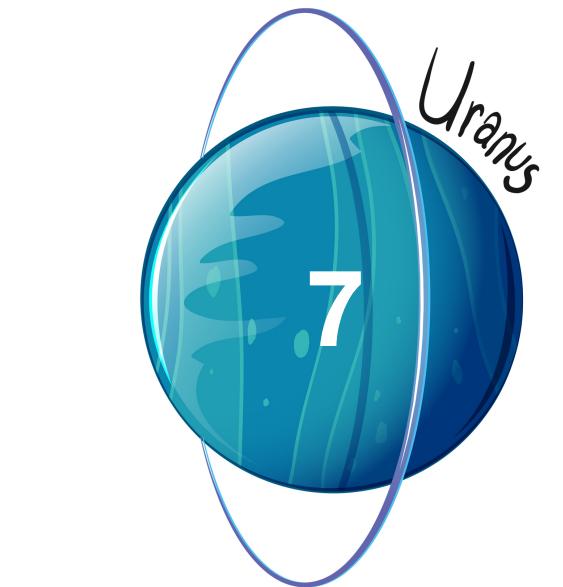
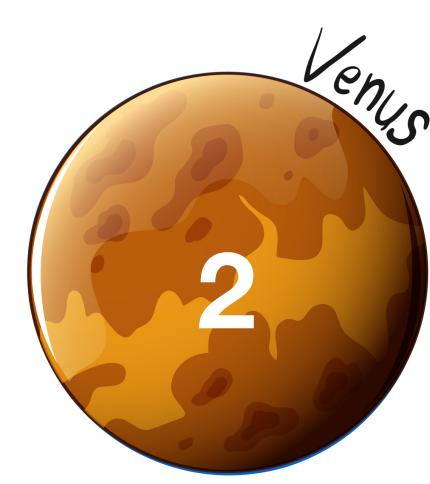


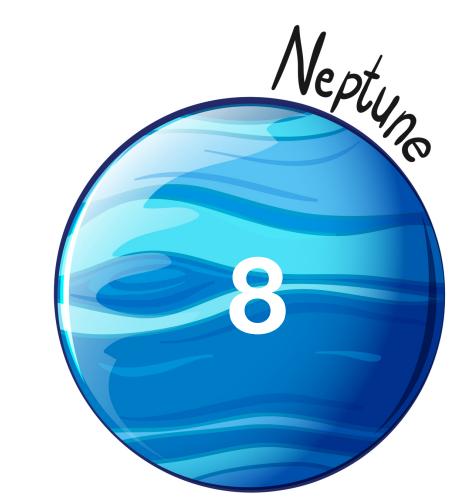
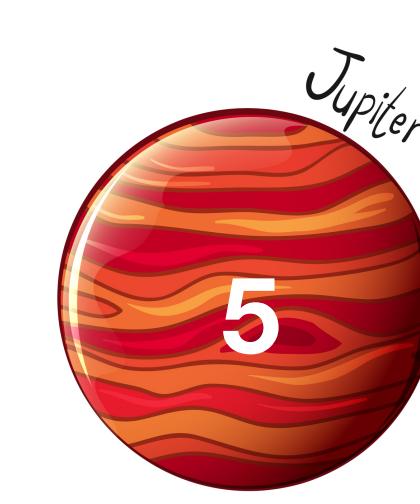
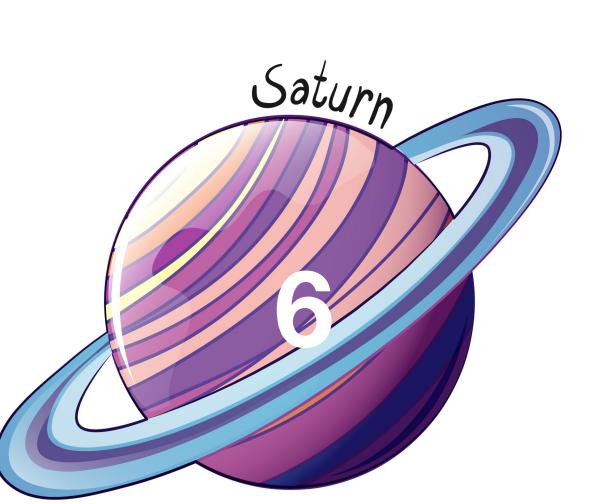
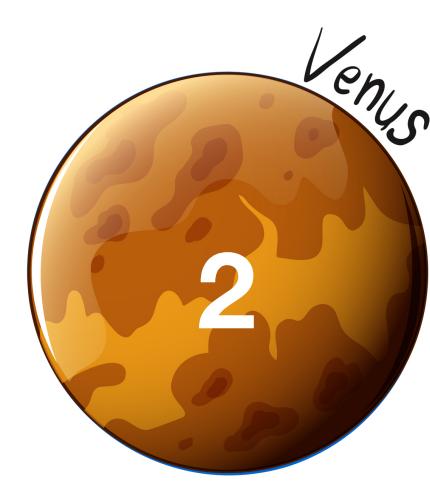


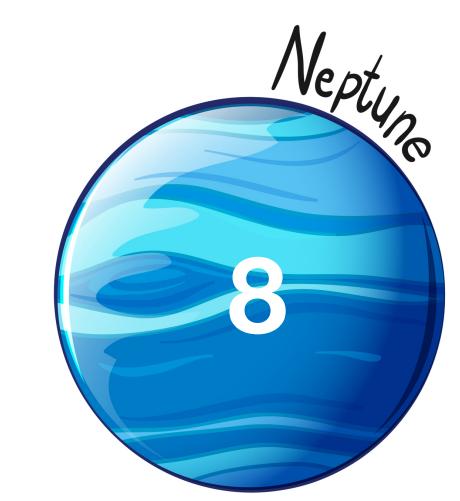
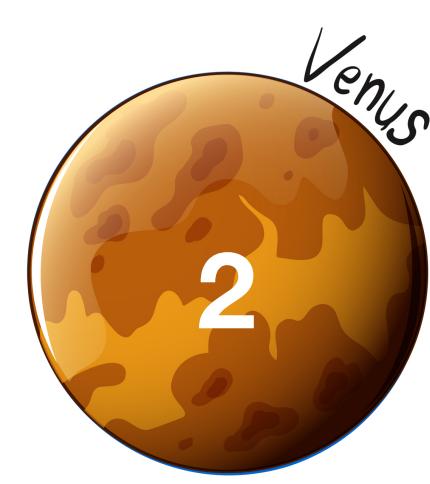


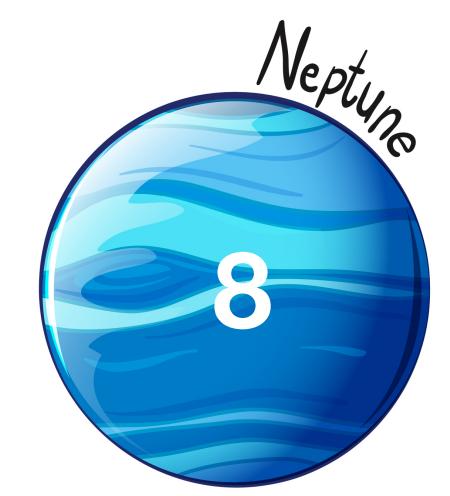
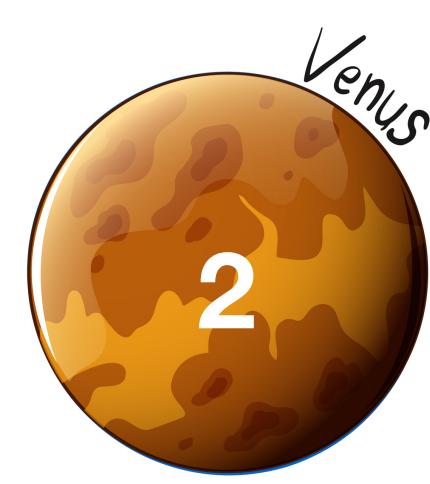




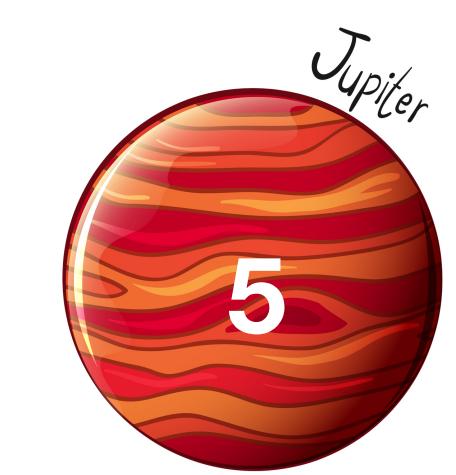
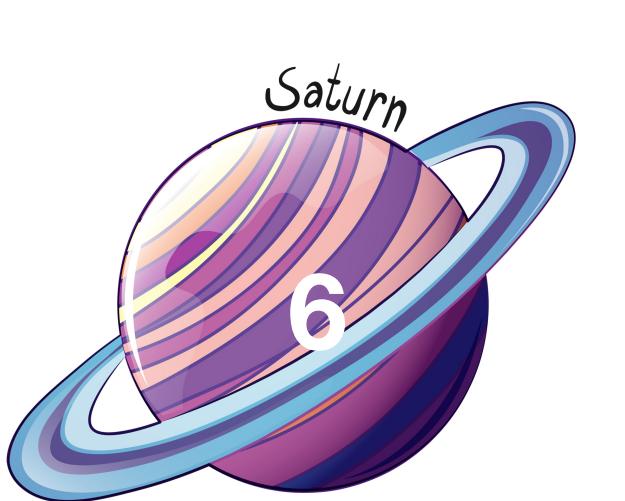




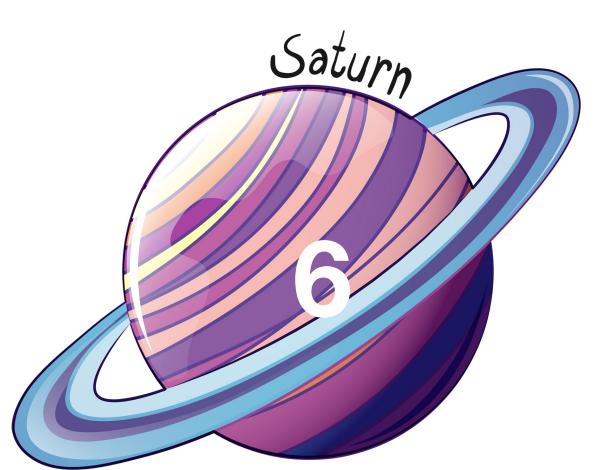




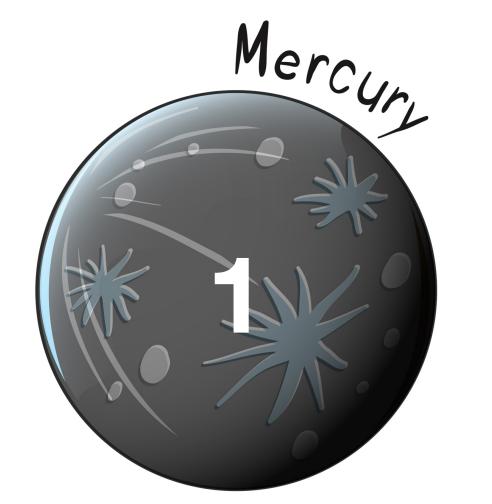
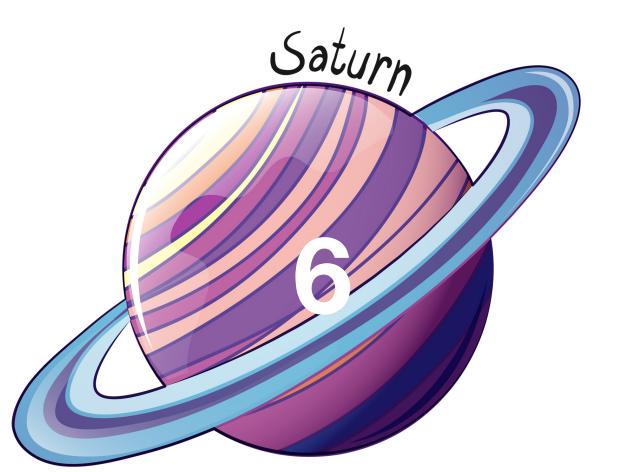


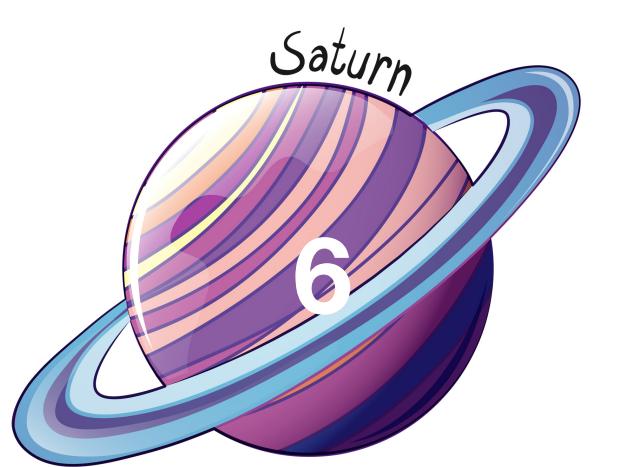






Selection Sort





Mercury

1

Venus

2

Saturn

6

Neptune

8

Mars

4

Earth

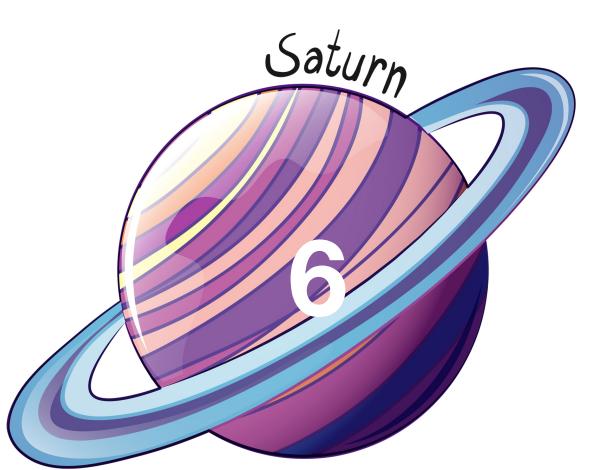
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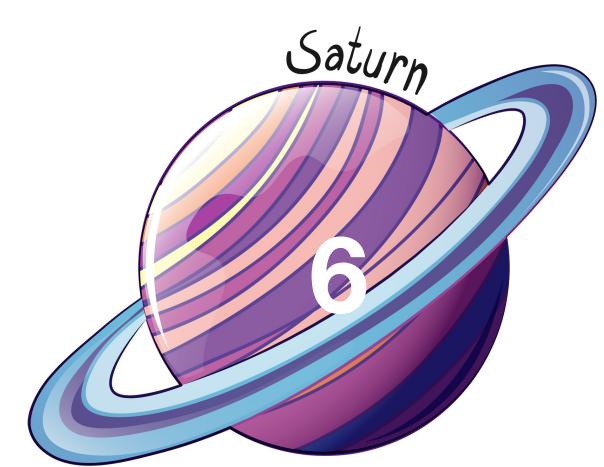
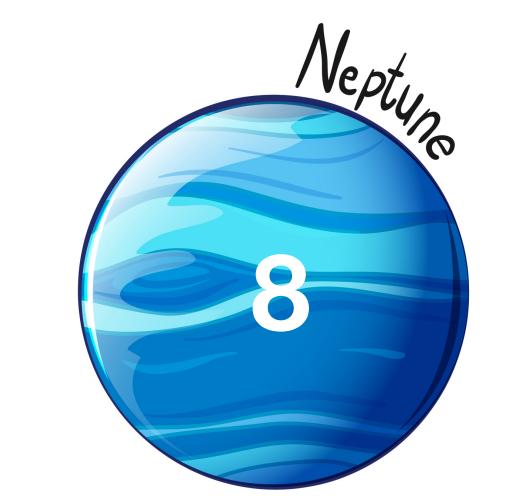
Uranus

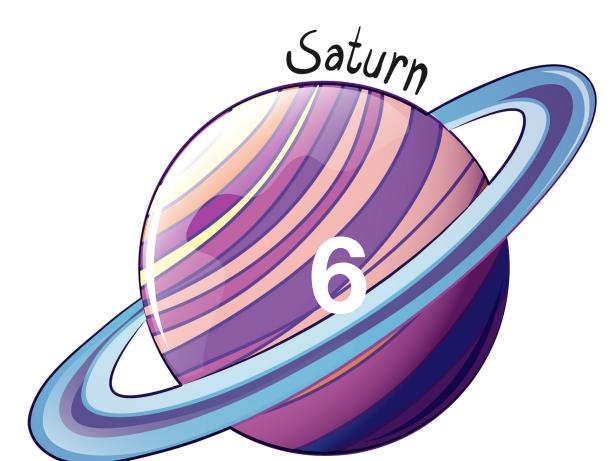
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Jupiter

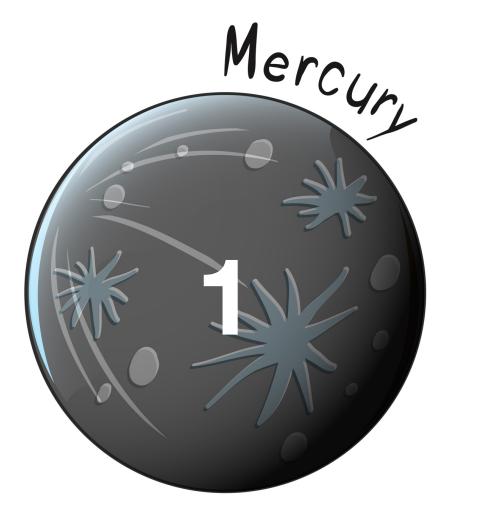
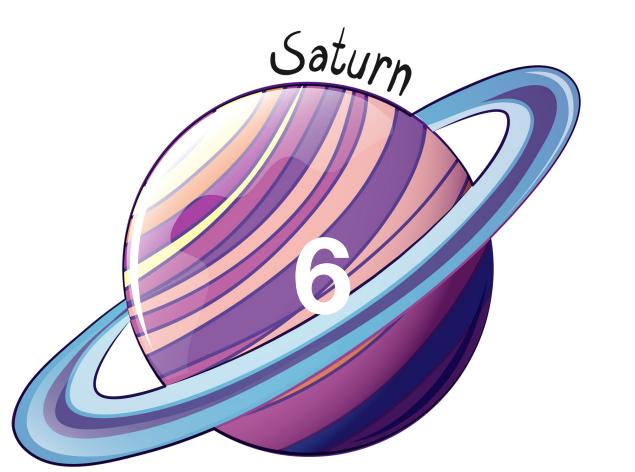
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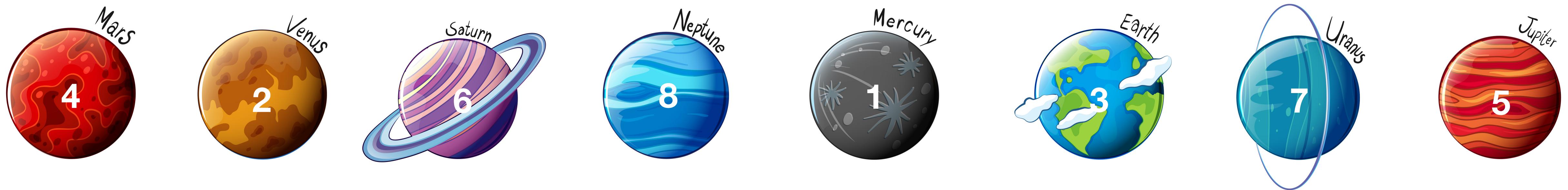
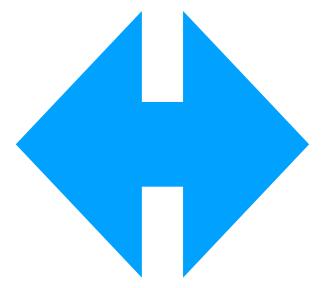


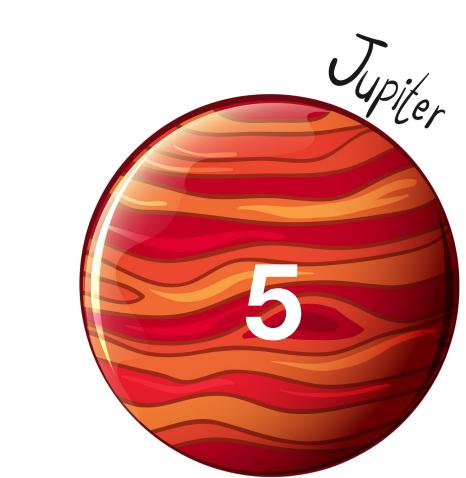
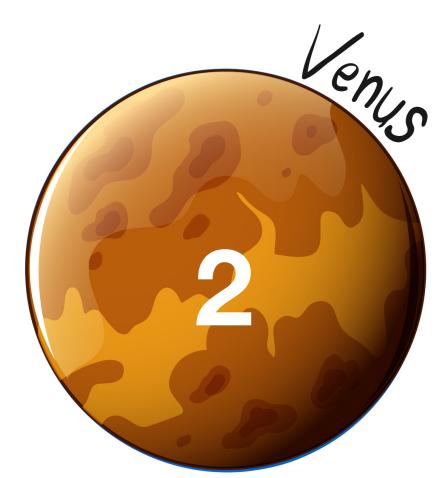


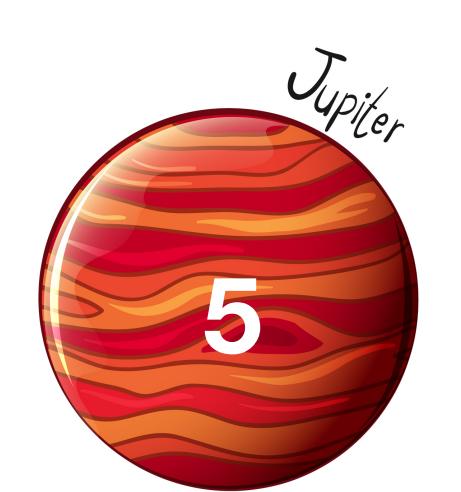
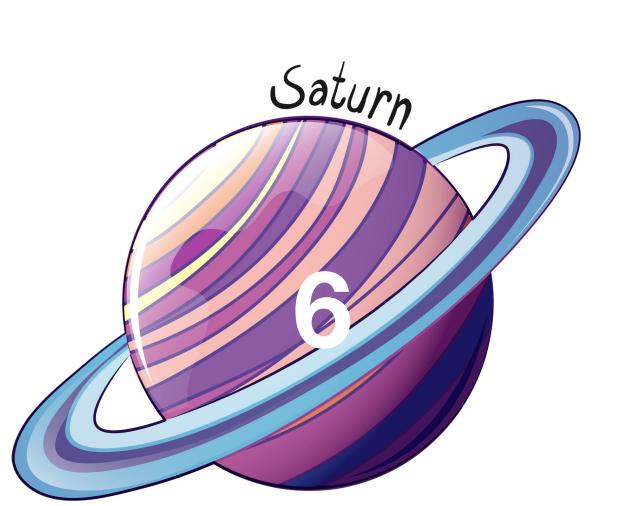


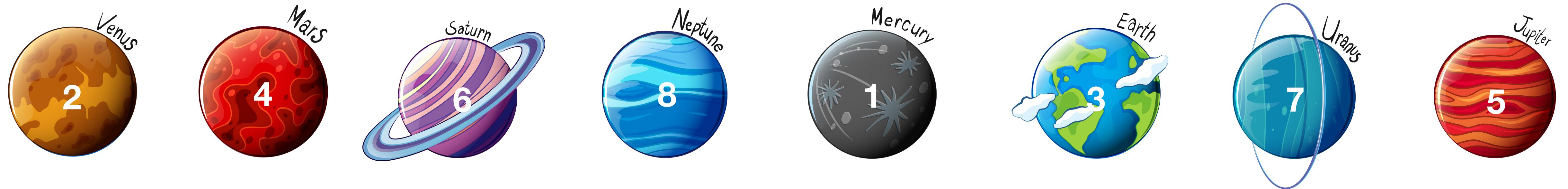
Insertion Sort

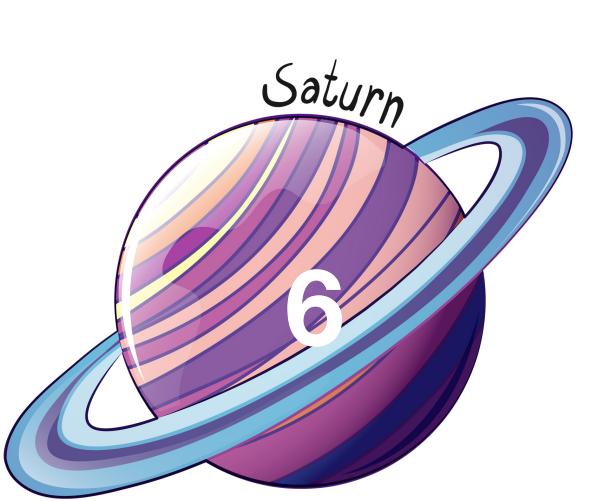


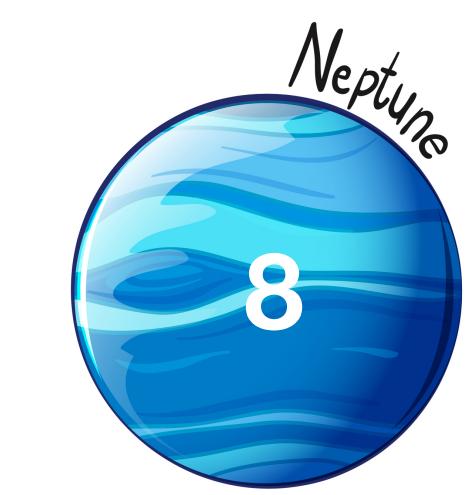
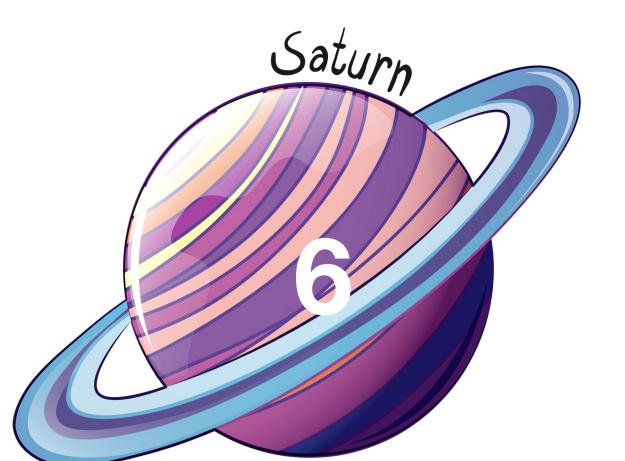


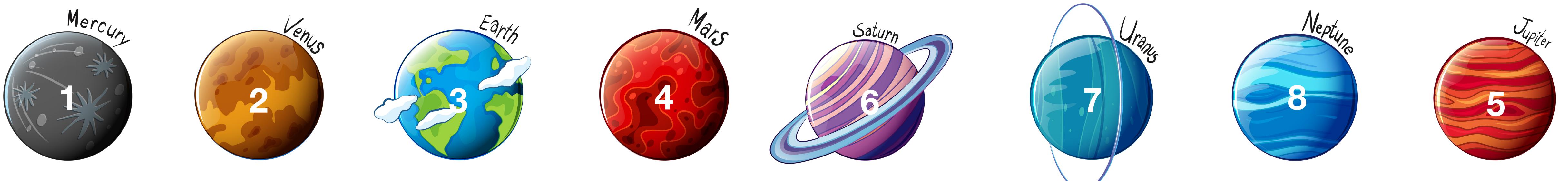


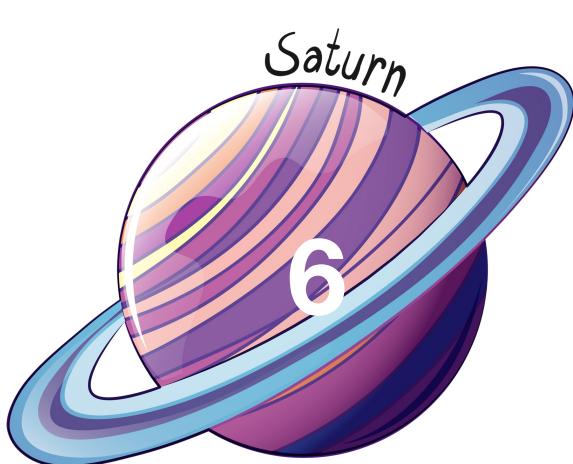












Complexity

8 7 6 5 4 3 2 1

(n-1)

8 7 6 5 4 3 2 1

(n-1) + (n-2)

8 7 6 5 4 3 2 1

(n-1) + (n-2) + ... + 1

8 7 6 5 4 3 2 1

$(n-1) + (n-2) + \dots + 1$

$n(n-1)/2$

8 7 6 5 4 3 2 1

$$(n-1) + (n-2) + \dots + 1$$

$$n(n-1)/2$$

$$(n^2 - n)/2$$

8 7 6 5 4 3 2 1

$$(n-1) + (n-2) + \dots + 1$$

$$n(n-1)/2$$

$$(n^2 - n)/2$$

$$n^2/2 - n/2$$

1,000,000

$$n^2/2 - n/2$$

$$n^2/2 - n/2$$

$$1,000,000^2/2 - 1,000,000/2$$

$$n^2/2 - n/2$$

$$1,000,000^2/2 - 1,000,000/2$$

$$500,000,000,000 - 500,000$$

$$n^2/2 - n/2$$

$$1,000,000^2/2 - 1,000,000/2$$

$$500,000,000,000 - 500,000$$

$$499,999,500,000$$

$$n^2/2 - n/2$$

$$O(n^2)$$

$O(n^2)$

$O(n \log n)$

$O(n)$

$O(\log n)$

$O(1)$

...

$\Omega(n^2)$

$\Omega(n \log n)$

$\Omega(n)$

$\Omega(\log n)$

$\Omega(1)$

...

Why do we care?

<https://www.toptal.com/developers/sorting-algorithms>

Sorting Algorithms Animations

The following animations illustrate how effectively data sets from different starting points can be sorted using different algorithms.



How to use: Press "Play all", or choose the ▶ button for the individual row/column to animate.

TRY ME!

▶	Play All	▶	Insertion	▶	Selection	▶	Bubble	▶	Shell	▶	Merge	▶	Heap	▶	Quick	▶	Quick3
▶	Random	▶		▶		▶		▶		▶		▶		▶		▶	
▶	Nearly Sorted	▶		▶		▶		▶		▶		▶		▶		▶	
▶	Reversed	▶		▶		▶		▶		▶		▶		▶		▶	
▶	Few Unique	▶		▶		▶		▶		▶		▶		▶		▶	

The page displays a grid of sorting algorithm visualizations. The columns represent different sorting algorithms: Play All, Insertion, Selection, Bubble, Shell, Merge, Heap, Quick, and Quick3. The rows represent different initial data sets: Random, Nearly Sorted, Reversed, and Few Unique. Each cell in the grid contains a small visualization of the data set being sorted by that specific algorithm. A green button labeled 'TRY ME!' is located in the top-left corner of the grid area.

<https://www.youtube.com/watch?v=t8g-iYGHpEA>



Merge Sort

```
1 pick up phone book
2 open to middle of phone book
3 look at names
4 if "Swift" is among names
5   call Taylor
6 else if "Swift" is earlier in book
7   open to middle of left half of book
8   go to line 3
9 else if "Swift" is later in book
10  open to middle of right half of book
11  go to line 3
12 else
13   give up
```

```
1 pick up phone book
2 open to middle of phone book
3 look at names
4 if "Swift" is among names
5   call Taylor
6 else if "Swift" is earlier in book
7   search for Swift in left half of book
8
9 else if "Swift" is later in book
10  search for Swift in right half of book
11
12 else
13   give up
```

on input of n elements

if n < 2

 return

else

 sort left half of elements

 sort right half of elements

 merge sorted halves

4 8 6 2 1 7 5 3

4

8

6

2

1

7

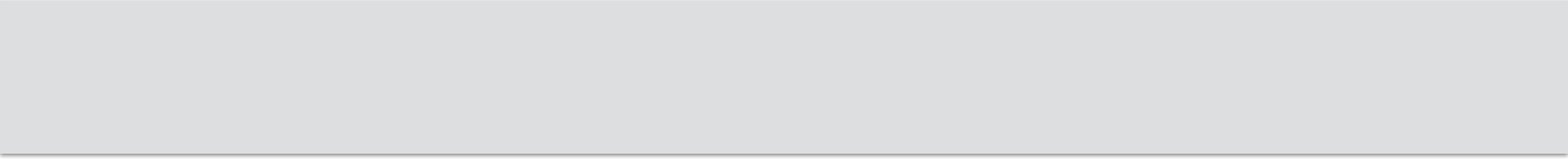
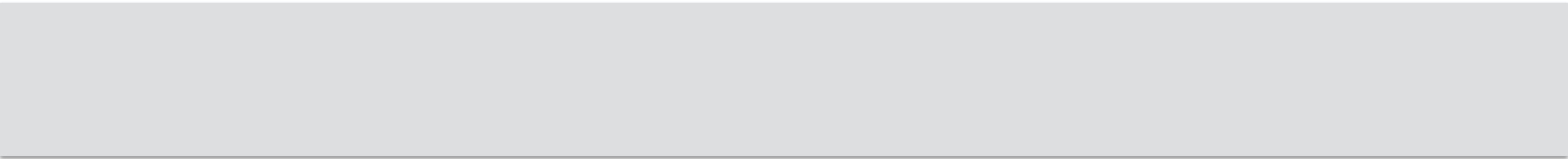
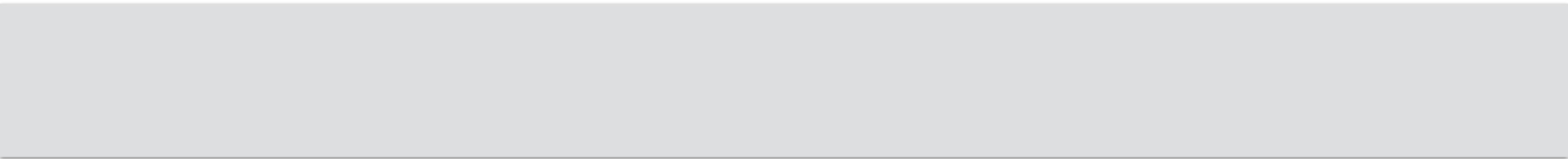
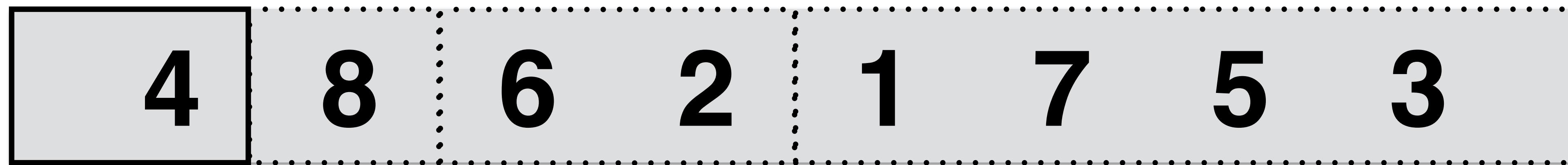
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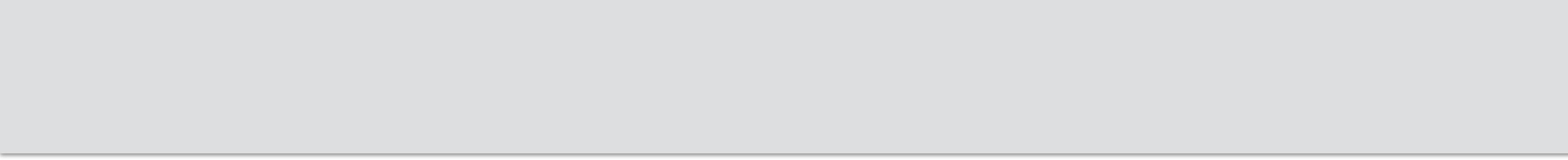
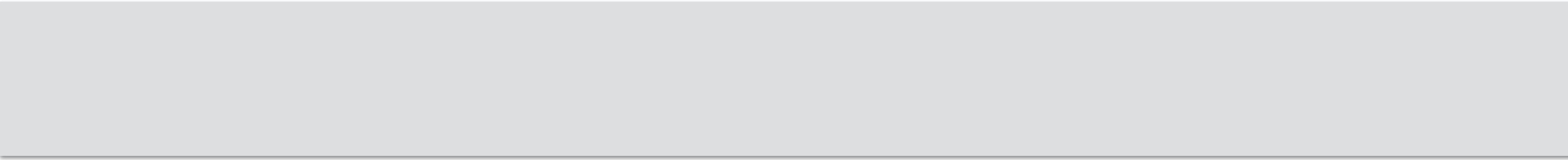
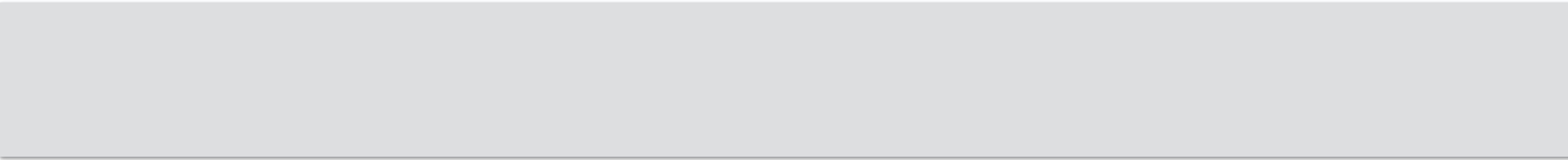
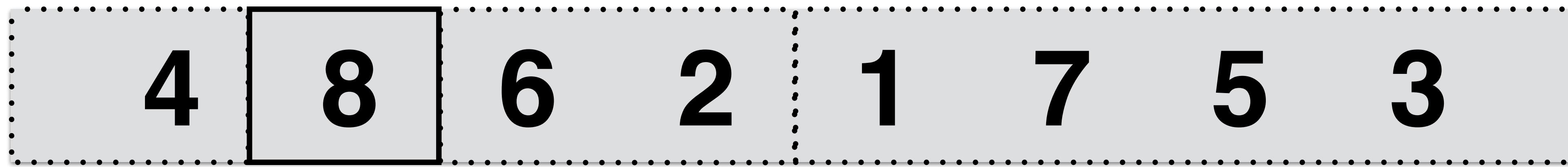
3

4 **8**

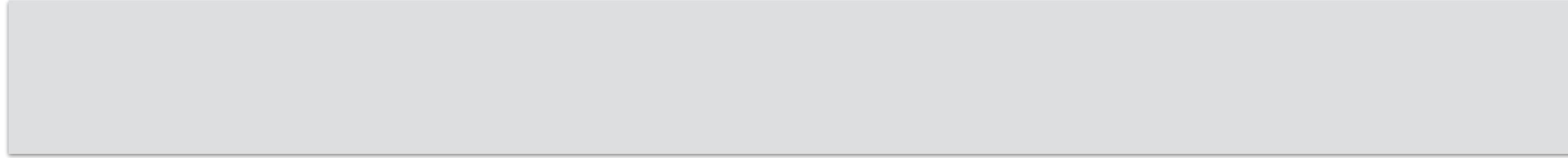
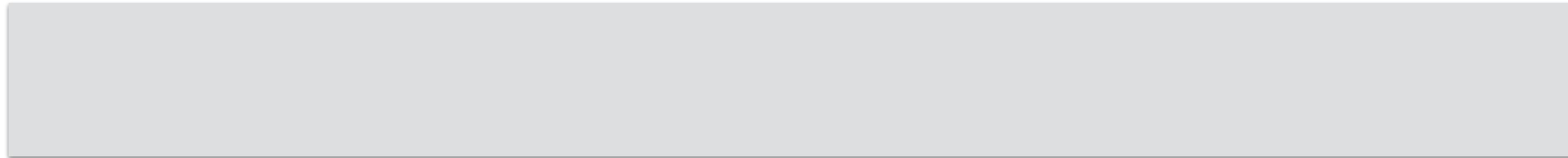
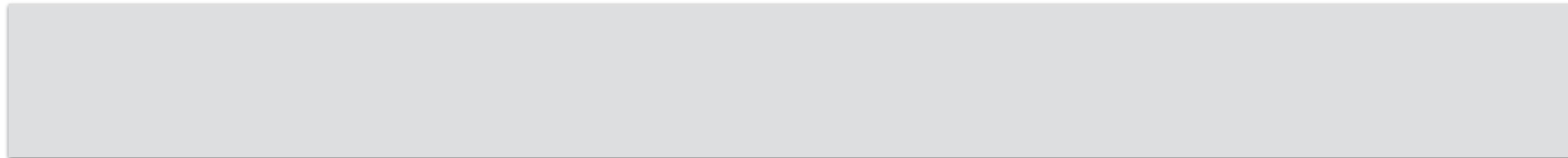
6 **2**

1 **7** **5** **3**

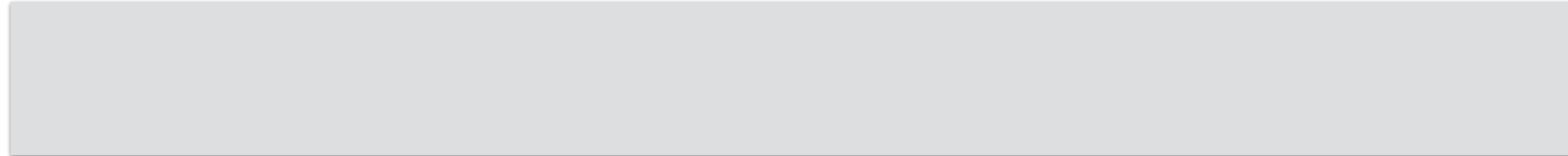
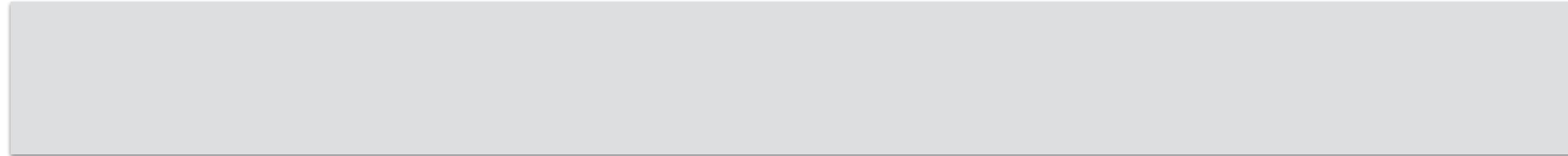
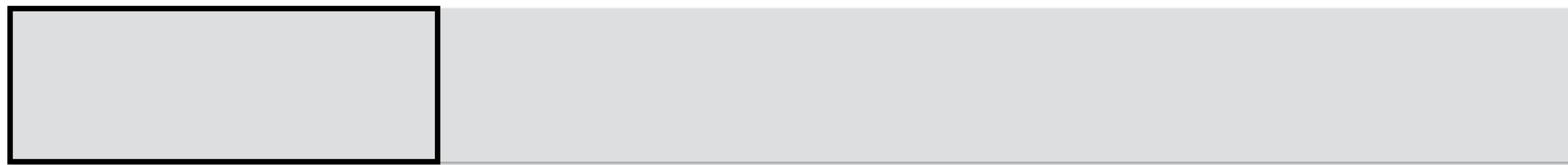


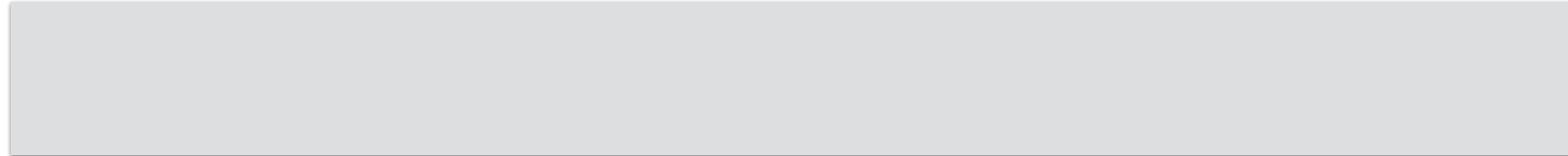
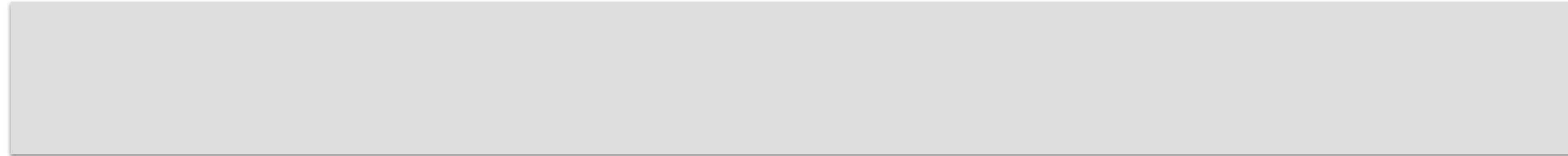
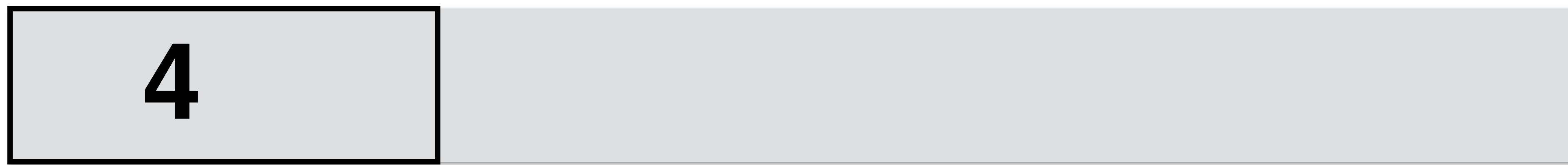
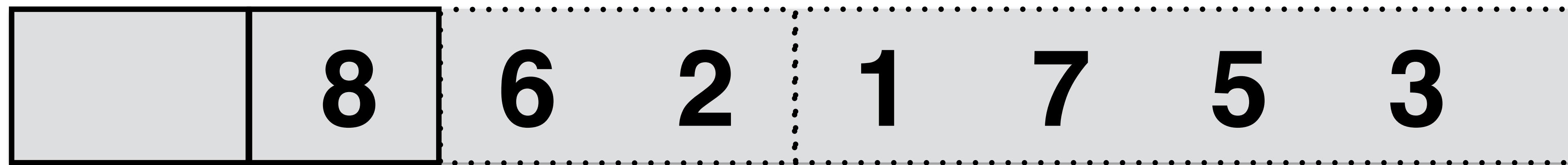


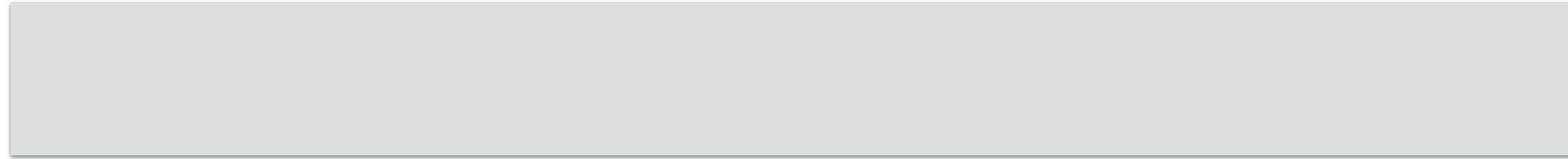
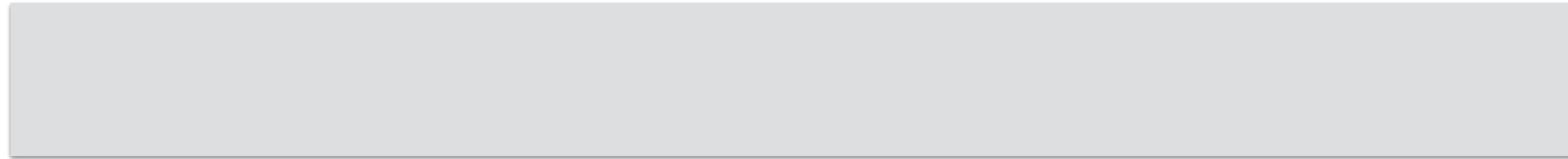
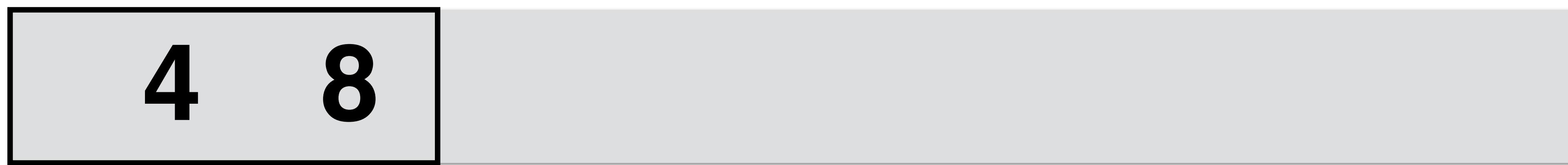
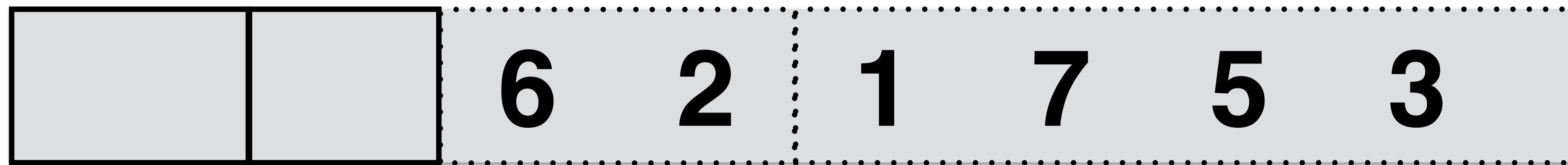
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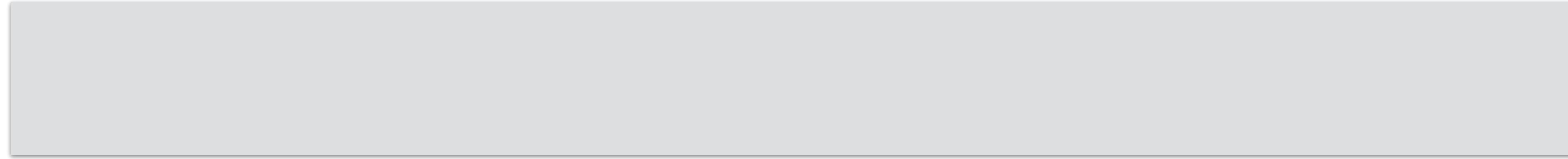
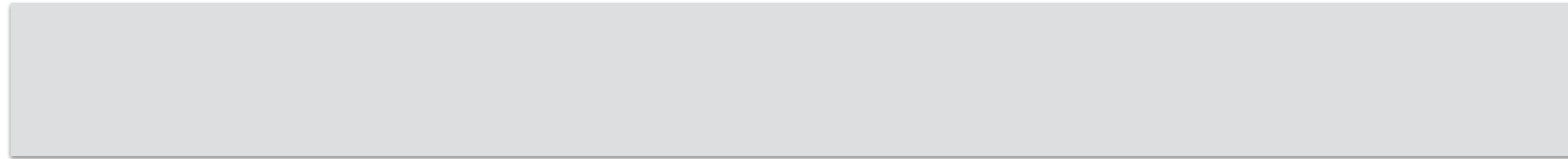
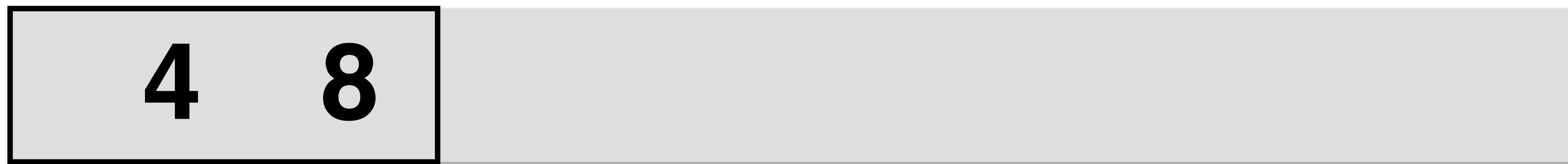
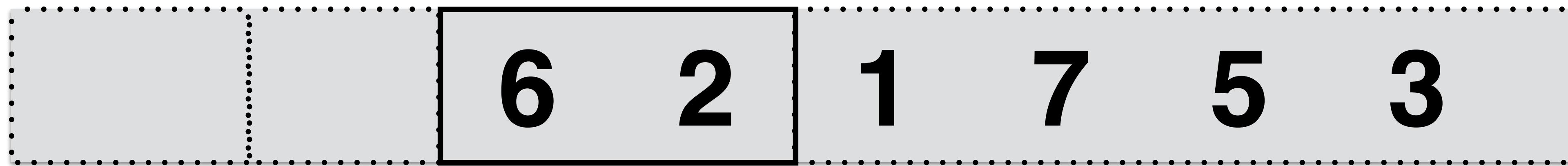


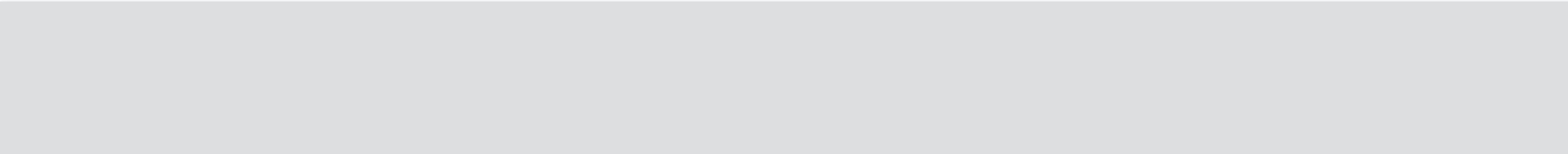
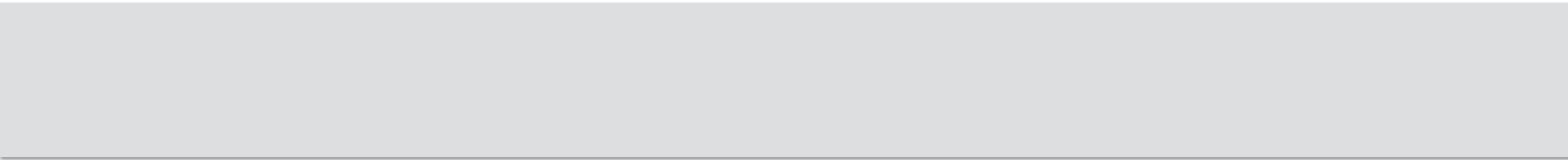
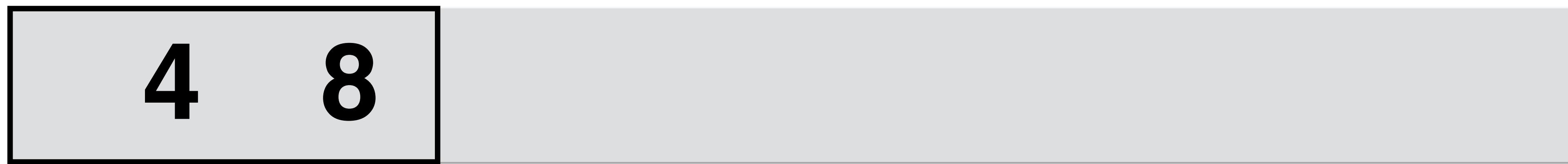
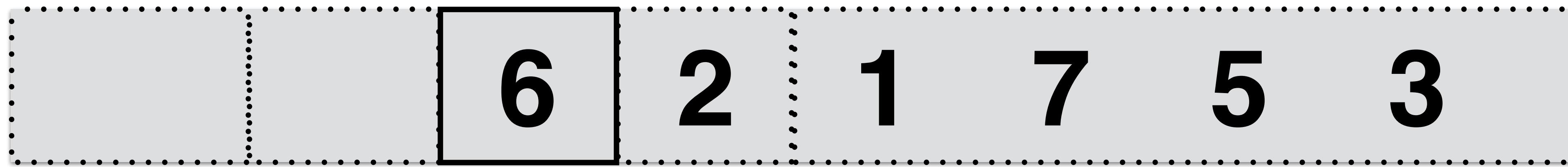
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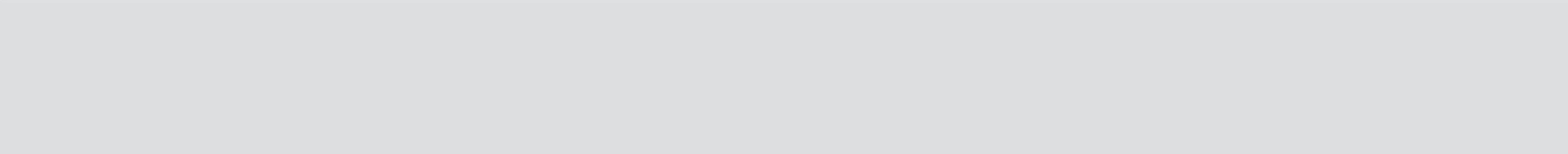
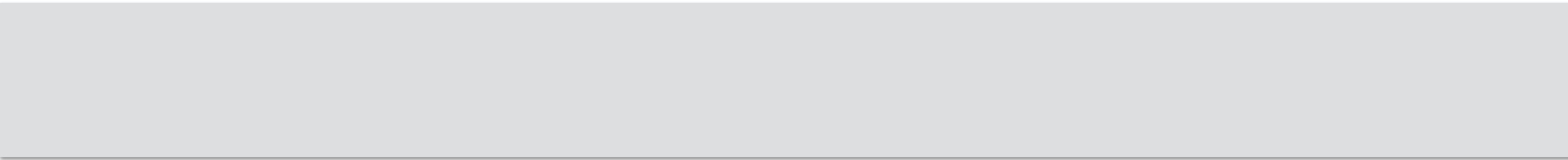
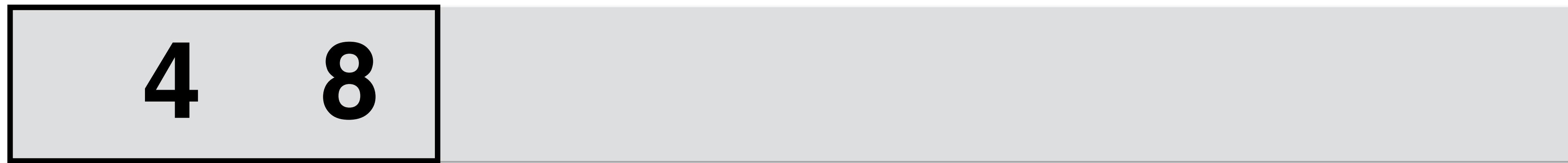
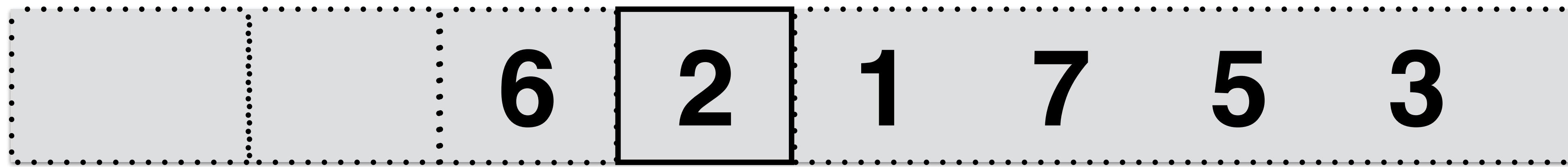


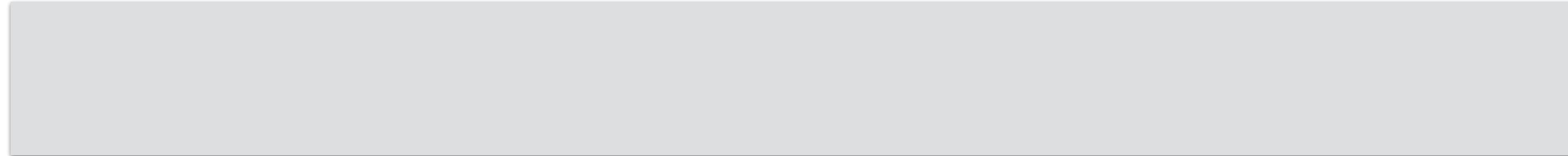
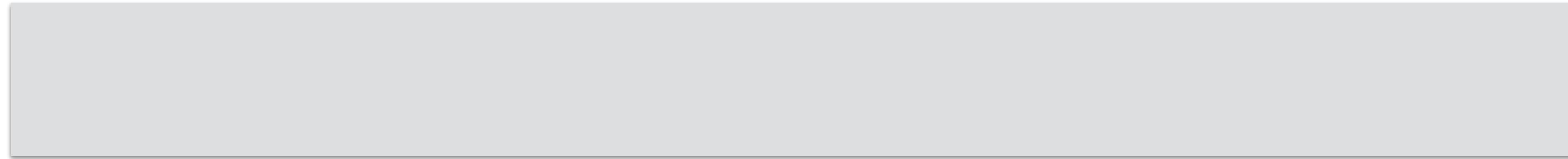
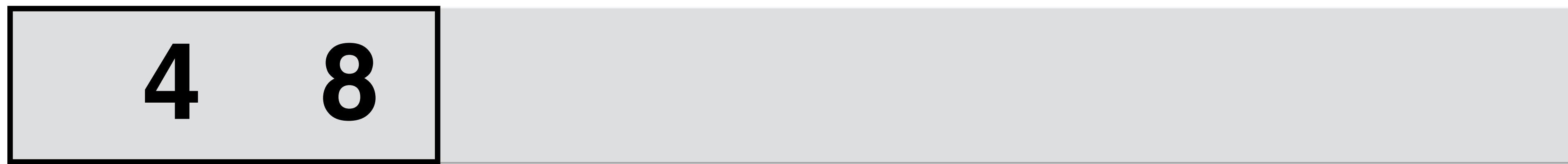
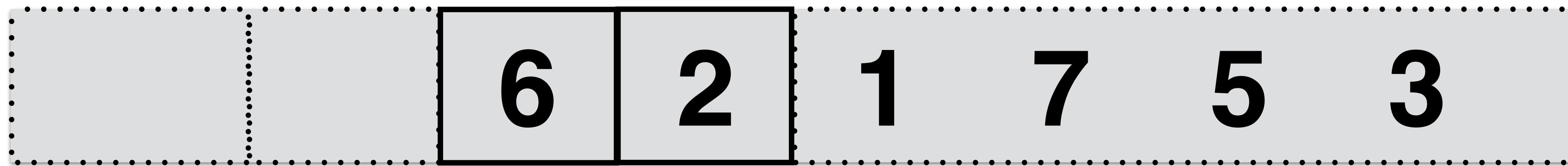


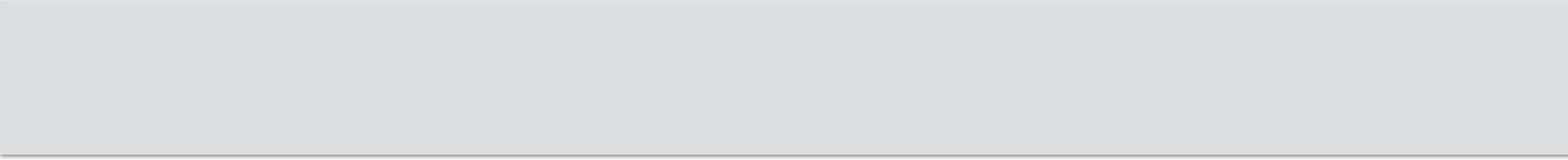
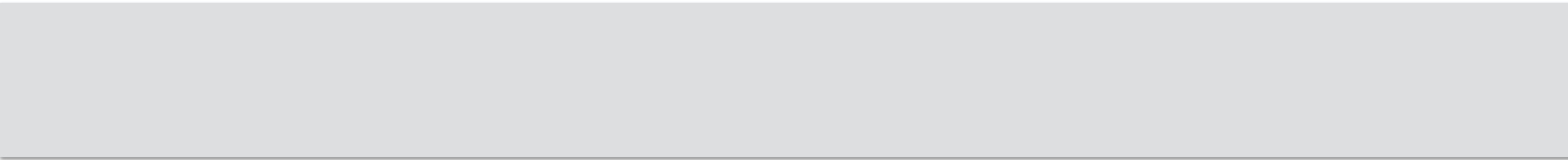
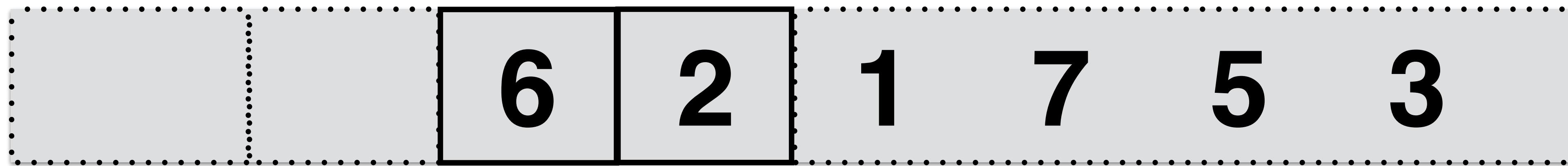


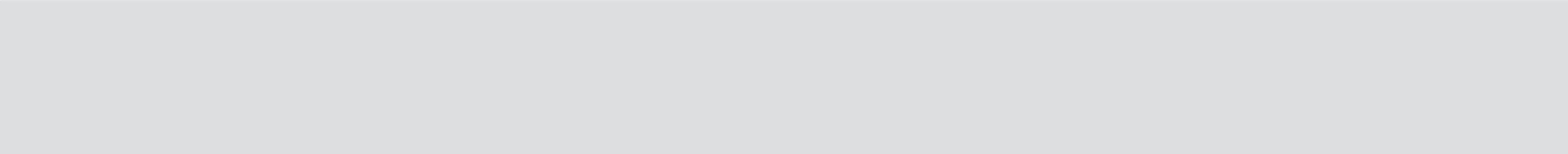
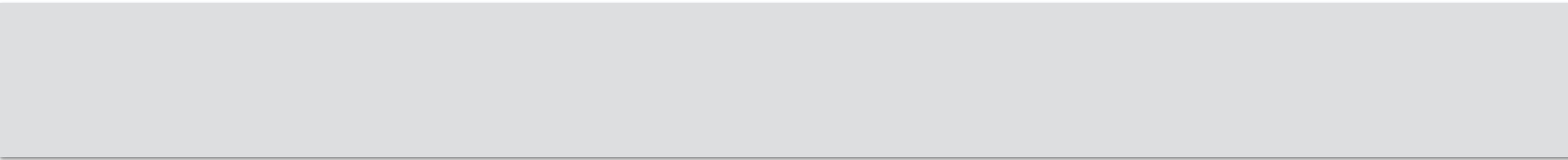
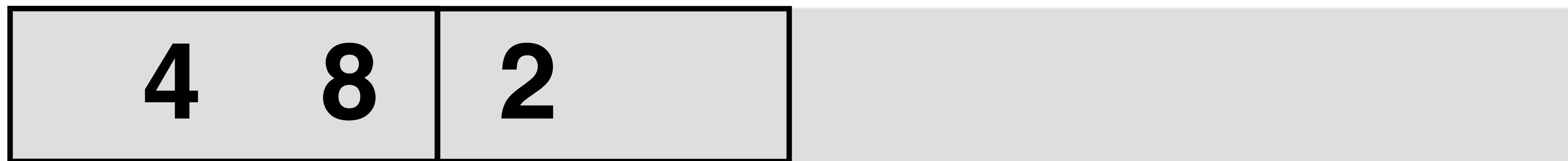
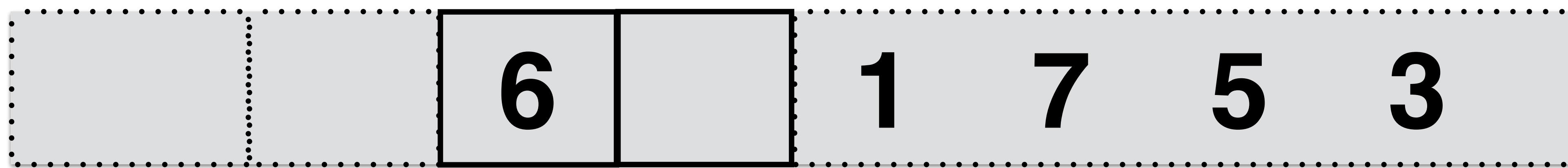


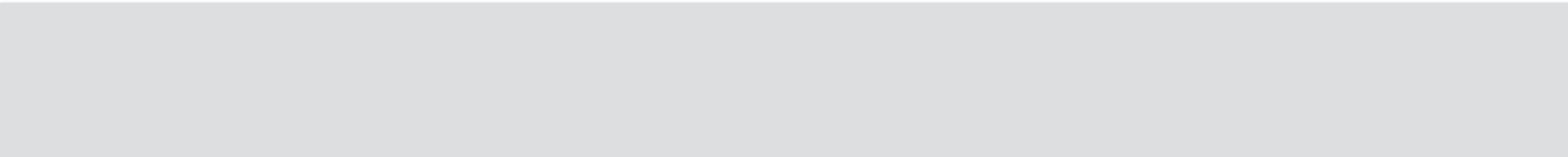
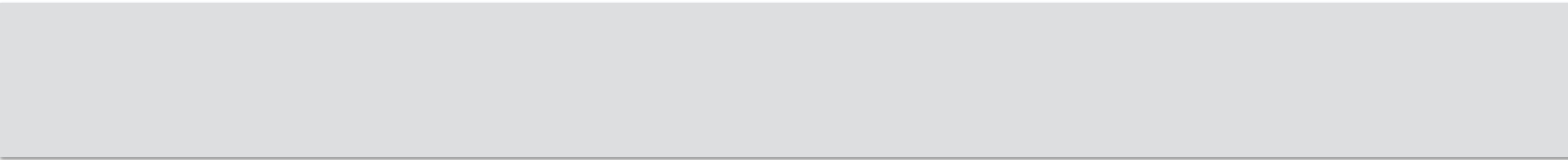
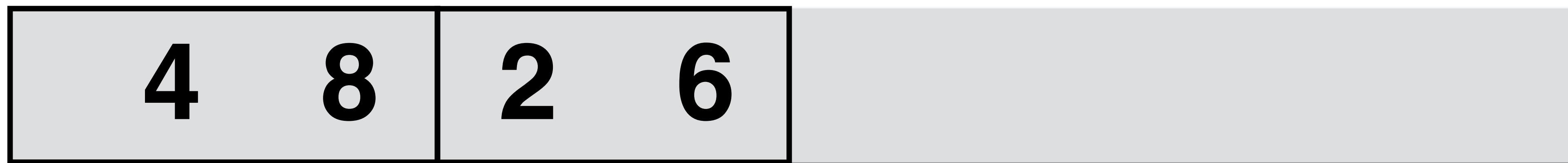
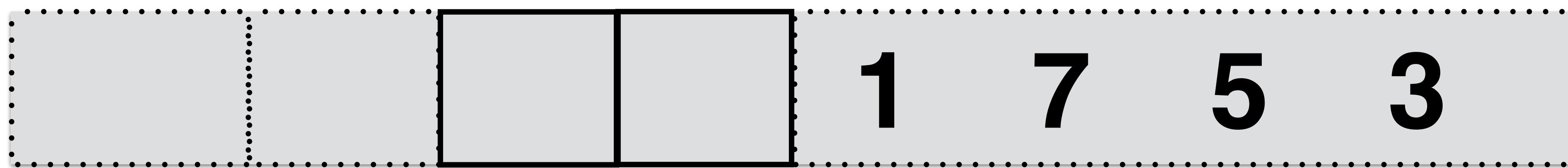


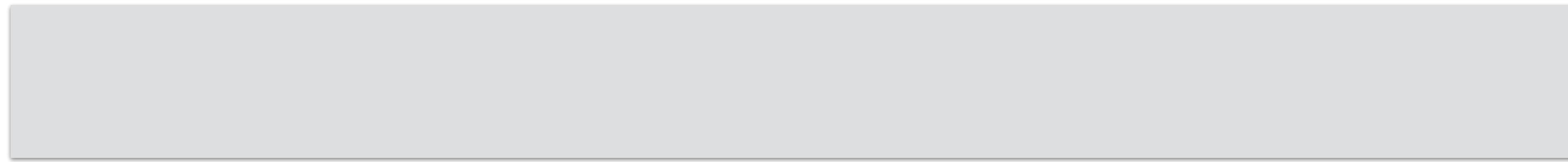
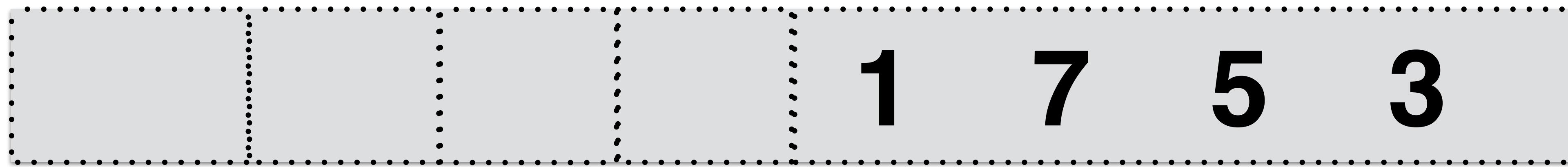










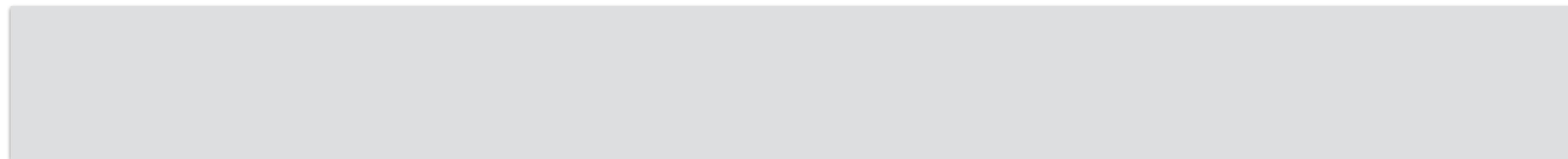
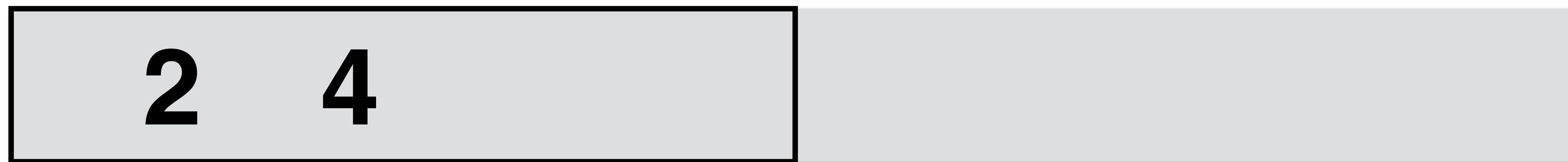
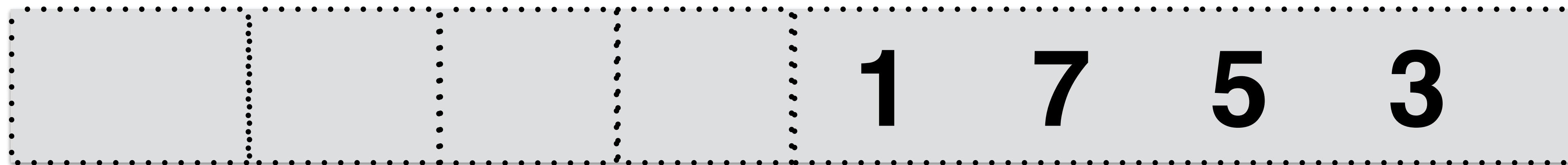


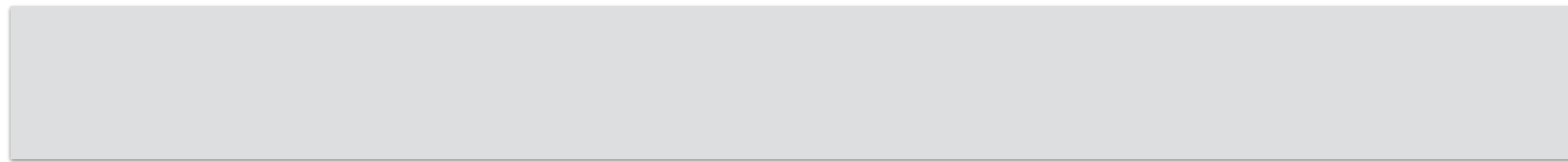
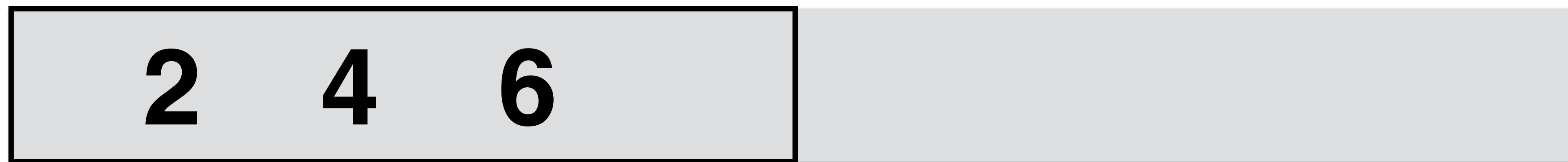
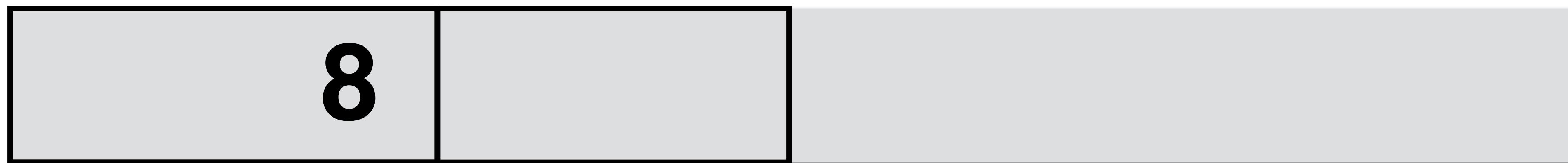
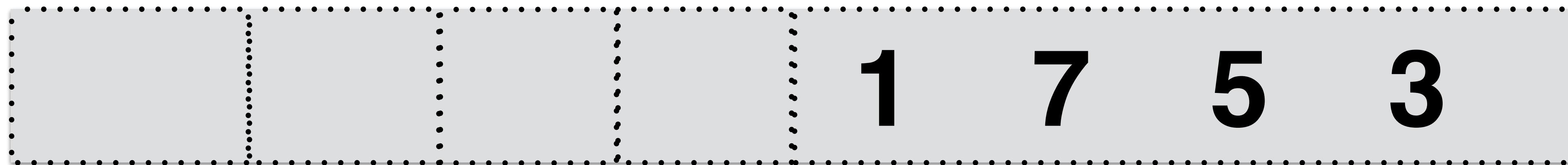
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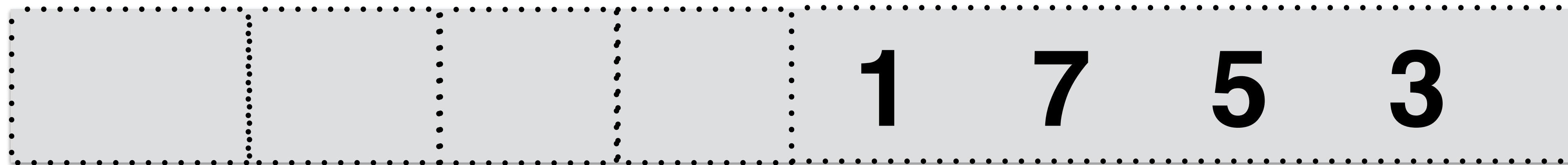
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4 8 | 6

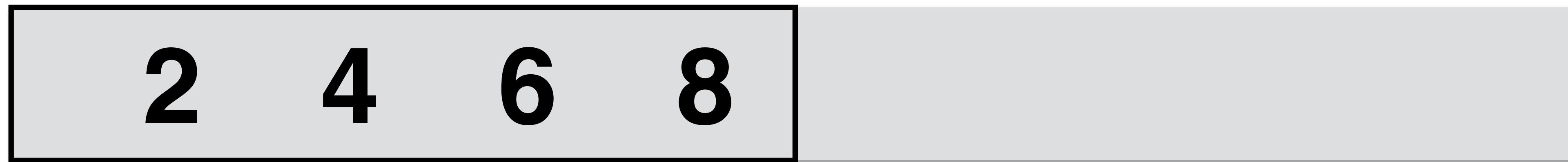
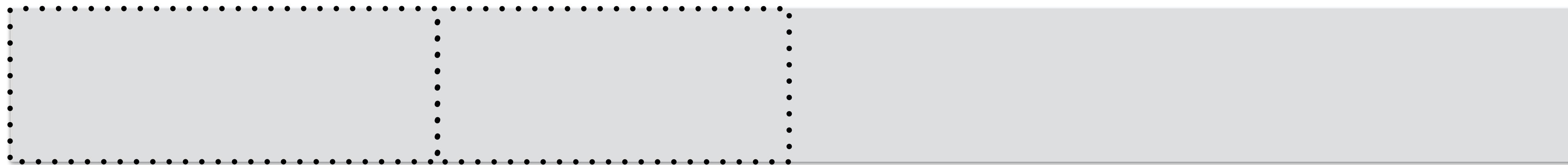
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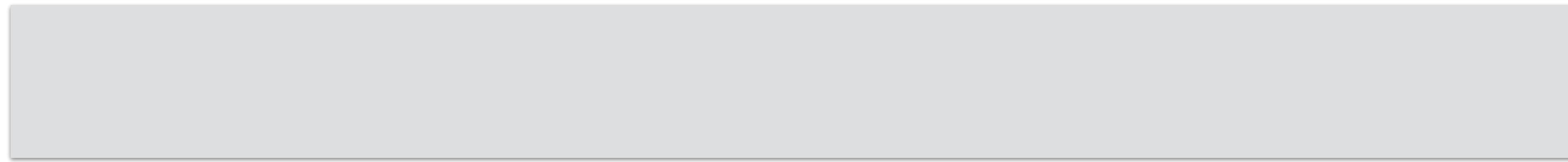


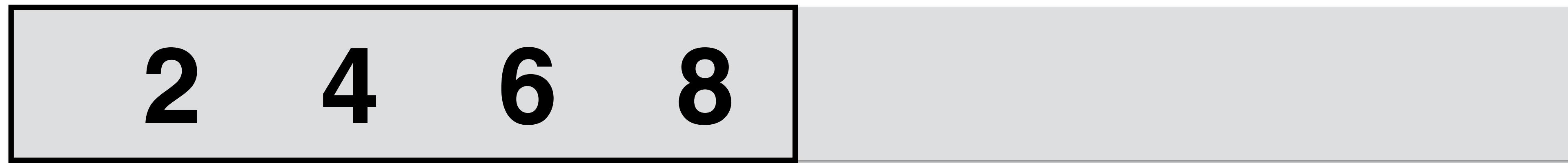
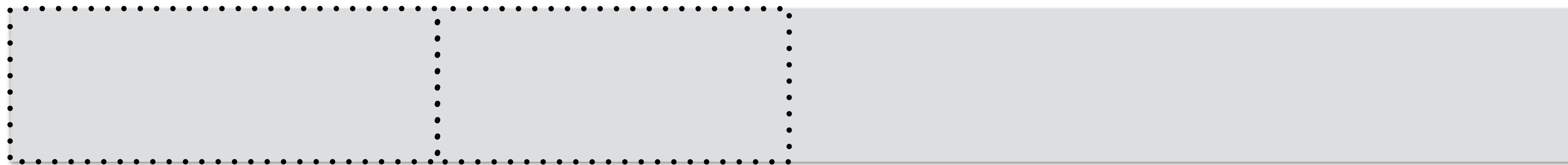
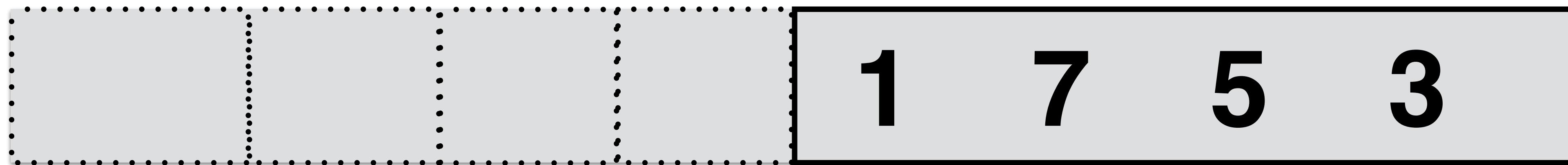


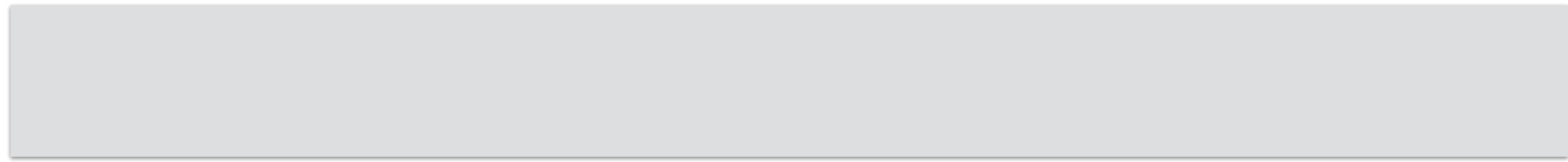
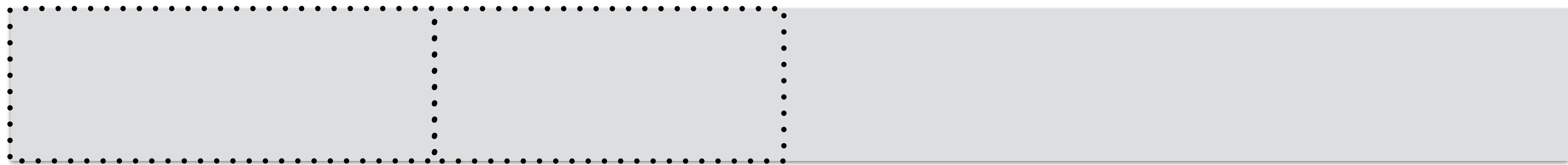
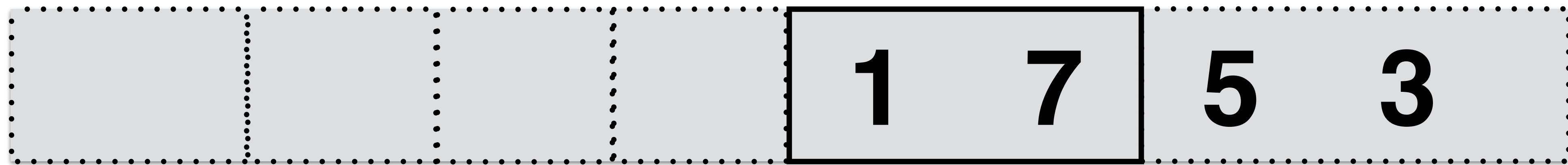
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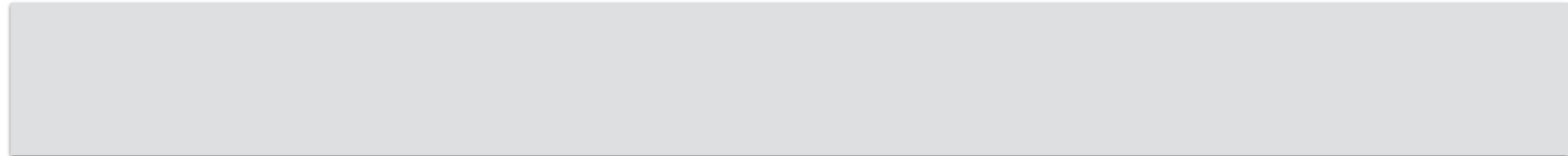
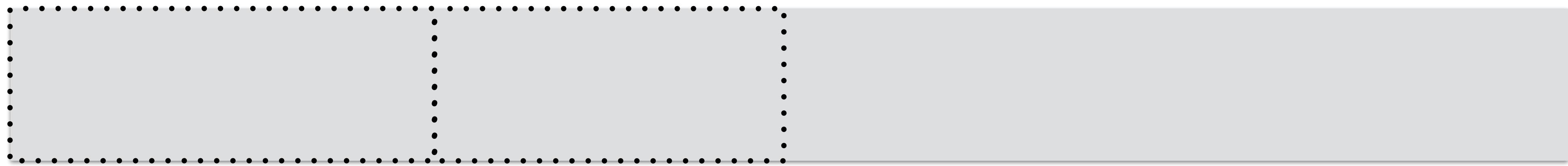
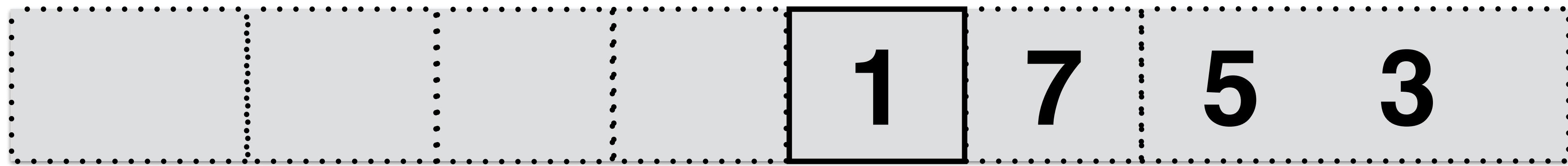


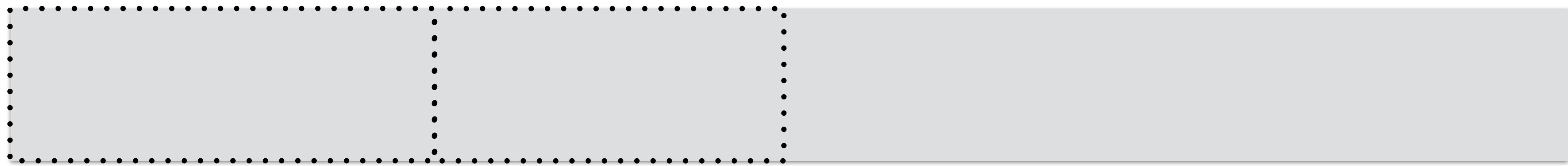
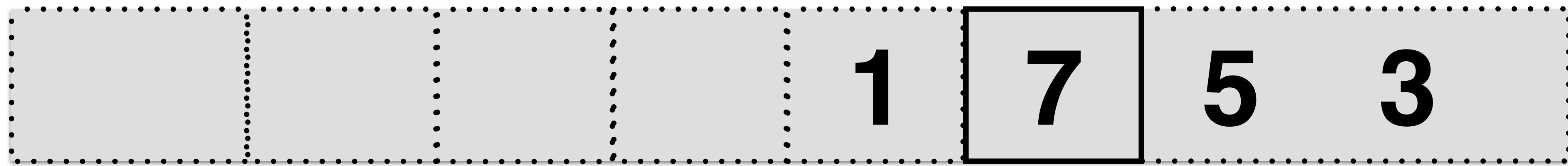
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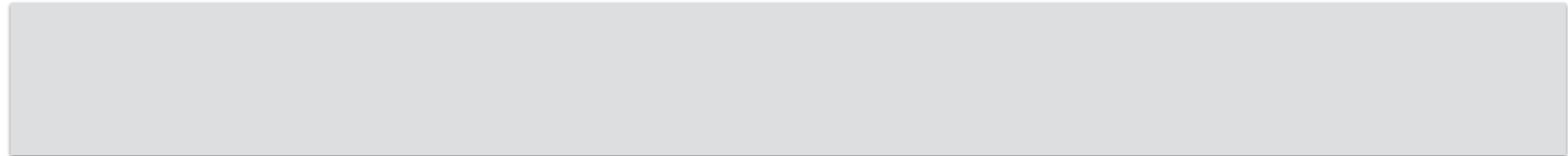
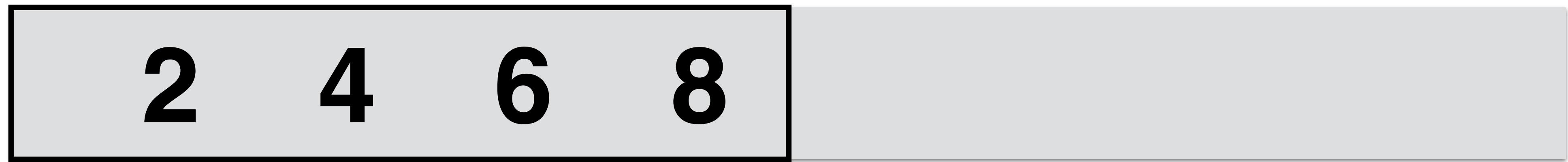
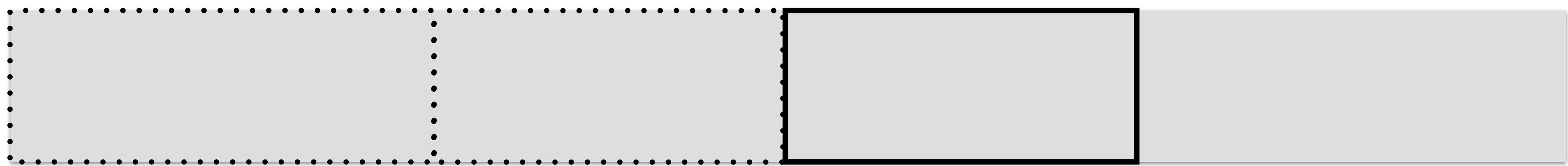
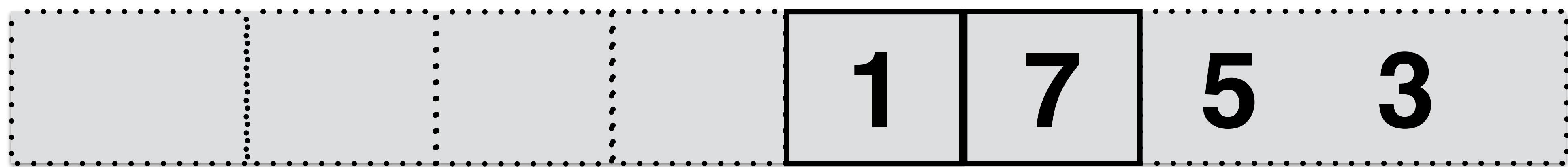


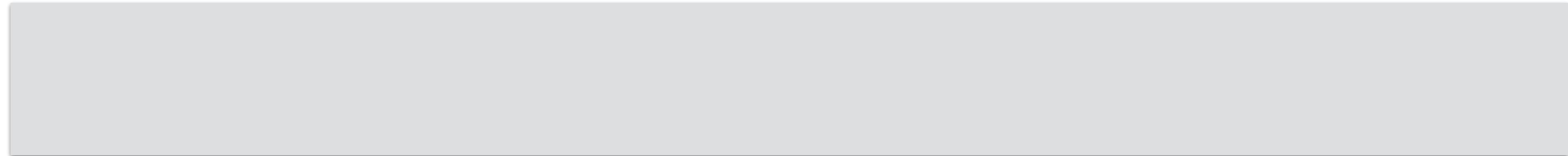
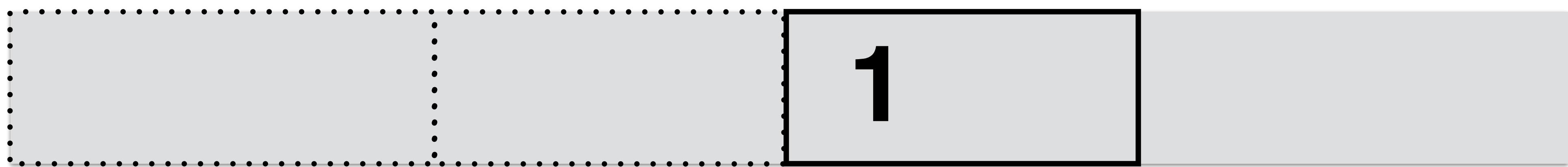
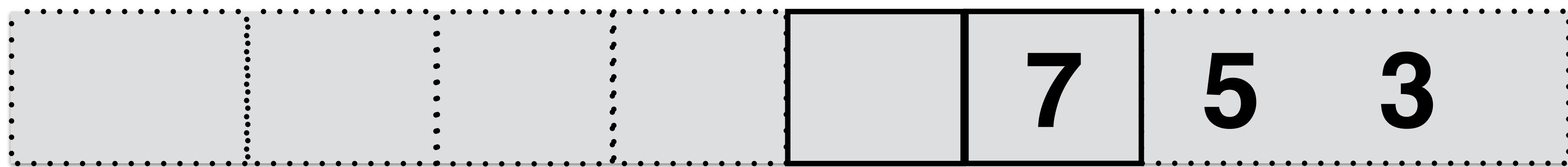


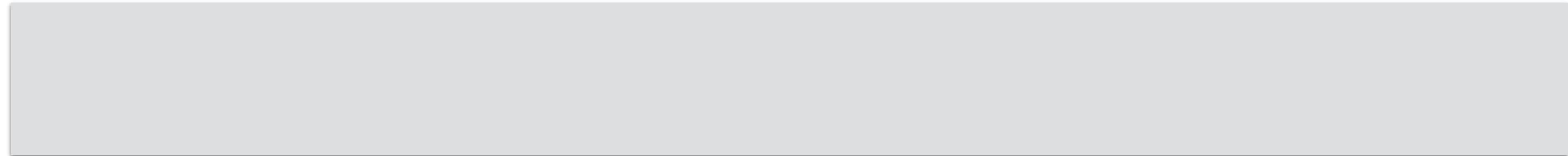
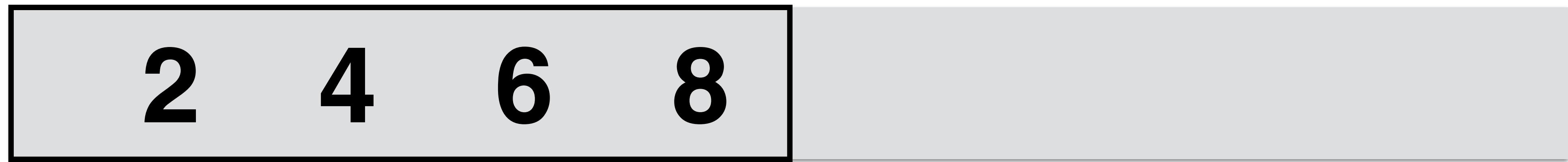
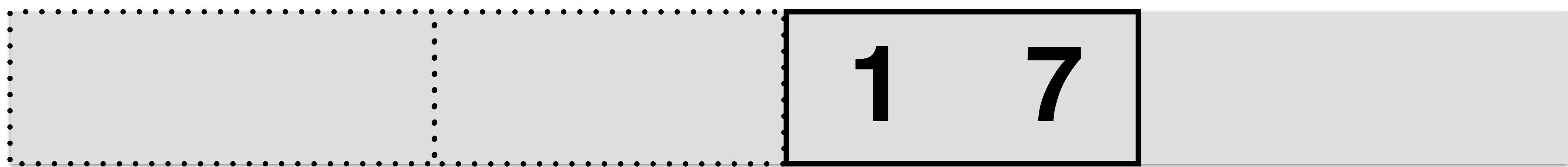
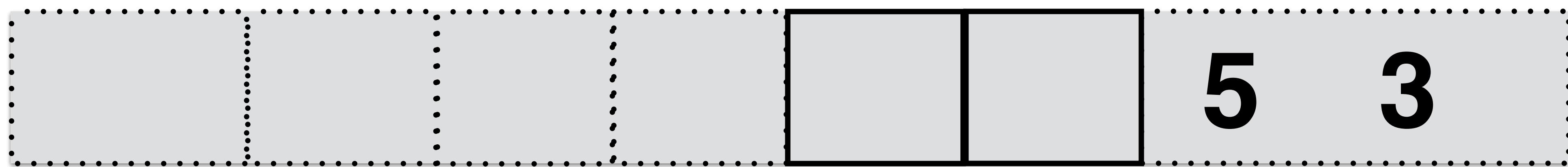












A diagram consisting of a horizontal dotted line with vertical dashed lines at regular intervals. A rectangular box with a black border is positioned on the right side of the line. Inside the box are the numbers 5 and 3.

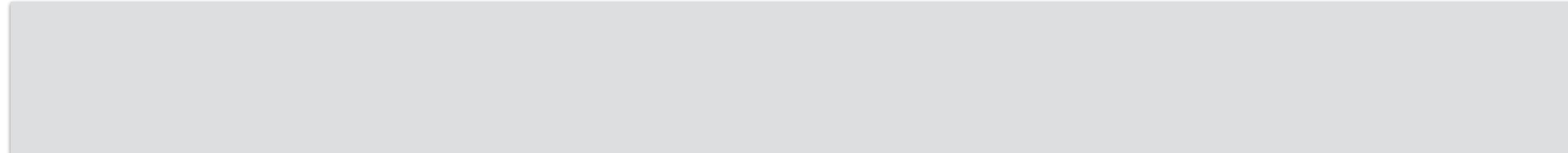
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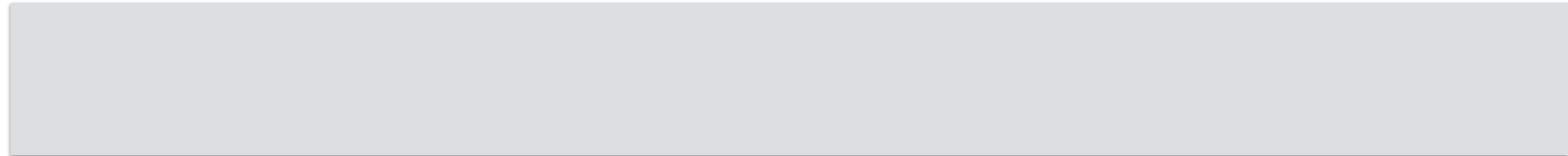
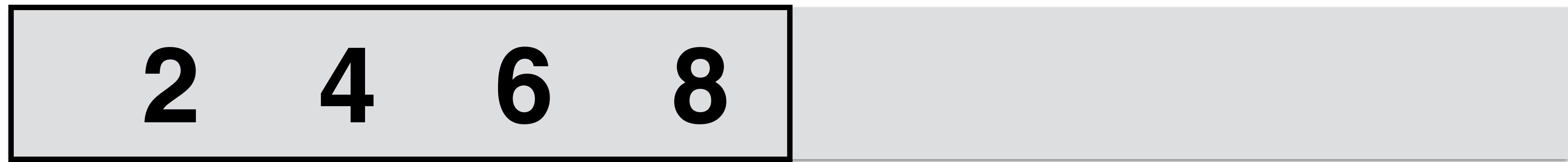
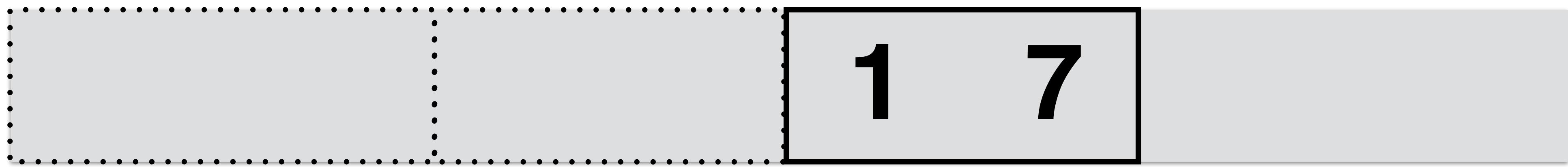
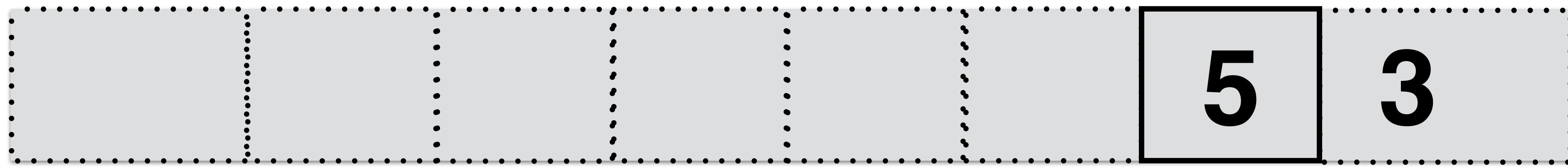
A diagram consisting of a horizontal dotted line with vertical dashed lines at regular intervals. A rectangular box with a black border is positioned on the right side of the line. Inside the box are the numbers 1 and 7.

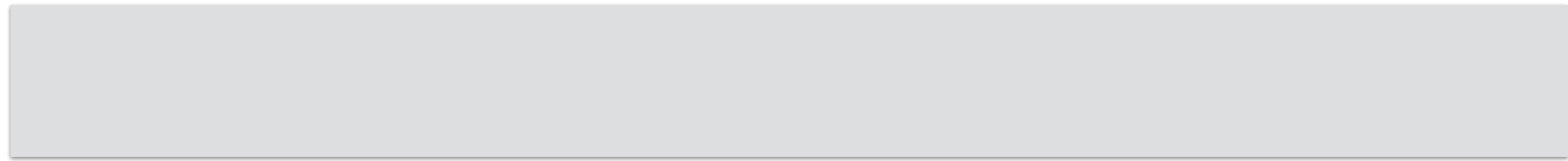
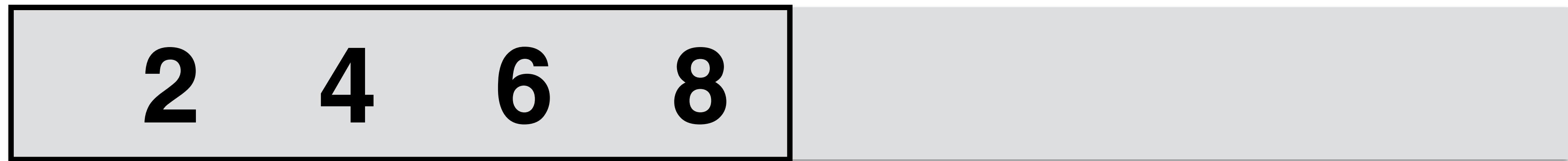
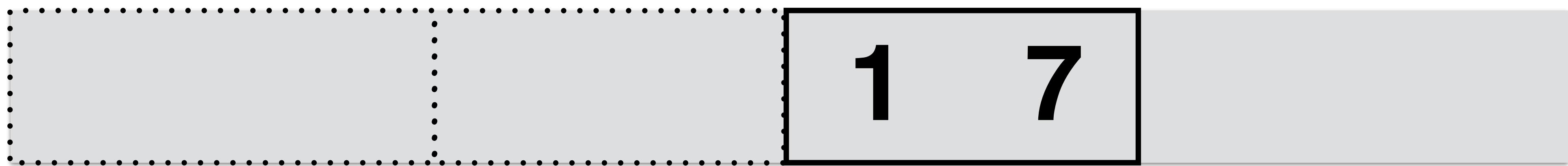
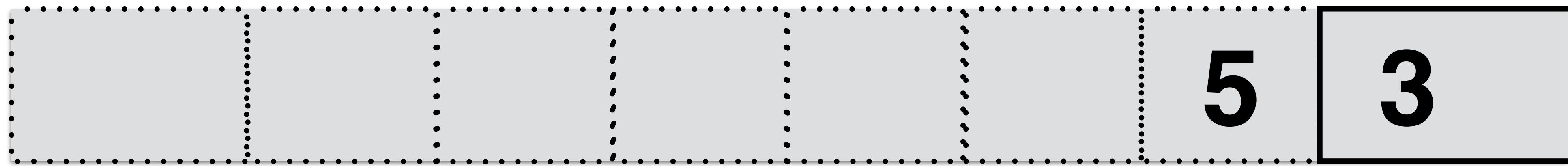
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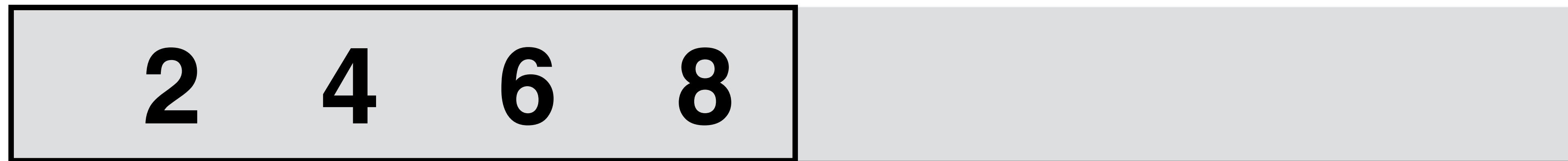
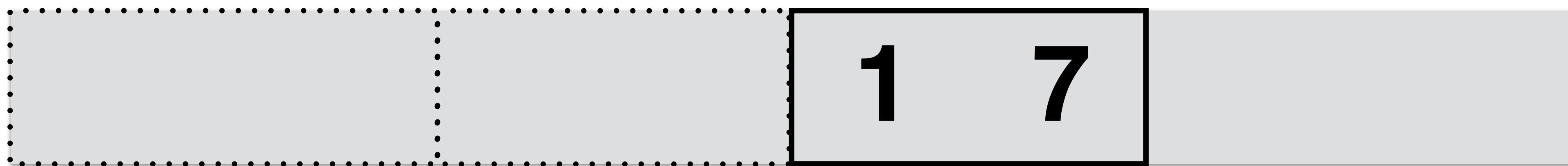
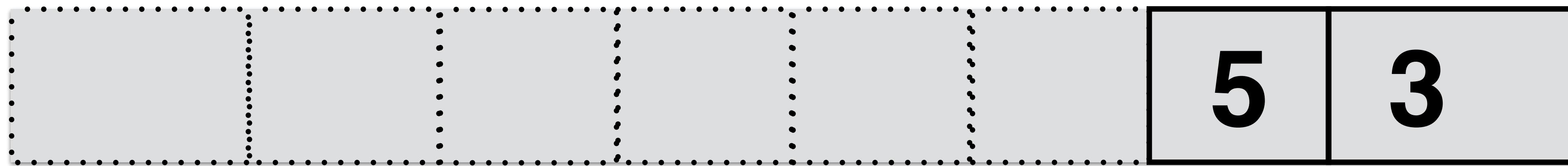
A diagram consisting of a horizontal dotted line with vertical dashed lines at regular intervals. A rectangular box with a black border is positioned on the left side of the line. Inside the box are the numbers 2, 4, 6, and 8. To the right of this box is a large, empty rectangular area.

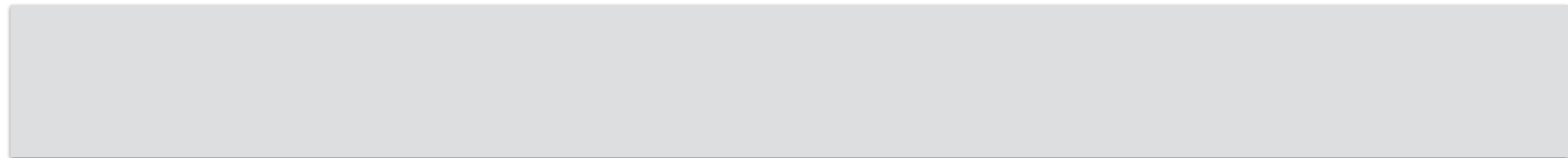
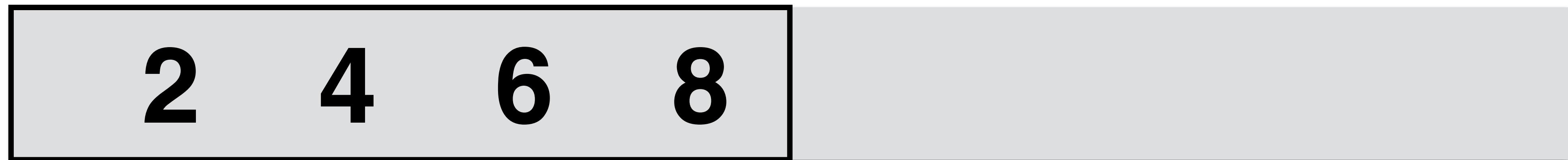
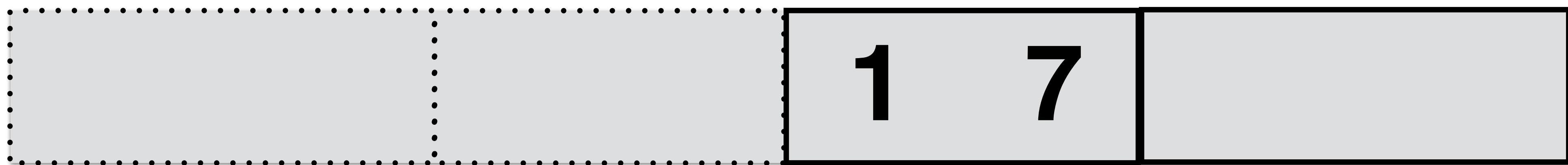
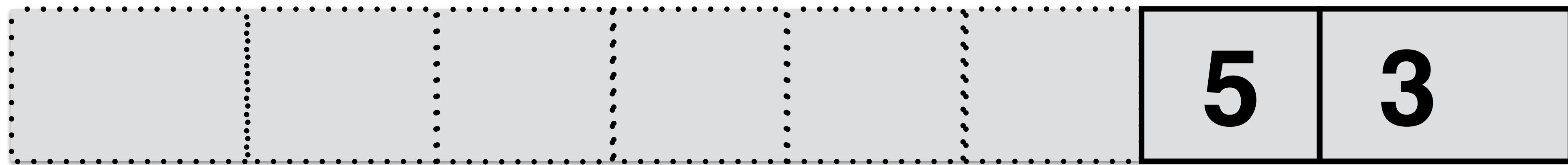
2 4 6 8

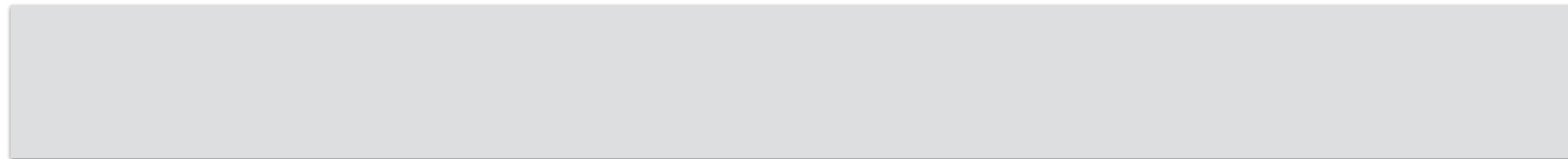
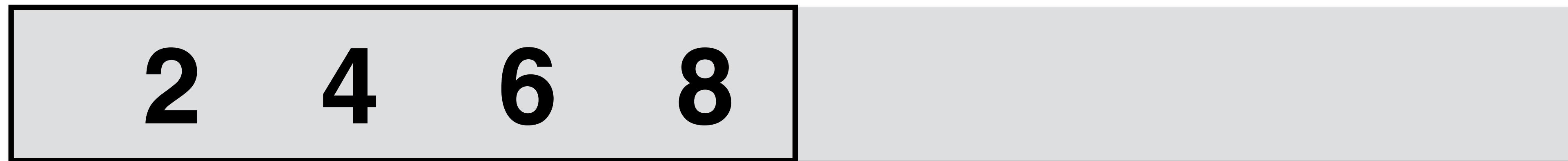
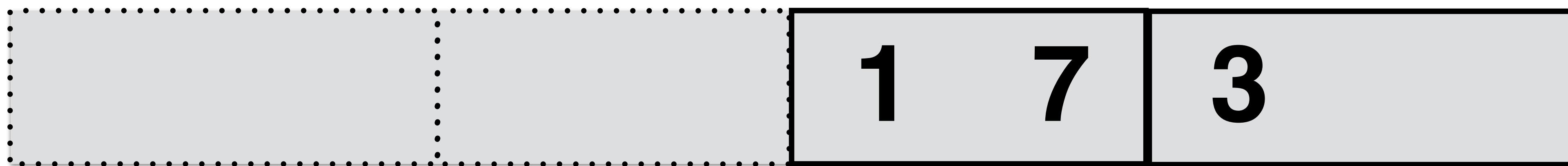
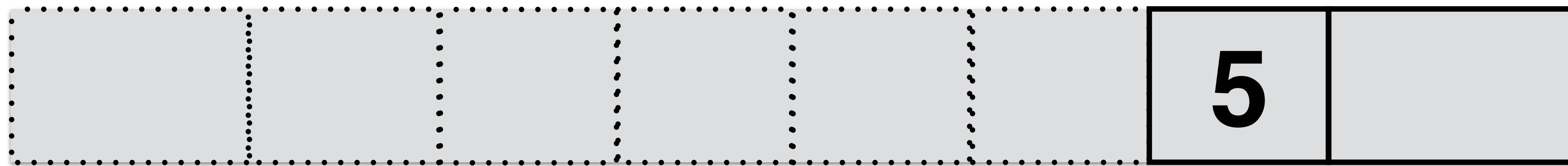


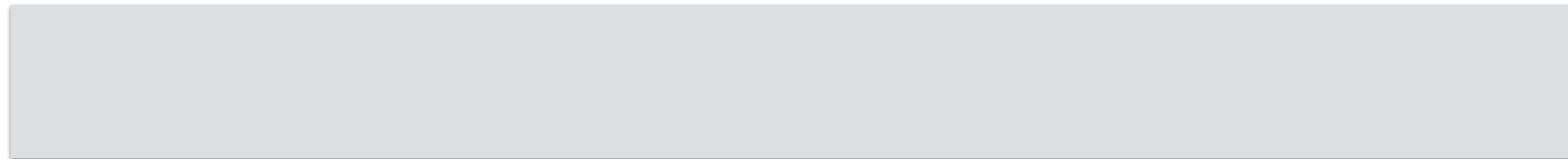
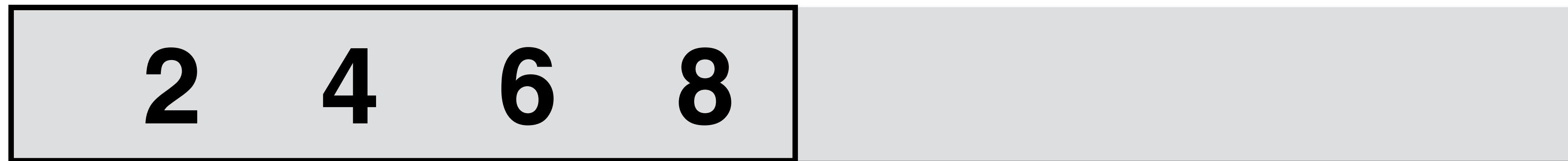
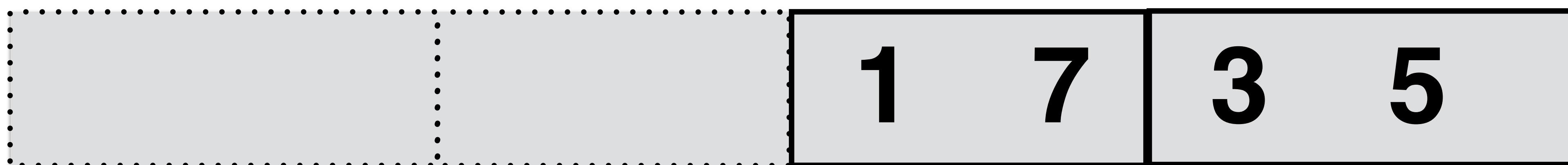
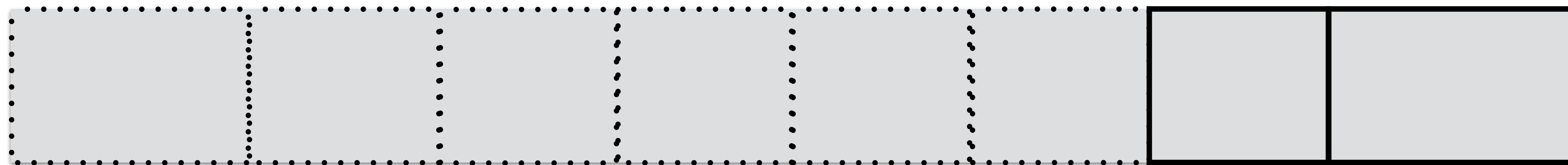


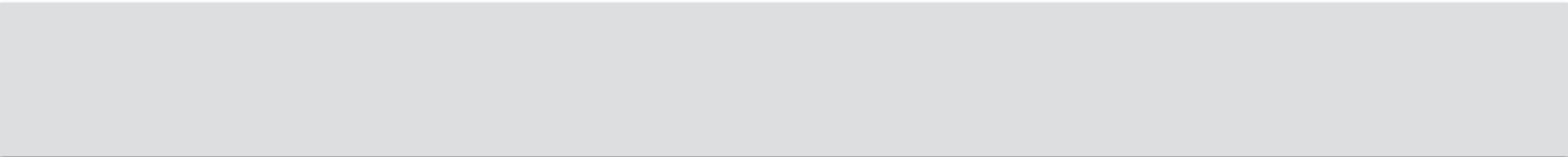
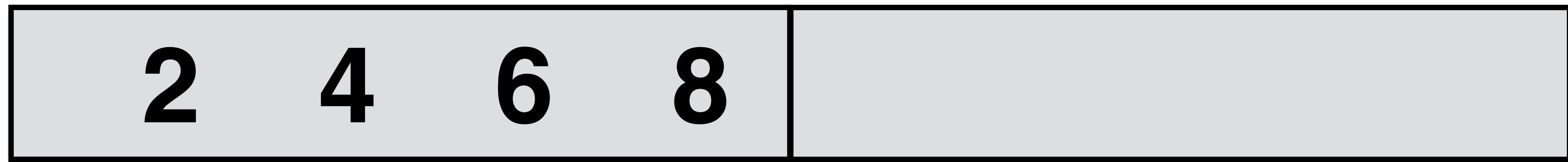
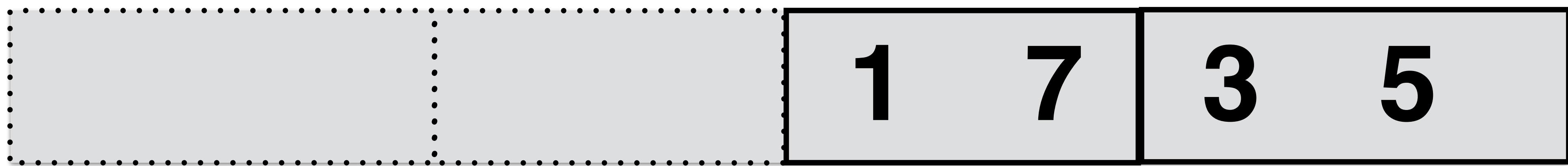
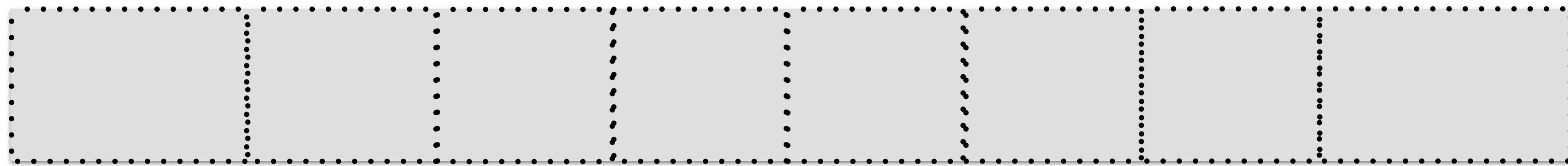


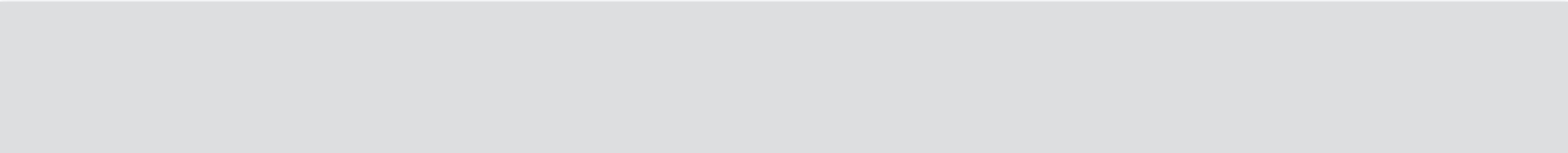
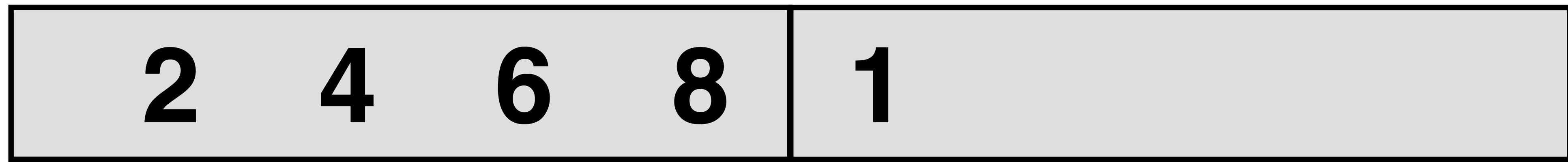
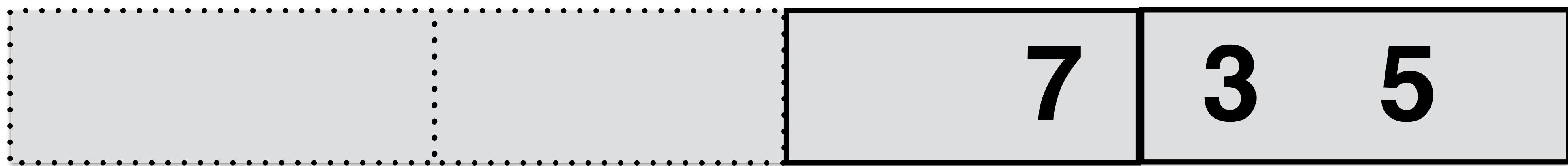
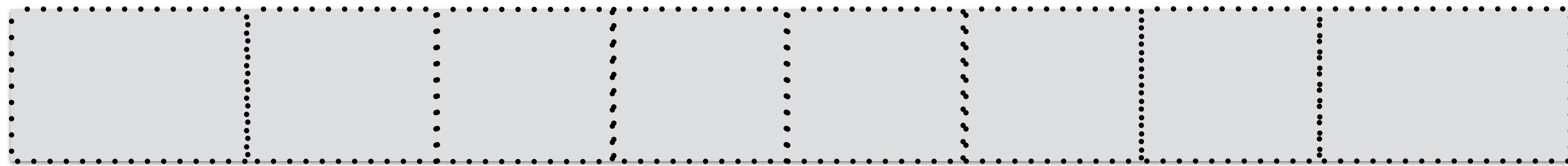


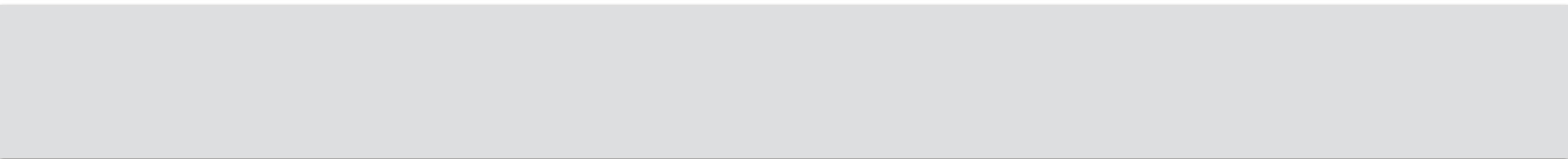
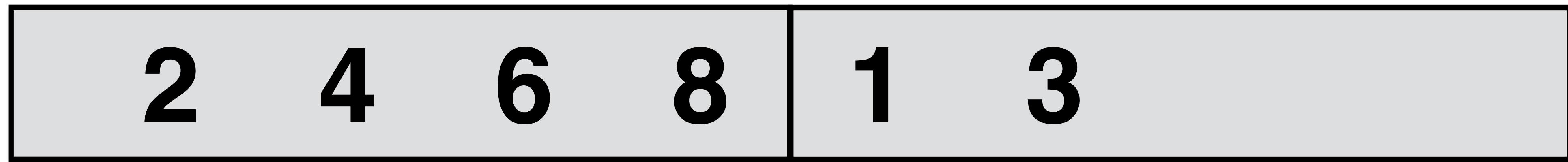
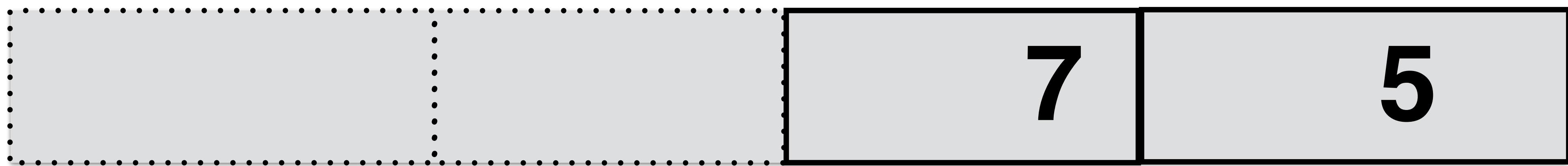
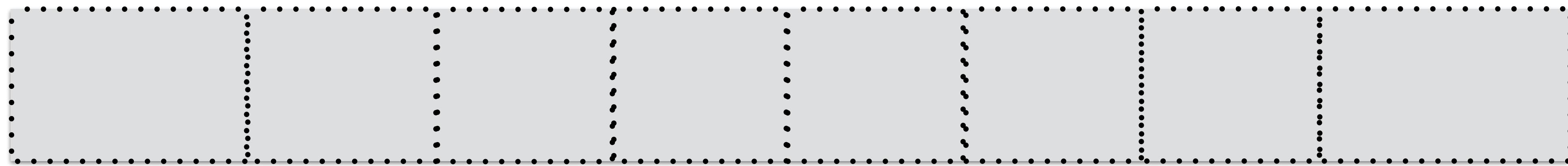


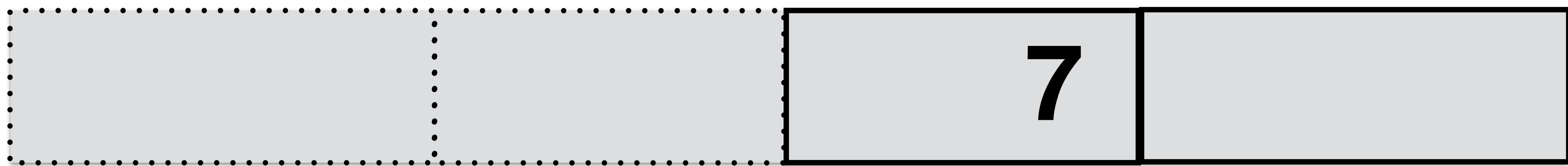
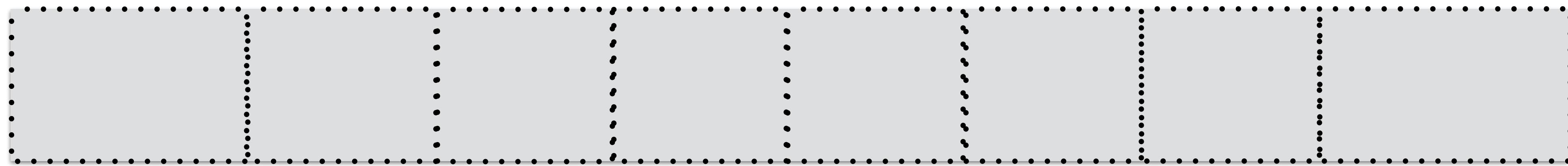




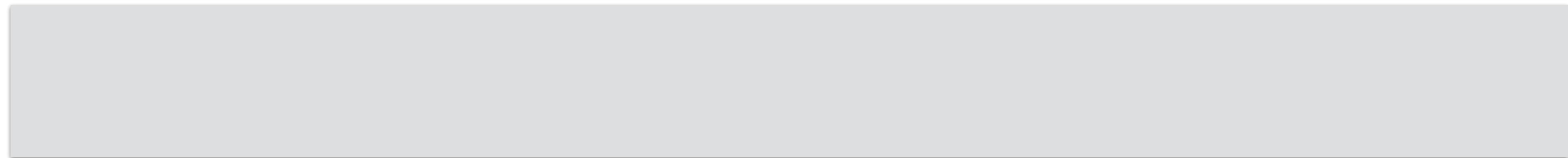


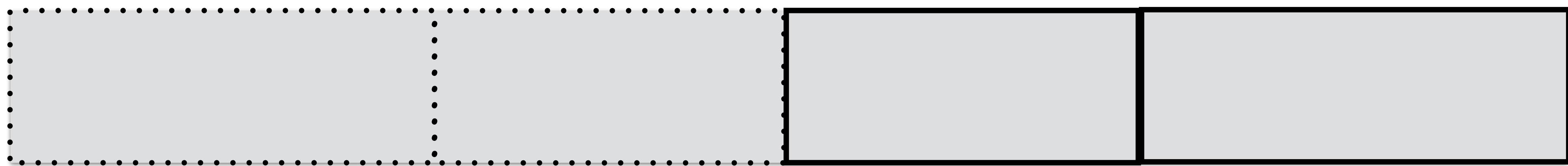
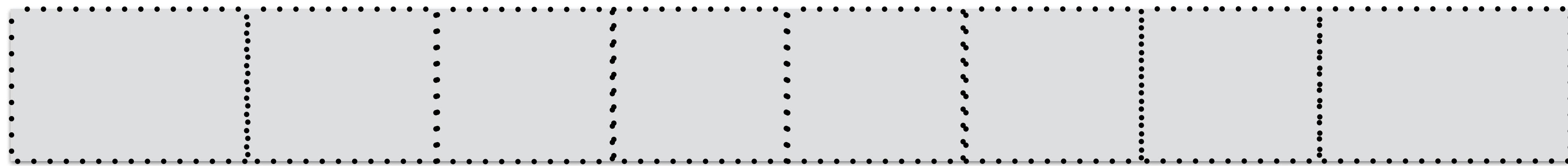




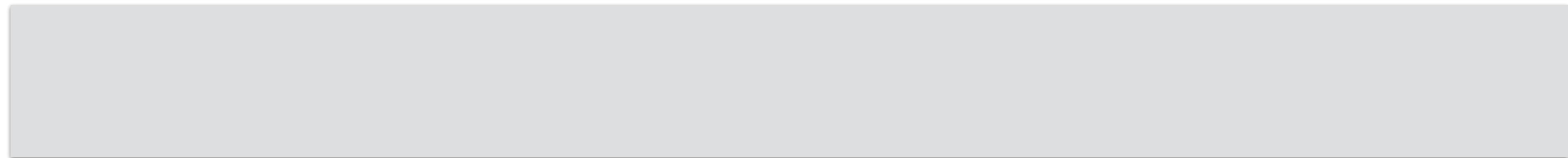


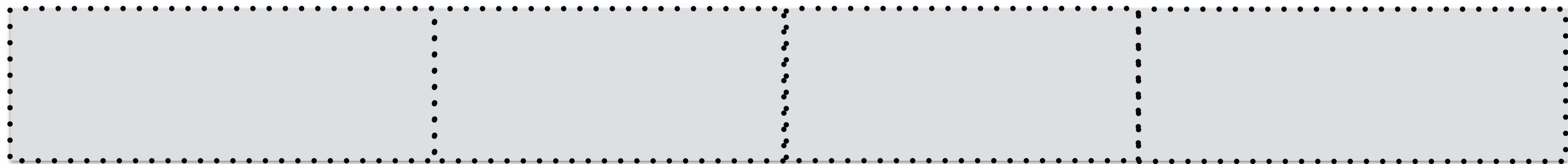
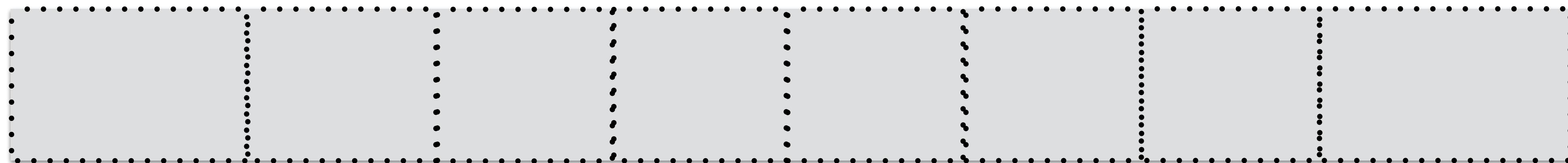
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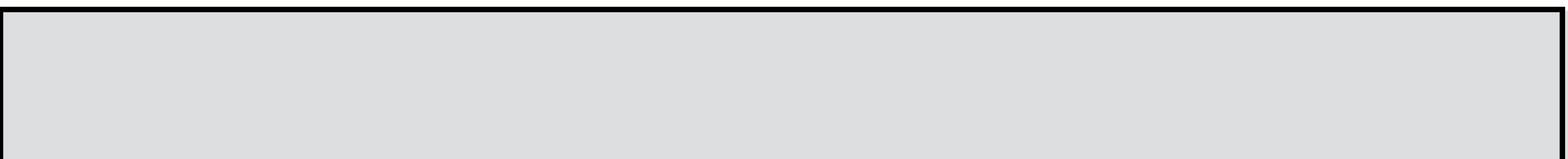


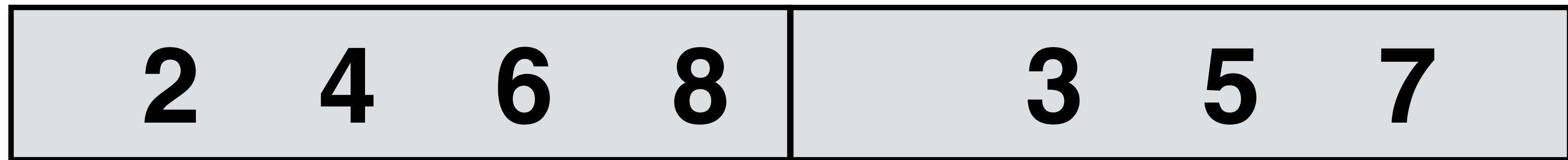
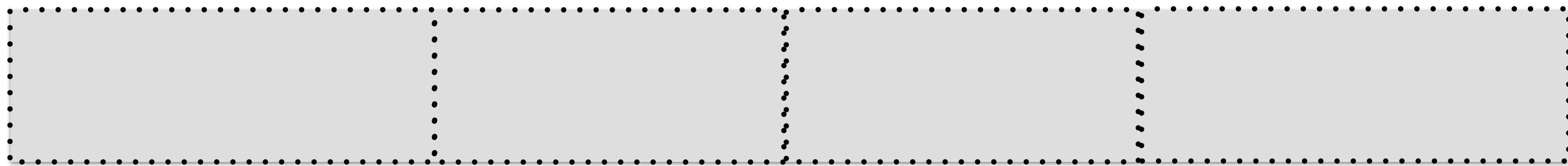
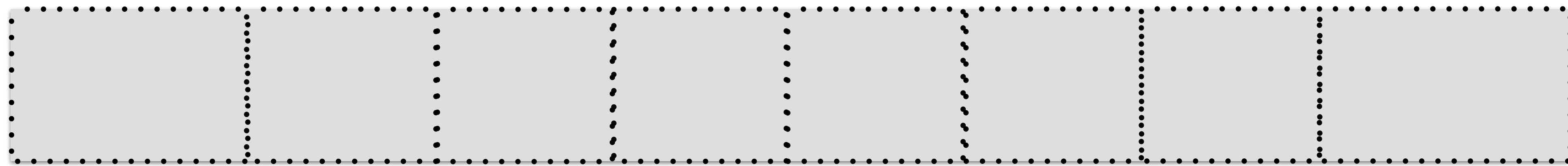
2	4	6	8	1	3	5	7
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2	4	6	8	1	3	5	7
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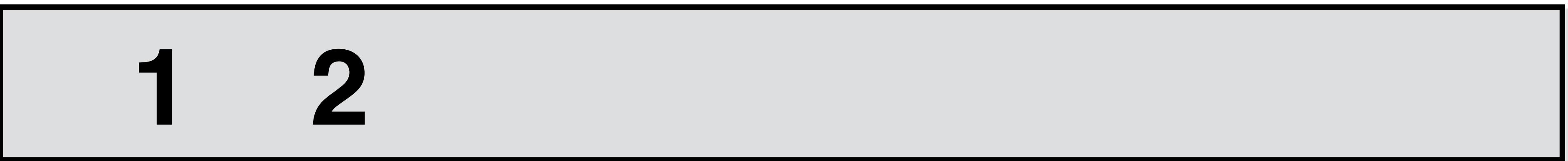
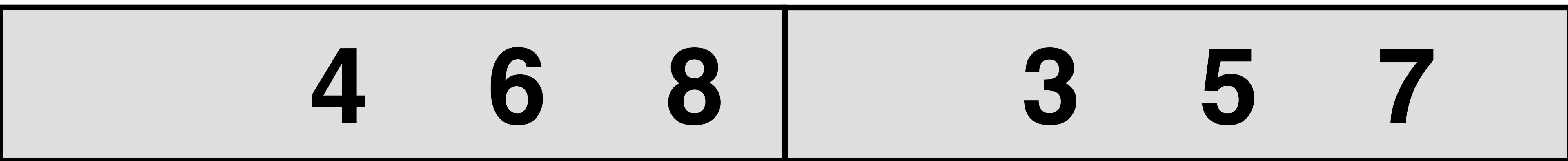
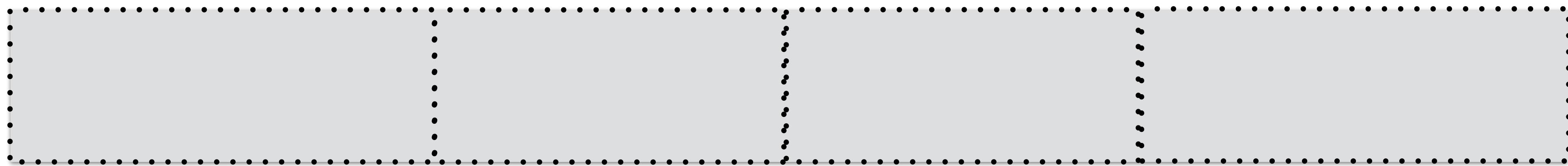
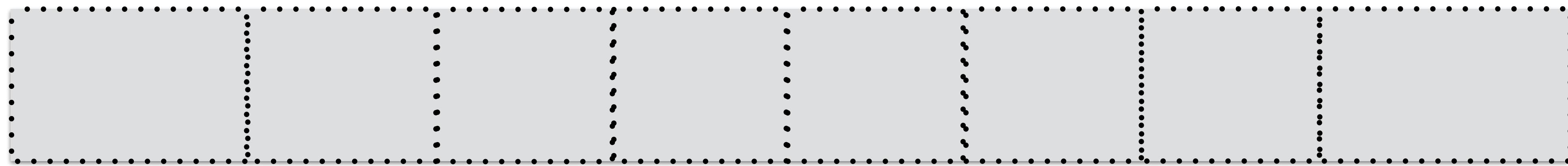


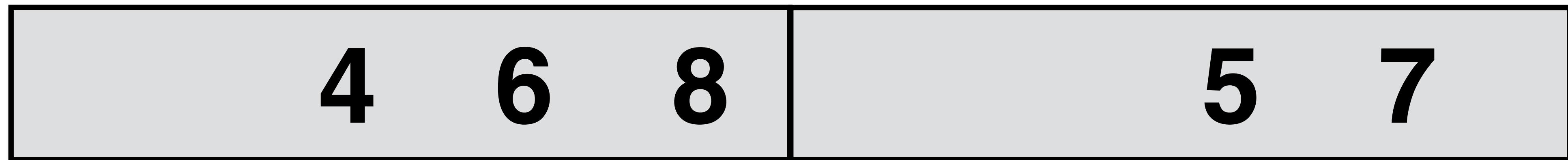
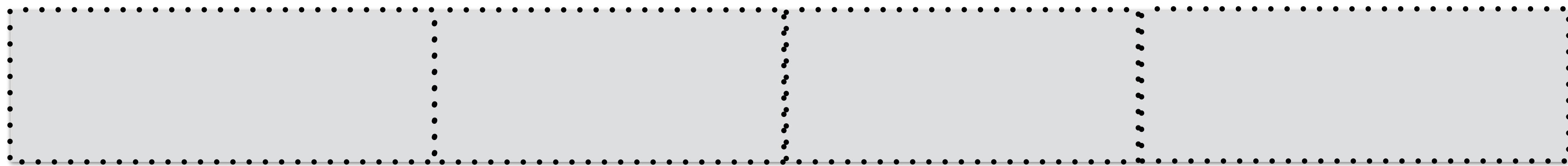
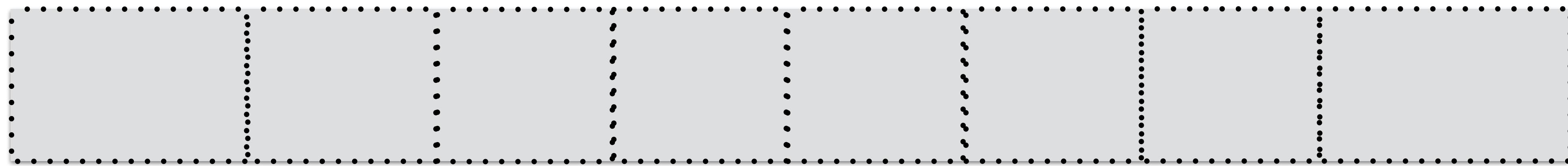


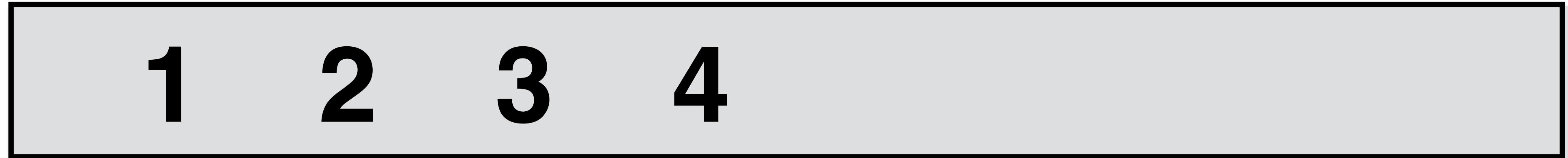
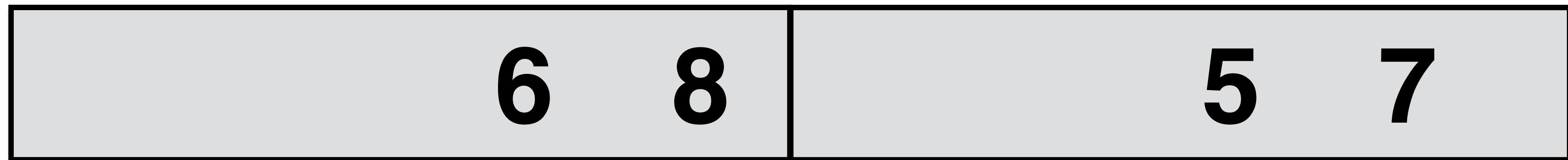
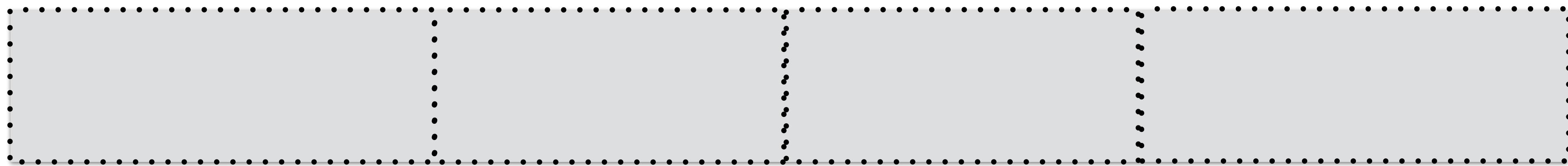
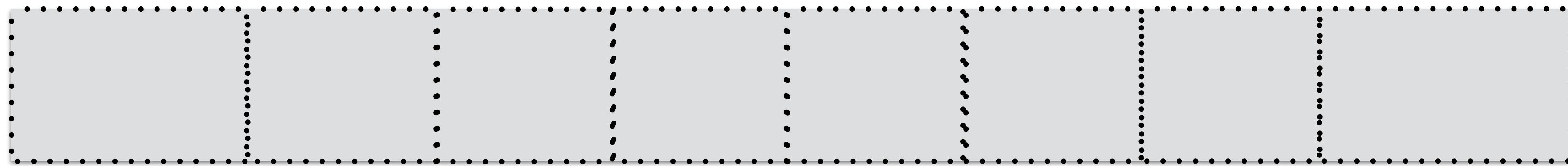
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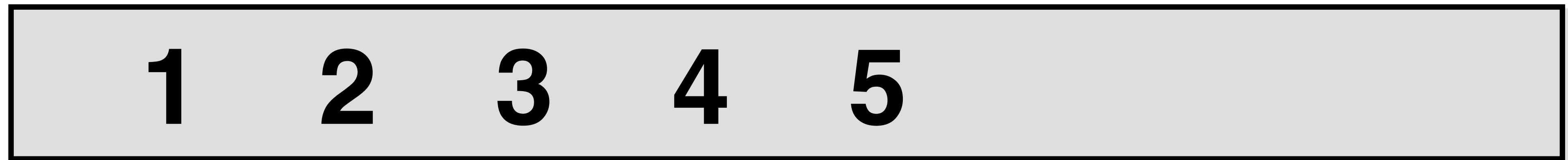
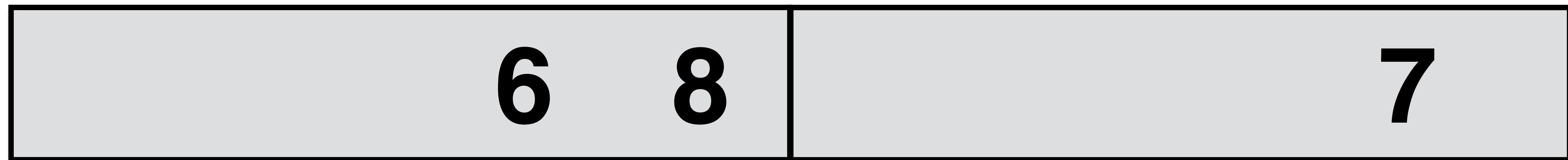
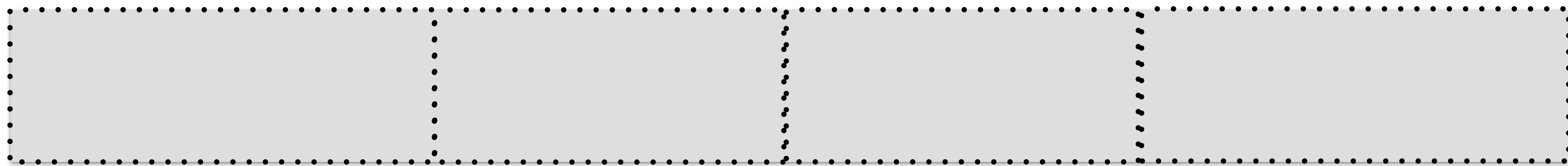
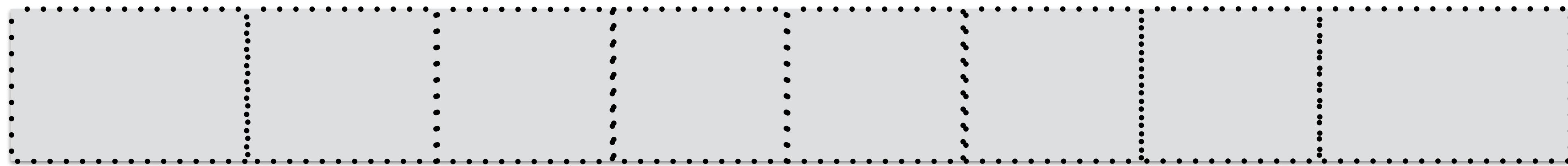


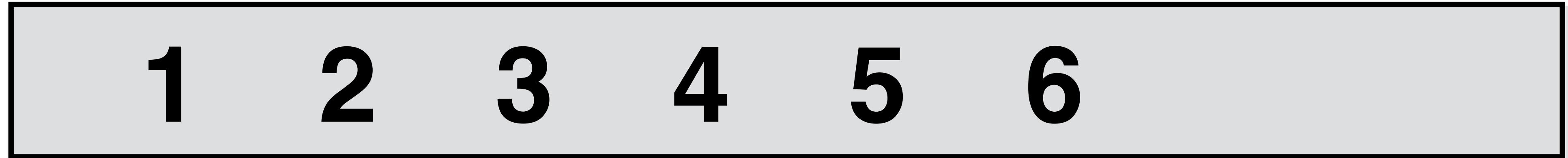
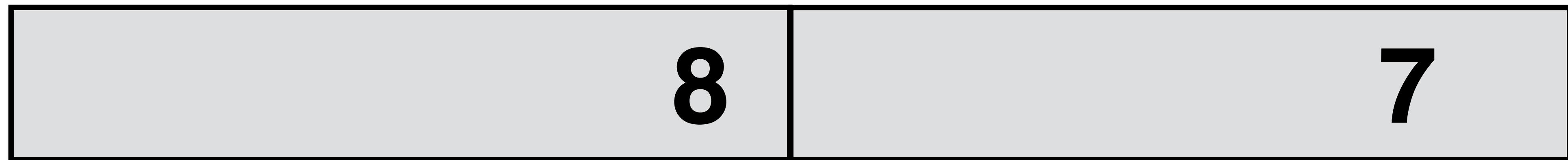
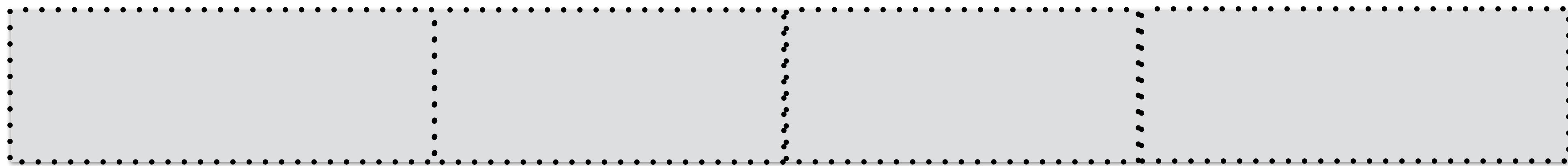
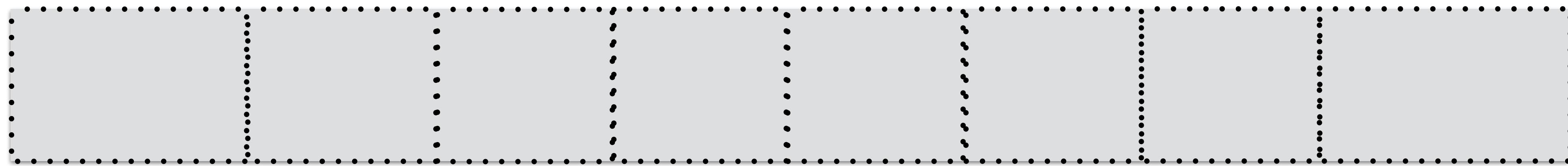
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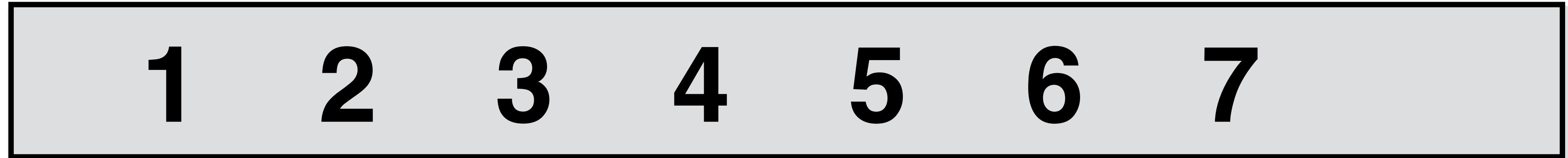
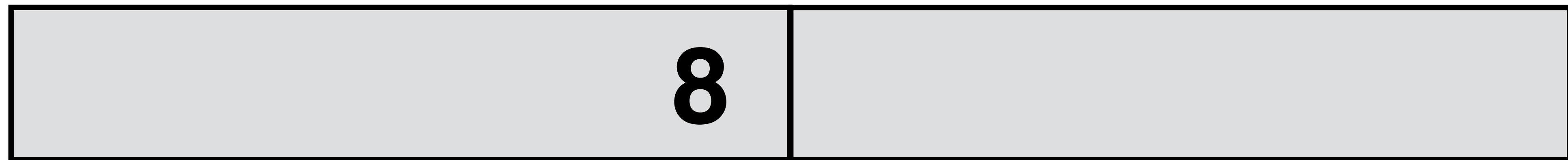
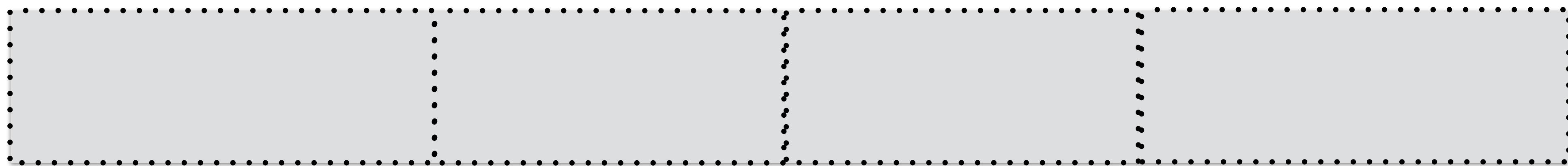
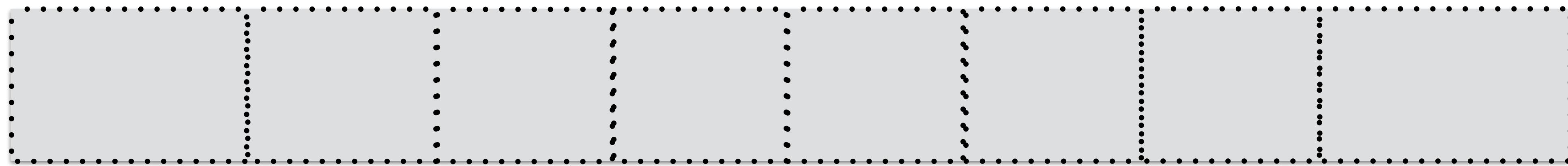


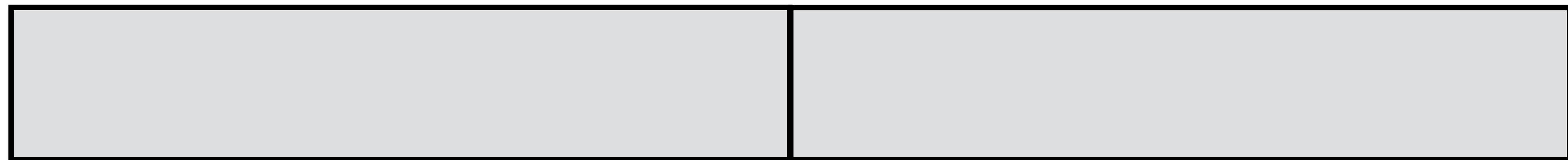
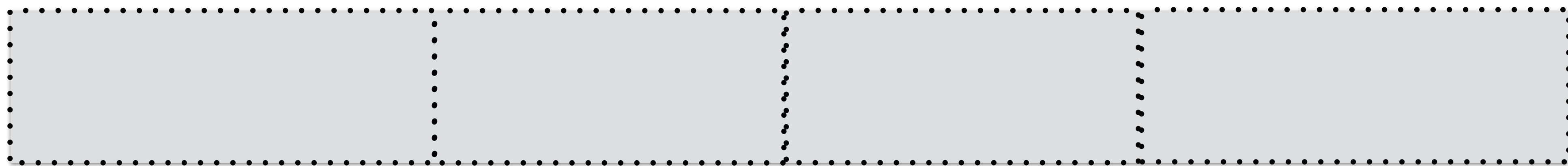
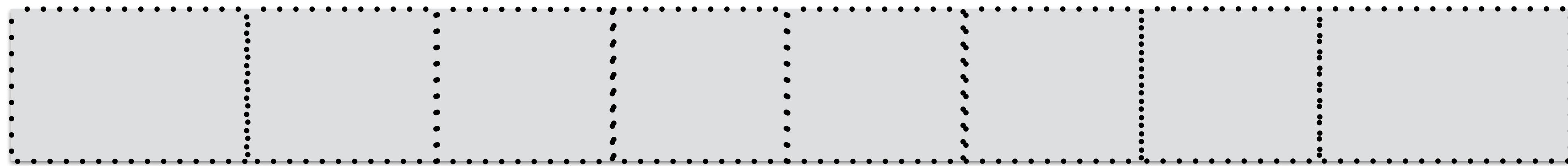






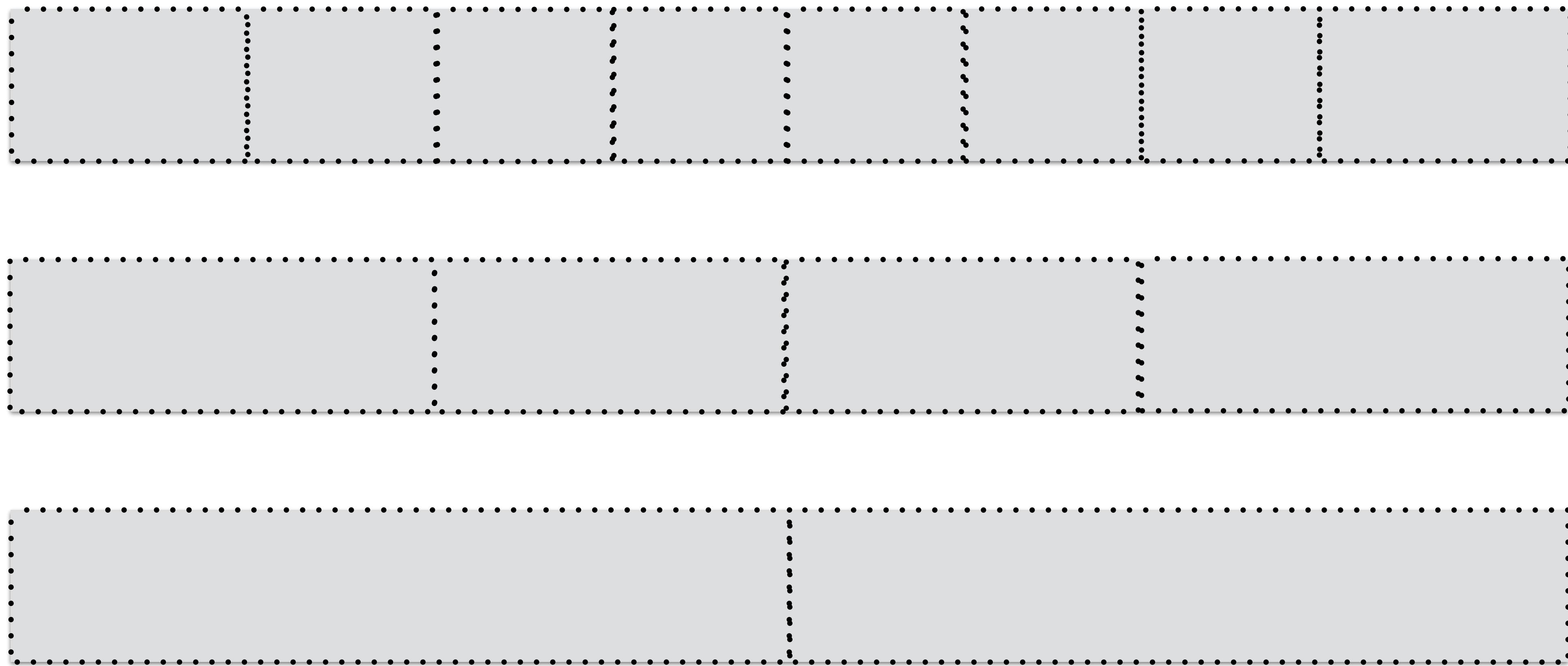






A horizontal bar divided into eight equal segments by vertical black lines. Each segment contains a black number from 1 to 8, corresponding to the segments from left to right.

1	2	3	4	5	6	7	8
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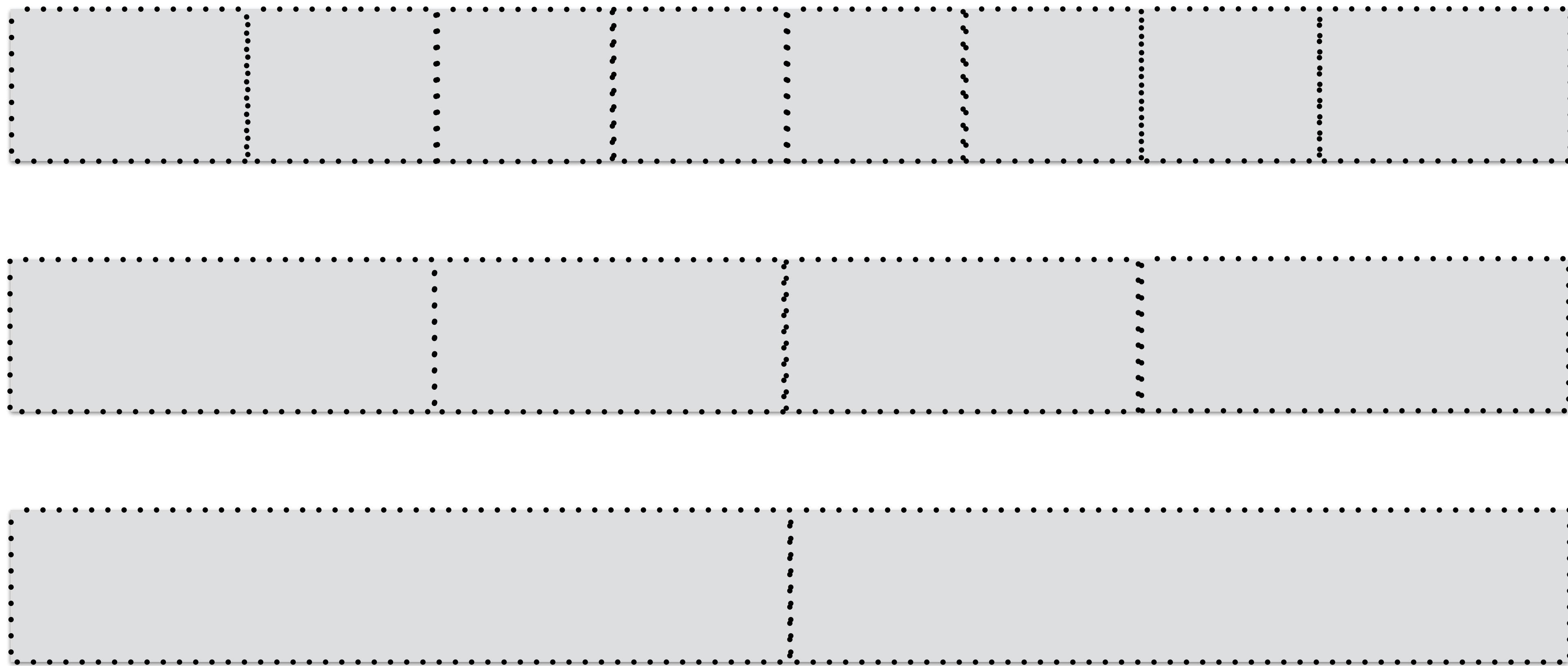


1 2 3 4 5 6 7 8

log n

$\log_2 n$

$$\log_2 8 = 3$$



1	2	3	4	5	6	7	8
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$O(n \log n)$

https://www.youtube.com/watch?v=k4RRi_ntQc8

