Lecture 22: Introduction to Unsteady Gas Dynamics

ENAE311H Aerodynamics I

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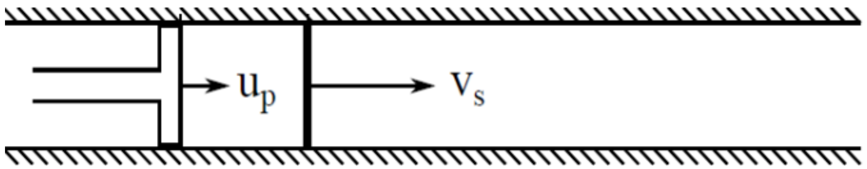
Propagating shock waves

Consider a constant area duct, with a piston that is impulsively started to u_p at $t = 0$ (with zero velocity before that).



We note the following:

- Since the flow won't have time to respond in a smooth fashion, a shock wave must propagate ahead of the piston at a speed $v_s > u_p$.
- The flow conditions behind the shock are uniform, so the fluid velocity must be equal to the fluid velocity, u_p.
- The shock speed will be precisely that required to accelerate the flow to u_p, according to the normal-shock relations.

Propagating shock waves

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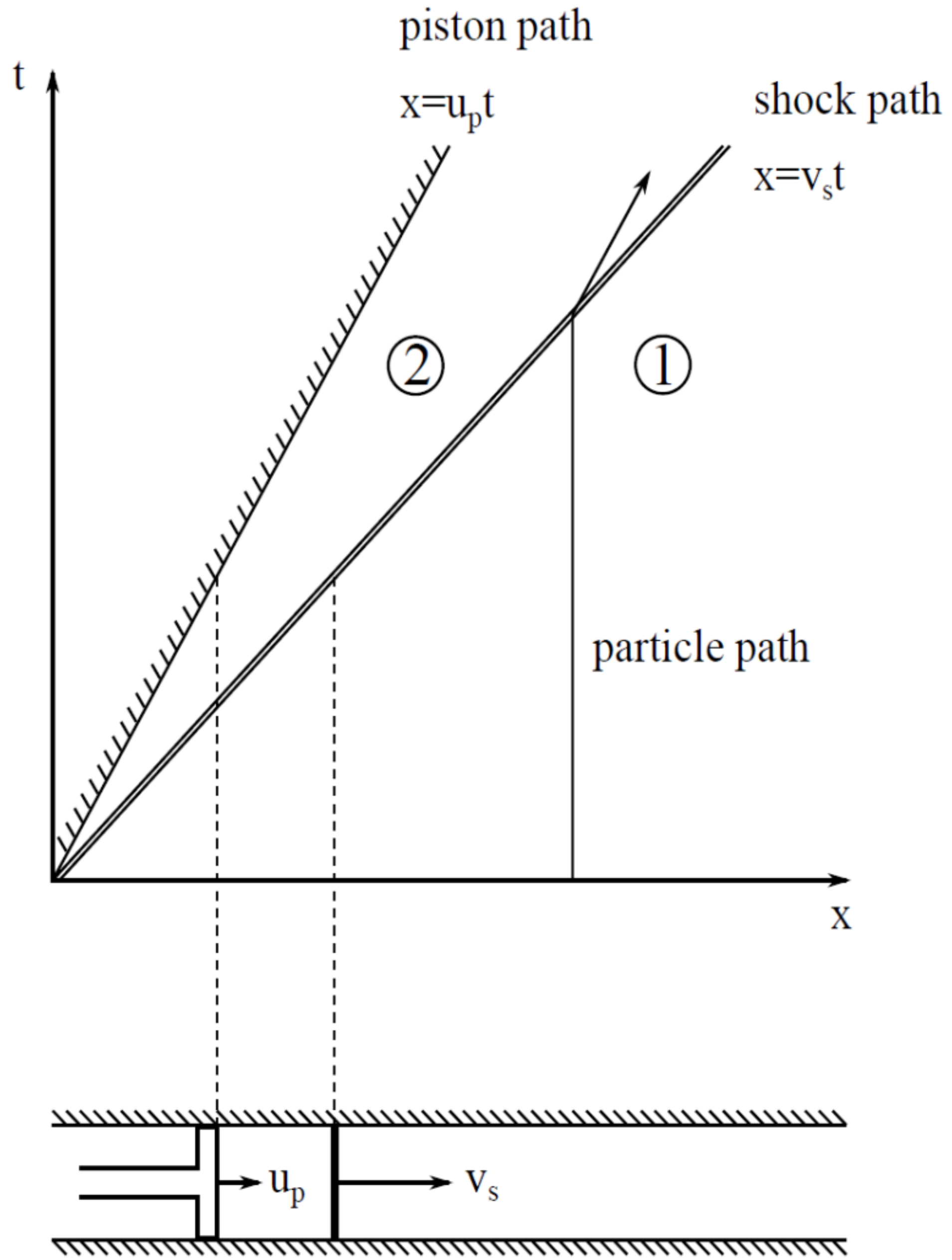
- A useful way to visualize such unsteady, one-dimensional flow phenomena is through an x-t diagram (note the similarity to the 2-D flow over a compression corner).
- Knowing the conditions in region 1 and the shock speed, v_s, all conditions in region 2 can be determined using the normal – shock relations. One further expression that will be useful relates the pressure jump to the piston speed:

$$u_p = a_1 \left(\frac{p_2}{p_1} - 1\right) \left(\frac{2/\gamma}{(\gamma+1)\frac{p_2}{p_1} + \gamma - 1}\right)^{1/2}.$$



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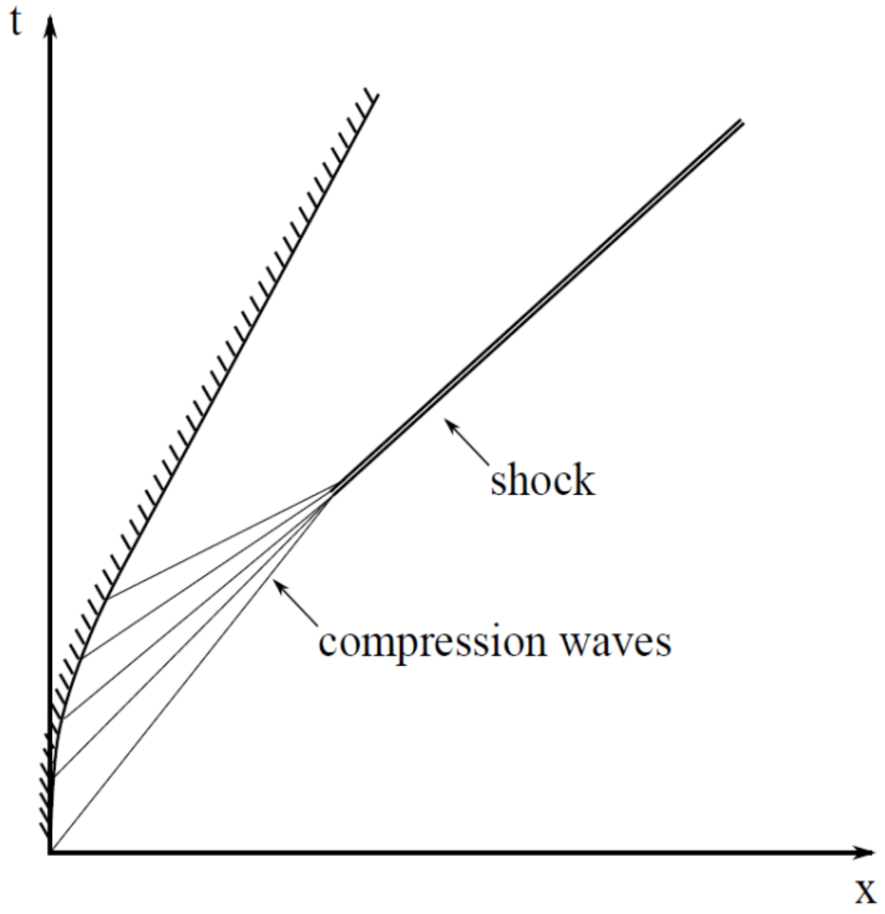
- Even if the piston acceleration is gradual, the compression waves will eventually coalesce to form a shock wave.



Propagating expansion waves

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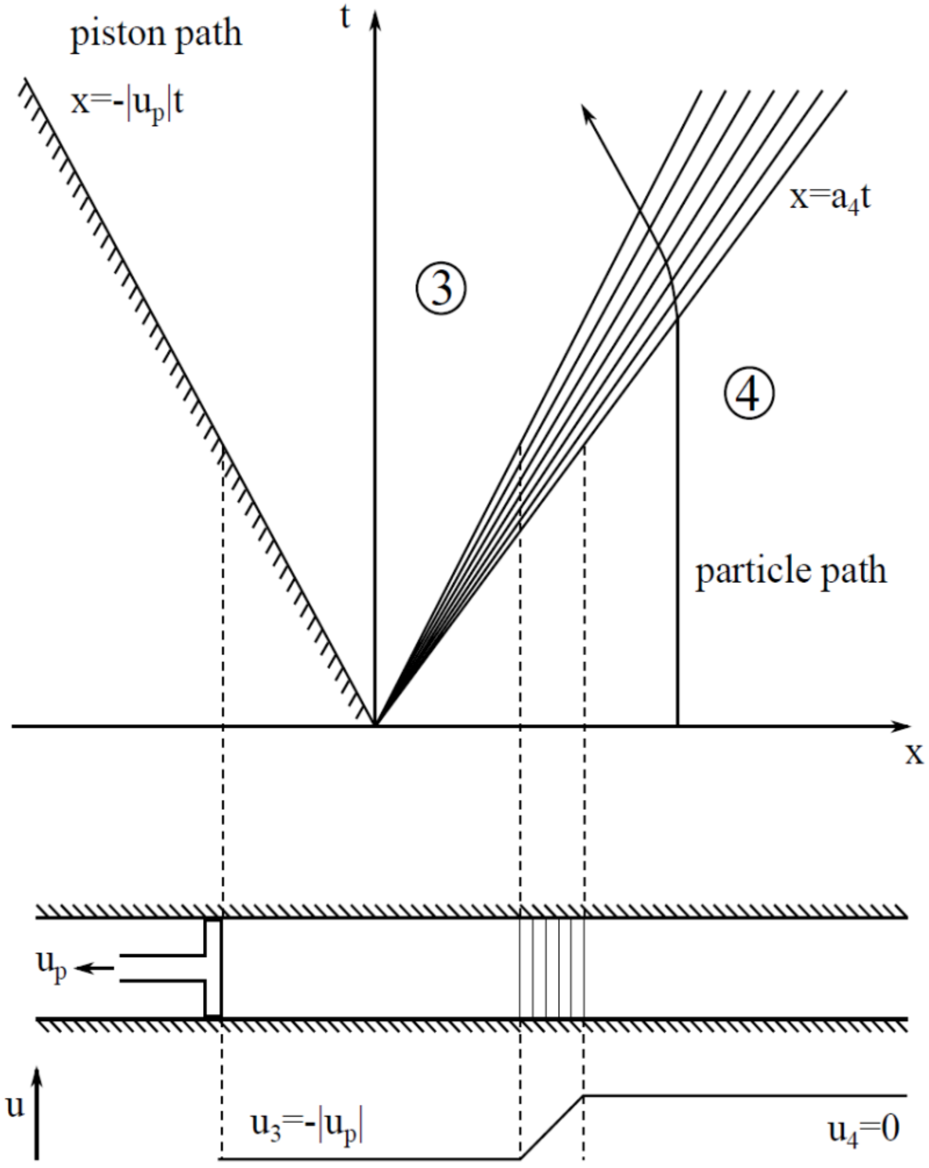
- An expansion wave must propagate into the fluid.
- Since a finite expansion wave is forbidden by the second law of thermodynamics, we must have a gradual (isentropic) expansion through a centered expansion fan. This can again be represented in an x-t diagram.
- The leading wave will have a propagation speed of a_4. For all other waves, the propagation speed (in the lab frame) is

$$c = a_4 + \frac{\gamma + 1}{2} u,$$

where u is the local fluid velocity. Since the final velocity matches the piston speed, the terminal wave propagates at

$$c_{terminal} = a_4 - \frac{\gamma + 1}{2} |u_p|.$$



Propagating expansion waves

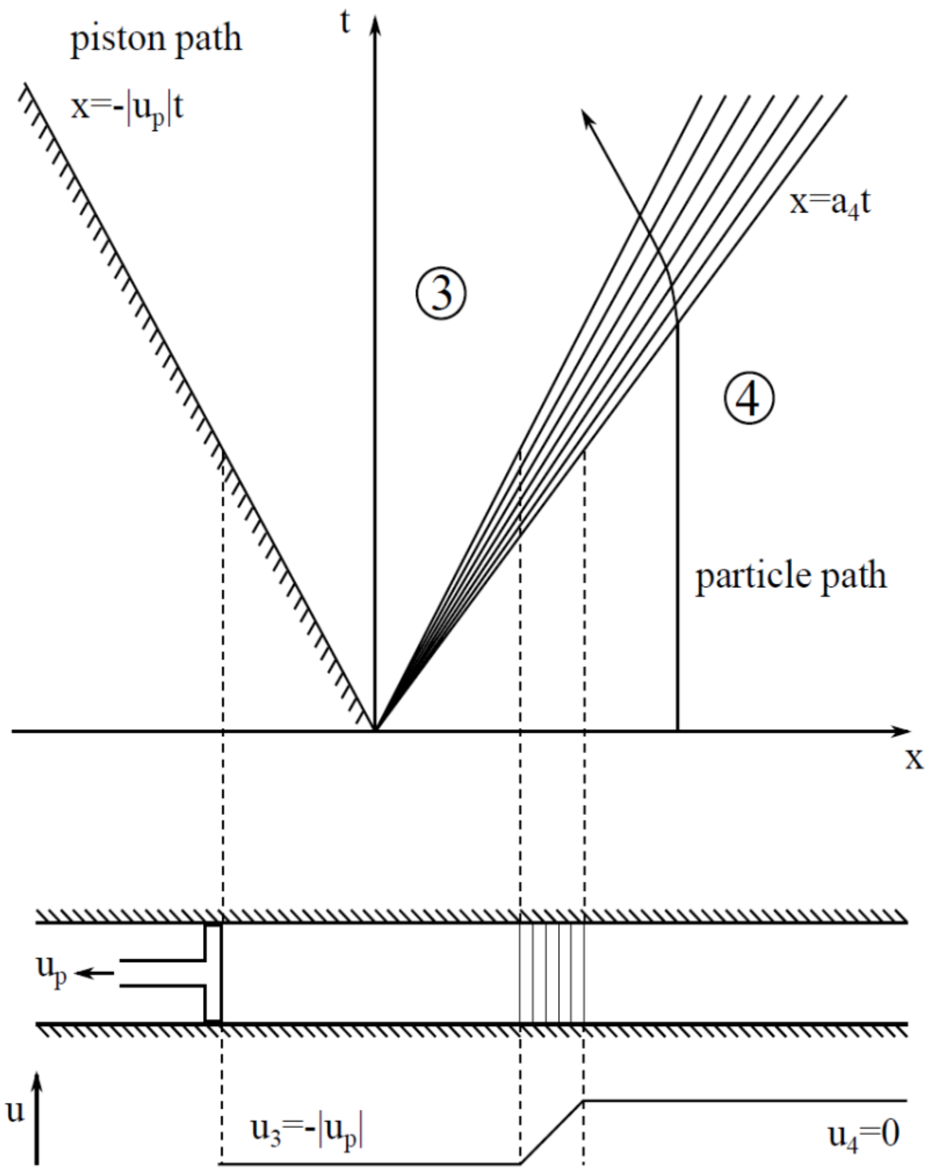
Now imagine that the piston is impulsively withdrawn away from the fluid at speed $|u_p|$, rather than accelerated into it.

We now note the following:

- The flow velocity decreases linearly through the expansion fan (and of course matches the piston speed at the trailing end).
- The strength of the expansion can be characterized in terms of the pre- and post-expansion pressures:

$$\frac{p_3}{p_4} = \left(1 - \frac{\gamma - 1}{2}\frac{|u_p|}{a_4}\right)^{2\gamma/(\gamma-1)} .$$

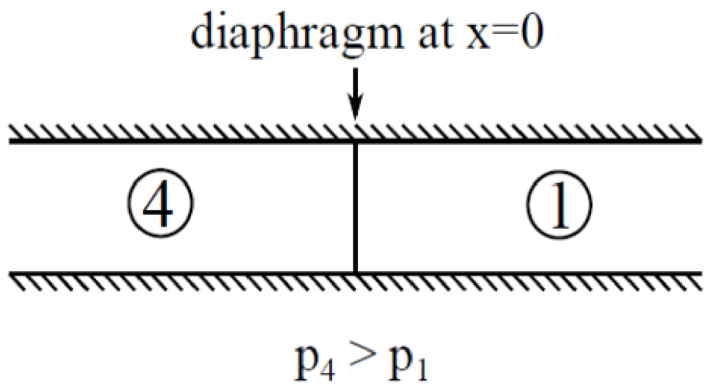
- Note that this entails a maximum velocity that the flow can achieve through an unsteady expansion.



Shock tubes

Now consider a constant area duct with high- and low-pressure regions (possibly of different compositions) separated by a diaphragm.

If the diaphragm is suddenly burst, the unsupported pressure difference will cause the fluid interface (also referred to as the *contact surface*) to start propagating towards the low-pressure region.



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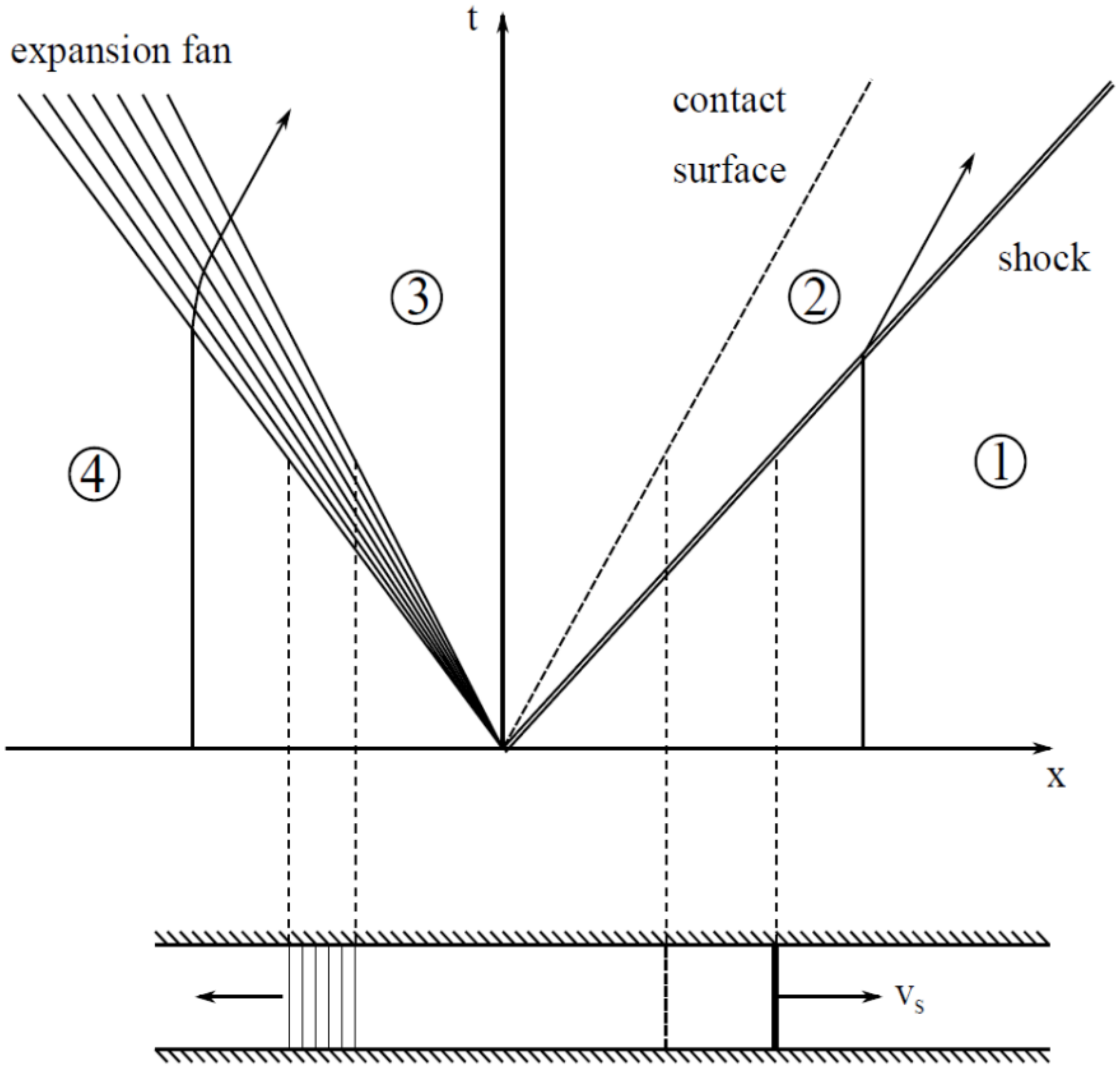
The contact surface will act as a fluid piston, causing a shock wave to propagate into the low-pressure region and an expansion wave into the high-pressure region. This can again be represented on an x-t diagram.

A contact surface is the 1-D, unsteady equivalent of a shear layer; the conditions across it are

$$\begin{aligned} p_2 &= p_3 \\ u_2 &= u_3 (= u_{cs}). \end{aligned}$$



Shock tubes

If we treat the contact surface as an equivalent fluid piston, we can rewrite our earlier equations for the propagating shock and expansion waves as:

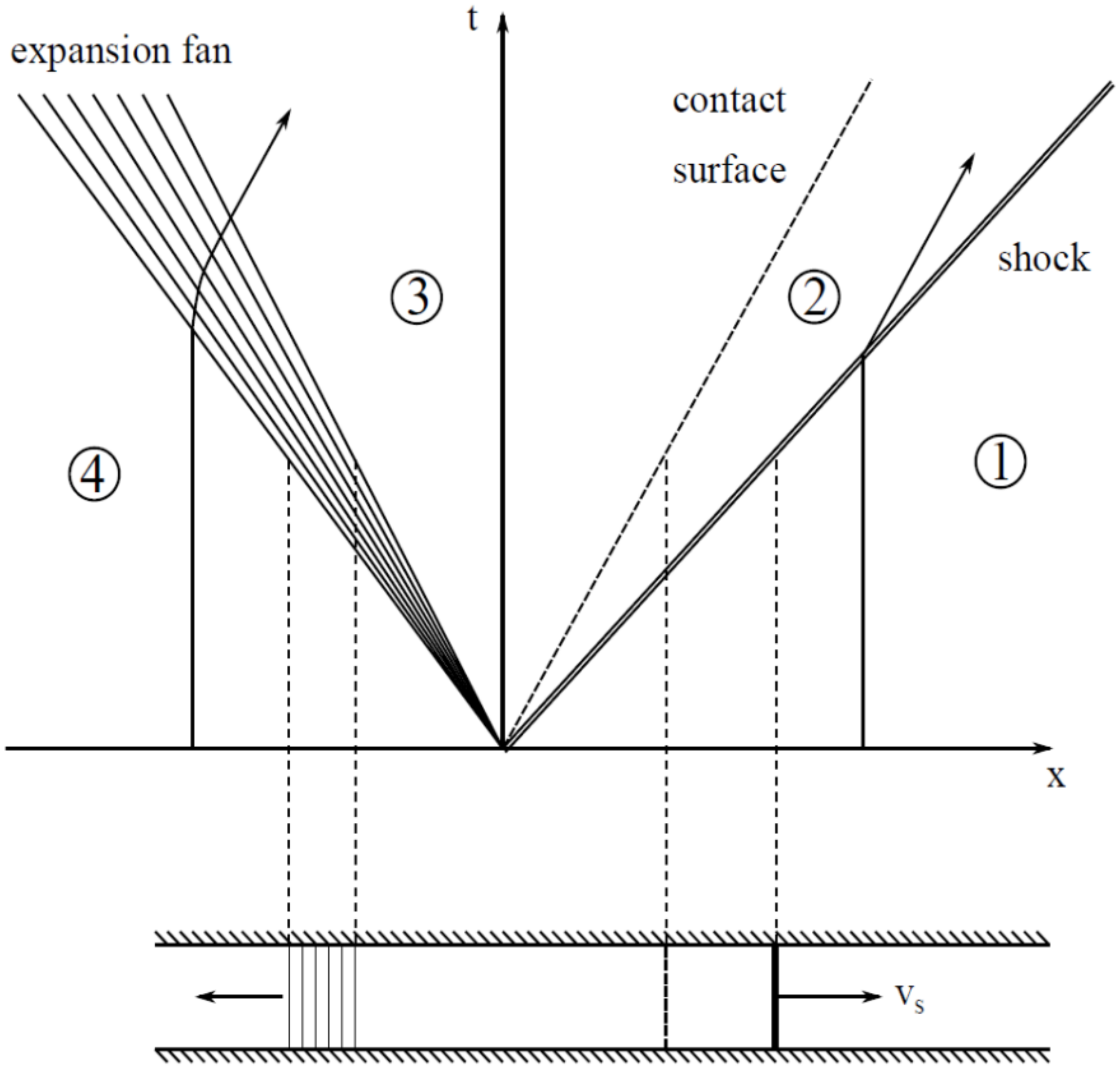
$$u_2 = a_1\left(\frac{p_2}{p_1}-1\right)\left(\frac{2/\gamma_1}{(\gamma_1+1)\frac{p_2}{p_1}+\gamma_1-1}\right)^{1/2}$$

$$u_3 = \frac{2a_4}{\gamma_4+1}\left[1-\left(\frac{p_3}{p_4}\right)^{(\gamma_4-1)/2\gamma_4}\right].$$

Matching pressures and velocities, we can then derive the shock-tube equation:

$$\boxed{\frac{p_4}{p_1}=\frac{p_2}{p_1}\left[1-\frac{(\gamma_4-1)\frac{a_1}{a_4}\left(\frac{p_2}{p_1}-1\right)}{\left\{2\gamma_1\left[2\gamma_1+(\gamma_1+1)\left(\frac{p_2}{p_1}-1\right)\right]\right\}^{1/2}}\right]^{-2\gamma_4/(\gamma_4-1)}.}$$

This gives the shock strength (p_2/p_1) implicitly as a function of the initial pressure ratio and sound speeds.



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Once we know p_2/p_1, the shock Mach number can be derived using the shock-jump relation

$$M_s = \left[1 + \frac{\gamma_1 + 1}{2\gamma_1}\left(\frac{p_2}{p_1} - 1\right)\right]^{1/2}.$$

All other properties in region 2 can then be easily derived.

Shock tubes are used for studying, e.g., unsteady flow phenomena and combustion ignition. They are also used to generate high-enthalpy conditions for shock tunnels.