University of Maryland at College Park

Dept. of Aerospace Engineering

ENAE 432: Aerospace Control Systems

Problem Set #5

Issued: 1 Mar. 2025
Due By: 7 Mar. 2025

Important: Use the `title` command in Matlab to add a title to all submitted plots which contains your name and the question number the plot belongs with. Submissions which lack this annotation on the Matlab generated graphs will receive no credit. These titling requirements are in addition to any other annotations required by the problems below. You can export figures as PDFs (or jpeg, png, etc) from the "Save As..." menu in the figure window for inclusion with the packet you upload to Gradescope

Matlab preamble:

The student version of Matlab, and all machines in the open clusters on campus, and the version on the VCL, contains a specialized toolbox for control system analysis. The most basic facility provided by this toolbox is the ability to easily define and manipulate transfer functions. The command `s=tf('s');` declares the variable `s` to be the same as the Laplace variable that appears in our transforms. Transfer functions can then be defined algebraically in terms of `s`, for example `G=1/(s+2)`. Once defined, you can add, subtract, multiply, etc different transfer functions:

```
>> G1=1/(s+2);
>> G2=s/(s^2+2*s+2);
>> G3=G1*G2

Transfer function:
          s
---------------------
s^3 + 4 s^2 + 6 s + 4

>> G4=G1+G2

Transfer function:
   2 s^2 + 4 s + 2
---------------------
s^3 + 4 s^2 + 6 s + 4
```

The command `zpk(G)` will show you the transfer function in the factored ZPK form, while `pole(G)` and `zero(G)` will more explicitly identify the poles and zeros of $G(s)$. You can obtain a unit step response of a transfer function `G` with the command `step(G)`, and the command `bode(G)` will draw the Bode diagrams of `G`. Finally, `evalfr(G,j*w)` will compute the complex number $G(j\omega)$ for a specified numerical value of `w`. You can then use the `abs` and `angle` commands to get the magnitude and angle respectively.

This is just an initial taste of Matlab functionality related to systems and control. We will start to use many of its other features as we learn the underlying theory.

Question 1:

For this problem, consider the family of transfer functions

$$G(s) = \frac{6(\tau s + 1)}{s^2 + 2s + 4}$$

a.) Use Matlab to generate the step response for this system in the three cases $\tau = 0$, $\tau = 1$, and $\tau = -1$. Use `hold on` to overlay all three responses on a single graph. Right-click on the resulting plot and use the "Characteristics" submenu to label the peak values and times. Click on the dots that appear to pop up a box with numerical details about each point.

b.) Qualitatively, how do the responses in a) agree with the class discussion regarding step responses for transfer functions containing zeros? Quantitatively, how do the numerical values for peak response and peak time agree with the class discussion in the specific case that $\tau = 0$?

Question 2:

a.) Repeat Question #1a if instead the demominator of $G(s)$ is $s^2 + 4s + 4$; label the settling times on the graph instead of the peaks.

b.) For the $\tau = 0$ case, how does the settling time compare with the approximation discussed in lecture? From the theory, would you expect to see any overshoot in the step response for this case?

c.) For the two cases where $\tau \neq 0$, do either exhibit overshoot? If so, how much? Is this overshoot associated with oscillations in the response?

Question 3:

Use Matlab to obtain the Bode diagrams for the transfer function:

$$G(s) = \frac{5000(s + .02)}{3(2 + s)(20 + s)^2}$$

Put a `grid` on your graphs, then answer the following questions as accurately as possible by inspecting the graphs carefully. Note that you can use the Matlab "Data cursor" (looks like a "+" in the plot toolbar) to help you pick points numerically from the graph. You can also increase the resolution of the data plotted by giving `bode` a second argument with a finer grid of frequency points than `bode` would automatically generate, for example: `w=logspace(-3,3,10000); bode(G,w)` (increase the 10000 to an even larger number to get better resolution if needed).

a.) If the input to the system is $u(t) = \sin(6t/10)$, what does the diagram predict the steady-state output of the system will be? Highlight the point(s) on each diagram you use to calculate this.

b.) Analytically verify your result in a.) by explicitly calculating $G(j\omega)$, $|G(j\omega)|$, and $\angle G(j\omega)$ for the appropriate value of ω.

c.) If the input to the system is $u(t) = 2\sin(70t + \pi/4)$, what do you expect the steady-state output of the system will be? Indicate the point(s) on each diagram you use to calculate this. Repeat b.) for this input.

d.) For an input of the form $u(t) = 2\sin(\omega t)$, approximately what range of frequencies ω would result in the largest amplitude oscillations in the steady-state output? Determine from the graph as exactly as possible the actual output amplitude at these frequencies. Verify using the technique in b.)

e.) For the input in d.), what value of ω will result in the output oscillations lagging the input by 90° ("lag"=negative phase shift)? What will the amplitude of the output oscillations be at this frequency? Use the plot to estimate, then the technique in b.) for precision.

Question 4:

a.) Give the Bode form for the transfer function in Question #3. Identify the Bode gain numerically. Discuss how and why this gain agrees with the "starting" (low frequency) magnitude shown on the left side of the Bode magnitude plot in Question #3.

b.) Use the `bodemag` function in Matlab to get just the Bode magnitude diagram for the transfer function in Question #3, put a `grid` on it, and print out this plot (use `orient landscape` just before printing to get a plot that fills the page horizontally). Sketch on top of this diagram the straight-line approximation using the technique described in class. Comment on the accuracy of this approximation.