Lecture 11 - finishing out-of-plane + Patched conics

Plane Change ellipse, not at periapsis

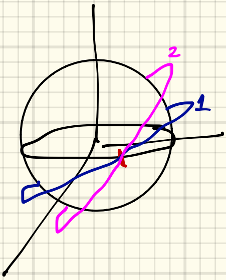
Initial orbit: $r_p = 8000$ km, $r_a = 12{,}000$ km, $\Omega_1 = 30°$, $i = 20°$, $\omega_1 = 20°$

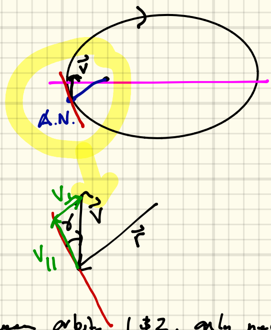
Final orbit: $r_p = 8000$ km, $r_a = 12{,}000$ km, $\Omega_2 = \Omega_1$, $i = 30°$, $\omega_2 = \omega_1$

Same a, e

↑ periapsis is not at the ascending node

These 2 orbits will intersect at the ascending & descending nodes





$V_\perp$ stays the same between orbits 1 & 2, only need to rotate the $V_{\parallel}$ component of velocity

$V_{\parallel} = V\cos\gamma$

$\Delta V^2 = v_1^2 + v_2^2 - 2v_1 v_2 \cos\theta$

$v_1 = v_2$ but we only need to rotate the $V_{\parallel}$ component of velocity

$\Delta V^2 = 2v_{\parallel}^2(1-\cos\theta)$

$= 2v^2\cos^2\gamma(1-\cos\theta)$

Angle between $(\vec{V}_{\parallel})_1$ & $(\vec{V}_{\parallel})_2 = \Delta i$

$\Delta V = v\cos\gamma\sqrt{2(1-\cos\Delta i)}$

Half-angle identity: $\sin\left(\frac{\Delta i}{2}\right) = \sqrt{\frac{1-\cos\Delta i}{2}}$

$$\boxed{\Delta V = 2v\cos\gamma\sin\left(\frac{\Delta i}{2}\right)}$$

Single-impulse, instantaneous

Maneuvers: Law of Cosines

- Where does the maneuver occur?
- What are v_1, v_2, θ?

Tangential:

- in-plane only
- $\theta = 0$
- Change the velocity magnitude
- a, e
- if not at r_p or r_a, ω, ν

Non-tangential:

in-plane:

- Change velocity direction and (maybe) magnitude
- $\theta = \gamma_2 - \gamma_1$
- Change: a, e, ω, ν

out-of-plane:

- Change the velocity direction and (maybe) magnitude
- Change any OE
- θ = complicated

Ex. w/ simple θ:

1. inclination change for circular orbit
2. a, e, i change where $\omega = 0$ or $180°$
3. inclination only change for $e \neq 0$, $\omega \neq 0, 180°$, $a_1 = a_2$, $e_1 = e_2$

$$\Delta v = 2 v \cos\gamma \sin(\Delta i / 2)$$

Patched Conics: interplanetary maneuvers

Execute a Hohmann transfer from Earth to Mars

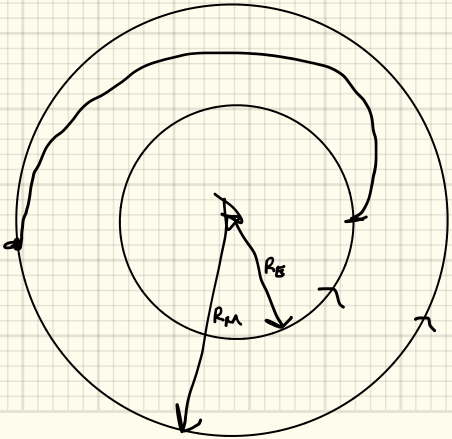
At Earth, S/c is initially on a parking orbit $r_{EP} = 7{,}000$ km

At Mars, parking orbit $r_{MP} = 7{,}000$ km (circular)

"Patching" together different Conic sections

Split the trajectory into 3 parts:

1. hyperbola to escape from Earth
2. ellipse about the Sun ← start here
3. hyperbola to get captured at Mars



R_E = radius of Earth's orbit about the Sun (circular)

R_M = radius of Mars' orbit (circular)

Calculate the velocity at periapsis & apoapsis of ellipse

$$\frac{V_{tp}^2}{2} - \frac{\mu_s}{R_E} = -\frac{\mu_s}{R_E + R_M} \qquad \mu = \mu_{sun}$$

$$\Rightarrow V_{tp}$$

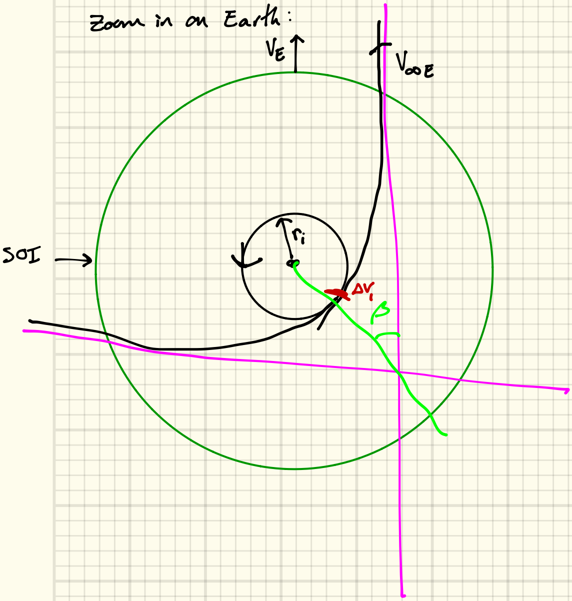
$$\frac{V_{ta}^2}{2} - \frac{\mu_s}{R_M} = -\frac{\mu_s}{R_E + R_M} \Rightarrow V_{ta}$$

Also, velocities of planets:

$$V_E = \sqrt{\frac{\mu_s}{R_E}}$$

$$V_M = \sqrt{\frac{\mu_s}{R_M}}$$

Zoom in on Earth:



r_i = radius of the initial parking orbit

SOI = Sphere of Influence

Only using Earth's gravity here

From the Earth's perspective, $r_{SOI} = \infty$

Assume periapsis of hyperbola is at r_i.

Execute a tangential maneuver to go from the circular parking orbit on to the hyperbola.

$$V_i = \sqrt{\frac{\mu_E}{r_i}}$$

$$\frac{V_{hp}^2}{2} - \frac{\mu_E}{r_i} = \frac{V_{\infty E}^2}{2} - \cancel{\frac{\mu_E}{r_{SOI}}} \to 0$$

V_{hp} = velocity of hyperbola at periapsis

$V_{\infty E} = V_{tp} - V_E$ (we know both of these from heliocentric phase on prior page)

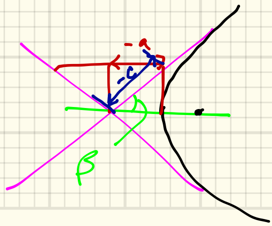
$$\Delta V_1 = V_{hp} - V_i$$

Given $V_{\infty E}$ & r_p (of the hyperbola), we can solve for the eccentricity.

$$r_p = \frac{h^2/\mu_E}{1 + e\cos\nu}, \quad \nu = 0 \text{ b/c periapsis.}$$

$$h = \frac{\mu_E\sqrt{e^2-1}}{V_{\infty E}}$$

$$\Rightarrow \boxed{e = 1 + \frac{r_p V_\infty^2}{\mu}}$$



$$\cos\beta = \frac{-a}{-c}$$

$$e = \frac{c}{a}$$

$$\boxed{\cos\beta = \frac{1}{e}}$$

Definition of sphere of influence:

$$\boxed{r_{SOI} = \left(\frac{m_s}{m_p}\right)^{2/5} a}$$

m_s = mass of the secondary (smaller)

m_p = mass of the primary (Sun, larger body)

a = semi-major axis of the secondary about the primary.

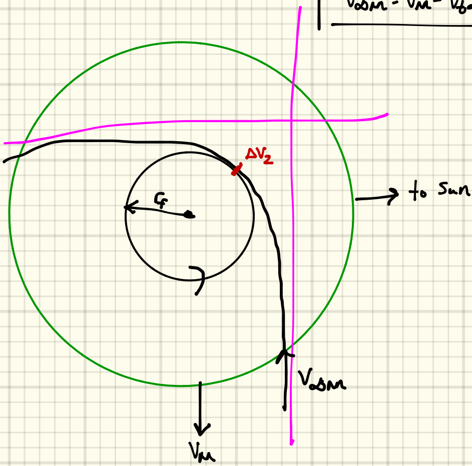
Arrival: Zoom in on Mars.

From the heliocentric phase, we know that s/c is traveling slower than Mars when it gets to Mars.

From the heliocentric phase, we know $V_{ta} = \sqrt{\frac{-2\mu_s}{R_E+R_M} + \frac{2\mu_s}{R_M}}$

$$V_M = \sqrt{\frac{\mu_s}{R_M}}$$

$$\boxed{V_{\infty M} = V_M - V_{ta}}$$



$V_{\infty M}$ goes in the opposite direction as V_M b/c the s/c is traveling slower than the planet. The planet "catches up" to the s/c.

$$V_f = \sqrt{\frac{\mu_M}{r_f}}$$

$$\frac{V_{ph}^2}{2} - \frac{\mu_M}{r_f} = \frac{V_{\infty M}^2}{2} - \cancelto{0}{\frac{\mu_M}{r_{SOI}}}$$

$\Delta V_2 = V_{ph} - V_f$.

Note: we are assuming that the hyperbola's periapsis is r_f.

Second simplifying assumption: Hohmann transfer for the heliocentric phase would follow the same procedure for a different transfer, but the orientation of V_∞ wrt V_E or V_M would be different.

Suppose the Hohmann transfer were from Mars to Earth: