# Lecture 8: TOF

<u>Time of Flight</u>: Time for the s/c to travel from point A to point B on a given orbit

The period for an ellipse: $\mathbb{P} = 2\pi\sqrt{\frac{a^3}{\mu}}$

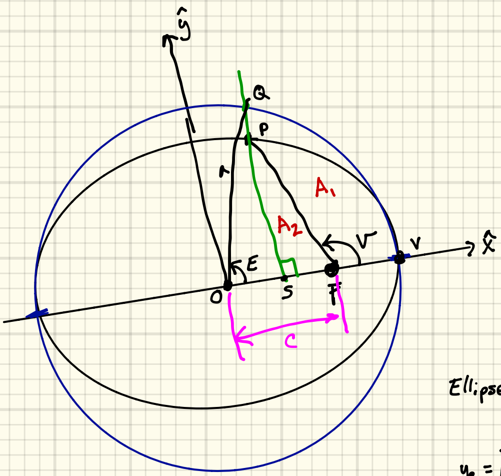
Kepler's 2nd law states that the line joining the planet to the sun sweeps out equal areas in equal times.

$$\frac{t-T}{A_1} = \frac{\mathbb{P}}{\underbrace{\pi a b}_{\text{Area of an ellipse}}}$$

$t$ = time when the s/c is at Point B

$T$ = time of the last periapsis passage

$A_1$ = area of the portion of the orbit from periapsis to point B.




E = eccentric anomaly

Need an expression for $A_1$.

$A_1 = \text{Area}(PSV) - A_2$

Base of $A_2 = c - a\cos E$

$= ea - a\cos E$

Note: $e = \frac{c}{a} \Rightarrow$

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$y_e = \sqrt{\frac{a^2b^2 - b^2x^2}{a^2}}$

$= \frac{b}{a}\sqrt{a^2 - x^2}$

Circle: $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$

$y_c = \sqrt{a^2 - x^2}$

$\Rightarrow \frac{y_e}{y_c} = \frac{b}{a}$

Height of $A_2 = \frac{b}{a}(a\sin E) = b\sin E$

Area $A_2$: $\frac{ba}{2}(e\sin E - \sin E\cos E)$

$\text{Area}(PSV) = \frac{b}{a}(\text{Area}(QSV))$

$\text{Area}(QSV) = \text{Area}(QOV) - \text{Area}(QOS)$

$\text{Area}(QOV) = \pi a^2 \frac{E}{2\pi} = \frac{1}{2} E a^2$ (E in RADIANS!!)

$\text{Area}(QOS) = \frac{1}{2}(a\cos E)(a\sin E)$

Plug into eqns for $A_1$: $A_1 = \frac{ba}{2}(E - e\sin E)$

Plug $A_1$ into Kepler's law eqn:

$$\frac{t-T}{\frac{ba}{2}(E-e\sin E)} = \frac{2\pi}{\pi ab}\sqrt{\frac{a^3}{\mu}}$$

$$\boxed{t-T=\sqrt{\frac{a^3}{\mu}}\,(E-e\sin E)}$$

Time of flight from periapsis to E.

Must use E in RADIANS!

Would like an expression for E as $f(\nu)$:

$$\boxed{\cos(E)=\frac{e+\cos\nu}{1+e\cos\nu}}$$

Half-plane check: if $\nu > 180°$, then $E > 180° = \pi$

$M = E - e\sin E$ = Mean Anomaly

$M = n(t-T)$

$n=\sqrt{\frac{\mu}{a^3}}$ = mean motion

avg angular rate as the s/c goes around the orbit

TOF between 2 arbitrary points on the orbit:

s/c @ $\nu_0, E_0$ @ $t_0$

s/c @ $\nu, E$ @ $t$

$t > t_0$

$$t-t_0=\sqrt{\frac{a^3}{\mu}}\left[2\pi k+(E-e\sin E)-(E_0-e\sin E_0)\right]$$

$k$ = # of times that the s/c passes thru periapsis.

Parabola: $$t-t_0=\frac{1}{2\sqrt{\mu}}\left[pD+\tfrac{1}{3}D^3)-(pD_0+\tfrac{1}{3}D_0^3)\right]$$

$D=\sqrt{p}\tan(\nu/2)$ = parabolic eccentric anomaly RAD

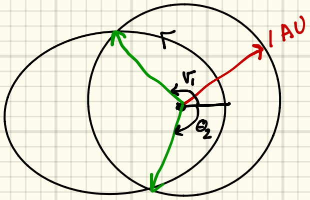
Hyperbola: $$t-t_0=\sqrt{\frac{(-a)^3}{\mu}}\left[(e\sinh F-F)-(e\sinh F_0-F_0)\right]$$

$$\cosh F=\frac{e+\cos\nu}{1+e\cos\nu}$$

if $\nu < \pi$, $F > 0$

else $F < 0$

Example: A probe in an elliptical orbit about the Sun. Perihelion is 0.5 AU, aphelion is 2.5 AU. How many days in each orbit is the s/c 1 AU or closer to the sun?




B/c the orbit is symmetric about the semi-major axis,

$$\nu_1 = \theta_2$$

To get the time when the s/c is 1 AU or closer to the Sun, Just Calculate TOF from $\nu=0$ to $\nu=\nu_1$ & multiply by 2.

$$t-T = \sqrt{\frac{a^3}{\mu}}\left[E - e\sin E\right] \qquad \cos E = \frac{e+\cos\nu}{1+e\cos\nu}$$

$$r(@\nu_1) = 1\text{ AU} = \frac{p}{1+e\cos\nu} \qquad p = a(1-e^2)$$

$$2a = r_p + r_a = 3\text{ AU} \Rightarrow a = 1.5\text{ AU} = \frac{3}{2}\text{ AU}$$

$$r_p = a(1-e) \Rightarrow e = 1 - \frac{r_p}{a} = 1 - \frac{1}{2}\cdot\frac{2}{3} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$r = \frac{a(1-e^2)}{1+e\cos\nu} \Rightarrow 1+e\cos\nu = \frac{a(1-e^2)}{r} \Rightarrow \frac{a(1-e^2)}{re} - \frac{1}{e} = \cos\nu$$

$$\cos\nu = \frac{a(1-e^2)}{re} - \frac{1}{e}$$
$$= \frac{3}{2}\frac{(1-4/9)\,3}{2} - \frac{3}{2} = -\frac{1}{4}$$

Plug $\cos\nu$ into expression for $\cos E \Rightarrow \cos E = \frac{1}{2} \Rightarrow E = 60° = \frac{60\pi}{180}$ rad

B/c we are using canonical units: $\mu = 1\ \frac{AU^3}{TU^2}$

Plug into $t-T$ eqn $\Rightarrow t-T = 0.863$ TU

TOF when s/c is $\leq 1$ AU from the Sun is $\underline{2(t-T) = 1.726\text{ TU}}$

Need to Convert from TU to days.

Knowing the dimensional value of $\mu_{sun}$ ($km^3/s^2$), we can solve for the conversion from TU to sec.

$1\text{ TU} = 5.024 \times 10^6$ sec

$\underline{\text{TOF} \simeq 100\text{ days}}$