Lecture 6 - Sketching Orbits

Inclination:

if $i = 0, 180°$ = equatorial $i = 90°$ = polar orbit

$0 < i < 90°$: Prograde

$90 < i < 180°$ = Retrograde

Eccentricity:

Can also be defined as a vector that points from the focus towards periapsis

Starting with an intermediate step from our derivation of the trajectory eqn:

$$\vec{r} \times \vec{h} = \mu \frac{\vec{r}}{r} + \vec{B} \qquad \vec{e} = \vec{B}/\mu \qquad r = \frac{p}{1 + e\cos\nu}$$

Solve for $\vec{B}$: $\vec{B} = \vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r}$

$$\vec{h} = \vec{r} \times \vec{v}$$

$$\mu \vec{e} = \vec{v} \times (\vec{r} \times \vec{v}) - \mu \frac{\vec{r}}{r}$$

$$= (\vec{v} \cdot \vec{v})\vec{r} - (\vec{r} \cdot \vec{v})\vec{v} - \mu \frac{\vec{r}}{r}$$

$$\boxed{\vec{e} = \frac{1}{\mu}\left[\left(v^2 - \frac{\mu}{r}\right)\vec{r} - (\vec{r} \cdot \vec{v})\vec{v}\right]}$$

Another expression for e:

$$p = a(1 - e^2) \Rightarrow e = \sqrt{1 - p/a}$$

$$p = h^2/\mu \;,\; \mathcal{E} = -\frac{\mu}{2a} \Rightarrow a = -\frac{\mu}{2\mathcal{E}}$$

$$e = \sqrt{1 + \frac{h^2/\mu}{\mu/2\mathcal{E}}} \qquad \Rightarrow \boxed{e = \sqrt{1 + \frac{2\mathcal{E}h^2}{\mu^2}}}$$

Other orbital elements: We talked about "Keplerian" orbital elements

If our orbit is in the equatorial plane, then we don't have an ascending node.

Π = longitude of periapsis: angle from $\hat{x}$ to $\hat{e}$

$\Pi = \Omega + \omega$

For a circular orbit, there is no periapsis.

u: argument of latitude: angle (in the plane of the orbit) between the ascending node & the position.

$$u = \omega + \nu$$

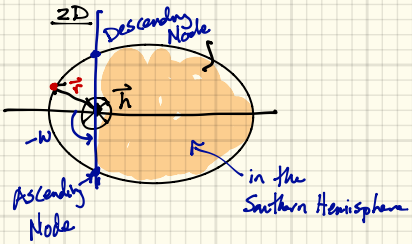
l: true longitude: angle from $\hat{x}$ to $\vec{r}$

$$l = \Omega + w + \nu$$

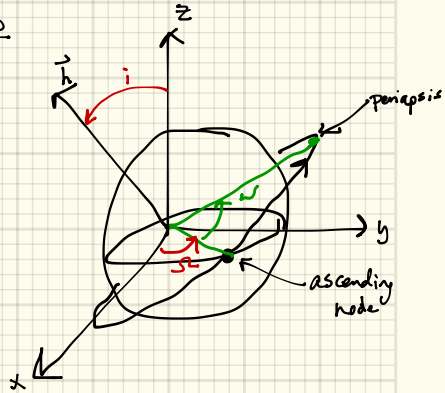
Sketching Orbits:

Earth orbit: $i = 20°$, $a = 20{,}000$ km, $w = 90°$, $e = 0.5$, $\Omega = 45°$, $\nu = 10°$

$$r_p = a(1-e) = 10{,}000 \text{ km} > 6378 \text{ km} \rightarrow \text{will not hit Earth}$$



3D

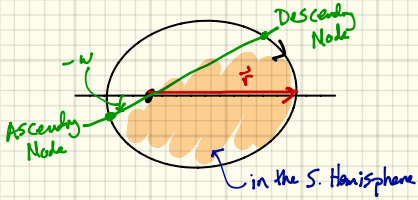


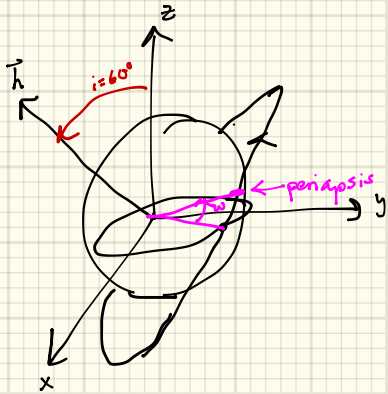
S/C is in the S. Hemisphere if:

$360° < w + \nu < 180$: S/C is in the N. Hemisphere

Earth orbiting:

$a = 20{,}000$ km, $e = 0.5$, $i = 60°$, $\Omega = 60°$, $w = 10°$, $\nu = 180°$





Earth orbit:

$a = 20{,}000$ km
$e = 0.5$
$i = 160°$

$\Omega = 45°$
$\omega = 0°$
$\nu = 10°$