UMJ-4741

B.Sc. IV Semester (NEP)

Examination, 2024

MATHEMATICS

Paper–I (Theory)

[Differential Equations]

Time : 3 Hours] **[Maximum Marks : 75**

Note : This question paper contains *two* sections A and B. Attempt any *five* questions each of 6 marks from Section A and any *three* questions each 15 marks from Section B.

SECTION–A

(5×6=30)

1. Solve $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$.

2. Solve $(x + 2y^3)dy = y\,dx$.

3. Find general and singular solution of the differential equation $(xp - y)^2 = p^2 - 1$.

4. Solve $(D^2 + D - 6)y = e^{2x}$.

5. Solve $\frac{d^2y}{dx^2} + 2\tan x \cdot \frac{dy}{dx} + 3y = \tan^2 x \sec x$.

6. Find the complete integral of $z^2(p^2z^2 + q^2) = 1$.

7. Classify the following partial differential equation of second order : $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$.

8. Solve using Laplace transforms :

$$\frac{d^2y}{dt^2} + y = t, \quad y(0) = 1, \quad \left.\frac{dy}{dt}\right|_{t=0} = -2.$$

SECTION–B

(3×15=45)

9. (a) Solve the following partial differential equation : $2r - s - 3t = 5e^{x-y}$.

   (b) Solve $s + p - q = z + xy$.

10. (a) Verify that the equation :

$(y - z)(y + z - 2x)dx + (z - x)(z + x - 2y)dy + (x - y)(x + y - 2z)dz = 0$ is exact and find the solution.

(b) Solve $\frac{dx}{y^2+z^2-x^2}=\frac{dy}{-2xy}=\frac{dz}{-2xz}$.

11. Find the series solution of the equation $x\frac{d^2y}{dx^2}+\frac{dy}{dx}+xy=0$.

12. (a) Find the orthogonal trajectories of the family curves

$$\frac{x^2}{(a^2+\lambda)}+\frac{y^2}{(b^2+\lambda)}=1,$$ where λ is the parameter.

(b) Solve $\frac{x\,dx+y\,dy}{x\,dy-y\,dx}=\sqrt{\left(\frac{a^2-x^2-y^2}{x^2+y^2}\right)}$.

13. (a) Find $L(e^{-t}\sin^2 t)$, L denote Laplace transform.

(b) Find $L^{-1}\left(\frac{6s-4}{s^2-4s+20}\right)$, L^{-1} denote the inverse Laplce transform.

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SECTION–A

(Short Answer Type Questions)

(5×6=30)

1. Solve :

(a) $\frac{dy}{dx}+2xy=e^{-x^2}$.

(b) $\cos^2 x.\left(\frac{dy}{dx}\right) + y = \tan x.$

2. Find the general and singular solution of :

$9p^2(2 - y)^2 = 4(3 - y)$, where $p = \frac{dy}{dx}$.

3. Solve : $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0.$

4. Solve : $(D^2 - 2D + 1)y = x\, e^x \sin x$, where $D = \frac{dy}{dx}$ is the differential operator.

5. Find the complete integral of :

(a) $z = pq.$

(b) $p^2 - 4q^2 = y^2 - x^2.$

6. Solve : $p + 3q = 5z + \tan(y - 3x).$

7. Solve using Laplace tranforms :

$(D^2 + 4)y = t$; $y(0) = 0$, $y'(0) = 0$ and $D = \frac{dy}{dx}$.

8. Find the Fourier transform of $F(x) = e^{-x^2/2}$.

SECTION–B

(Long Answer Type Questions)

(3×15=45)

9. Solve the following partial differential equations :

(a) $2r - 5s + 2t = 24(y - x)$.

(b) $4r - 4s + t = 16 \log(x + 2y)$.

10. Solve the following differential equations :

(a) $(yz + z^2)dx - xzdy + xydz = 0$.

(b) $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}$.

11. Solve using the method of variation of parameters :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x.$$

12. (a) Solve : $y = 2px + y^2p^3$, $p = \frac{dy}{dx}$.

(b) Find the orthogenal trajectories of the system of curves $r^n = a^n \cos n\theta$, a being the parameter.

13. (a) Find $L[(t + 3)^2 e^t]$; L denote Laplace tranforms.

(b) Find $L^{-1}\left\{\frac{3s-8}{4s^2+25}\right\}$, where L^{-1} denote the inverse laplace transfroms.
