Topic 5: Computational Fluid Dynamics – Particle Suspension

The simplest type of fluid system has a single homogeneous component. In practical applications, fluids are rarely so simple. Often, there are particles of different species mixed into the fluid system.

In a *suspension*, the particles mixed into the fluid are much larger than the fluid molecules. Thus they must be treated as individual solid objects with boundaries in contact with the surrounding fluid. The difference between this situation and the dynamics of fluid in a region with bounding walls is that the fluid affects the motion of the suspended particles.

There are many interesting applications of suspensions:

- colloids in chemistry and chemical engineering,
- dust in the atmosphere,
- snow mixed into air in an avalanche,
- nutrients transported in blood, and
- sedimentation of particles is various fluid systems.

Single particle in a fluid

A single particle in a fluid experiences the same types of forces that a fluid element experiences:

- pressure forces due to pressure gradients in the fluid,
- shearing forces due to fluid viscosity, and
- body forces, such as gravity.

If the particle is very small, then gravity can be neglected and the particle undergoes random *Browninan motion* caused by viscous forces (which are ultimately molecular in origin), in addition to being transported along with the fluid flow.

If the particle is relatively large, then pressure and body forces usually dominate.

The Péclet number

$$\mathrm{Pe} = \frac{Ua}{D}\,,$$

where U is the speed of the particle relative to the fluid, a is its radius, and D is the thermal diffusion coefficient for motion of the particle in the fluid, can be used to distinguish between these two regimes: if $\mathrm{Pe} \ll 1$ diffusion dominates and the motion is Brownian, whereas if $\mathrm{Pe} \gg 1$ pressure dominates and the motion is *convective*.

Spherical particle motion at low Reynolds number

Recall that the Reynolds number is

$$\mathrm{Re} = \frac{UL}{\nu} ,$$

where U is a typical fluid velocity, L a typical linear dimension, and $\nu = \eta/\rho$ is the kinematic viscosity coefficient. If the fluid is very viscous (large ν) and/or slowly moving, then the nonlinear convective term $\mathbf{v} \cdot \nabla \mathbf{v}$ in the Navier-Stokes equations can be neglected, and one obtains the *Stokes equations*

$$\frac{\partial \mathbf{v}}{\partial t} = -\nabla P + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{v} + \mathbf{g} .$$

Stokes was able to solve these equations for flow past a spherical object and determine the *drag force*

$$F_D = 6\pi\eta a U ,$$

where η is the dynamic viscosity coefficient, a is the radius of the sphere, and U is the speed of the fluid far away from the sphere. This result can be used to compute the *settling speed* or *Stokes velocity* of a spherical particle in a fluid

$$U_{\mathrm{Stokes}} = \frac{2}{9} \frac{a^2(\rho_{\mathrm{particle}} - \rho)g}{\eta} .$$

There are very few situations in which such simple analytical results can be obtained. The presence of a particle in the fluid influences the flow over a large volume: typically, the flow pattern decays slowing $\sim \frac{1}{r}$ with distance r from the particle. Thus, if there are many particles present, the flow pattern is not easy to determine even if the particles positions are fixed. In fact, the flow pattern determines the motion of the particles, so a complex set of coupled equations for the fluid and the suspension need to be solved.

Immersed Boundary Method for Fluid-Particle Equations

Many numerical methods have been invented for solving for the motion of particles suspended in fluids. An efficient method, which has been used widely in biological applications, was presented by Fogelson and Peskin in a paper entitled *A Fast Numerical Method for Solving the Three-Dimensional Stokes' Equations in the Presence of Suspended Particles*, published in *J. Computational Physics* **79**, 50 (1988). This method is now called the *Immersed Boundary Method*.

An outline of the method is as follows:

- The fluid is assumed to be incompressible, and the 3 Navier-Stokes equations and the continuity equation are discretized on a regular lattice or mesh in space.

- The grid spacing is chosen to be smaller than the size of a suspended particle, so a particle volume contains several grid points. A particle is represented by a shape, e.g., a sphere in 3-D or a disc in 2-D which encloses a set of grid points. As the particle moves, the grid points associated with the shape will change.

- The no-slip boundary condition at the surface of a particle is approximated using the body-force term in the Navier-Stokes equation to to make the fluid at the particle surface move as if a boundary condition were being imposed. This is the essential idea introduced by Fogelson and Peskin.

- The shape which represents a particle moves under the action of pressure and gravity, just as in the Stokes solution for single sphere. In computing the motion of the particle shape, the body forces introduced to implement the no-slip boundary condition must be cancelled by introducing additional terms into the equations of motion.

- If there are several particles in the suspension then collisions between particles must be taken into account. An appropriate form for collisional forces must be introduced: for example, the collisions can be assumed to be hard-sphere elastic, or a *soft* force from a Lennard-Jones type of potential can be assumed as in the Molecular Dynamics topic.