GMI23G Logic and Mathematics for Computer Science P3/2022
Homework Math Problem Set 2 Solution

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Problem 2.1

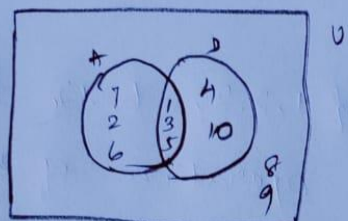
a) The set A = {1, 2, 3, 5, 6, 7} is not closed under the operation of addition because there is atleast one result which is not an element of Set A. The below addition chart contains the element 4, 8, 9, 10, 11, 12, 13, 14 none of them are not in Set A.

	1	2	3	5	6	7
1	2	3	4	6	7	8
2	3	4	5	7	8	9
3	4	5	6	8	9	10
5	6	7	8	10	11	12
6	7	8	9	11	12	13
7	8	9	10	12	13	14

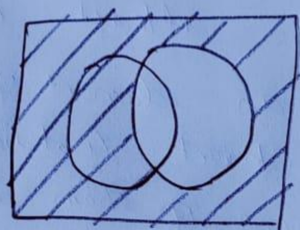
b) $A = \{1, 2, 3, 5, 6, 7\}$ $B = \{1, 3, 4, 5, 10\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

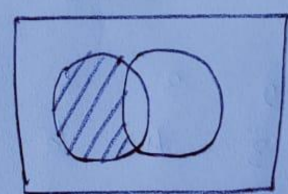
$A \cap B = \{1, 3, 5\}$



$B' =$



$A \cap B' =$



$C = A \cap B'$

$C = \{2, 6, 7\}$

Cardinality = 3

2.2) HIPPOPOTAMUS (3)

$$nP_r = \frac{n!}{(n_1! n_2! \dots n_k!)}$$

$n = 12$

$H=1$, $I=1$, $P=3$, $O=2$, $T=1$, $A=1$, $M=1$, $U=1$ $S=1$

$$= \frac{12!}{1! \times 1! \times 3! \times 2! \times 1! \times 1! \times 1! \times 1! \times 1!}$$

$$= \frac{12!}{3! \times 2!}$$

$$= \frac{479001600}{12} = 39916800 \text{ ways.}$$

Problem 2.3)

$n = 32$

$m = 4$ $\quad k = \frac{n}{m} = \frac{32}{4} = 8$

The No. of ways n distinct things can be divided among the distinct group. is

$$\frac{n!}{(k!)^m}$$

$= C(32,8) \times C(24 \times 8) \times C(16,8) \times 1$

$= 10518300 \times 735471 \times 12870 \times 1$

$= 99561092450391000$ ways.

Problem 2.4)

Let 4 Mens be M_1, M_2, M_3, M_4

Let 3 womens be W_1, W_2, W_3

	M_1	M_2	M_3	M_4	W_1	W_2	W_3
M_1	—	$(M_1 M_2)$	$(M_1 M_3)$	$(M_1 M_4)$	$(M_1 W_1)$	$(M_1 W_2)$	$(M_1 W_3)$
M_2	$(M_1 M_2)$	—	$(M_2 M_3)$	$(M_2 M_4)$	$(M_2 W_1)$	$(M_2 W_2)$	$(M_2 W_3)$
M_3	$(M_3 M_1)$	$(M_3 M_2)$	—	$(M_3 M_4)$	$(M_3 W_1)$	$(M_3 W_2)$	$(M_3 W_3)$
M_4	$(M_4 M_1)$	$(M_4 M_2)$	$(M_4 M_3)$	—	$(M_4 W_1)$	$(M_4 W_2)$	$(M_4 W_3)$

By looking at the above table
(M_1, M_2) (M_1, M_3) (M_1, M_4) $(M_2 M_3)$ $(M_2 M_4)$ $(M_3 M_4)$
repeated twice so it is subracted. ~~with~~
$4C_2$ from $C(4,1)\,C(6,1)$. For example pair of (M_1, M_2) is
Same as (M_2, M_1) so this kind of repetation should be eliminated.
Hence Correct answer is $C(4,1)\,C(6,1) - C(4,2)$

So $C(4,1)\,c(6,1) - C(4,2)$

$4 \times 6 - 6$ → above 6 repeted outcome

$24 - 6$
$= 18$ ways.

In other ways it can be written as $= 4C_1 \times 3C_1 + 4C_2 \times 3C_0$
$= 18$

Problem 2.5)

5

Expand $(5x-3y)^4$ using the binomial theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

where $\binom{n}{k} = \frac{n!}{(n-k!)k!}$ & $n! = 1.2 \dots n$

$a = 5x$, $b = -3y$, $n = 4$

$$(5x-3y)^4 = \sum_{k=0}^{4} \binom{4}{k} (5x)^{4-k} (-3y)^k$$

Now Calculate the product for every value of k from 0 to 4

$k=0$ $\binom{4}{0}(5x)^{4-0}(-3y)^0 = \frac{4!}{(4-0)!0!}(5x)^{4-0}(-3y)^0$

$= 6525x^4$

$k=1$: $\binom{4}{1}(5x)^{4-1}(-3y)^1 = \frac{4!}{(4-1)!1!}(5x)^{4-1}(-3y)^1 = -1500x^3y$

$k=2$ $\binom{4}{2}(5x)^{4-2}(-3y)^2 = \frac{4!}{(4-2)!2!}(5x)^{4-2}(-3y)^2 = 1350x^2y^2$

$k=3$ $\binom{4}{3}(5x)^{4-3}(-3y)^3 = \frac{4!}{(4-3)!3!}(5x)^{4-3}(-3y)^3 = -540xy^3$

$k=4 = \binom{4}{4}(5x)^{4-4}(-3y)^4 = \frac{4!}{(4-4)!4!}(5x)^{4-4}(-3y)^4 = 81y^4$

Thus Answer: $625x^4 - 1500x^3y + 1350x^2y^2 - 540xy^3 + 81y^4$

Problem 2.6

Proove by binomial theorem

$c(n,0) - c(n,1) + c(n,2) - \dots + (-1)^n c(n,n) = 0$

$(a+b)^n = c(n,0)\, a^n b^0 + c(n,1)\, a^{n-1} b^1 + \dots + c(n,n) a^0 b^n$

Taking first & last value.

$c(n,0) = c(n,0)\, a^n b^0$

anything power 0 = 1

so $b^0 = 1$

$a^n = 1$

according to exponent of 1 rule

$n^1 = n$

here a & n = 1

so $a^n = 1$

$a = 1$

$c(n,n)(-1)^n = c(n,n)\, a^0 b^n$

By equaling above it is found that $b = -1$

In this case a can be any number becaus $a^0 = 1$

It is found that $a = 1$ & $b = -1$

by using a & b in formula we get

$(a+b)^n = c(n,0) - c(n,1) + c(n,2) - \dots (-1)^n c(n,n) = 0$

Replace $a = 1$ & $b = -1$

$(1-1)^n = 0$

Hence Prooved.

Problem2.7)

(7)

$P(S) = 0.76$ $\quad$ $P(S') = 0.24$

$P(W) = 0.31$

$P(C) = 0.44$ $\quad$ $P(C') = 0.56$

The day being windy but neither sunny nor cloudy.

$= P(W \cap S' \cap C')$

$= 0.31 \times 0.24 \times 0.56$

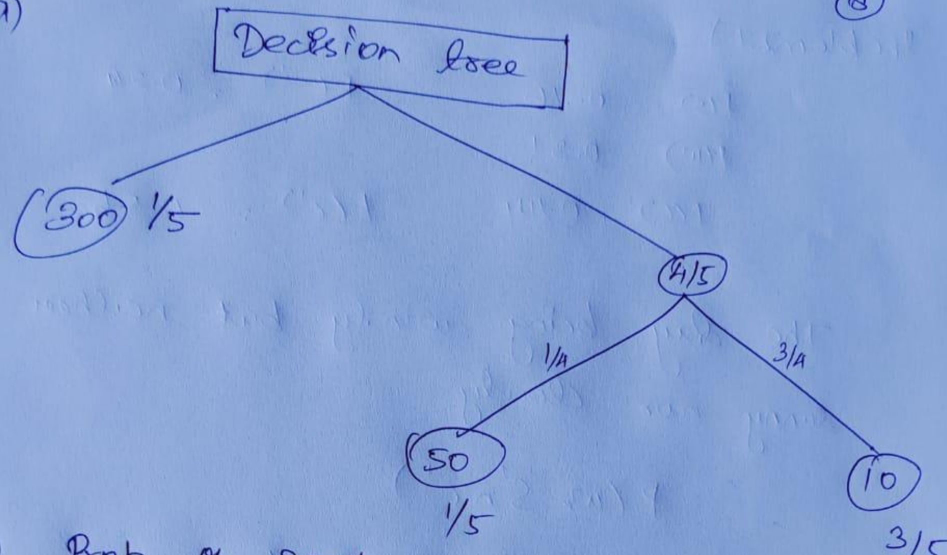
$= 0.041664$

Probability is 0.041664 day being windy but neither sunny nor cloudy

Problem 2.8

(8)

a)



b) Prob of 300\$ = 1/5 as per the above tree

In 5 outcome prob of winning 300\$ is 1/5

c) Prob of 50 \$ = 1/5 as per the above tree

$= \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$

d) Prob of winning 10 \$ = 3/5 as per the above tree.

$= \frac{3}{4} \times \frac{4}{5} = 3/5$

e) Expected amount of Money that you will Make in the game is

$$\left(300 \times \frac{1}{5}\right) + \left(50 \times 1/5\right) + \left(10 \times 3/5\right)$$

$= 60 + 10 + 6$

$= 76$ \$