GMI23G Logic and Mathematics for Computer Science P3/2022

Homework Math Problem Set 3 Solution

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Problem 3.1)

a) $f(m,n)$ for $(m,n) = (3,5)$ & $(4,2)$

given $f(m,n) = 2^m 3^n$

for $(3,5)$ $2^3 \times 3^5$

$= 8 \times 243$

$= 1944$

$f(m,n) = (4,2) = 2^4 \times 3^2$

$= 16 \times 9$

$= 144$

b) f is one to one As per the book pg No 389 to show the function f is 1-1

assume $f(s_1) = f(s_2)$

$s_1 = s_2$

given that $f(m,n) = 2^m 3^n$

so then $f(x,y) = f(m,n)$

$2^x 3^y = 2^m 3^n$

$2^{x-m} = 3^{n-y}$

here both 3 & 2 are prime Numbers.

so it is not possible for two (m,n) domin values to map to same element in the codomin.

It can be shown with one condition.

b) continuous ②

$$2^0 = 3^0 = 1$$

$x - m = 0$; so $x = m$

$n - y = 0$; $n = y$

Hence prooved that $f(x,y) = f(m,n)$

so function f is one to one.

c) Let us assume there exists the 7 in the codomin. In onto every element in the codomin there must exists an element in domin

If we use any of the numbers in $(2^m \times 3^n)$ we cant get ~~8~~ 7 because 2 & 3 are prime numbers & 7 is also a prime number as per the condition g: ~~N₀ × N₀ → N₊~~

$N_+ = \{1, 2 \ldots\}$

As 7 is not maped to any element in the domin so it is not onto.

Problem 3.1)

d) g is not a injective

given $g(m,n) = 2^m 4^n$

4 is factorized by 2, 4^n can be written as 2^m

$$\cancel{2^1 \times 4^3 = 2^5 \times 4^1}$$

$$g(m,n) = 2^m 4^n$$

$$g(0,3) = 2^0 \times 4^3$$
$$= 64$$

$$g(6,0) = 2^6 \times 4^0$$
$$= 64$$

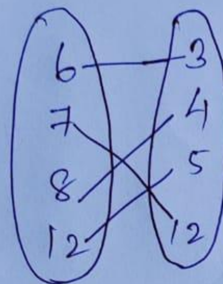
$$g(0,3) = g(6,0)$$

so g is not an injection

Hence prooved.

Problem 3.2

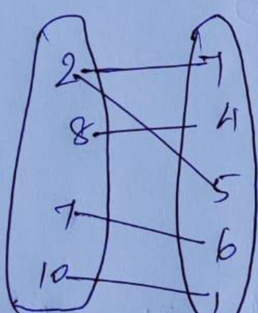
a) $P = \{(12,5), (8,4), (6,3), (7,12)\}$



One to One

each element in a domain matches to one element in a codomain.

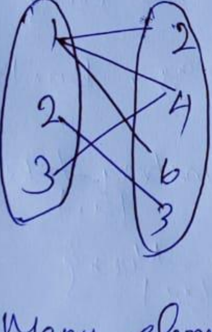
b)



One to Many

here element 2 in domin maps to Many element 7 & 5 in codomin

c)

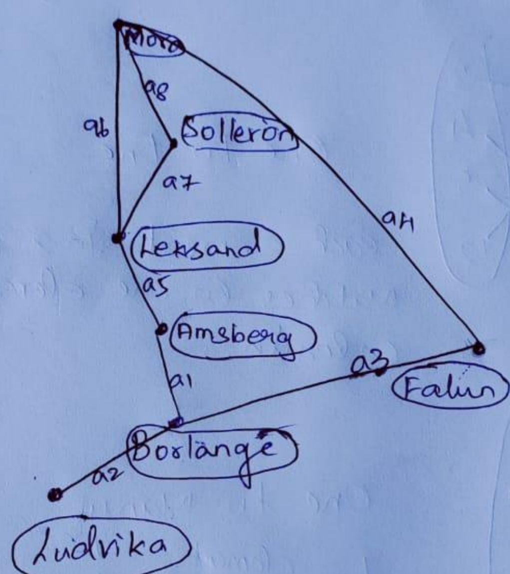


Many to Many

here 1 in domin Matches to Many element 2, 4, 6 in Codomin & 4 in codomin matches to Many element 1 & 3 in domin.

Problem 3.3 ⑤

a



Nodes (vertices): {Borlänge, Falun, Mora, Amsberg, Leksand, Ludvika, Solleron}

Arcs (edges) : {Borlänge - Amsberg (a_1), Borlänge - Ludvika (a_2), Borlange - Falun (a_3), Falun - Mora (a_4), Amsberg - Leksand (a_5), Leksand - Mora $(a6)$, Leksand - Solleron $(a7)$, Solleron - Mora $(a8)$}

b)

for Non Directional Matri

		B	F	M	A	Le	Lu	S
		0	1	2	3	4	5	6
B Borlänge	0	0	1	0	1	0	1	0
F Falun	1	1	0	1	0	0	0	0
M Mora	2	0	1	0	0	1	0	1
A Amsberg	3	1	0	0	0	1	0	0
Le leksand	4	0	0	1	1	0	0	1
LU Ludvika	5	1	0	0	0	0	0	0
S Solleron	6	0	0	1	0	1	0	0

b) $A = \begin{bmatrix} 0&1&0&1&0&1&0 \\ 1&0&1&0&0&0&0 \\ 0&1&0&0&1&0&1 \\ 1&0&0&0&1&0&0 \\ 0&0&1&1&0&0&1 \\ 1&0&0&0&0&0&0 \\ 0&0&1&0&1&0&0 \end{bmatrix}$

(c) $A^2 = A \times A$ $\quad$ $A^3 = A \times A \times A$ or $A^2 \times A$

$$A^2 = \begin{bmatrix} 0&1&0&1&0&1&0 \\ 1&0&1&0&0&0&0 \\ 0&1&0&0&1&0&1 \\ 1&0&0&0&1&0&0 \\ 0&0&1&1&0&0&1 \\ 1&0&0&0&0&0&0 \\ 0&0&1&0&1&0&0 \end{bmatrix} \times \begin{bmatrix} 0&1&0&1&0&1&0 \\ 1&0&1&0&0&0&0 \\ 0&1&0&0&1&0&1 \\ 1&0&0&0&1&0&0 \\ 0&0&1&1&0&0&1 \\ 1&0&0&0&0&0&0 \\ 0&0&1&0&1&0&0 \end{bmatrix}$$

(7)

$$\begin{bmatrix} 1+1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1+1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1+1+1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1+1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1+1+1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 3 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 2 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 3 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 0 & 4 & 1 & 4 & 1 & 3 & 2 \\ 4 & 0 & 4 & 1 & 2 & 0 & 1 \\ 1 & 4 & 2 & 2 & 5 & 1 & 4 \\ 4 & 1 & 2 & 0 & 4 & 0 & 1 \\ 1 & 2 & 5 & 4 & 2 & 1 & 4 \\ 3 & 0 & 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 4 & 1 & 4 & 0 & 2 \end{bmatrix}$$

d) Meaning of A^3 (8)

A^3 shows the No. of paths of length 3 for every set of Nodes.

Length of Path is the No. of edges in the Path. In a Adjecency Matrix A where $A[i][j]$ will be 1 if edge between i & j otherwise 0.
The No of path of the length k between i & j can be found by A^k.

Problem 3.4)
Modular Multiplicative Inverse.
As per the Tom St Denis & Greg Rose the Modular Inverse of a number refers to the Modular Multiplicative Inverse. For any Integer such that $(a, P) = 1$ there exists the another Integer b such that $ab \equiv 1 \pmod{P}$. Here the Integer b is Multiplicative Inverse of a which is shown as $b = a^{-1}$

$$a * b * \text{Modulo } b = 0$$

36 Mod 85 ⑨

$85 = 36 \times 2 + 13$
$36 = 13 \times 2 + 10$
$13 = 10 \times 1 + 3$
$10 = 3 \times 3 + 1$

$1 = 10 - 3 \times 3$
$3 = 13 - (10 \times 1)$
$3 = 13 - 10$
$10 = 36 - (13 \times 2)$
$13 = 85 - (36 \times 2)$

$$
\begin{aligned}
1 &= 10 - (3)(3) \\
&= 36 - (13 \times 2) - 3(3) \\
&= 36 - (13 \times 2) - 3(13 - 10) \\
&= 36 - (13 \times 2) - 3(13) + 3(10) \\
&= 36 - 2(13) - 3(13) + 3(36 - (13 \times 2)) \\
&= 36 - 2(13) - 3(13) + 3(36) - 3((13)(2)) \\
&= 36 - 2(13) - 3(13) + 3(36) - 6(13) \\
&= 4(36) - 11(13) \\
&= 4(36) - 11(85 - 36 \times 2) \\
&= 4(36) - 11(85) + 22(36) \\
&= 26(36) - 11(85)
\end{aligned}
$$

$a \times b$ Modulo $b = 0$

The Modular Multiplicative Inverse of 36 Modulo 85 is 26.