Homework 4 (Due April 30, 2019, Tuesday, 11:59pm)

Structural Vector Autoregression

Instructions: Please type your answers in Microsoft Word and upload your solution to Canvas.

Data: Please use the data "macro_data_us.xlsx" from your Homework 3, which is of quarterly frequency and covers 1960Q1-2018Q4. Please use "gretl" to conduct the SVAR analysis. For instructions of gretl, refer to the class notes on Canvas (econ4304-topic5-var-gretl.pdf).

Variable list:

"**inflation**": inflation of CPI

"**growth**": growth of real GDP

"**r**": federal fund rate / the short run interest rate

"**cp**": inflation of commodity price index

"**m2**": growth of money supply, defined as the log-difference of M2

"**u**": Civilian Unemployment Rate

"**nu**": natural rate of unemployment

Some background: In this exercise, we wish to study how the "monetary policy shock", such as an unexpected increase of interest rate by the central bank, affects the economy, such as inflation and output growth. We will first consider a VAR(8) model for the three variables $(inflation, growth, r)$.

$$\begin{pmatrix} inflation_t \\ growth_t \\ r_t \end{pmatrix} = c + \Phi_1 \begin{pmatrix} inflation_{t-1} \\ growth_{t-1} \\ r_{t-1} \end{pmatrix} + \cdots + \Phi_8 \begin{pmatrix} inflation_{t-8} \\ growth_{t-8} \\ r_{t-8} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{pmatrix}$$

We know that in the above reduced-form VAR, the error term e_{jt} is a linear function of structural shocks, such as monetary policy shock and productivity shock, etc. To identify the monetary policy shock, there are lots of possible methods. One method is to assume that <u>"current quarter monetary policy shock" does not affect current quarter inflation and output</u>, which suggests ordering interest rate the **last** in the recursive VAR.

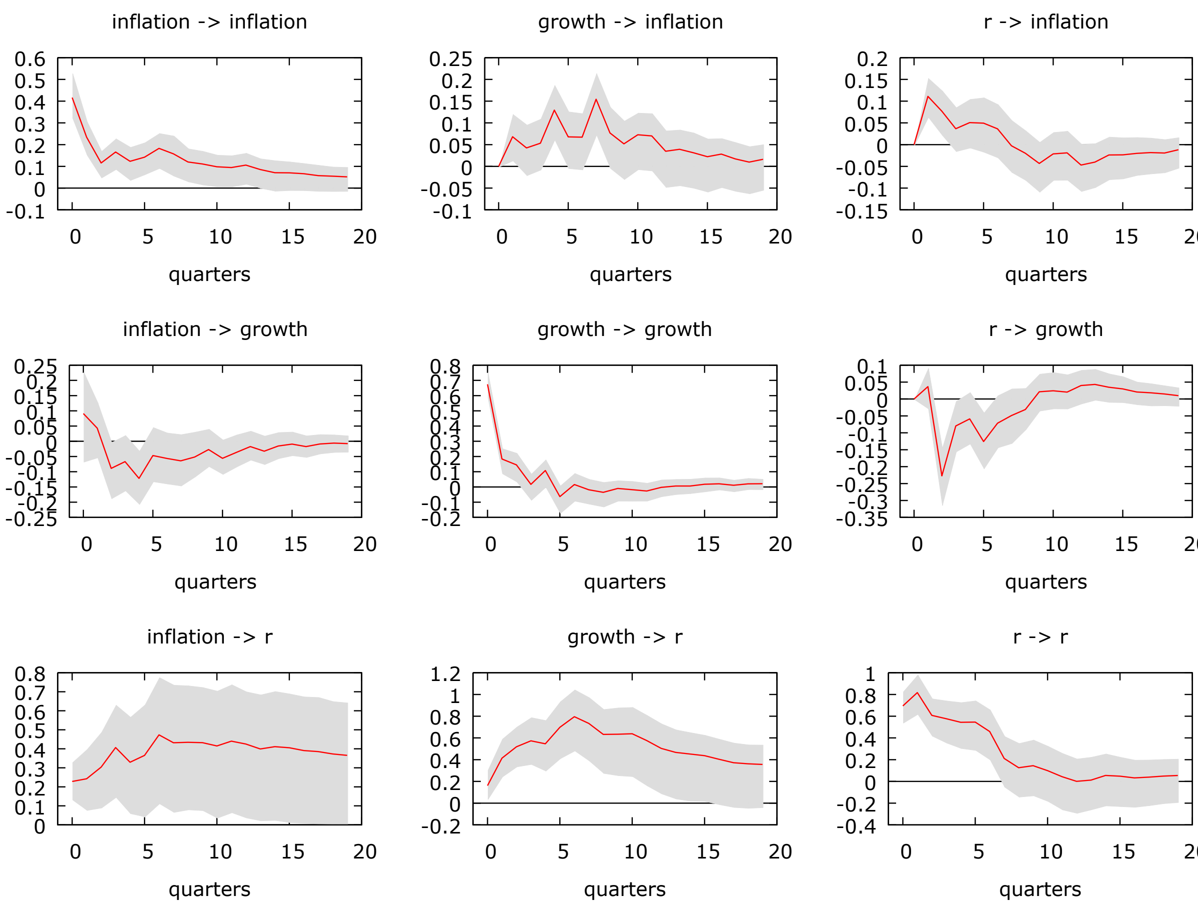
Because we are only interested in the monetary policy shock, so it does not matter whether we rank inflation before or after growth. The detail of the ordering is described in the following equation:

$$\begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \\ u_{rt} \end{pmatrix}$$

The above equation says that both current inflation and current growth **do not respond to current** monetary policy shock "u_{rt}" (i.e., in the 3×3 matrix the first two entries of the last column are zero.) The shock u_{1t} can be interpreted as the collection of all possible shocks that affect current inflation. The shock u_{2t} is the shock (other than u_{1t}) that affects current growth but is independent of u_{1t}. By assumption, both u_{1t} and u_{2t} do not include the monetary policy shock. The monetary policy responds to all shocks, including u_{1t}, u_{2t}, and the monetary policy shock u_{rt}.

Remark: u_{1t} and u_{2t} together include all the current shocks that affect current inflation and growth. The remaining shocks which do not affect current inflation or growth, but affect current interest rate, are represented by u_{rt}, and is treated as the monetary policy shock.

After estimating the VAR(8) using my old data set (which covers up to 2013Q4), we report the impulse responses below: (shaded area is the 90% confidence band)



We will only interpret the **last column** of the above graph, which provides how one unit increase of u_{rt} affects inflation and growth over time. For example, the subpanel with title "r->growth" means how the structural shock from the interest rate equation affects growth.

Some puzzling empirical observation (row 1 column 3 in the above graph): After a surprise hike in interest rate (increase of $\mathrm{u_{rt}}$, or tightening/contractionary monetary policy shock), the inflation will increase (statistically significant) for several quarters. On the other hand, the macro theory says that the inflation should decrease after a tightening monetary policy. This puzzle is called the “**price puzzle**”, first studied by the Nobel laureate Chris Sims in 1992.

Some background of economic theory: A contractionary monetary policy shock is a surprise hike in short run interest rate, or equivalently **a surprise reduction of the money supply** (Think about the Rederal Reserve, who suddenly announces that they will stop buying more bonds, such as the corporate bonds and the treasury bonds, from the market, which is effectively withdrawing money from circulation or reducing the money supply. You may check the meaning of QE (Quantitative Easing) from the online resource. In the same time, less demand of the bonds from the Fed implies a higher interest rate or yield of the bond, which we also call a lower price of the bonds.). One way or the other, by contractionary/tightening monetary policy, we mean reducing money supply, or increasing the interest rate.

[Likewise, the expansionary/loosening monetary policy means increasing money supply, or reducing the interest rate **in general**. Note that here we use the phrase “in general”, because sometimes, the Fed can increase the money supply without lowering the interest rate. One case in point is the recent financial crisis episode, in which the interest rate has reached its lower bound, or zero. The Fed can still effectively loosen the monetary policy by injecting more cash into the market (the **Quantitative Easing** in the form of buying bonds on a regular basis), or put in different words, increasing money supply, without further lowering the interest rate – they cannot lower it further now that the interest rate is already zero. Negative interest rate is very rare if not impossible because holding cash will at least give you zero interest rate.]

In the IS-LM diagram, this is reflected as a left shift of the LM curve. In the short run, the output will be below the potential output/equilibrium output (i.e., the level of output at full employment, excluding those who voluntarily leave work), the interest rate will be higher than the original level, and the employment will be lower. (You may draw a diagram in a scratch paper, which is a very good way of organizing ideas and thinking about economic issues. Later when you are very familiar with such diagrams, you may even draw the diagram in your mind without resorting to any paper work, and such good habit will pay off in your future career and study.) So **in the short run**, the output is below the potential output. Now you may draw a diagram of AD-AS curve (remember that IS-LM paradigm determines the aggregate demand curve, or the downward sloping AD curve, and the labor market equilibrium determines the aggregate supply curve, or the upward sloping AS curve. The intersection of AD-AS determines the market equilibrium. In econ and finance, we always assume that the economy is at some equilibrium location.). For any given price level, the demand is lower, implying that the AD curve shifts to the left **in the short run**, which implies the equilibrium price level with adjust to a lower level --- this means that in the short run, we expect to see inflation lower than before. Starting from a hypothetical zero inflation equilibrium, this means in the short run, we should see negative inflation (i.e., deflation) following a contractionary monetary policy. The output will also be lower temporarily.

In the **medium run**, what happens is that the firms will hire back more people due to the fact that the firm is able to run under large capacity (the natural output requires more labor). This boosts the production and further lowers the prices. Due to the reduction in prices, the LM curve moves back towards the original location, in the same time, the AS curve moves downward along the AD curve to achieve the natural output and a lower price level. Note that the real supply of money, which is the ratio of money supply and price level, remains exactly the same as the original level. Before reaching the new equilibrium with a lower price and same natural output, the inflation keeps being negative, because the price level continues to decrease. But the inflation can go up or down, due to the different possible speed that the AS curve is adjusting to the new equilibrium along the AD curve. To quantitatively characterize such dynamic movement, such IS-LM and AS-AD analysis is not enough. And this is one of the most crucial reasons we need to consider a dynamic model, and use the VAR to analyze the impulse response function.

Now go back to the previous impulse response function graph, you can see that the implication of contractionary monetary policy to growth agrees with the theory, but as for the inflation, we have the “prize puzzle”.

Possible reasons of the observed “price puzzle”: beyond lagged inflation, there might be **other factors** that can signal future inflation, such as the commodity price. The u_{rt} in the original VAR could be a combination of such signals and monetary policy shock. The “price puzzle” indicates that there might be a problem with the empirical VAR model which produces the results.

For introduction of commodity price, refer to “http://en.wikipedia.org/wiki/Commodity_price_index”.

<u>**Empirical Exercise: Description**</u>

In the following exercises, you are asked to run a VAR(8) for four variables $(inflation, growth, cp, r)$, where *cp* is the commodity price inflation. A natural assumption on top of the previous ones is that $(inflation, growth, cp)$ **do not respond to current** monetary policy shocks. This is a widely used assumption in the literature on “prize puzzle”.

Let the reduced-form VAR be

$$\begin{pmatrix} inflation_t \\ growth_t \\ cp_t \\ r_t \end{pmatrix} = c + \Phi_1 \begin{pmatrix} inflation_{t-1} \\ growth_{t-1} \\ cp_{t-1} \\ r_{t-1} \end{pmatrix} + \cdots + \Phi_8 \begin{pmatrix} inflation_{t-8} \\ growth_{t-8} \\ cp_{t-8} \\ r_{t-8} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{pmatrix}$$

Let the structural shocks be $u_{1t}, u_{2t}, u_{3t}, u_{rt}$, where u_{3t} is the structural shock of commo dity price inflation. The recursive VAR implies that

$$\begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \begin{pmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{rt} \end{pmatrix}$$

Exercise part 1.

(1.1) Estimate the VAR(8) model for $(inflation, growth, cp, r)$. Report the Granger Causality test results for all four variables. No need to report the regression table.

Equation 1: Inflation
F-tests of zero restrictions:

All lags of inflation	F(8, 195) = 9.0595 [0.0000]
All lags of growth	F(8, 195) = 1.4207 [0.1898]
All lags of r	F(8, 195) = 2.9103 [0.0043]
All lags of cp	F(8, 195) = 2.9927 [0.0035]
All vars, lag 8	F(4, 195) = 1.0031 [0.4071]

→ Based on this, inflation is not granger-caused by growth (p > 0,05), but there's evidence that it is granger-caused by r and cp at the 5% level (p < 0.05).

Equation 2: growth
F-tests of zero restrictions:

All lags of inflation	F(8, 195) = 0.82138 [0.5846]
All lags of growth	F(8, 195) = 4.4597 [0.0001]
All lags of r	F(8, 195) = 2.3744 [0.0184]
All lags of cp	F(8, 195) = 1.0089 [0.4306]
All vars, lag 8	F(4, 195) = 0.39948 [0.8089]

→ Based on this, growth is not granger-caused by inflation (p > 0,05) (consistent with above), and it also is not granger-caused by cp (p > 0.05), but there's evidence that it is granger-caused by r at the 5% level (p < 0.05).

Equation 3: r
F-tests of zero restrictions:

All lags of inflation	F(8, 195) = 5.7629 [0.0000]

All lags of growth	F(8, 195) = 5.2868 [0.0000]
All lags of r	F(8, 195) = 143.91 [0.0000]
All lags of cp	F(8, 195) = 4.4796 [0.0001]
All vars, lag 8	F(4, 195) = 0.59099 [0.6696]

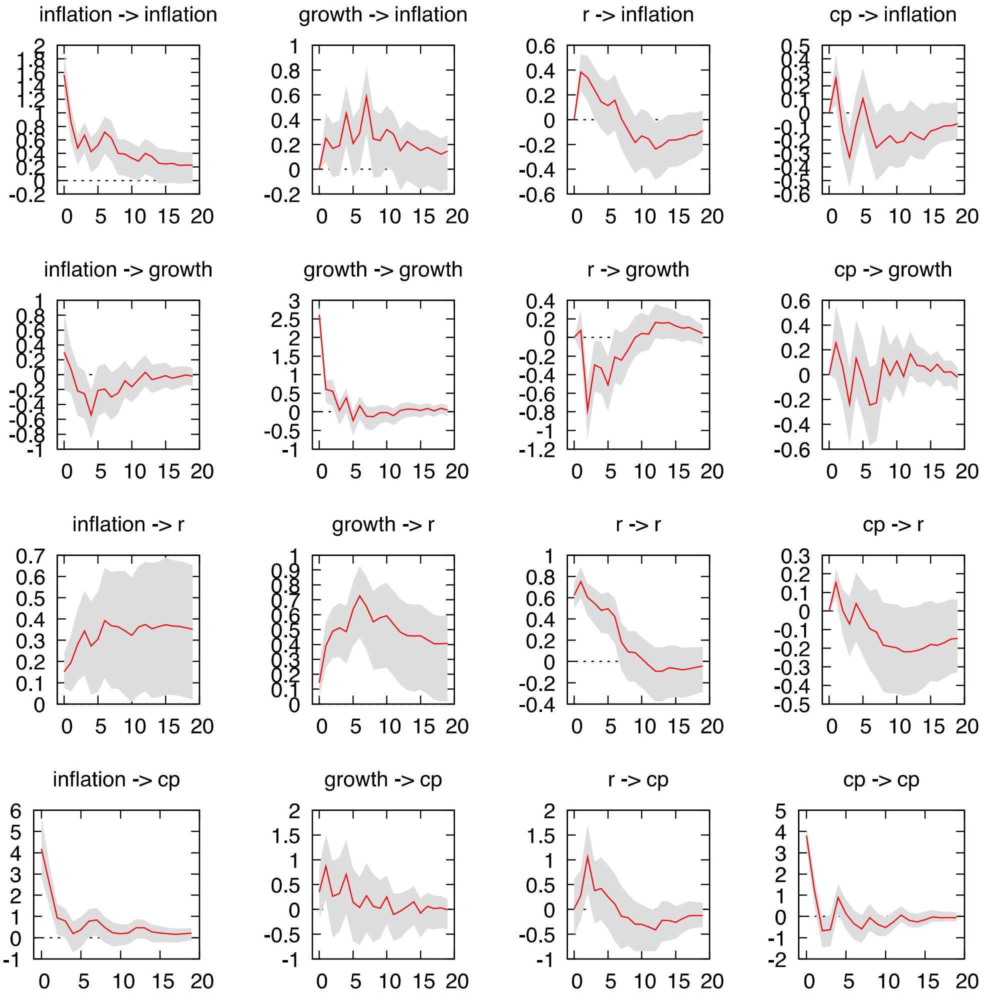
→ Based on this, r is granger-caused by all inflation, cp, and growth at the 5% level ($p < 0.05$)

Equation 4: cp
F-tests of zero restrictions:

All lags of inflation	F(8, 195) = 2.8328 [0.0054]
All lags of growth	F(8, 195) = 0.80687 [0.5972]
All lags of r	F(8, 195) = 1.0102 [0.4296]
All lags of cp	F(8, 195) = 4.2778 [0.0001]
All vars, lag 8	F(4, 195) = 0.19080 [0.9430]

→ Based on this, cp is granger-caused by inflation at the 5% level ($p < 0.05$), but not by growth and r ($p > 0.05$)

(1.2) Copy paste the graph of impulse responses along with the 90% confidence intervals below.

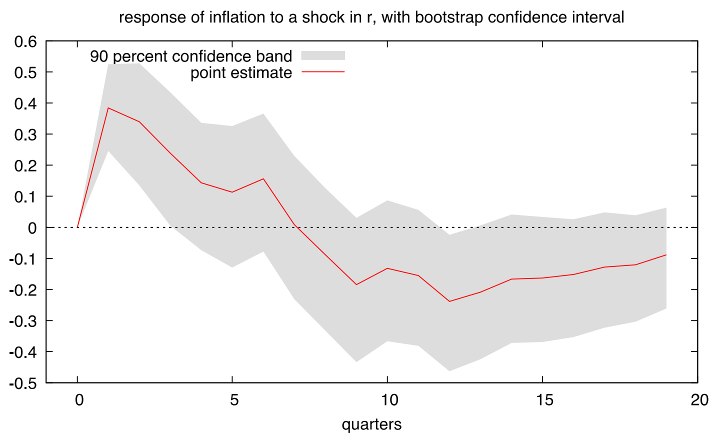


(1.3) Qualitatively describe the change of inflation and output over time, given a one unit positive shock to the interest rate. ($\Delta u_{rt} = 1$.)

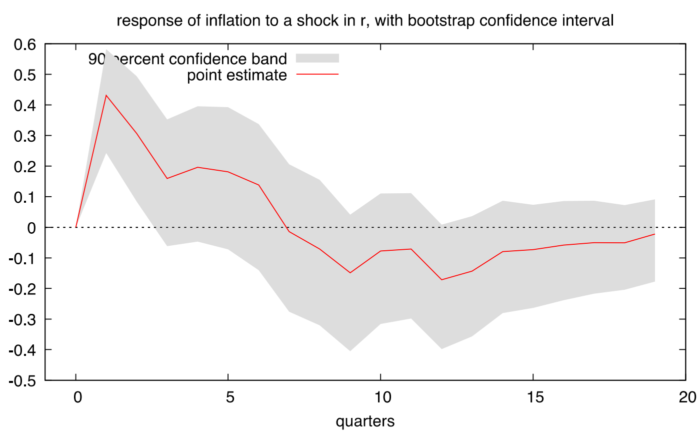
We look at the effects of one-unit change to interest rate by looking at the third column of the IRF. For instance, an increase in r will increase inflation for a few quarters but will decrease it again after a longer period. The effect to growth is the inverse. There is declining growth for the first few quarters, but growth increases afterwards.

(1.4) According to your answer in (1.3), does adding the commodity prices solve or mitigate the “price puzzle”?

VAR(8) with commodity price



VAR(8) without commodity price



It looks like adding commodity prices mitigates the price puzzle in the impulse response function. The behavior does not change too much, but overall, with the addition of commodity price, inflation decreases much faster and does not increase as much in the first few quarters.

(1.5) Briefly discuss possible reasons behind your findings in (1.4). (Use your intuition. There is no standard answer. You just need to make your reasoning logically consistent.)

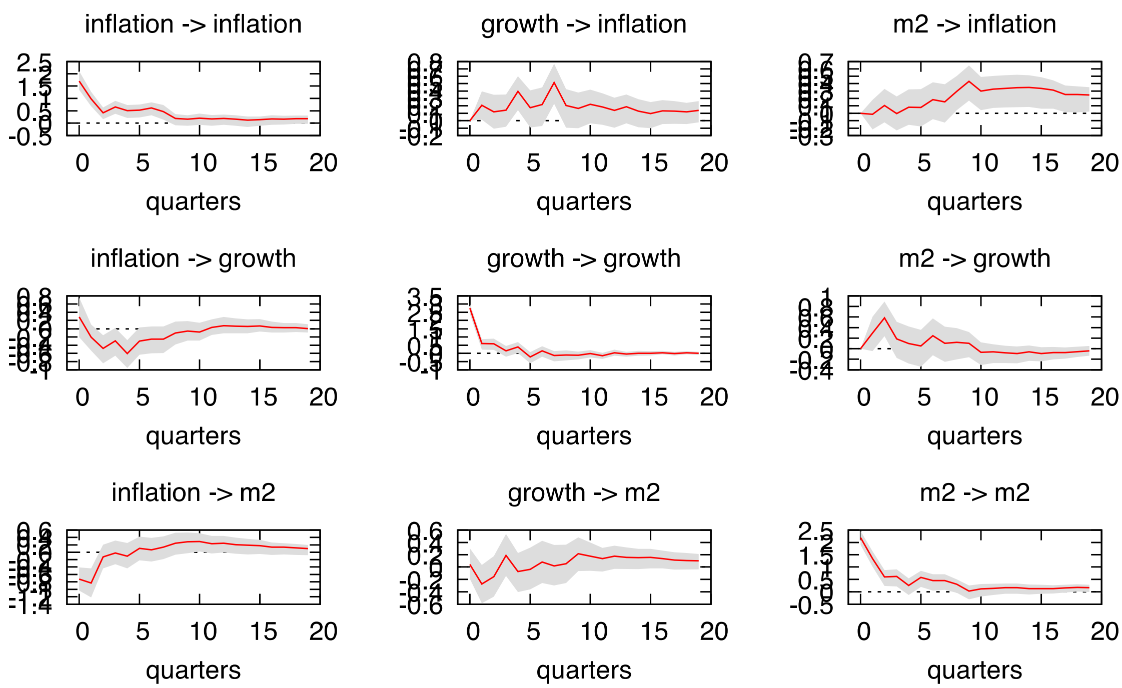
Hint: After the 2007 financial crisis, the Fed has kept the interest rate close to its zero bound for a long time. Can the interest rate still be a monetary policy instrument for such periods?

There could be a few possible reasons. Commodity pricing helps with predicting future inflation, which are “ignored” in the IRF without the prices, and so it did mitigate the price puzzle. Furthermore, as we see in the VAR’s Granger causality test, they do have bi-directional causality, which somewhat explains that the lags of inflation and commodity prices help explain each other. However, even though commodity prices mitigate the price puzzle, the behavior is still somewhat similar to the IRF without the commodity prices. This may be because interest rate cannot just be a monetary policy instrument, and it is also possible for inflation to be mostly influenced by money supply without changing interest rate policy.

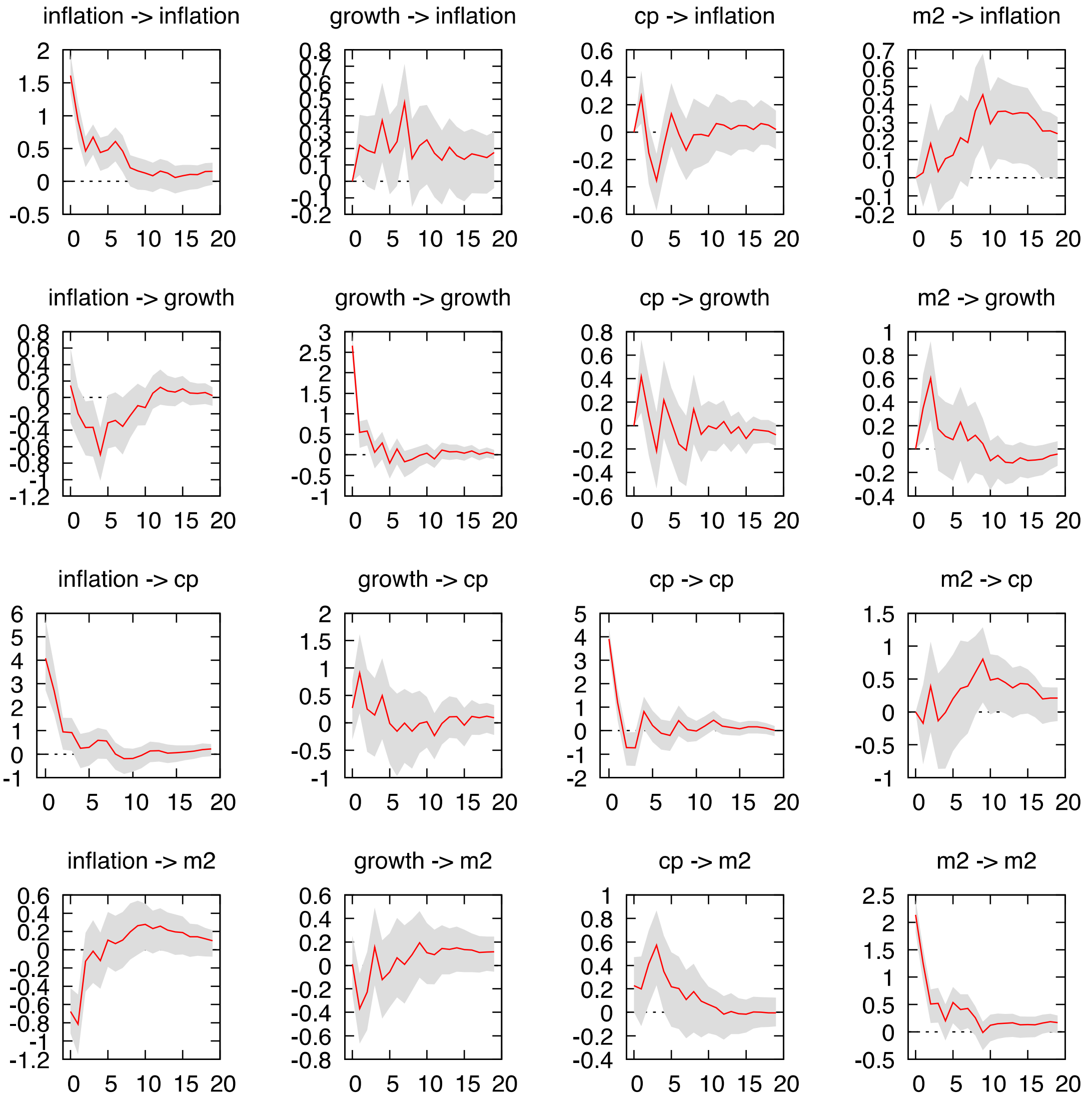
Exercise part 2.

In this exercise, you are asked to run a VAR(8) for four variables $(inflation, growth, cp, m2)$. That is, you replace the interest rate by the growth of M2. According to our theoretical argument in page 3, an expansionary monetary policy is effectively an increase of money supply (M2), and a contractionary monetary policy is a reduction of M2. So a positive shock to m2 (the growth of M2) means a surprise expansionary monetary policy. (Note that in this context, a positive shock for the m2 equation is not the contractionary monetary policy but an expansionary one). The following exercises assume that the monetary policy shock does not contemporaneously affect the other economic variables in the VAR system.

(2.1) Estimate a VAR(8) model for $(inflation, growth, m2)$. Copy paste the graph of impulse responses along with the 90% confidence intervals below.



(2.2) Estimate a VAR(8) model for $(inflation, growth, cp, m2)$. Copy paste the graph of impulse responses along with the 90% confidence intervals below.



(2.3) Briefly describe whether “price puzzle” still exists in the VAR regressions in (2.1) and (2.2). Do the responses of output make economic sense?

The price puzzle doesn’t exist anymore (m2->inflation). The responses make sense for inflation – when money supply increases, inflation will increase in the short run. In the medium run, they might decrease/stagnate to reflect lower prices.

The responses of output also make sense. It is the inverse relation to r->growth from Part (1), which already made sense then. When money supply increases, output will increase above the natural level in the short run, but will go back to the natural level in the medium and long run.

Analytical Exercise

Other issues around the VAR analysis include structural breaks in the regression model as well as in the variances of macroeconomic variables. We do not have the tool to analyze the changing variance so far. However, we know how to analyze the structural breaks in the regression coefficients.

Consider a two-equation recursive VAR(1):

$$\mathrm{y}_{1\mathrm{t}} = a_{11}y_{1,t-1} + a_{12}y_{2,t-1} + \varepsilon_{1t}$$
$$\mathrm{y}_{2\mathrm{t}} = a_{21}y_{1,t-1} + a_{22}y_{2,t-1} + ay_{1t} + \varepsilon_{2\mathrm{t}}$$

Assume that $\mathrm{var}(\varepsilon_{\mathrm{it}}) = 1, \mathrm{i} = 1,2$. Suppose we conducted QLR test for each equation and found a structural break for $\mathrm{y}_{1\mathrm{t}}$'s equation at time T_0, but no structural break for $\mathrm{y}_{2\mathrm{t}}$'s equation.

(3.1) Briefly explain whether there is there any structural break in the impulse response functions $\frac{\partial \mathrm{y}_{\mathrm{i,t+h}}}{\partial \varepsilon_{\mathrm{jt}}}, i, j = 1,2.$

There might be a structural break in the impulse response function. Through structural breaks in y1t, a11 and a12 may have different coefficients between [1,T0] and [T0,T].

Generally speaking, one change in e1t will cause a one-unit increase in y1t.

Suppose now coefficients are different between the two periods. One-unit change of y1t will subsequently cause *a* unit of change in y2t. At t+1, one-unit change of y1t will also lead to a11 units of change to y1,t+1 and a11+a21 units of change to y2,t+1

From here, we can see that a11 might be different from the original model, depending on which model we now choose due to the structural break.

(3.2) Suppose we run the reduced-form VAR regression. Do you expect to find any structural break in either equation?

We should find structural breaks in either equation. The error terms in the reduced-form VAR will be a linear function of all shocks, and so if there is a structural break in the first equation of the recursive VAR (y1t), we should also find one in the second equation (y2t).

(3.3) Define a time dummy $\mathrm{D}_\mathrm{t} = 0, if\ t < T_0; D_t = 1, if\ t \geq T_0$. The equation for $\mathrm{y}_{1\mathrm{t}}$ during $[1, \mathrm{T}_0)$ will differ from the equation during $[\mathrm{T}_0, T]$. How would you revise the original recursive VAR(1) model so that you may estimate the impulse response function using the new model? Hint: the new model will use D_t to define some new regressors to augment the regression functions of the original VAR(1).

$$\mathrm{y}_{1\mathrm{t}} = a_{11}y_{1,t-1} + a_{12}y_{2,t-1} + \varepsilon_{1t}$$
$$\mathrm{y}_{2\mathrm{t}} = a_{21}y_{1,t-1} + a_{22}y_{2,t-1} + ay_{1t} + \varepsilon_{2\mathrm{t}}$$

If there is a structural break in T0, then the revised equation will be:

$$y_{1t} = a'_{11}y_{1,t-1} + a'_{12}y_{2,t-1} + \varepsilon_{1t}, if\ t < T_0$$

$$y_{1t} = a''_{11}y_{1,t-1} + a''_{12}y_{2,t-1} + \varepsilon_{1t}\ , if\ t \geq T_0$$

Now, we define a dummy variable for:

$D_t = 0\ if\ t < T_0$

$D_t = 1\ if\ t \geq T_0$

The one-equation is:

$$y_{1t} = a''_{11}y_{1,t-1} + (a''_{11} - a'_{11})D_t y_{1,t-1} + a''_{12}y_{2,t-1} + (a''_{12} - a'_{12})D_t y_{2,t-1} + \varepsilon_{1t}$$

(3.4) Use your notation for the VAR(1) in (3.3) to provide the impulse response functions $\frac{\partial y_{2,t+h}}{\partial \varepsilon_{1t}}, h = 0,1,2$. Hint: the impulse response functions depend on the regression coefficients as well as the time dummy.

Rewriting the two equations:

$$y_{1t} = a''_{11}y_{1,t-1} + (a''_{11} - a'_{11})D_t y_{1,t-1} + a''_{12}y_{2,t-1} + (a''_{12} - a'_{12})D_t y_{2,t-1} + \varepsilon_{1t}$$

$$y_{2t} = a_{21}y_{1,t-1} + a_{22}y_{2,t-1} + ay_{1t} + \varepsilon_{2t}$$

At h=0:

$\Delta\varepsilon_{1t} = 1 \rightarrow \Delta y_{1t} = 1 \rightarrow \Delta y_{2t} = a$

At h=1:

$\Delta y_{1,t+1} = a''_{11} + (a''_{11} - a'_{11})D_t + a''_{12} * a + (a''_{12} - a'_{12})D_t * a$

$\Delta y_{2,t+1} = a_{21} + a_{22} * a + a * (a''_{11} + (a''_{11} - a'_{11})D_t + a''_{12} * a + (a''_{12} - a'_{12})D_t * a)$

At h=2:
(Some equations not re-written, refer to above)

$$\Delta y_{1,t+2} = a''_{11} * \left(\Delta y_{1,t+1}\right) + (a''_{11} - a'_{11})D_t * \left(\Delta y_{1,t+1}\right) + a''_{12} * \left(\Delta y_{2,t+1}\right) + (a''_{12} - a'_{12})D_t\left(\Delta y_{1,t+1}\right)$$

$$\Delta y_{2,t+2} = a_{21} * \left(\Delta y_{1,t+1}\right) + a_{22}\left(\Delta y_{2,t+1}\right) + a * \Delta y_{1,t+2}$$

Some background of VAR analysis with structural breaks in macroeconomics

Drifts vs volatilities -- A debate between Tom Sargent and Chris Sims

Remark: this part is not an exercise. It is for after-class reading. Hopefully it can spike some ideas regarding the term project.

- Sargent: the VAR coefficients have experienced a structural break.
- Sims: the VAR coefficients remain constant over time. It is the variance of the error term that has experienced structural breaks.

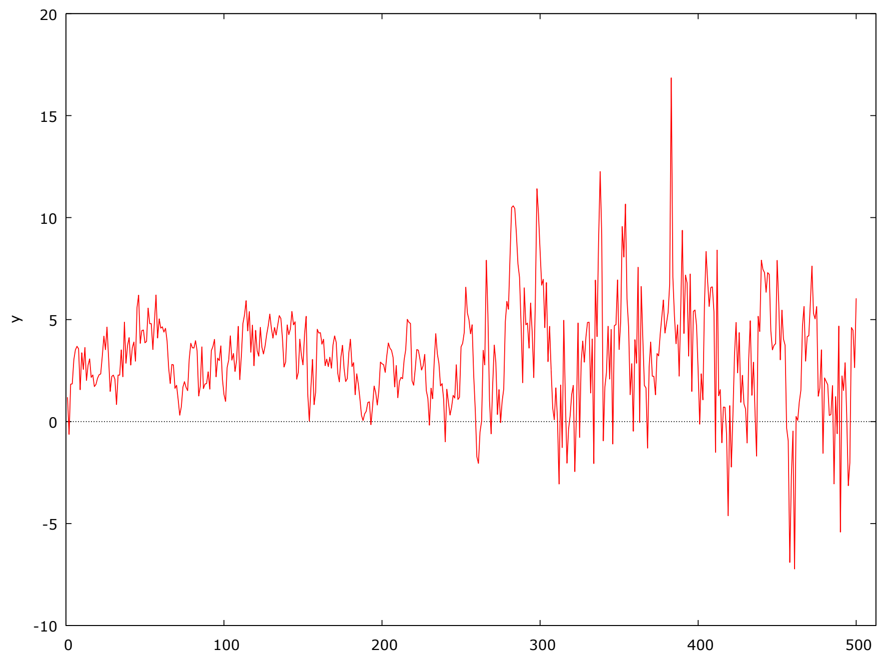
An experiment:

Simulate a time series according to AR(2), with a structural break in the variance of the error term.

The gretl script is below:

```
nulldata 500       # sample size

scalar n = $nobs

setobs 1 1 --time-series   # claim the data set is a time series

set seed 20190306          # set a seed if you want to get same results

# initialize the series

series u = normal()

series y = normal()

series dd = (obs>249)      # simulate a time dummy

series y = 1+ 0.5*y(-1) + 0.2*y(-2) + u + 2*dd*u     # simulate AR(2)
```

Then the variance of the regression error is 1 for t ≤ 249, and is 9 for t ≥ 250. The time series plot looks like this: (remember there is no break in the AR coefficients.)



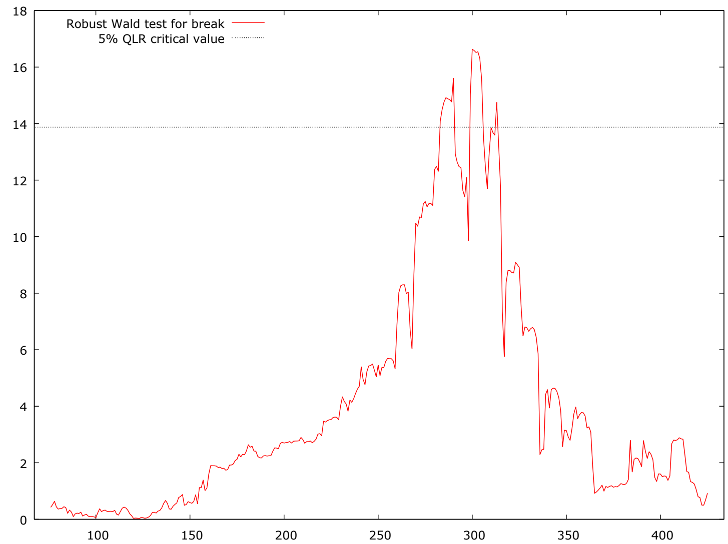
Now run a QLR test for the AR(2) regression model:

Model 1: OLS, using observations 3-500 (T = 498)
Dependent variable: y
HAC standard errors, bandwidth 5 (Bartlett kernel)

	Coefficient	*Std. Error*	*t-ratio*	*p-value*	
const	1.04686	0.163482	6.403	<0.0001	***
y_1	0.426065	0.0596131	7.147	<0.0001	***
y_2	0.245415	0.0576814	4.255	<0.0001	***

Mean dependent var	3.157030	S.D. dependent var	2.604693
Sum squared resid	2153.753	S.E. of regression	2.085909
R-squared	0.361256	Adjusted R-squared	0.358676
F(2, 495)	106.7004	P-value(F)	2.96e-39
Log-likelihood	−1071.259	Akaike criterion	2148.518
Schwarz criterion	2161.149	Hannan-Quinn	2153.475
rho	0.005737	Durbin-Watson	1.984737

QLR test for structural break -
Null hypothesis: no structural break
Test statistic: chi-square(3) = 16.6309 at observation 300
with asymptotic p-value = 0.0160543



We did find evidence of a break in the AR coefficients at 5% level. However, this is a spurious finding.

The takeaway from this simulation is that, if there was a change in the variances in the error terms, the QLR structural break test will possibly provide misleading results. The intuition is that the larger variances will cloud the truth in the finite sample.