② $T(n) = aT(n/b) + f(n)$ $a \geq 1$; $b > 1$;

$f(n)$ is asymptotically ~~increasing~~ non-decreasing function;

$$T(n) = \begin{cases} \Theta(n^{\log_b a}) & \text{if } f(n) \in O(n^{\log_b a - \varepsilon});\ \varepsilon > 0; \\ \Theta(n^{\log_b a} \log n) & \text{if } f(n) \in \Theta(n^{\log_b a}) \\ \Theta(f(n)) & \text{if } f(n) \in \Omega(n^{\log_b a + \varepsilon}) \text{ and } af(n/b) < cf(n) \\ & \varepsilon > 0; \quad c < 1 \end{cases}$$

1) $x_1(n) = 0.5x_1\left(\frac{n}{2}\right) + \frac{1}{n}$ – can not be solved. using MT.

$f(n) = n^{-1}$ – decreasing function

2)

$x_2(n) = 3x_2\left(\frac{n}{4}\right) + n\log n$

$f(n) \in \Omega(n^{\log_4 3 + \varepsilon}) \rightarrow n\log n \in \Omega(n^{\log_4 3 + \varepsilon})$

$x_2(n) \in \Theta(f(n))$

$x_2(n) \in \Theta(n\log(n))$

3) $x_3(n) = 3x_3(n/3) + n/2$

$f(n) \in \Theta(n^{\log_3 3}) \rightarrow \frac{n}{2} \in \Theta(n)$

$x_3(n) \in \Theta(n\log n)$

4) $x_4(n) = 6x_4\left(\frac{n}{3}\right) + n^2\log n$

$f(n) \in \Omega(n^{\log_3 6 + \varepsilon})$ ✓

$\cancel{6}^{2} \frac{n^2}{\cancel{9}_3} \log\frac{n}{3} < \frac{2}{3} n^2 \log n$ ✓

$x_4(n) \in \Theta(n^2\log(n))$

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5) $X_5(n) = 4X_5\left(\frac{n}{2}\right) + \frac{n}{\log n}$;

$f(n) \in O(n^{\log_2 4} + \varepsilon)$

$\frac{n}{\log n} \in O(n^{2+\varepsilon})$

$\exists\ c, n_0$ s.t $\frac{n}{\log n} \leq c n^{2+\varepsilon}\ \ \forall\ n > n_0$

$n_0 = 1$
$c = 2$

$X_5(n) = \Theta(n^2)$

6) $X_6(n) = 2^n X_6\left(\frac{n}{2}\right) + n^n$ – can't be solved using MT

a is not constant
$f(n)$ is not polinomial

a) $T(1)=1$ (3)

$T(n)=T(n-1)+2n-1=T(n-2)+2n-2-1+2n-1=$

$\underline{T(n-2)+2\cdot 2n-2-2}$

$=T(n-3)+2n-4-1+2\cdot 2n-2-2=\underline{T(n-3)+3\cdot 2n-6-3}$

$=T(n-4)+2n-6-1+3\cdot 2n-6-3=\underline{T(n-4)+4\cdot 2n-12-4}$

$=T(1)+(n-1)\cdot 2n-(n-1)^2$

$=1+2n^2-2n-(n^2-2n+1)=\cancel{1}+2n^2-\cancel{2n}-n^2+\cancel{2n}-\cancel{1}=$

$=n^2$

$\underline{T(n)=n^2}$ ✓

b) $T(1)=0$

$T(n)=T(n-1)+1$

$=T(n-2)+2$

$=T(n-3)+3$

$=T(1)+n-1$

$T(n)=0+n-1$

$\underline{T(n)=n-1}$ ✓

c) $T(1)=0$

$T(n)=T(n-1)+2$

$=T(n-2)+2\cdot 2$

$=T(n-3)+3\cdot 2$

$=T(n-4)+4\cdot 2$

$=T(1)+(n-1)\cdot 2$

$=0+2n-2$

$\underline{T(n)=2n-2}$ ✓

5) findRottenWalnut (list[0..N-1])

```
find RottenWalnut (list[0..N-1])
    if N<2
        return failure
    return findRottenWalnutH (list, 0, N-1)
```

```
findRottenWalnutH (list[0..N-1], low, high)   # -1 - failure
    if low == high
        return low
    mid = (high + 1 + low)/2
    compare = compareScales (list[low..mid-1 if (high to low is even)
                                       else mid], list[mid.. high])

    if compare == 0
        return (-1) if (low.. high even) or (not even and list[mid]=list[mid-1])
                    else (mid)
    else if compare == 1
        return findRottenWalnutH (list, low, mid-1 if (low.. high even)
                                                   else mid)
    else
        return findRottenWalnut(list, mid, high)
```

Best: If N is odd and rotten walnut is in the middle or it is not in the array at all

$B(n) = 1 \in \Theta(1)$

Worst: If N is odd and $RW = L[N/2-1]$ or $RW = L[N/2+1]$, or N is even and $RW = L[N/2]$ or $RW = L[N/2-1]$; or $RW = L[0]$ or $L[N-1]$

Suppose $n = 2^{k-1}$ then $k = \log_2(n+1)$ which is the number of steps in the worst case

$W(n) \in \Theta(\log(n))$

⑥ a) $T_1(n) = 3T_1(n-1)$ for $n>1$

$T_1(1) = 4$

$T_1(2) = 12$

$T_1(3) = 36$

$T_1(4) = 108$

$T_1(n) = 4 \cdot 3^{n-1}$

Base case: $n=1$; $T_1(1) = 4 \cdot 3^{1-1} = 4 \cdot 1 = 4$ ✓

Assume: $T_1(n-1) = 4 \cdot 3^{(n-2)}$ is true.

Then $T_1(n) = 3T_1(n-1) = 3 \cdot 4 \cdot 3^{n-2} = \cancel{3} \cdot 4 \cdot \frac{3^{n-1}}{\cancel{3}} = 4 \cdot 3^{n-1}$ ✓

$T_2(n) = \underline{T_2(n-1) + n} \Rightarrow$; for $n>1$

$\underline{T_2(0) = 0}$

$\Rightarrow T_2(n-2) + n - 1 + n = \underline{T_2(n-2) + 2n - 1}$

$= T_2(n-3) + n - 2 + 2n - 1 = \underline{T_2(n-3) + 3n - 3}$

$= T_2(n-4) + n - 3 + 3n - 3 = \underline{T_2(n-4) + 4n - 6}$

$= T_2(n-5) + n - 4 + 4n - 6 = \underline{T_2(n-5) + 5n - 10}$

$= T_2(n-n) + n \cdot n - \frac{(n-1)(n-1+1)}{2}$

$= T_2(0) + n^2 - \frac{n(n-1)}{2}$

$= 0 + n^2 - \frac{n^2 - n}{2} = n^2 - \frac{n^2}{2} + \frac{n}{2} = \frac{n^2}{2} + \frac{n}{2} = \frac{n^2 + n}{2} = \frac{n(n+1)}{2}$ ✓

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$T_3(n) = T(n/2) + n$ for $n > 1$; $n = 2^k$

$T_3(1) = 0$, $k = 0$

$k=1$, $T_3(2) = T(1) + 2 = 2$

$k=2$, $T_3(4) = T(2) + 4 = 6$

$k=3$, $T_3(8) = 6 + 8 = 14$

$k=4$, $T_3(16) = 14 + 16 = 30$

$T_3(n) = 2n - 2$

Base: $T_3(1) = 2 \cdot 1 - 2 = 0$

Assume: $T_3(\frac{n}{2}) = 2\frac{n}{2} - 2 = n - 2$

Then: $T_3(n) = T(n/2) + n = n - 2 + n = 2n - 2$ ✓