Model Question and Answer
Fundamental of Electrical Engineering (FY Btech 2022 course)

Unit 03: AC Circuits

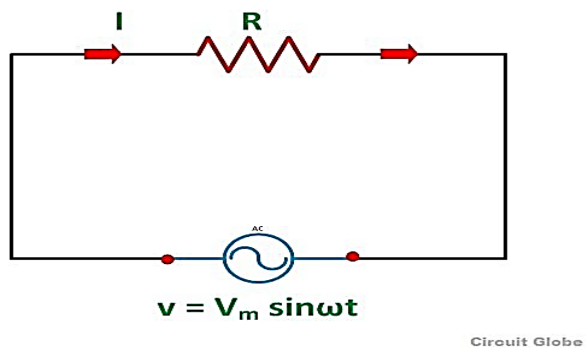
Q1:	A sinusoidal voltage is applied across pure resistive circuit. Derive expression for voltage, current and power of the circuit	(6)

Ans

Let the alternating voltage applied across the circuit be given by the equation

$$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

Then the instantaneous value of current flowing through the resistor shown in the figure below will be:



$$i = \frac{v}{R} = \frac{V_m}{R}\sin\omega t \ldots\ldots\ldots(2)$$

The value of current will be maximum when ωt= 90° or sinωt = 1

Putting the value of sinωt in equation (2) we will get

$$i = I_m \sin\omega t \ldots\ldots\ldots.(3)$$

Therefore, the instantaneous power in a purely resistive circuit is given by the equation shown below:

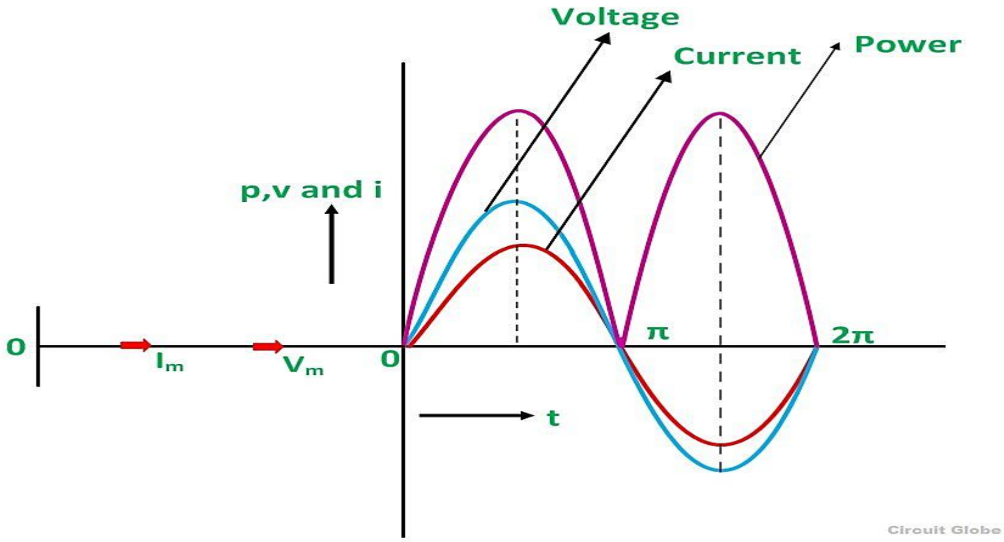
Instantaneous power, p= vi

$$p = (V_m \sin\omega t)(I_m \sin\omega t)$$

$$p = \frac{V_m I_m}{2}\, 2\sin^2\omega t = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}(1 - \cos 2\omega t)$$

$$p = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos 2\omega t$$

	The average power consumed in the circuit over a complete cycle is given by $$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} - \text{ average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\omega t \ldots\ldots..(4)$$ As the value of cosωt is zero. So, putting the value of cosωt in equation (4) the value of power will be given by $$P = V_{r.m.s} I_{r.m.s} - 0$$ Where, P – average power $V_{r.m.s}$ – root mean square value of supply voltage $I_{r.m.s}$ – root mean square value of the current Hence, the power in a purely resistive circuit is given by: $$P = VI$$ The voltage and the current in the purely resistive circuit are in phase with each other having **no phase difference** with phase angle zero. The alternating quantity reaches their peak value at the interval of the same time period that is the rise and fall of the voltage and current occurs at the same time.	
Q2:	A sinusoidal voltage is applied across pure resistive circuit. Draw the waveforms and phasor of the circuit.	(6)
Ans	Let the alternating voltage applied across and current flowing through purely resistive circuit be given by the equation $$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1) \qquad i = \frac{v}{R} = \frac{V_m}{R} \sin\omega t \ldots\ldots\ldots(2)$$ From equation (1) and (3), it is clear that there is no phase difference between the applied voltage and the current flowing through a purely resistive circuit, i.e. phase angle between voltage and current is **zero**. Hence, in an AC circuit containing pure resistance, the current is in phase with the voltage as shown in the waveform figure below.	



Waveform and Phasor Diagram of Pure Resistive Circuit

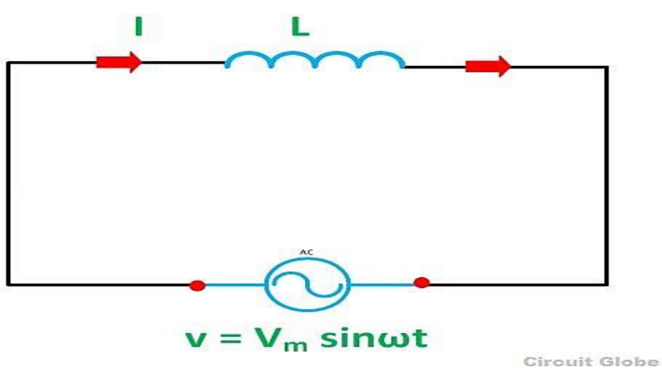
Q3:	A sinusoidal voltage is applied across pure inductive circuit. Derive expression for voltage, current and power of the circuit	(6)

Ans

Let the alternating voltage applied to purely inductive circuit is given by the equation:

$$v = V_m \mathrm{Sin}\omega t \text{ ....(1)}$$

The circuit containing pure inductance is shown below:



Circuit Diagram of pure Inductive Circuit

As a result, an alternating current i flows through the inductance which induces an emf in it. The equation is shown below:

$$e = -L\frac{di}{dt}$$

The emf which is induced in the circuit is equal and opposite to the applied voltage. Hence, the equation becomes,

$$v = -e \;\ldots\ldots\ldots..(2)$$

Putting the value of e in equation (2) we will get the equation as

$$v = -\left(-L\,\frac{di}{dt}\right) \quad \text{or}$$

$$V_m \mathrm{Sin}\omega t = L\,\frac{di}{dt} \quad \text{or}$$

$$di = \frac{V_m}{L}\,\sin\omega t\; dt \;\ldots\ldots\ldots\ldots(3)$$

Integrating both sides of the equation (3), we will get

$$\int di = \int \frac{V_m}{L}\,\sin\omega t\, dt \quad \text{or}$$

$$i = \frac{V_m}{\omega L}\,(-\cos\omega t) \quad \text{or}$$

$$i = \frac{V_m}{\omega L}\,\sin(\omega t - \pi/2) = \frac{V_m}{X_L}\,\sin(\omega t - \pi/2) \;\ldots\ldots\ldots..(4)$$

where,
$X_L = \omega L$ is the opposition offered to the flow of alternating current by a pure inductance and is called inductive reactance.

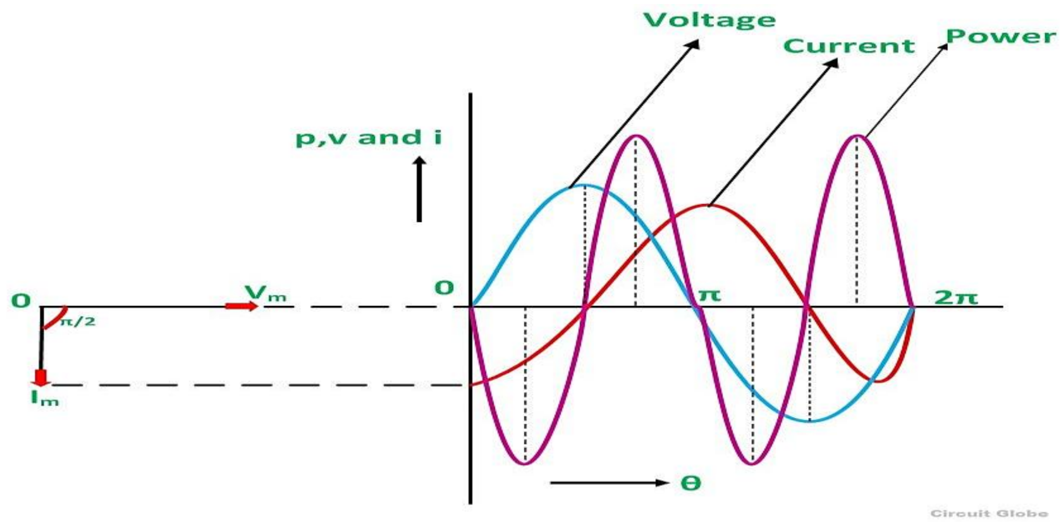
The value of current will be maximum when $\sin(\omega t - \pi/2) = 1$

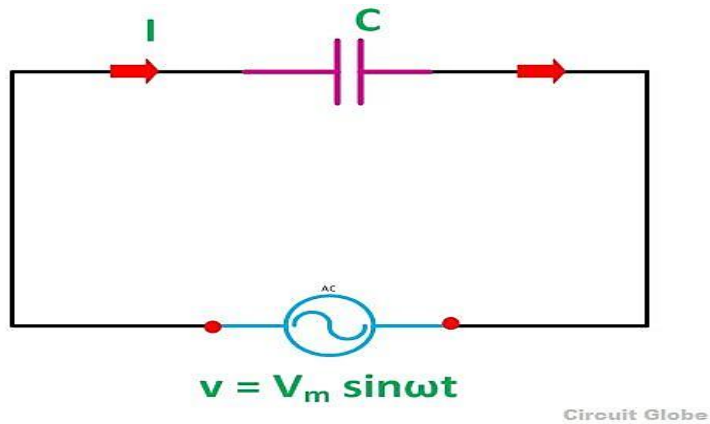
Therefore,

$$I_m = \frac{V_m}{X_L} \;\ldots\ldots\ldots\ldots(5)$$

Substituting this value in I_m from the equation (5) and putting it in equation (4) we will get

	$i = I_m \sin(\omega t - \pi/2)$ **Power in Pure Inductive Circuit** Instantaneous power in the inductive circuit is given by $p = vi$ $P = (V_m \sin\omega t)(I_m \sin(\omega t + \pi/2)$ $P = V_m I_m \sin\omega t \cos\omega t$ $P = \frac{V_m I_m}{2} 2 \sin\omega t \cos\omega t$ $P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\omega t$ or $P = 0$ Hence, the average power consumed in a purely inductive circuit is zero. The average power in one alteration, i.e., in a half cycle is zero, as the negative and positive loop is under power curve is the same. In the purely inductive circuit, during the first quarter cycle, the power supplied by the source, is stored in the magnetic field set up around the coil. In the next quarter cycle, the magnetic field diminishes and the power that was stored in the first quarter cycle is returned to the source.This process continues in every cycle, and thus, no power is consumed in the circuit.	
Q4:	A sinusoidal voltage is applied across pure inductive circuit. Draw waveforms and phasor of circuit	(4)
Ans	The current in the pure inductive AC circuit lags the voltage by 90 degrees. The waveform, power curve and phasor diagram of a purely inductive circuit is shown below	

	 *Phasor Diagram and Waveform of Pure Inductive Circuit* The voltage, current and power waveform are shown in blue, red and pink colours respectively. When the values of voltage and current are at its peak as a positive value, the power is also positive and similarly, when the voltage and current give negative waveform the power will also become negative. This is because of the phase difference between voltage and current. When the voltage drops, the value of the current changes. When the value of current is at its maximum or peak value of the voltage at that instance of time will be zero, and therefore, the voltage and current are out of phase with each other by an angle of 90 degrees.	
Q5:	A sinusoidal voltage is applied across pure capacitive circuit. Derive expression for voltage, current and power of the circuit	(6)
Ans	Let the alternating voltage applied to pure capacitive circuit is given by the equation: $v = V_m \sin\omega t$ ……………(1) Charge of the capacitor at any instant of time is given as: $q = Cv$ … … … … (2)	



Circuit Diagram of pure Capacitor Circuit

Current flowing through the circuit is given by the equation:

$$i = \frac{d}{dt} q$$

Putting the value of q from the equation (2) in equation (3) we will get

$$i = \frac{d}{dt} (Cv) \dots\dots\dots\dots (3)$$

Now, putting the value of v from the equation (1) in the equation (3) we will get

$$i = \frac{d}{dt} C\, V_m \text{Sin}\omega t = C\, V_m \frac{d}{dt} \sin\omega t \quad \text{or}$$

$$i = \omega\, C\, V_m \cos\omega t = \frac{V_m}{1/\omega C} \sin(\omega t + \pi/2) \quad \text{or}$$

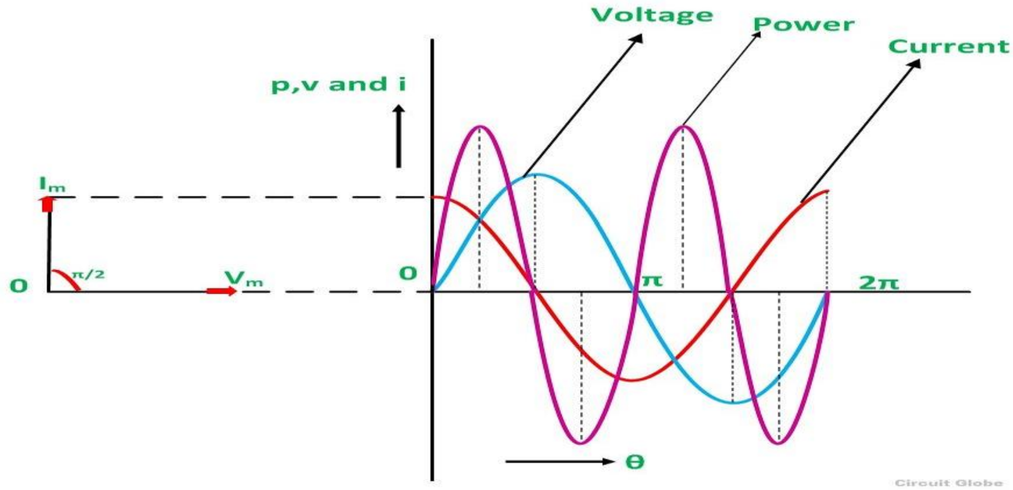
$$i = \frac{V_m}{X_C} \sin(\omega t + \pi/2) \dots\dots\dots\dots (4)$$

Where $X_c = 1/\omega C$ is the opposition offered to the flow of alternating current by a pure capacitor and is called **Capacitive Reactance.**

The value of current will be maximum when $\sin(\omega t + \pi/2) = 1$. Therefore, the value of maximum current I_m will be given as:

$$I_m = \frac{V_m}{X_C}$$

	Substituting the value of I_m in the equation (4) we will get: $$i = I_m \sin(\omega t + \pi/2)$$ **Power in Pure Capacitor Circuit** Instantaneous power is given by p = vi $$P = (V_m \sin\omega t)(I_m \sin(\omega t + \pi/2))$$ $$P = V_m I_m \sin\omega t \cos\omega t$$ $$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\,\omega t \quad \text{or}$$ $$P = 0$$ Hence, from the above equation, it is clear that the average power in the capacitive circuit is zero. The average power in a half cycle is zero as the positive and negative loop area in the waveform shown are same. In the first quarter cycle, the power which is supplied by the source is stored in the electric field set up between the capacitor plates. In the another or next quarter cycle, the electric field diminishes, and thus the power stored in the field is returned to the source. This process is repeated continuously and, therefore, no power is consumed by the capacitor circuit.	
Q6:	A sinusoidal voltage is applied across pure capacitive circuit. Draw waveforms and phasor of circuit.	(6)
Ans	In the pure capacitor circuit, the current flowing through the capacitor leads the voltage by an angle of 90 degrees. The phasor diagram and the waveform of voltage, current and power are shown below:	



Phasor Diagram and Waveform of Pure Capacitor Circuit

The red colour shows current, blue colour is for voltage curve, and the pink colour indicates a power curve in the above waveform.

When the voltage is increased, the capacitor gets charged and reaches or attains its maximum value and, therefore, a positive half cycle is obtained. Further when the voltage level decreases the capacitor gets discharged, and the negative half cycle is formed.

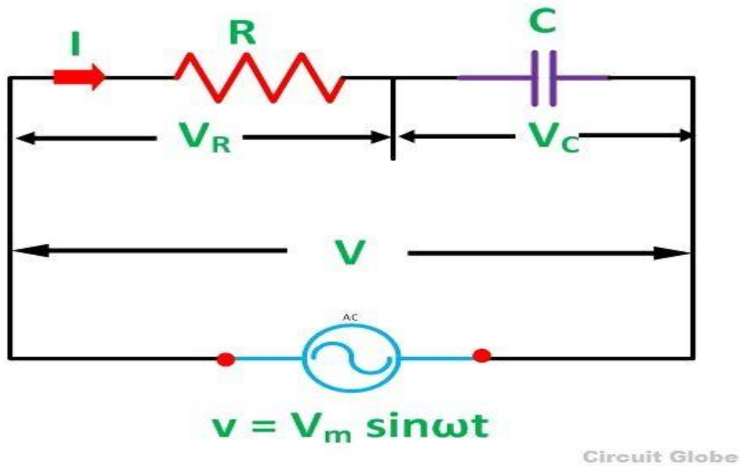
when the voltage attains its maximum value, the value of the current is zero that means there is no flow of current at that time.

When the value of voltage is decreased and reaches a value π, the value of voltage starts getting negative, and the current attains its peak value. As a result, the capacitor starts discharging. This cycle of charging and discharging of the capacitor continues.

The values of voltage and current are not maximised at the same time because of the phase difference as they are out of phase with each other by an angle of 90 degrees.

The phasor diagram is also shown in the waveform indicating that the current (I_m) leads the voltage (V_m) by an angle of π/2.

Q7:	Draw circuit diagram and phasor diagram for The RC Series circuit. Also explain the steps to draw phasor diagram.	(4)
Ans	The RC Series circuit is shown in the figure below:	

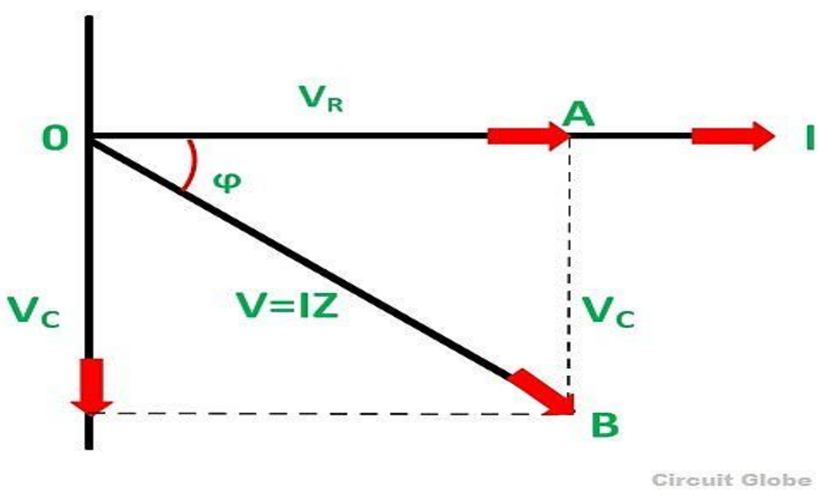


Where,

V_R – voltage across the resistance R
V_C – voltage across capacitor C
V – total voltage across the RC Series circuit

Phasor Diagram of RC Series Circuit

The phasor diagram of the RC series circuit is shown below:



Steps to draw a Phasor Diagram

The following steps are used to draw the phasor diagram of RC Series circuit

Take the current I (r.m.s value) as a reference vector
Voltage drop in resistance VR = IR is taken in phase with the current vector

Voltage drop in capacitive reactance VC = IXC is drawn 90 degrees behind the current vector, as current leads voltage by 90 degrees (in the pure capacitive circuit)
The vector sum of the two voltage drops is equal to the applied voltage V (r.m.s value).

Now, $V_R = I_R$ and $V_C = IX_C$

Where $X_C = I/2\pi fC$

In right triangle OAB,

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + X_C^2}$$

Z is the total opposition offered to the flow of alternating current by an RC series circuit and is called **impedance** of the circuit. It is measured in ohms (Ω).
Phase angle

From the phasor diagram shown above, it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the **phase angle**.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

If the alternating voltage applied across the circuit is given by the equation

$$v = V_m \mathrm{Sin}\omega t \;\ldots\ldots\ldots\ldots\ldots(1)$$

Then,

$$i = I_m \sin(\omega t + \varphi) \ldots \ldots \ldots . (2)$$

Therefore, the instantaneous power is given by p = vi

Putting the value of v and i from the equation (1) and (2) in p = vi

$$P = (V_m \mathrm{Sin}\omega t) x\, I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t + \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi)$$

The average power consumed in the circuit over a complete cycle is given by:

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V\, I \cos\varphi$$

Where cosϕ is called the **power factor** of the circuit.

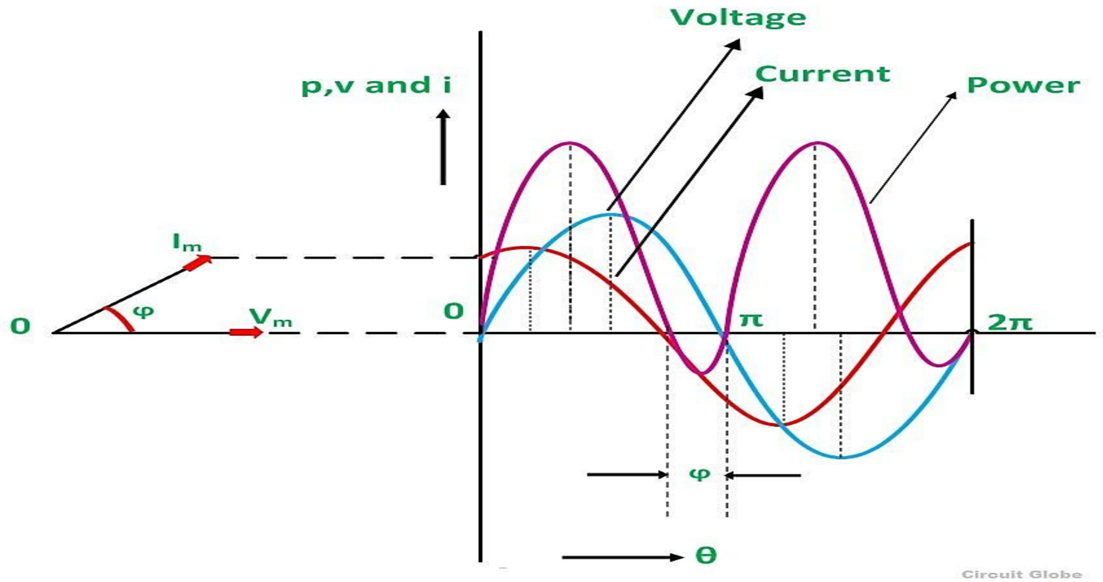
$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \ldots \ldots \ldots \ldots . (3)$$

Putting the value of V and cosϕ from the equation (3) the value of power will be

$$P = (IZ)(I)(R/Z) = I^2 R \ldots \ldots \ldots . (4)$$

From the equation (4) it is clear that the power is actually consumed by the resistance only and the capacitor does not consume any power in the circuit.

Q8:	A sinusoidal voltage is applied across **RC Series Circuit**. Draw waveforms and phasor of	(6)

	circuit.	
Ans	**Waveform and Power Curve of the RC Series Circuit** The waveform and power curve of the RC circuit is shown below:  The various points on the power curve are obtained from the product of the instantaneous value of voltage and current. The power is negative between the angle (180° – ϕ) and 180° and between (360° -ϕ) and 360° and in the rest of the cycle, the power is positive. Since the area under the positive loops is greater than that under the negative loops, therefore the net power over a complete cycle is **positive**.	
Q9:	Explain the steps to draw phasor diagram of RL series circuit and draw it with proper notations.	(6)

Ans

Steps to draw the Phasor Diagram of RL Series Circuit

The following steps are given below which are followed to draw the phasor diagram step by step:

- Current I is taken as a reference.
- The Voltage drop across the resistance $V_R = I_R$ is drawn in phase with the current I.
- The voltage drop across the inductive reactance $V_L = IX_L$ is drawn ahead of the current I. As the current lags voltage by an angle of 90 degrees in the pure Inductive circuit.
- The vector sum of the two voltages drops V_R and V_L is equal to the applied voltage V.

Now,

In right-angle triangle OAB

$V_R = I_R$ and $V_L = IX_L$ where $X_L = 2\pi fL$

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

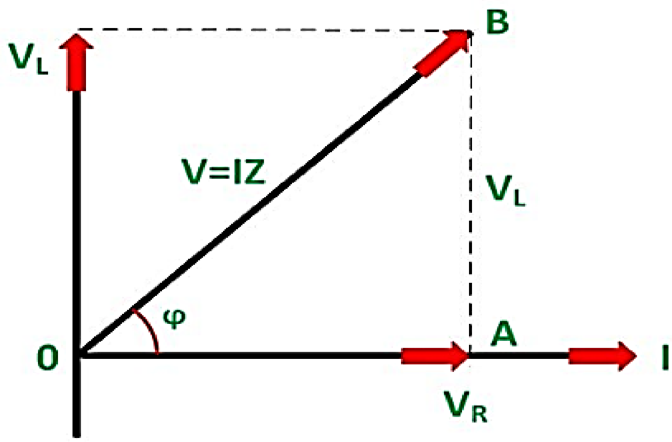
$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$

$$I = L = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + X_L^2}$$

Z is the total opposition offered to the flow of alternating current by an RL Series circuit and is called impedance of the circuit. It is measured in ohms (Ω).

	Phasor Diagram of the RL Series Circuit The phasor diagram of the RL Series circuit is shown below:  	
Q10:	Explain phase angle, power and wave forms for RL series circuit.	(4)
Ans	**Phase Angle** In RL Series circuit the current lags the voltage by 90 degrees angle known as phase angle. It is given by the equation: $$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$ $$\varphi = \tan^{-1}\frac{X_L}{R}$$ **Power in R L Series Circuit** If the alternating voltage applied across the circuit is given by the equation: $$v = V_m \text{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$ The equation of current I is given as: $$i = I_m \sin(\omega t - \varphi) \ldots\ldots\ldots\ldots(2)$$	

Then the instantaneous power is given by the equation:

$$p = v\,i \;\dots\dots\dots(3)$$

Putting the value of v and i from the equation (1) and (2) in the equation (3) we will get

$$P = (V_m \mathrm{Sin}\omega t) x\, I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2}\, 2\sin(\omega t - \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\left[\cos\varphi - \cos(2\omega t - \varphi)\right]$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below:

$$P = \text{average of } \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s}\cos\varphi = V\,I\cos\varphi$$

Where cosϕ is called the power factor of the circuit.

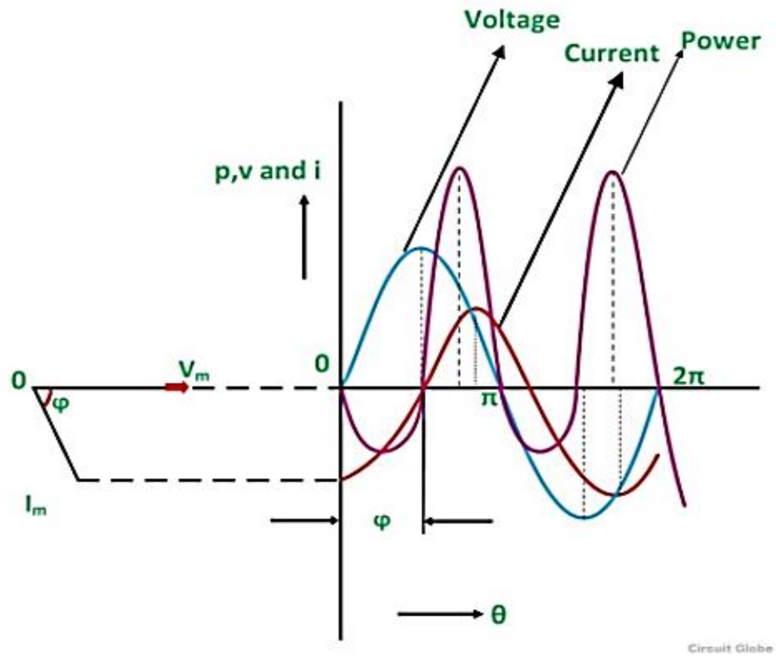
$$\cos\varphi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \dots\dots\dots\dots(4)$$

The power factor is defined as the ratio of resistance to the impedance of an AC Circuit.

Putting the value of V and cosϕ from the equation (4) the value of power will be:

$$P = (IZ)(I)(R/Z) = I^2 R \dots\dots\dots(5)$$

From equation (5) it can be concluded that the inductor does not consume any power in the circuit.

	Waveform and Power Curve of the RL Series Circuit The **waveform** and **power curve** of the RL series circuit is shown below:  The various points on the power curve are obtained by the product of voltage and current. If you analyze the curve carefully, it is seen that the power is negative between angle 0 and ϕ and between 180 degrees and (180 + ϕ) and during the rest of the cycle the power is positive. The current lags the voltage and thus they are not in phase with each other.	
Q11:	Explain the steps to draw phasor diagram of RLC series circuit and draw it with proper notations.	(6)
Ans	**Steps to draw the Phasor Diagram of the RLC Series Circuit** ✓ Take current I as the reference as shown in the figure above ✓ The voltage across the inductor L that is V_L is drawn leads the current I by a 90-degree angle. ✓ The voltage across the capacitor c that is V_c is drawn lagging the current I by a 90-degree angle because in capacitive load the current leads the voltage by an angle of 90 degrees. ✓ The two vector V_L and V_C are opposite to each other.	

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

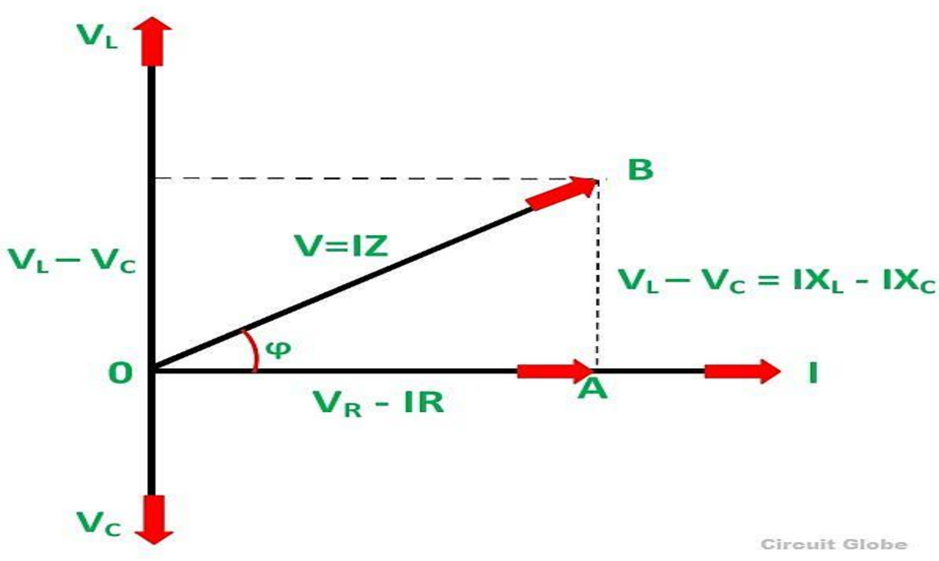
Where,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

It is the total opposition offered to the flow of current by an RLC Circuit and is known as **Impedance** of the circuit.

Phasor Diagram of RLC Series Circuit

The phasor diagram of the RLC series circuit when the circuit is acting as an inductive circuit that means ($V_L > V_C$) is shown below and if ($V_L < V_C$) the circuit will behave as a capacitive circuit.



Q12:	Explain phase angle and power factor for RLC series circuit.	(6)
Ans		

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

Power in RLC Series Circuit

The product of voltage and current is defined as power.

$$P = VI\cos\varphi = I^2R$$

Where cosϕ is the power factor of the circuit and is expressed as:

$$\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$$

The three cases of RLC Series Circuit

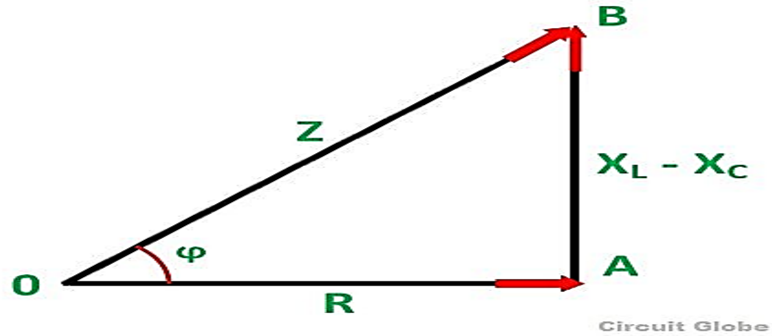
- ✔ When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- ✔ When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- ✔ When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of the power factor is **unity.**

Q13:	Explain impedance triangle in RLC series circuit; when (X_L > X_C) and ((X_L< X_C). Also give applications of RLC series circuit.	(4)

Ans

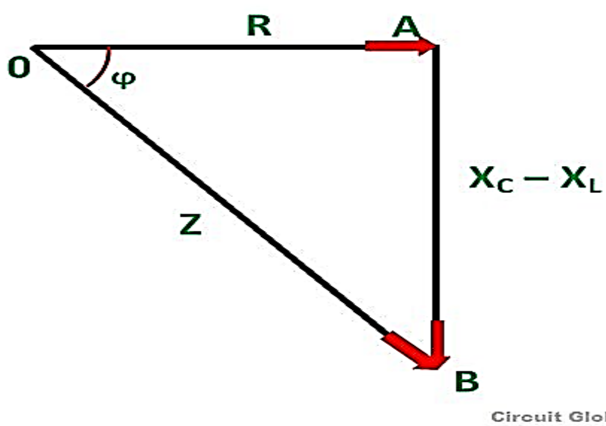
Impedance Triangle of RLC Series Circuit

When the quantities of the phasor diagram are divided by the common factor I then the right angle triangle is obtained known as impedance triangle. The impedance triangle of the RL series circuit, when ($X_L > X_C$) is shown below:



If the inductive reactance is greater than the capacitive reactance than the circuit reactance is inductive giving a **lagging phase angle**.

Impedance triangle is shown below when the circuit acts as an RC series circuit ($X_L < X_C$)



When the capacitive reactance is greater than the inductive reactance the overall circuit reactance acts as capacitive and the phase angle will be leading.

Applications of RLC Series Circuit

The following are the application of the RLC circuit:

- It acts as a variable tuned circuit
- It acts as a low pass, high pass, bandpass, bandstop filters depending upon the type of frequency.
- The circuit also works as an oscillator
- Voltage multiplier and pulse discharge circuit

Q14:	Explain Admittance method to solve parallel AC circuit.	(6)

Ans

Admittance

The reciprocal of the impedance of an AC circuit is known as Admittance of the circuit. Since impedance is the total opposition offered to the flow of alternating current in an AC circuit.

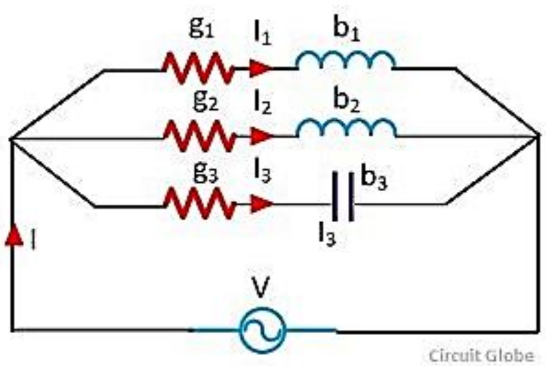
Therefore, Admittance is defined as the effective ability of the circuit due to which it allows the alternating current to flow through it. It is represented by (Y). The old unit of admittance is **mho** (℧). Its new unit is **Siemens.**

The circuit has an impedance of one ohm has an admittance of one Siemens. The old unit was mho.

$$Y = \frac{1}{Z}$$

Application of Admittance Method

Consider the 3-branched circuit shown in the figure below. Total conductance is found by merely adding the conductance of three branches. Similarly, total susceptance is found by algebraically adding the individual susceptance of different branches.



Total conductance $G = g_1 + g_2 + g_3 + \ldots..$

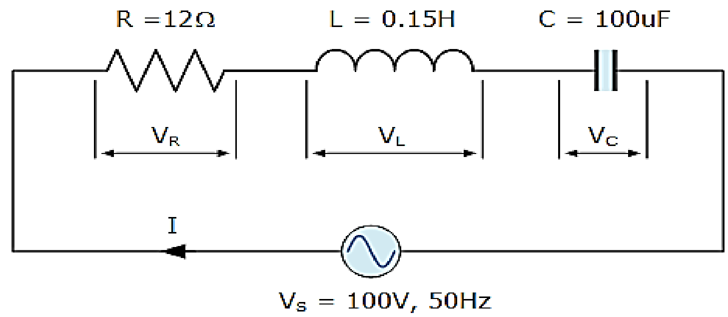
Total susceptance $B = (-b_1) + (-b_2) + b_3 \ldots.$

Total admittance $Y = (G_2 + B_2)$

Total current $I = VY$; Power Factor $\cos\Phi = G / Y$

Q15:	A series RLC circuit containing a resistance of 12Ω, an inductance of 0.15H and a capacitor of 100uF are connected in series across a 100V, 50Hz supply. Calculate the total circuit impedance, the circuits current, power factor and draw the voltage phasor diagram.	(6)

Ans



Inductive Reactance, X_L.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

Capacitive Reactance, X_C.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

Circuit Impedance, Z.

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Circuits Current, I.

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14\,\text{Amps}$$

Voltages across the Series RLC Circuit, V_R, V_L, V_C.

$$V_R = I \times R = 5.14 \times 12 = 61.7 \text{ volts}$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2 \text{ volts}$$

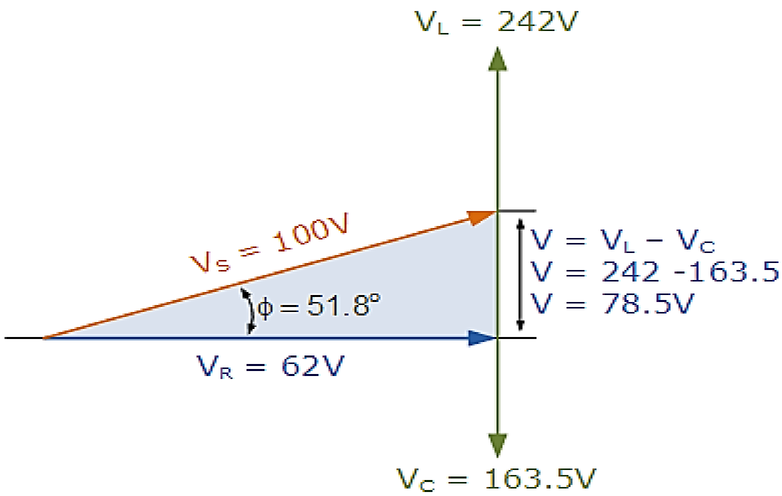
$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5 \text{ volts}$$

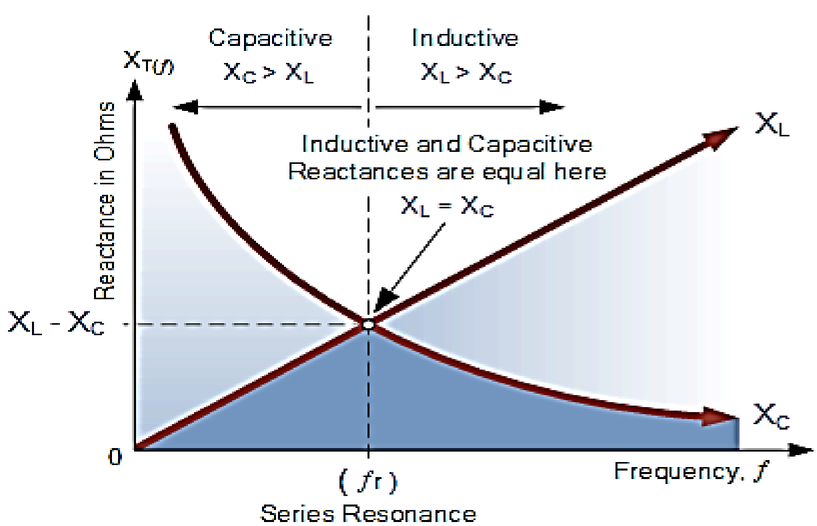
Circuits Power factor and Phase Angle, θ.

$$\cos\phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619$$

$$\therefore \cos^{-1} 0.619 = 51.8^{0} \text{ lagging}$$

Phasor diagram



Q16:	What is resonance frequency? Describe series resonance in RLC series circuit.	(6)
Ans	**Series Resonance Frequency** 	

where: f_r is in Hertz, L is in Henries and C is in Farads.

Electrical resonance occurs in an AC circuit when the two reactances which are opposite and equal cancel each other out as $X_L = X_C$ and the point on the graph at which this happens is were the two reactance curves cross each other. In a series resonant circuit, the resonant frequency, f_r point can be calculated as follows.

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}}\,(\mathrm{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}\,(\mathrm{rads})$$

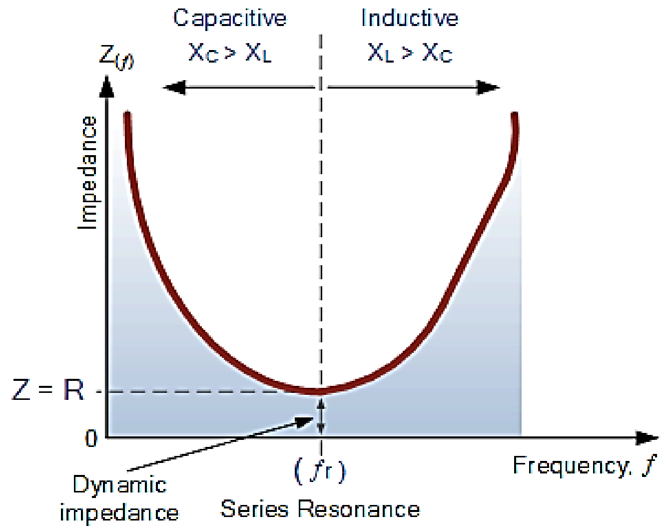
We can see then that at resonance, the two reactances cancel each other out thereby making a series LC combination act as a short circuit with the only opposition to current flow in a series resonance circuit being the resistance, R. In complex form, the resonant frequency is the frequency at which the total impedance of a series RLC circuit becomes purely *"real"*, that is no imaginary impedance's exist. This is because at resonance they are cancelled out. So the total impedance of the series circuit becomes just the value of the resistance and therefore: Z = R.

Then at resonance the impedance of the series circuit is at its minimum value and equal only to the resistance, R of the circuit. The circuit impedance at resonance is called the "dynamic impedance" of the circuit and depending upon the frequency, X_C (typically at high frequencies) or X_L (typically at low frequencies) will dominate either side of resonance as shown below.

Q17:	With proper diagrams explain impedance and current at series resonance condition.	(6)

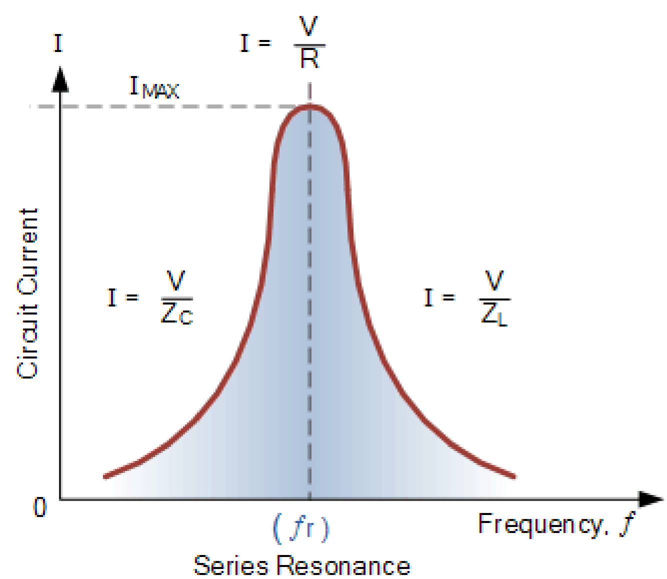
Ans

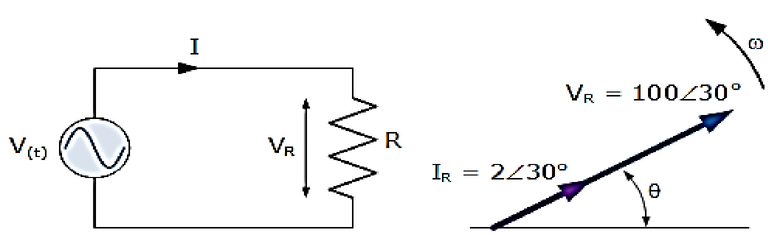
Impedance in a Series Resonance Circuit



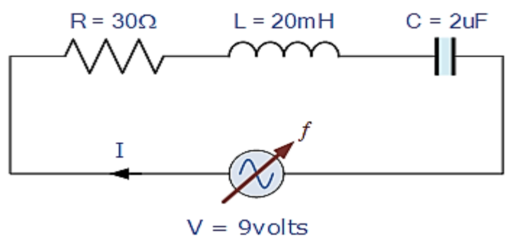
Then in a **series resonance** circuit as $V_L = -V_C$ the resulting reactive voltages are zero and all the supply voltage is dropped across the resistor. Therefore, $V_R = V_{supply}$ and it is for this reason that series resonance circuits are known as voltage resonance circuits, (as opposed to parallel resonance circuits which are current resonance circuits).

Series Circuit Current at Resonance



	The frequency response curve of a series resonance circuit shows that the magnitude of the current is a function of frequency and plotting this onto a graph shows us that the response starts at near to zero, reaches maximum value at the resonance frequency when $I_{MAX} = I_R$ and then drops again to nearly zero as f becomes infinite. The result of this is that the magnitudes of the voltages across the inductor, L and the capacitor, C can become many times larger than the supply voltage, even at resonance but as they are equal and at opposition they cancel each other out. As a series resonance circuit only functions on resonant frequency, this type of circuit is also known as an **Acceptor Circuit** because at resonance, the impedance of the circuit is at its minimum so easily accepts the current whose frequency is equal to its resonant frequency.	
Q18	A sinusoidal voltage supply defined as: $V(t) = 100 \times \cos(\omega t + 30^o)$ is connected to a pure resistance of 50 Ohms. Determine its impedance and the peak value of the current flowing through the circuit. Draw the corresponding phasor diagram.	(6)
Ans	The sinusoidal voltage across the resistance will be the same as for the supply in a purely resistive circuit. Converting this voltage from the time-domain expression into the phasor-domain expression gives us: $V_{R(t)} = 100\cos(\omega t + 30^\circ) \quad \Rightarrow \quad V_R = 100\angle 30^\circ \text{ volts}$ Applying Ohms Law gives us: $I_R = \frac{V_R}{R} = \frac{100\angle 30^o}{50\Omega} = 2\angle 30^o \text{ Amps}$ The corresponding phasor diagram will therefore be:  	

Q.19 A series resonance network consisting of a resistor of 30Ω, a capacitor of 2uF and an inductor of 20mH is connected across a sinusoidal supply voltage which has a constant output of 9 volts at all frequencies. Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies. (6)



1.Resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times2\times10^{-6}}} = 796\text{Hz}$$

2.Circuit current at resonance

$$I = \frac{V}{R} = \frac{9}{30} = 0.3\text{A or } 300\text{mA}$$

3. Inductive Reactance at Resonance, X_L

$$X_L = 2\pi f L = 2\pi\times796\times0.02 = 100\Omega$$

4. Voltages across the inductor and the capacitor, V_L, V_C

$$V_L = V_C$$
$$V_L = I\times X_L = 300\text{mA}\times100\Omega$$
$$V_L = 30\text{volts}$$

Note: the supply voltage may be only 9 volts, but at resonance, the reactive voltages across the capacitor, V_C and the inductor, V_L are 30 volts peak!

5. Quality factor, Q

$$Q = \frac{X_L}{R} = \frac{100}{30} = 3.33$$

6. Bandwidth, BW

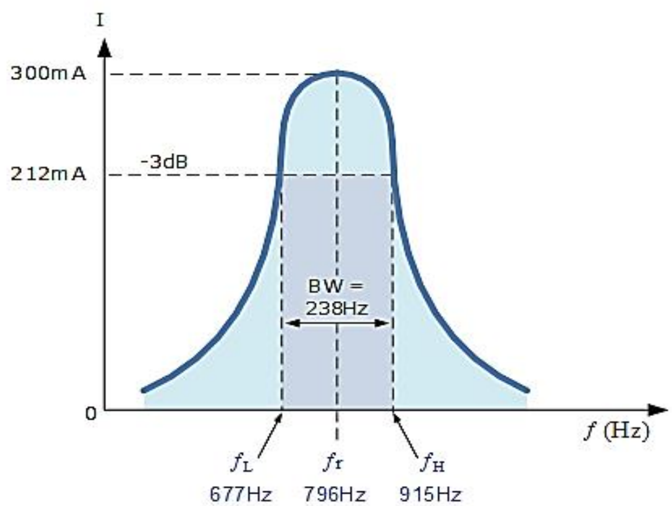
$$BW = \frac{f_r}{Q} = \frac{796}{3.33} = 238\text{Hz}$$

7. The upper and lower -3dB frequency points, f_H and f_L

$$f_L = f_r - \frac{1}{2}BW = 796 - \frac{1}{2}(238) = 677\text{Hz}$$

$$f_H = f_r + \frac{1}{2}BW = 796 + \frac{1}{2}(238) = 915\text{Hz}$$

8. Current Waveform



Q20	A series circuit consists of a resistance of 4Ω, an inductance of 500mH and a variable capacitance connected across a 100V, 50Hz supply. Calculate the capacitance require to produce a series resonance condition, and the voltages generated across both the inductor and the capacitor at the point of resonance.	(6)

Ans	Resonant Frequency, f_r $$X_L = 2\pi f L = 2\pi\times 50\times 0.5 = 157.1\Omega$$ $$\text{at resonance: } X_C = X_L = 157.1\Omega$$ $$\therefore C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi.50.157.1} = 20.3\mu F$$ Voltages across the inductor and the capacitor, V_L, V_C $$I_S = \frac{V}{R} = \frac{100}{4} = 25\text{Amps}$$ $$\text{at Resonance: } V_L = V_C$$ $V_L = I\times X_L = 25\times 157.1$ Thus $V_L = 3{,}927.5\text{volts or } 3.9kV$ and $V_C = 3{,}927.5\text{volts or } 3.9kV$	