Linear Algebra I

Exercise 5

Aviv Vaknin
316017128

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2.1 Calculate A^2, B^2, AB and BA

$$A^2 = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

2.2

$$A^k = \begin{bmatrix} 1 & k & (A^{k-1} + (k-1)) & 0 & 0 \\ 0 & 1 & k & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & k \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B^k = \begin{bmatrix} 1 & 0 & k & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & x & y \\ 0 & 0 & 0 & z & w \end{bmatrix}$$

When x, y, z and w alternate between $-1, 0$ and 1.

4.3 Find P, R and show P as a multiplication of elementary matrices

$$R = \begin{bmatrix} 1 & 0 & 15 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 3 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5 Decide whether the matrix is invertible or not. If it is invertible, find its inverse.

5.1

U_1 is invertible.

$$U_1^{-1} = \begin{bmatrix} \frac{9}{14} & -\frac{1}{14} & -\frac{3}{14} \\ \frac{10}{7} & -\frac{5}{7} & -\frac{1}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

5.2

U_2 is not invertible, because its rref is:

$$rref(U_2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \neq I_3$$

5.3

U_3 is not invertible, because its rref is:

$$rref(U_3) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \neq I_3$$

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