

## 4 Find the *RREF* form for the given matrices:

### 4.1

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 3 & 5 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$

Working on the first column, we'll apply  $A_2 - 2A_1$ , and  $A_3 - 4A_1$ :

$$\begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & -1 & 5 & 6 \\ 0 & 0 & 13 & 16 \end{bmatrix}$$

For the second column, we'll apply  $A_2 \cdot (-1)$ , and then  $A_1 - 2A_2$ :

$$\begin{bmatrix} 1 & 0 & 10 & 11 \\ 0 & 1 & -5 & -6 \\ 0 & 0 & 13 & 16 \end{bmatrix}$$

For the third column, we'll apply  $A_3 \cdot \frac{1}{13}$ ,  $A_1 - 10A_3$  and  $A_2 + 5A_3$ , and therefore:

$$rref(A) = \begin{bmatrix} 1 & 0 & 0 & -\frac{17}{13} \\ 0 & 1 & 0 & \frac{2}{13} \\ 0 & 0 & 1 & \frac{16}{13} \end{bmatrix}$$

### 4.2

$$B = \begin{bmatrix} 0 & 0 & 1 & 3 & -2 \\ 0 & 1 & 2 & 6 & 0 \\ 0 & 2 & 3 & 8 & 2 \\ 0 & 1 & 1 & 3 & 3 \end{bmatrix}$$

We'll start with the second column, and replace  $B_1$  with  $B_2$ .

Then we'll use  $B_1$  to zero out the rest of the column, namely  $B_3 - 2B_1$  and  $B_4 - B_1$ :

$$\begin{bmatrix} 0 & 1 & 2 & 6 & 0 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & -1 & -4 & 2 \\ 0 & 0 & -1 & -3 & 3 \end{bmatrix}$$

Now, for the third column, we'll apply  $B_1 - 2B_2$ ,  $B_3 + B_2$  and  $B_4 + B_2$ .

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For the fourth column, we'll add  $3B_3$  to  $B_2$ , and then multiply  $B_3$  by  $(-1)$ .

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For the last column, we'll apply  $B_1 - 4B_4$  and  $B_2 + 2B_4$ , and therefore:

$$rref(B) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

### 4.3

$$C = \begin{bmatrix} 0 & 3 & 1 \\ 5 & -4 & 2 \\ 2 & 2 & 7 \\ 1 & -1 & 0 \\ 0 & 5 & 3 \end{bmatrix}$$

We'll start with the first column, and replace  $C_4$  with  $C_1$ , and then apply  $C_2 - 5C_1$  and  $C_3 - 2C_1$ :

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 5 & 3 \end{bmatrix}$$

Now, for the second column, we'll apply  $C_1 + C_2$ ,  $C_4 - 3C_2$  and  $C_5 - 5C_2$ :

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & -5 \\ 0 & 0 & -7 \end{bmatrix}$$

For the third and last column, we'll replace  $C_3$  and  $C_4$ , multiply  $C_3$  by  $-\frac{1}{5}$  and then apply  $C_1 - 2C_3$ ,  $C_2 - 2C_3$  and  $C_5 + 7C_3$ , and get:

$$rref(C) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

## 8 Find all of the solutions, $\mathbb{F} = Q$

First, let's convert the equations system into a matrix:

$$A = \left[ \begin{array}{ccc|c} \frac{1}{3} & 2 & -6 & 0 \\ -4 & 0 & 5 & 0 \\ -3 & 6 & -13 & 0 \\ -\frac{7}{3} & 2 & -\frac{8}{3} & 0 \end{array} \right]$$

And then find its rref:

$$\text{rref}(A) = \left[ \begin{array}{ccc|c} 1 & 0 & -\frac{5}{4} & 0 \\ 0 & 1 & -\frac{67}{24} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Therefore, we've received the following system:

$$\begin{cases} X_1 - \frac{5}{4}X_3 = 0 \\ X_2 - \frac{67}{24}x_3 = 0 \end{cases}$$

We'll mark  $x_3$  as  $t_1$ , therefore:

$$\begin{aligned} x_1 &= \frac{5}{4}t_1 \\ x_2 &= \frac{67}{24}t_1 \end{aligned}$$

$$\mathbb{S} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \left\{ \begin{bmatrix} \frac{5}{4}t_1 \\ \frac{67}{24}t_1 \\ t_1 \end{bmatrix} : t_1 \in Q \right\}$$