Discrete Math

Exercise 4

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1 Prove or disprove

1.1

The statement is **true**.
That is because if R is antisymmetric, then by subtracting elements from it, we cannot make it symmetric, but only by adding elements to it.
As an example, if R contains the elements (a, b) and (c, d), it doesn't matter how many elements we'll reduce from R, it'll still be antisymmetric (even if it'll be the empty-set).

1.2

The claim is **false**. An example:

$$R = \left\{ (1,2), (2,1), (3,4) \right\}$$
$$S = \left\{ (3,4) \right\}$$
$$R \backslash S = \left\{ (1,2), (2,1) \right\}$$

We can see that both R and $R \backslash S$ are symmetric.

1.3

The statement is **false**.
As an example, let:

$$S = \left\{ (1,2) \right\}$$
$$R = \left\{ (2,3) \right\}$$
$$S \cup R = \left\{ (1,2), (2,3) \right\}$$

We can see that both S and R are transitive.
However, $S \cup R$ is not transitive as $(1,3) \notin S \cup R$, which contradicts the definition of transitivity.

2

We'll split R into 5 partitions according to their sum: 0, 1, 2, 3, 4 and 5.

$$\begin{aligned}
[0] \;&-\; ((0,0,0,0),(0,0,0,0)) - 1 \text{ element} \\
[1] \;&-\; ((1,0,0,0),(1,0,0,0)) - 16 \text{ elements} \\
[2] \;&-\; ((1,1,0,0),(1,1,0,0)) - 36 \text{ elements} \\
[3] \;&-\; ((1,1,1,0),(1,1,1,0)) - 16 \text{ elements} \\
[4] \;&-\; ((1,1,1,1),(1,1,1,1)) - 1 \text{ element}
\end{aligned}$$

3

We'll split R into 6 partitions according to the number of times each color exists in the square:

$$\begin{aligned}
[\text{2White1Red1Blue}] \;&-\; (\text{White-Blue-White-Red}) - 4 \text{ elements} \\
[\text{2Blue1Red1White}] \;&-\; (\text{Blue-White-Blue-Red}) - 4 \text{ elements} \\
[\text{2Red1White1Blue}] \;&-\; (\text{Red-White-Red-Blue}) - 4 \text{ elements} \\
[\text{2White0Red2Blue}] \;&-\; (\text{White-Blue-White-Blue}) - 2 \text{ elements} \\
[\text{2White2Red0Blue}] \;&-\; (\text{White-Red-White-Red}) - 2 \text{ elements} \\
[\text{0White2Red2Blue}] \;&-\; (\text{Red-Blue-Red-Blue}) - 2 \text{ elements}
\end{aligned}$$

4

Question	Reflexive	Symmetric	Antisymmetric	Transitive
a	Yes	No	Yes	No
b	Yes	Yes	No	No
c	No	No	Yes	Yes
d	No	Yes	No	No

5

Question	Reflexive	Symmetric	Antisymmetric	Transitive
a	Yes	No	No	Yes
b	No	No	Yes	No