3 Calculate AC, BC, ABC and BAC

$$AC = \begin{bmatrix} c_1^1 & c_2^1 & c_3^1 \\ c_1^3 & c_2^3 & c_3^3 \\ c_1^2 & c_2^2 & c_3^2 \end{bmatrix}$$

$$BC = \begin{bmatrix} c_1^3 & c_2^3 & c_3^3 \\ c_1^1 & c_2^1 & c_3^1 \\ c_1^2 & c_2^2 & c_3^2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} c_1^3 & c_2^3 & c_3^3 \\ c_1^2 & c_2^2 & c_3^2 \\ c_1^1 & c_2^1 & c_3^1 \end{bmatrix}$$

$$B(AC) = \begin{bmatrix} c_1^2 & c_2^2 & c_3^2 \\ c_1^1 & c_2^1 & c_3^1 \\ c_1^3 & c_2^3 & c_3^3 \end{bmatrix}$$

6 Prove that

$$A(\lambda B) = B(\lambda A) = \lambda(AB)$$

We'll prove this by showing:

$$[A(\lambda B)]_j^i = [B(A\lambda)]_j^i = [\lambda(AB)]_j^i$$

Therefore, if for all of the matrices, all of the cells are identical, the matrices are the same.

$$[A(\lambda B)]_j^i = \sum_{k=1}^{n} a_k^i (b_j^k \lambda)$$

$$[B(A\lambda)]_j^i = \sum_{k=1}^{n} b_j^k (a_k^i \lambda)$$

$$[\lambda(AB)]_j^i = \lambda(\sum_{k=1}^{n} a_k^i b_j^k)$$

As all of the operations are happening inside $\mathbb{F}$, we can factor the λ, and therefore:

$$[A(\lambda B)]_j^i = \sum_{k=1}^{n} a_k^i (b_j^k \lambda) = \lambda(\sum_{k=1}^{n} a_k^i b_j^k)$$

$$[B(A\lambda)]_j^i = \sum_{k=1}^{n} b_j^k (a_k^i \lambda) = \lambda(\sum_{k=1}^{n} a_k^i b_j^k)$$

Therefore, we've shown that:

$$[A(\lambda B)]_j^i = [B(A\lambda)]_j^i = [\lambda(AB)]_j^i$$

And therefore:

$$A(\lambda B) = B(\lambda A) = \lambda(AB)$$

$\square$

7 Prove that

$$A(\lambda C + D) = O$$

According to question **5**:

$$A(\lambda C + D) = A\lambda C + AD$$

According to question **6**:

$$A\lambda C + AD = \lambda \cdot AC + AD$$

It is given that $AC = AD = O$, therefore:

$$\lambda \cdot AC + AD = \lambda \cdot O + O$$

According to the properties of matrix scalar multiplication:

$$a \cdot O = O$$

Therefore:

$$\lambda \cdot O + O = O + O = O$$

$\square$

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