1.

In order to show that S is a field, we'll need to prove the following ***binary operator*** properties:

1. In S, there are two members, zero - 0_S and one - 1_S
2. S supports the addition and multiplication binary operators
3. Every member in S can be negated, i.e. for every x there is $-x$
4. For every member in S that is not 0_S, $\exists x^{-1} \in S$, it is called the multiplicative inverse of x

In addition, the mentioned binary operators should satisfy the following properties, referred to as ***field axioms***:

1. Associativity of addition(A1) and multiplication(M1):
$$a + (b + c) = (a + b) + c$$
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$
2. Commutativity of addition(A2) and multiplication(M2):
$$a + b = b + a$$
$$a \cdot b = b \cdot a$$
3. Additive identity(A3) and multiplicative identity(M3)
$$a + 0 = a$$
$$a \cdot 1 = a$$
4. Additive inverse(A4) and multiplicative inverse(M4)
$$a + (-a) = 0$$
$$a \cdot a^-1 = 1$$
5. Distributivity(D)
$$a(b + c) = (a \cdot b) + (a \cdot c)$$

2.

Prove: $((a+b)+c)+d = (a+b)+(c+d) = a+(b+(c+d))$

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