Discrete Math

Exercise 3

Aviv Vaknin
316017128

1 Create an injective and surjective function from $\mathbb{N}$ to the natural numbers that can be divided by 5

We'll define:

$$f : N \longrightarrow M$$
$$f(n) = 5n$$

We'll define f's inverse function:

$$f^{-1} : M \longrightarrow N$$
$$f^{-1}(n) = \frac{n}{5}$$

Now, we need to show that f is injective and surjective.
We'll do that by showing that:

$$f \circ f^{-1} = id_M$$
$$f^{-1} \circ f = id_N$$

$$f \circ f^{-1}(n) = f(f^{-1}(n)) = f(\frac{n}{5}) = 5 \cdot \frac{n}{5} = n$$
$$f^{-1} \circ f(n) = f^{-1}(f(n)) = f^{-1}(5n) = \frac{5n}{5} = n$$

Therefore, we've shown that:

$$f \circ f^{-1} = id_M$$
$$f^{-1} \circ f = id_N$$

$\square$

2 Create a bijective function from $\mathbb{N}$ to $\mathbb{N} \times \{1, 2, 3\}$

$$F(n) = \begin{cases} (\frac{n}{3}, 3) & 3|n \\ (\frac{n+1}{3}, 2) & 3|n+1 \\ (\frac{n+2}{3}, 1) & 3|n+2 \end{cases}$$

I am not sure how to prove this is bijective, but we can easily see that it is bijective, as for every different $n \in \mathbb{N}$ a different ordered-pair will return. $\square$

3 Prove that the following sets are not countable

3.1

Let's look at:

$$B = \left\{ a + b\sqrt{2} : \ a, b \in Z \right\}$$

We can see that it includes all of the integers($\mathbb{Z}$) and some multiplicitive variations of $\sqrt{2}$.
Therefore, we can create a bijective function F such that:

$$F : \mathbb{N} \longrightarrow B$$

Hence, B is countable.
Therefore, if we deduct a countable set from a non-countable set, the result will be necessarily a non-countable set.

3.2

$\left\{0, 1\right\}^{\mathbb{N}}$ is not countable, because it is a set of all of infinite sequences, as shown in lecture. In the given set - B, there is no difference, as $f^{-1}(0)$ and $f^{-1}(1)$ are infinite, and that means that there are infinite sequences.

4 Are the following sets countable?

4.1

A is not countable because we can still create an infinite number of sequences of $\left\{0, 1\right\}$.

4.2

A is countable because it can be viewed as a countable union on a countable number of finite sets.
As we've seen in the lecture, this type of union result in a countable set.

5 Build a bijective function from (0,2) to (0,1)∪(2,3)

$$F(n) = \begin{cases} n & 0 < n < 1 \\ n + 1 & 1 < n < 2 \end{cases}$$

6 Build a bijective $f : A \longrightarrow B$ or prove that it cannot exist

This function cannot exist.
It seems like B is "twice" as "big" as A.
For example, we can take $1 \longrightarrow (1, 0)$, and $-1 \longrightarrow (-1, 0)$.
However, that only fills half of the tuples of B.
This way, we'll only be able to build an injective function that will do it, but it will **not** be bijective.