7 Find the solution set for:

$$X + Y - Z = -1$$

$$X - Y - Z = -1$$

First, let's isolate X:

$$X = Z - Y - 1$$

$$X = Z + Y - 1$$

From the above, we can conclude that $Y = 0$. If we change one of the original equations, we'll get:

$$X = Z - 1$$

Therefore, the solution set for X, Y, Z is:

$$S = \left\{ \begin{pmatrix} z-1 \\ y \\ z \end{pmatrix} : y, z \in \mathbb{R} \right\}$$

$\blacksquare$

8 Show that the inverse of the inheritance rule is not true.

The equations system:

$$x^1 + x^2 = 1$$

$$x^2 - x^3 = 3$$

We can generate the equation L, which is a linear form of the mentioned two equations, by multiplying the first equation by 3, and the second one by (-1).

$$L : 3x^1 + 2x^2 + x^3 = 0$$

As we can easily see, the following solution to L is not a solution to the equations system:

$$x^1 = 0$$

$$x^2 = 0$$

$$x^3 = 0$$

$\blacksquare$

10 Are the two equation systems equivalent? if yes, write every equation as a linear form of the other system.

10.1 Showing the systems are equivalent

10.1.1 Right-hand side system

First, let's find the solutions set of the right-hand side system.
We can easily see from the first equation that $X^1 = X^3$, in addition, let's declare $t^1 = X^3$.
Now, from the second equation, we get:

$$X^2 = -3t^1$$

This leads us to the following solutions set for the right-hand side system:

$$S = \left\{ \begin{pmatrix} t^1 \\ -3t^1 \\ t^1 \end{pmatrix} : t^1 \in \mathbb{R} \right\}$$

10.1.2 Left-hand side system

Let's find the linear form of $L_2 - 2L_3$:

$$X^2 + 3x^3 = 0$$

Which leads us to:

$$X^2 = -3x^3$$

In addition, we'll mark X^3 as t^1, i.e.:

$$X^2 = -3t^1$$

Now, we'll find the linear form of $L_2 - 2L_1$:

$$3x^1 - 3t^1 = 0$$

Which leads to:

$$3x^1 = 3t^1$$

$$x^1 = t^1$$

This leads us to the following solutions set for the left-hand side system:

$$S = \left\{ \begin{pmatrix} t^1 \\ -3t^1 \\ t^1 \end{pmatrix} : t^1 \in \mathbb{R} \right\}$$

As we can see, the two systems have identical solution sets, thus, they're equivalent.

10.2 Write as a linear form of the other system

10.2.1 Left-hand side system as a linear form of the other system

$$-X^1 + X^2 + 4X^3 = 0$$

If we multiply the first and second equations by (-1) and (1), respectively, we'll receive the intended equation.

$$X^1 + 3X^2 + 8X^3 = 0$$

If we multiply the first and second equations by (1) and (3), respectively, we'll receive the intended equation.

$$\frac{1}{2}X^1 + X^2 + \frac{5}{2}X^3 = 0$$

If we multiply the first and second equations by $(\frac{1}{2})$ and (1), respectively, we'll receive the intended equation.

10.2.2 Right-hand side system as a linear form of the other system

$$X^1 - X^3 = 0$$

If we multiply the first and second equations by (-1) and (1), respectively, we'll receive the intended equation.