2EL1720 – Distributions and operators

Instructors: Pauline Lafitte
Department: DÉPARTEMENT MATHÉMATIQUES
Language of instruction: FRANCAIS
Campus: CAMPUS DE PARIS - SACLAY
Workload (HEE): 60
On-site hours (HPE): 35,00
Elective Category : Fundamental Sciences
Advanced level : Yes

Description

The goal of this theoretical course is to go back to the sources of the concepts of functional analysis that were introduced in the Analysis and Partial Differential Equations courses in first year.
Historically, the distributions and the operators were introduced to provide a formal mathematical frame for problems arising in Physics. In this way, the concepts of functions were generalized into a theory that allows to treat rigorously fundamental questions of analysis (exchanging limits, exchanging limits and integrals, Fourier transform)...
These concepts provide an answer to the main question : in which functional space do we have to search for the solution of the problem so that it is well-posed, that is, it admits one and only one solution that depends continuously on the data ? In particular, the concept of (general) topology on such spaces plays an essential role in the study of the question of continuity and, more generally, the question of convergence. Depending on the considered cases, they can be defined by a distance, a norm, a family of semi-norms...
In the general frame of the stochastic processes (or random functions), the distributions and the operators are the basic mathematical tools to study Gaussian processes or extensions of the classical Brownian motion. The concepts introduced in this course constitute the basis of the spectral or integral representation of these processes, which allow their fine study (geometric property, Markov property, definition of a stochastic integral, etc.)

Quarter number

SG6

Prerequisites (in terms of CS courses)

CIPPDE

Syllabus

This course of fundamental mathematics is organized around the theoretical study of the following notions:

- Hahn-Banach's theorems
- Unbounded operators
- Weak topologies
- Advanced concepts of distributions

For each of these subjects, the main results are rigorously proved on the blackboard.

Class components (lecture, labs, etc.)
Lectures during which the concepts and results are introduced and proved on the blackboard. These are complemented by tutorials.

Grading
Homeworks, Personal project, Written midterm exam: 1.5 hr, Written final exam: 2 hr ; no documents allowed.

Course support, bibliography
Partial solutions of the exercises.

Resources
Lecturer: Pauline Lafitte
Labs: Alexandre Richard
One lab group

Learning outcomes covered on the course
Mastering the theoretical bases of functional analysis: the students will have all the required qualities of rigorous reasoning that are necessary for modeling and analyzing mathematically.

Description of the skills acquired at the end of the course
C1: Analyze, design and build complex systems with scientific, technological, human and economic components
C2.1: Deepen a field of engineering sciences or a scientific discipline
C2.2: Import knowledge from other fields or disciplines
C2.3: Identify and acquire independently new knowledge and skills
C7.1: Structure ideas and arguments, be synthetic (assumptions, objectives, expected results, approach, and value created)

This course gives an important basis for students who want to attend a Master 2 connected to fundamental mathematics (for instance in Analysis, Partial Differential Equations or Probability).