

## 1 Nominal correlations.

Measurements		CVW/%	IIW/%	MIW/%	RI/%	Error
AXS	103.00 ± 3.87	40.00	40.00	40.00	40.00	3.87
BXS	98.00 ± 3.16	60.00	60.00	60.00	60.00	3.16
Correlations	—	—	0	—	—	—
BLUE <sub>xs</sub>	100.00 ± 2.45	100.00	100.00	100.00	100.00	2.45

Table 1: BLUE of the combination ( $\chi^2/\text{ndof} = 1.00/1$ ). For each input measurement  $i$  the following are listed: the central value weight  $\text{CVW}_i$  or  $\lambda_i$ , the intrinsic information weight  $\text{IIW}_i$ , the marginal information weight  $\text{MIW}_i$ , the relative importance  $\text{RI}_i$ . The intrinsic information weight  $\text{IIW}_{\text{corr}}$  of correlations is also shown on a separate row.

OffDiag & ErrSrc	Error	OffDiag
BXS / AXS	0	0
ErrSrc	0	GlobFact 0

Table 2: Normalised Fisher information derivatives  $1/I \cdot dI/dX$  for the combination under consideration. The derivatives in the table are computed with respect to scale factors  $X$ , representing the ratio of a given correlation to its "current" value in the combination under consideration, and all normalized by the information  $I$  for the "current" values of correlations. They are computed for the "current" values of correlations (in this case: nominal correlations). Color boxes indicate normalised derivatives greater than 0.05 (yellow), 0.10 (orange) and 0.15 (red). The last column and last row list information derivatives when the same rescaling factor is used for a given off-diagonal element or error source, which are equal to the sums of individual derivatives in each row and column, respectively.

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 3: Full input covariance between measurements (summed over error sources).

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 4: Partial input covariance between measurements. Error source #0: Error.

## 2 Modified correlations.

### 2.1 Summary of results.

Combination	BLUE	Error	$\chi^2/\text{ndof}$
Nominal correlations	$100.00 \pm 2.45$	2.45	1.00/1
Minimize by global factor	$100.00 \pm 2.45$	2.45	1.00/1
Minimize by error sources	$100.00 \pm 2.45$	2.45	1.00/1
Minimize by off-diagonal elements	$100.00 \pm 2.45$	2.45	1.00/1
Remove negative CVWs	$100.00 \pm 2.45$	2.45	1.00/1
Onionize	$100.00 \pm 2.45$	2.45	1.00/1
NO correlations	$100.00 \pm 2.45$	2.45	1.00/1

Table 5: Summary table. BLUE's of the combinations performed with nominal and modified correlations.

## 2.2 Minimize correlations by a global rescaling factor.

Measurements		CVW/%	IIW/%	MIW/%	RI/%	Error
AXS	103.00 ± 3.87	40.00	40.00	40.00	40.00	3.87
BXS	98.00 ± 3.16	60.00	60.00	60.00	60.00	3.16
Correlations	—	—	0	—	—	—
BLUE <sub>xs</sub>	100.00 ± 2.45	100.00	100.00	100.00	100.00	2.45

Table 6: BLUE of the combination ( $\chi^2/\text{ndof} = 1.00/1$ ). For each input measurement  $i$  the following are listed: the central value weight  $\text{CVW}_i$  or  $\lambda_i$ , the intrinsic information weight  $\text{IIW}_i$ , the marginal information weight  $\text{MIW}_i$ , the relative importance  $\text{RI}_i$ . The intrinsic information weight  $\text{IIW}_{\text{corr}}$  of correlations is also shown on a separate row.

Parameter name	ParID	Parameter value ScaleFactor X @MIN	1/ $I^{\text{nom}} \cdot dI/dX$			Fixed or Variable
			@0	@MIN	@1	
GlobalScaleFact	#0	1.0000 ± N/A	0	0	0	FIXED

Table 7: Normalised Fisher information derivatives  $1/I^{\text{nom}} \cdot dI/dX$  (before and after minimization) and minimization results. The derivatives in the table are computed with respect to scale factors X, representing the ratio of a given correlation to the corresponding nominal correlation, and all normalized by the information  $I^{\text{nom}}$  at nominal correlations ("@1"). They are computed at three different values of the scale factors X: for nominal values of all correlations (i.e. when all scale factors are 1: "@1"), for correlations all equal to zero (i.e. when all scale factors are 0: "@0") and for the scale factors minimizing Fisher information ("@MIN"). In the minimization, the scale factors X were varied (between 0 and 1, starting at 1) unless  $dI/dX@0 == dI/dX@1 == 0$ . No minimization was attempted in this case as all parameters were kept fixed.

OffDiag & ErrSrc	Error	OffDiag
BXS / AXS	0	0
ErrSrc	0	GlobFact 0

Table 8: Normalised Fisher information derivatives  $1/I \cdot dI/dX$  for the combination under consideration. The derivatives in the table are computed with respect to scale factors X, representing the ratio of a given correlation to its "current" value in the combination under consideration, and all normalized by the information I for the "current" values of correlations. They are computed for the "current" values of correlations (in this case: correlations in minimization by global factor). Color boxes indicate normalised derivatives greater than 0.05 (yellow), 0.10 (orange) and 0.15 (red). The last column and last row list information derivatives when the same rescaling factor is used for a given off-diagonal element or error source, which are equal to the sums of individual derivatives in each row and column, respectively.

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 9: Full input covariance between measurements (summed over error sources). Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 10: Partial input covariance between measurements. Error source #0: Error. Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

## 2.3 Minimize correlations by one factor per error source.

Measurements		CVW/%	IIW/%	MIW/%	RI/%	Error
AXS	103.00 ± 3.87	40.00	40.00	40.00	40.00	3.87
BXS	98.00 ± 3.16	60.00	60.00	60.00	60.00	3.16
Correlations	—	—	0	—	—	—
BLUE <sub>xs</sub>	100.00 ± 2.45	100.00	100.00	100.00	100.00	2.45

Table 11: BLUE of the combination ( $\chi^2/\text{ndof} = 1.00/1$ ). For each input measurement  $i$  the following are listed: the central value weight  $\text{CVW}_i$  or  $\lambda_i$ , the intrinsic information weight  $\text{IIW}_i$ , the marginal information weight  $\text{MIW}_i$ , the relative importance  $\text{RI}_i$ . The intrinsic information weight  $\text{IIW}_{\text{corr}}$  of correlations is also shown on a separate row.

Parameter name	ParID	Parameter value ScaleFactor X @MIN	1/ $I^{\text{nom}} \cdot dI/dX$			Fixed or Variable
			@0	@MIN	@1	
Error	#0	1.0000 ± N/A	0	0	0	FIXED

Table 12: Normalised Fisher information derivatives  $1/I^{\text{nom}} \cdot dI/dX$  (before and after minimization) and minimization results. The derivatives in the table are computed with respect to scale factors X, representing the ratio of a given correlation to the corresponding nominal correlation, and all normalized by the information  $I^{\text{nom}}$  at nominal correlations ("@1"). They are computed at three different values of the scale factors X: for nominal values of all correlations (i.e. when all scale factors are 1: "@1"), for correlations all equal to zero (i.e. when all scale factors are 0: "@0") and for the scale factors minimizing Fisher information ("@MIN"). In the minimization, the scale factors X were varied (between 0 and 1, starting at 1) unless  $dI/dX@0 == dI/dX@1 == 0$ . No minimization was attempted in this case as all parameters were kept fixed.

OffDiag & ErrSrc	Error	OffDiag
BXS / AXS	0	0
ErrSrc	0	GlobFact 0

Table 13: Normalised Fisher information derivatives  $1/I \cdot dI/dX$  for the combination under consideration. The derivatives in the table are computed with respect to scale factors X, representing the ratio of a given correlation to its "current" value in the combination under consideration, and all normalized by the information I for the "current" values of correlations. They are computed for the "current" values of correlations (in this case: correlations in minimization by error source). Color boxes indicate normalised derivatives greater than 0.05 (yellow), 0.10 (orange) and 0.15 (red). The last column and last row list information derivatives when the same rescaling factor is used for a given off-diagonal element or error source, which are equal to the sums of individual derivatives in each row and column, respectively.

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 14: Full input covariance between measurements (summed over error sources). Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 15: Partial input covariance between measurements. Error source #0: Error. Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

## 2.4 Minimize correlations by one factor per off-diagonal element.

Measurements		CVW/%	IIW/%	MIW/%	RI/%	Error
AXS	103.00 ± 3.87	40.00	40.00	40.00	40.00	3.87
BXS	98.00 ± 3.16	60.00	60.00	60.00	60.00	3.16
Correlations	—	—	0	—	—	—
BLUE <sub>xs</sub>	100.00 ± 2.45	100.00	100.00	100.00	100.00	2.45

Table 16: BLUE of the combination ( $\chi^2/\text{ndof} = 1.00/1$ ). For each input measurement  $i$  the following are listed: the central value weight  $\text{CVW}_i$  or  $\lambda_i$ , the intrinsic information weight  $\text{IIW}_i$ , the marginal information weight  $\text{MIW}_i$ , the relative importance  $\text{RI}_i$ . The intrinsic information weight  $\text{IIW}_{\text{corr}}$  of correlations is also shown on a separate row.

Parameter name	ParID	Parameter value ScaleFactor X @MIN	1/ $I^{\text{nom}} \cdot dI/dX$			Fixed or Variable
			@0	@MIN	@1	
BXS/AXS	#0	1.0000 ± N/A	0	0	0	FIXED

Table 17: Normalised Fisher information derivatives  $1/I^{\text{nom}} \cdot dI/dX$  (before and after minimization) and minimization results. The derivatives in the table are computed with respect to scale factors X, representing the ratio of a given correlation to the corresponding nominal correlation, and all normalized by the information  $I^{\text{nom}}$  at nominal correlations ("@1"). They are computed at three different values of the scale factors X: for nominal values of all correlations (i.e. when all scale factors are 1: "@1"), for correlations all equal to zero (i.e. when all scale factors are 0: "@0") and for the scale factors minimizing Fisher information ("@MIN"). In the minimization, the scale factors X were varied (between 0 and 1, starting at onionized covariances) unless  $dI/dX@0 == dI/dX@1 == 0$ . No minimization was attempted in this case as all parameters were kept fixed.

OffDiag & ErrSrc	Error	OffDiag
BXS / AXS	0	0
ErrSrc	0	GlobFact 0

Table 18: Normalised Fisher information derivatives  $1/I \cdot dI/dX$  for the combination under consideration. The derivatives in the table are computed with respect to scale factors X, representing the ratio of a given correlation to its "current" value in the combination under consideration, and all normalized by the information I for the "current" values of correlations. They are computed for the "current" values of correlations (in this case: correlations in minimization by off-diagonal elements). Color boxes indicate normalised derivatives greater than 0.05 (yellow), 0.10 (orange) and 0.15 (red). The last column and last row list information derivatives when the same rescaling factor is used for a given off-diagonal element or error source, which are equal to the sums of individual derivatives in each row and column, respectively.

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 19: Full input covariance between measurements (summed over error sources). Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 20: Partial input covariance between measurements. Error source #0: Error. Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).



## 2.5 Remove measurements with negative central value weights.

Measurements		CVW/%	IIW/%	MIW/%	RI/%	Error
AXS	103.00 ± 3.87	40.00	40.00	40.00	40.00	3.87
BXS	98.00 ± 3.16	60.00	60.00	60.00	60.00	3.16
Correlations	—	—	0	—	—	—
BLUE <sub>xs</sub>	100.00 ± 2.45	100.00	100.00	100.00	100.00	2.45

Table 21: BLUE of the combination ( $\chi^2/\text{ndof} = 1.00/1$ ). For each input measurement  $i$  the following are listed: the central value weight  $\text{CVW}_i$  or  $\lambda_i$ , the intrinsic information weight  $\text{IIW}_i$ , the marginal information weight  $\text{MIW}_i$ , the relative importance  $\text{RI}_i$ . The intrinsic information weight  $\text{IIW}_{\text{corr}}$  of correlations is also shown on a separate row.

N meas in BLUE	Measurement removed in iteration			BLUE	Error	$\chi^2/\text{ndof}$
	Removed	CVW/%	MIW/%			
2	NONE	N/A	N/A	100.00 ± 2.45	2.45	1.00/1

Table 22: From the original combination of 2 with nominal correlations, a new combination where all remaining 2 measurements have central value weights  $\text{CVW} > 0$  was derived by removing measurements iteratively. At each step of the iteration, the measurement with the most negative  $\text{CVW} \leq 0$  in the combination with N measurements was removed until all remaining measurements had  $\text{CVW} > 0$  in the combination of N-1 measurements. No measurements were removed in this case as no measurements with  $\text{CVW} \leq 0$  were found in the original combination of 2 measurements. For each iteration and for the final result, the results of the BLUE and the name, CVW and MIW of the measurement removed in that iteration are displayed.

OffDiag & ErrSrc	Error	OffDiag
BXS / AXS	0	0
ErrSrc	0	GlobFact 0

Table 23: Normalised Fisher information derivatives  $1/I \cdot dI/dX$  for the combination under consideration. The derivatives in the table are computed with respect to scale factors X, representing the ratio of a given correlation to its "current" value in the combination under consideration, and all normalized by the information I for the "current" values of correlations. They are computed for the "current" values of correlations (in this case: correlations in combination with  $\text{CVW} > 0$  measurements). Color boxes indicate normalised derivatives greater than 0.05 (yellow), 0.10 (orange) and 0.15 (red). The last column and last row list information derivatives when the same rescaling factor is used for a given off-diagonal element or error source, which are equal to the sums of individual derivatives in each row and column, respectively.

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 24: Full input covariance between measurements (summed over error sources).

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 25: Partial input covariance between measurements. Error source #0: Error.

## 2.6 Onionize correlations.

Measurements	CVW/%	IIW/%	MIW/%	RI/%	Error
AXS	103.00 $\pm$ 3.87	40.00	40.00	40.00	3.87
BXS	98.00 $\pm$ 3.16	60.00	60.00	60.00	3.16
Correlations	—	—	0	—	—
BLUE <sub>xs</sub>	100.00 $\pm$ 2.45	100.00	100.00	100.00	2.45

Table 26: BLUE of the combination ( $\chi^2/\text{ndof} = 1.00/1$ ). For each input measurement  $i$  the following are listed: the central value weight  $\text{CVW}_i$  or  $\lambda_i$ , the intrinsic information weight  $\text{IIW}_i$ , the marginal information weight  $\text{MIW}_i$ , the relative importance  $\text{RI}_i$ . The intrinsic information weight  $\text{IIW}_{\text{corr}}$  of correlations is also shown on a separate row.

OffDiag & ErrSrc	Error	OffDiag
BXS / AXS	0	0
ErrSrc	0	GlobFact 0

Table 27: Normalised Fisher information derivatives  $1/I \cdot dI/dX$  for the combination under consideration. The derivatives in the table are computed with respect to scale factors  $X$ , representing the ratio of a given correlation to its "current" value in the combination under consideration, and all normalized by the information  $I$  for the "current" values of correlations. They are computed for the "current" values of correlations (in this case: correlations in onionization 1st recipe). Color boxes indicate normalised derivatives greater than 0.05 (yellow), 0.10 (orange) and 0.15 (red). The last column and last row list information derivatives when the same rescaling factor is used for a given off-diagonal element or error source, which are equal to the sums of individual derivatives in each row and column, respectively.

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 28: Full input covariance between measurements (summed over error sources). Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 29: Partial input covariance between measurements. Error source #0: Error. Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

## 2.7 Zero correlations.

Measurements	CVW/%	IIW/%	MIW/%	RI/%	Error
AXS	103.00 $\pm$ 3.87	40.00	40.00	40.00	3.87
BXS	98.00 $\pm$ 3.16	60.00	60.00	60.00	3.16
Correlations	—	—	0	—	—
BLUE <sub>xs</sub>	100.00 $\pm$ 2.45	100.00	100.00	100.00	2.45

Table 30: BLUE of the combination ( $\chi^2/\text{ndof} = 1.00/1$ ). For each input measurement  $i$  the following are listed: the central value weight  $\text{CVW}_i$  or  $\lambda_i$ , the intrinsic information weight  $\text{IIW}_i$ , the marginal information weight  $\text{MIW}_i$ , the relative importance  $\text{RI}_i$ . The intrinsic information weight  $\text{IIW}_{\text{corr}}$  of correlations is also shown on a separate row.

OffDiag & ErrSrc	Error	OffDiag
BXS / AXS	0	0
ErrSrc	0	GlobFact 0

Table 31: Normalised Fisher information derivatives  $1/I \cdot dI/dX$  for the combination under consideration. The derivatives in the table are computed with respect to scale factors  $X$ , representing the ratio of a given correlation to its "current" value in the combination under consideration, and all normalized by the information  $I$  for the "current" values of correlations. They are computed for the "current" values of correlations (in this case: zero correlations). Color boxes indicate normalised derivatives greater than 0.05 (yellow), 0.10 (orange) and 0.15 (red). The last column and last row list information derivatives when the same rescaling factor is used for a given off-diagonal element or error source, which are equal to the sums of individual derivatives in each row and column, respectively.

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 32: Full input covariance between measurements (summed over error sources). Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

$$\left( \begin{array}{c|cc} & \text{AXS} & \text{BXS} \\ \hline \text{AXS} & 15.00 & 0 \\ \text{BXS} & 0 & 10.00 \end{array} \right)$$

Table 33: Partial input covariance between measurements. Error source #0: Error. Color boxes indicate covariances lower than nominal values by a factor up to 2 (green), up to 3 (cyan) or greater than 3 (blue).

## Appendix A1. Input data.

```

1 #=====
2 #---- BlueFin input data file -----
3 #=====
4
5 # The file is expected to have the following format.
6 # Blank lines and lines with only empty spaces are ignored.
7 # Lines starting by '#' are reserved for comments and are ignored.
8 # Data lines are composed of fields separated by one or more empty spaces.
9 # Fields cannot contain empty spaces, with the exception of the title line.
10
11 # The next line must have 2 fields: 'TITLE' and the title of the
12 # BlueFin combination, which must be enclosed within double quotes
13 # and may contain only alphanumeric characters or spaces or hyphens.
14 TITLE "Cross section example XSE"
15
16 # The next line must have 2 fields: 'NOBS' and the number of observables.
17 NOBS 1
18
19 # The next line must have 2 fields: 'NMEA' and the number of measurements.
20 NMEA 2
21
22 # The next line must have 2 fields: 'NERR' and the number of error sources.
23 NERR 1
24
25 # The next NERR+3 lines must have NMEA+1 fields in this format:
26 # - in the 1st line: 'MEANAME' followed by NMEA distinct measurement names
27 #   (measurement names may contain only alphanumeric characters or spaces);
28 # - in the 2nd line: 'OBSNAME' followed by the NMEA names (with NOBS distinct
29 #   values) of the observables measured by the corresponding measurements
30 #   (observable names may contain only alphanumeric characters or spaces
31 #   and should preferably be at most 3 characters long);
32 # - in the 3rd line: 'MEAVAL' followed by the NMEA measured central values;
33 # - in each of the last NERR lines: the error source name followed by the
34 #   NMEA partial errors for each measurement due to the given error source
35 #   (error source names may contain only alphanumeric characters or spaces).
36 MEANAME      AXS      BXS
37 OBSNAME      xs      xs
38 MEAVAL      103.0000  98.0000
39 Error       3.8730   3.1623
40
41 # The next NMEA*(NMEA-1)/2+1 rows must have NERR+2 fields in this format:
42 # - in the 1st line: 'CMEA1' 'CMEA2' (correlations between 2 measurements)
43 #   followed by the NERR error source names in the same order used above;
44 # - in each of the NMEA*(NMEA-1)/2 last lines: the names of two distinct
45 #   measurements followed by the NERR correlations between the partial
46 #   errors on the two measurements due to corresponding error source.
47 # Measurements must appear in the same order listed above.
48 CMEA1 CMEA2 Error
49 AXS   BXS   0.0000

```

Input data file: `xsePaper1.bfin`.