Problem Set 2 Corrections

October 6, 2020

1 1. (Based on Rice 3.18) Let X and Y have the joint density function f(x,y) = kxy,0 y x 2 or 0 elsewhere.

- A) Describe the region over which the density if positive and use it in determining limits of integration to answer the following questions.
- B) Find k.
- C) Find the marginal densities of X and Y.
- D) Find the conditional densities of Y given X and X given Y.

I did this problem pretty much correctly. I messed up my integration on part C with a silly mistake, but I had the right logic for this problem.

2 2. (Based on Rice 3.20) If X1 is uniform on [0,1], and, conditional on X1, X2 is uniform on [X1,2], find the joint and marginal distributions of X1 and X2.

I forgot to incorprate the 1/2 constant in my integral for this problem. Other than that it's good.

3 4. (Based on Rice 3.71) Let X1, ..., Xn be independent RVs all with the same density f. Find an expression for the probability that the interval [X(1),∞) encompasses at least 100v% of the probability mass of density f. Note: remember that the notation X(1) refers to the first order statistic.

I forgot to take the inverse CDF on this problem.

4 5. (Based on Rice 4.31) Let X be uniformly distributed on the interval [1,4]. Find E[1/X]. Is E[1/X] = 1/E[X]? Note: find E[X] using the definition of expectation, don't just plug in the expectation of a U(a,b) RV.

I got everything right on this problem.

5 6. (Based on Rice 4.49) Two independent measurements, X and Y, are taken of a quantity . E[X] = E[Y] = , but x and y are unequal. The two measurements are combined by means of a weighted average to give:

$$Z = X + (1-)Y$$

5.1 where is a scalar and 0 1.

- A Show that E[Z] = .
- B If X and Y are not independent, what is E[Z]?
- C What is Var(Z)? Does this result hold if X and Y are not independent?
- D Find in terms of X and Y to minimize Var(Z).
- E Under what circumstances is it better (i.e., minimizing variance) to use the average (X + Y)/2 than either X or Y alone?

Only missed a half point on this question due to not going into detail on why b is the same as a. I needed to comment on independence.

6 Problem 7. (Based on Rice 4.57) If X and Y are independent random variables, find E[XY] and Var(XY) in terms of the means and variances of X and Y. Hints: 4.1.1 Corollary A, realize that E[X2] = V ar(X) + E[X]2

I got everything right on this problem.