QBS 120 - Problem Set 1

Problems for Rice, Chapter 1:

- Scientists discover the life form in the clouds of Venus that is responsible for unusual levels of phosphine. To their surprise, the genetic material is also encoded using DNA but with five possible bases (A,C,T,G,B) rather than 4 for life on Earth (A,C,T,G). Also, this life form uses 2 base codons rather than 3 base codons. Assume the values of the two independent bases in one codon are measured. Compute answers to the following using R:
  - How many unique codons are possible? How does this compare the number of unique 3 base codons for Earth DNA?
  - What are the sequences of these unique 2 base codons?
  - List the elements of the event E_1 that both bases are the same.
  - List the elements of the event E_2 that both bases are different.
  - List the elements of the event E_3 that the first base is B.
  - List the elements of the event $E_1 \cap E_3$.
  - List the elements of the event $E_1 \cup E_3$.
- Five people want to play a card game so they discard the two cards from a 52 card deck and are each dealt 10 cards. How many unique deals are possible? If there were four playing instead and each was dealt 13 cards, how many unique deals are possible? Before computing, which number do you think will be larger?
- Rice, 1.18 (changed so that probability that at least one defect turns up is 0.8 instead of 0.9): A lot of n items contains k defectives, and m are selected randomly and inspected. How should the value of m be chosen so that the probability that at least one defective item turns up is 0.80? Apply your answer to (a) $n = 1000, k = 10$ and (b) $n = 1000, k = 100$. Notes: solve for m numerically using R; hint: remember that $P(A) = 1 - P(A^C)$; for case b), use n=1000, k=100 to ensure a numerical solution without more complex simplifications/approximations.
- Rice 1.49 (changed for 2 coins and 3 tosses): Two fair coins are simultaneously tossed three times.
  - a. What is the probability of two or more heads given that there was at least one head?
  - b. What is the probability of two or more heads given that there was at least one tail?
- Rice 1.75 (changed so that each individual has either 0, 1 or 2 progeny instead of 0 or 2): A population starts with one member; at a time $t = 1$, it either has 1 progeny with probability p, 2 progeny with probability $2p$, or dies. Also, we know that $3p < 1$. If it successfully reproduces, then its children behave independently with the same alternatives at time $t = 2$. What is the probability that there are no members in the third generation? For what value

of p is this probability equal to 0.5? Note: can solve for p numerically; hint: use law of total probability.

Problems for Rice, Chapter 2:

- What is a random variable? Define both discrete and continuous RVs in terms of the sample space.
- One of the observed values in an experiment is 13.4, which is modeled as a RV during analysis. How can a fixed number be modeled as a RV? Where is the random component?
- Rice 2.9 (solve without using R then confirm through visualization): For what values of p is a 2 out of 3 majority decoder better than transmission of the message once? (here p is probability of success on one transmission)
- Rice 2.40 (changed so that $f(x) = cx(x+2)$ for $0 \leq x \leq 2$ and 0 otherwise; also asking to plot the CDF): Suppose X has the density function $f(x) = cx(x+2)$ for $0 \leq x \leq 2$ and $f(x) = 0$ otherwise.
  - a) Find c
  - b) Find the CDF
  - c) What is $P(0.1 \leq x \leq 0.5)$?
  - d) Plot the CDF.
- Rice 2.52 (numbers adjusted; for part a) you may only use R functions for the standard normal, i.e., $\mathcal{N}(0,1)$): Suppose that in a certain population, individual's heights are approximately normally distributed with parameters $\mu = 60$ inches and $\sigma = 4$ inches.
  - a) What proportion of the population is over 6 ft tall?
  - b) What is the distribution of heights if they are expressed in centimeters?
  - c) In meters?
- Rice 2.67 (For part c), confirm simulation approach using R rweibull() function with $\alpha = \beta = 2$. Hint: use inverse of transform from part b) to go from exponential to Weibull; need to first get exponential from uniform.) The Weibull CDF is:

$$F(x) = 1 - e^{-(x/\alpha)^\beta}, x \geq 0, \alpha > 0, \beta > 0$$

  - a) Find the density function.
  - b) Show that if W follows a Weibull distribution, then $X = (W/\alpha)^\beta$ follows an exponential distribution.
  - c) How could Weibull RVs be generated from a uniform random number generator?

Note that answers to most odd numbered problems are in the back of Rice. For many of the Rice problems, there may exist full solutions on the web (or from prior years of QBS 120). OK to check your solutions against existing answers but you are expected to make a valid attempt before checking one of the solutions. Copying down an existing solution is not OK. The goal is to understand how to solve the problems rather than just having the correct answer on the submitted problem set.