Laser 2: Gaussian Beam Optics

PURPOSE: To understand the fundamentals of Gaussian beam optics as they apply to lasers.

READINGS: For details on the theory behind this experiment refer to the readings in the binder of readings in the laboratory, especially the excerpts from the Melles Griot catalog. Melles Griot, along with Newport Corporation, is a leading manufacturer of top quality optical components. Their catalog is a storehouse of information. You will also find an excellent tutorial at the Newport web site: http://www.newport.com/tutorials/Gaussian_Beam_Optics.html.

EQUIPMENT: Linear, high-speed, charge-coupled photodiode array; power supply; He-Ne laser; variable optical attenuator; HP storage oscilloscope; optical bench; lenses

INTRODUCTION: One usually thinks of a laser beam as a perfectly collimated beam of light rays with the beam energy uniformly spread across the cross section of the beam. This is not an adequate picture for discussing the propagation of a laser beam over any appreciable distance because diffraction causes the light waves to spread transversely as they propagate, Fig. 1.

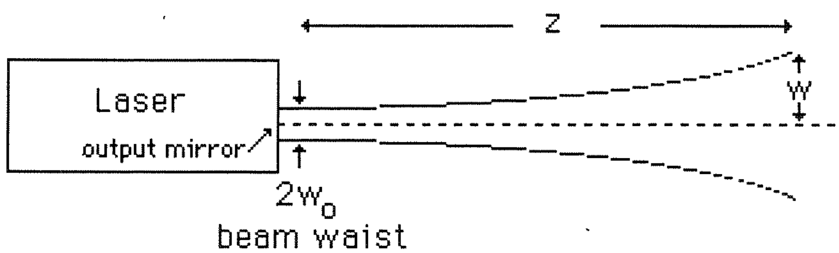

Figure 1. Divergence of a Laser Beam

Additionally, the energy (irradiance) profile of a laser beam is typically not uniform. For the most commonly used He-Ne lasers (operating in the TEM_{00} mode) the irradiance (the power carried by the beam across a unit area perpendicular to the beam = W/m^2) is given by a Gaussian function:

$$I(r) = I_o e^{-2r^2/w^2} = \frac{2P}{\pi w^2} e^{-2r^2/w^2}, \qquad (1)$$

where w is defined as the distance out from the center axis of the beam where the irradiance drops to $1/e^2$ of its value on axis. P is the total power in the beam. r is the transverse distance from the central axis. w depends on the distance z the beam has propagated from the beam waist. w_o is the beam radius at the waist. [The beam waist is defined as the point where the beam wave front was last flat (as opposed to spherical at other locations).] For a hemispherical laser cavity such as you will use in the experiment, the waist is located at the focal point of the spherical mirror (45 cm), which is roughly the location of the output mirror. w_o is related to w by

$$w(z) = w_o\left[1+\left(\frac{\lambda z}{\pi w_o^2}\right)^2\right]^{1/2} = w_o\left[1+\left(\frac{z}{z_R}\right)^2\right]^{1/2} \tag{2}$$

where we have defined a new parameter, called the Rayleigh range,

$$z_R = {}^{\pi w_o^2}\!/_{\lambda}, \tag{3}$$

which combines the wavelength and waist radius into a single parameter and completely describes the divergence of the Gaussian beam. Note that the Rayleigh range is the distance from the beam waist to the point at which the beam radius has increased to $\sqrt{2}w_o$. For a 633 nm red He-Ne laser with a waist of 0.4 mm, $z_R \approx 0.8$ m.

When $z >> z_R$, Eq. (2) simplifies to $w = w_o z/z_R$ and the laser beam diverges at a constant angle

$$\theta = \frac{w}{z} = \frac{w_o}{z_R} = \frac{\lambda}{\pi w_o} \tag{4}$$

Note that the smaller the Rayleigh range, the more rapidly the beam diverges.

In this experiment you will use a CCD detector array to measure how the irradiance varies across the beam for several values of $z >> z_R$. You will set up a spreadsheet and fit your data for each z to Eq. (1), which will yield values for $w(z)$. Then you will use Eqs. (4) and (3) to determine w_o and calculate z_R.

To illustrate how to use Gaussian beam optics we will consider how to make a beam expander, which in its simplest form can simply be an astronomical telescope operated backward. The reason beam expanders are useful is that if you want to project a laser beam over a long distance -- such as for accurately measuring the distance to the moon where you time how long it takes a laser pulse to travel to the moon and back -- you do not want the beam to expand so much you can't detect it when it returns. We see from Eq. (4) that the smaller the waist radius, the more rapidly the beam diverges. Thus to project the beam you must first expand its waist radius to reduce the divergence at greater distances. To understand the beam expander we first review the operation of an astronomical telescope.

Astronomical telescope: In order to understand how a beam expander works, it is useful to review the astronomical telescope. The telescope, as shown in Fig. 2, uses a long focal length (f_o) objective lens to focus (parallel) rays from a distant object on its focal plane. The short focal length (f_e) eye piece lens is placed so that this image lies on its focal plane (i.e. the two lenses

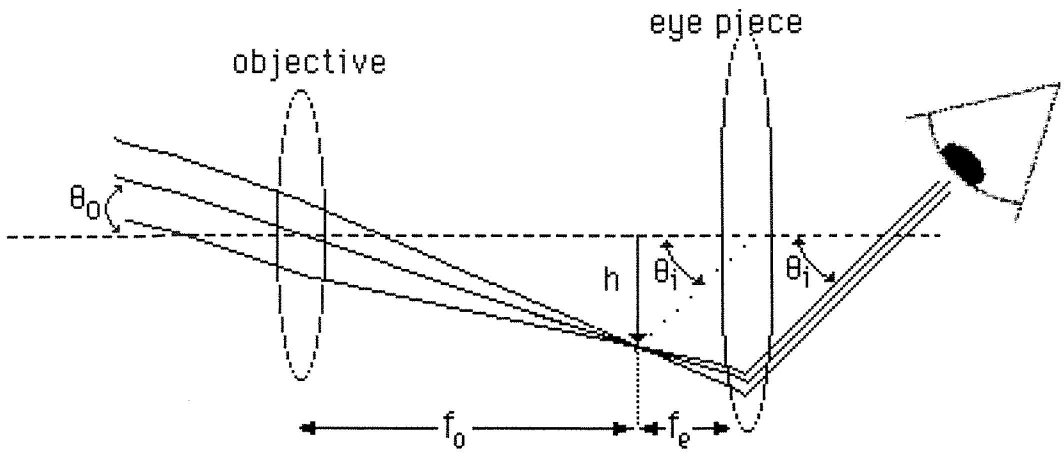

Figure 2. The Astronomical Telescope

are separated by $f_o + f_e$). The rays reaching the eye are again parallel, but appear to subtend a much larger angle than the original object. From Fig. 2 it is easy to see that the angular magnification is

$$m = \frac{\theta_i}{\theta_o} = \frac{h/f_e}{h/f_o} = \frac{f_o}{f_e}.$$

Thus the astronomical telescope converts a narrowly diverging beam into a widely diverging one. By interchanging the object and eye we see that a diverging beam can be made to diverge less strongly.

Beam expander: Because Gaussian beams do not follow the rules of ray optics, we cannot use the lens equation to design a beam expander. However, as discussed in the Melles Griot catalog (pp 18-5,6), if you consider the object to be the beam waist of the incoming beam and the image to be the beam waist after the beam passes through the lens, then you can use a modified lens equation:

$$\frac{1}{s/f + (z_R/f)^2/(s/f - 1)} + \frac{1}{s''/f} = 1, \tag{5}$$

where s is the object (waist) distance from the lens, s" is the image (waist) distance, and f is the focal length of the lens. You will need two additional equations for the magnification and the Rayleigh range of the image beam waist:

$$m = \frac{w_o''}{w_o} = \frac{1}{\sqrt{\left[1 - \frac{s}{f}\right]^2 + \left(\frac{z_R}{f}\right)^2}} \tag{6}$$

and

$$z_R'' = m^2 z_R. \tag{7}$$

If you have several lenses you apply Eqs. (5)-(7) to each lens in succession with the image of the first becoming the objection of the second, etc.

Let's now apply this to an inverted astronomical telescope with the focal length of the first lens being 5 cm and the second 40 cm. In the astronomical telescope the two lenses are separated by the sum of the focal lengths of the two lenses -- 45 cm in this case. We assume we have a red He-Ne laser (633 nm) with beam waist radius of 0.4 mm. We first use Eq. (3) to get $Z_R = 0.80$ m. For the first lens, f = 5 cm, and the beam waist for the laser is close to the exit of the laser. We put the lens as close to the laser as possible and assume s = 0. Then using Eq. (5) we get

$$\frac{s''}{f} = \frac{1}{1 + f^2 / z_R^{\,2}} = 0.996.$$

[Notice the difference here with ordinary ray optics -- where would the image be if we put the object right at the lens?]

Thus it would seem we have simply moved the waist out by 5 cm. But note that the magnification is now

$$m = \frac{w_o''}{w_o} = \frac{1}{\sqrt{1 + \left(\frac{z_R}{f}\right)^2}} = 0.0613$$

or $w_o'' = 0.0245$ mm and $z_R'' = 0.300$ cm. Instead of expanding the beam we have compressed it, but now we will expand it with the second lens. We place the two lenses 45 cm apart so that the waist (s") is at the focal point of the second lens. Then using s = 40 cm, or s/f = 1, in Eq. (5) we see the waist is again at the focal point of the lens (i.e. 40 cm in front of the second lens) and using Eq. (6) we find:

$$m'' = \frac{w_o''''}{w_o''} = \frac{f}{z_R} = 133$$

or $w_o'''' = 3.27$ mm and $z_R'''' = 5330$ cm. We then see from the definition of the Rayleigh range that after traveling through the expander, the beam will travel 53.3 m from the waist before the radius increases by a further $\sqrt{2}$ factor (to 4.62 mm). Or put another way, after traveling 1000 m, the expanded beam would have a beam radius of 61.4 mm while the original beam would have a radius of 500 mm. [We have used the asymptotic relation $z/z_R = w/w_o$.]

PROCEDURE: This experiment has two parts. You will first use a linear CCD (charge coupled device) detector to study the shape of the cross section of the laser beam and how it diverges as the beam propagates. Then you will set up a beam expander and compare the divergence of the expanded beam with that of the original laser beam.

CCD Detector: The detector consists of 2048 p-n junction photodiodes arranged as a linear array with 14 micrometer spacing between adjacent elements. When a photon strikes one of the elements, it releases some charge which is stored in the element. The amount of charge stored during the measuring period on each element (which is proportional to the irradiance) is read out one element after another at a rate of 6 μs per element. Thus the entire array can be read in 12 ms. The read out appears as a series of pulses 6 μs apart whose amplitude is proportional to the irradiance which can be observed with an oscilloscope. The scope display of irradiance versus time can be converted to irradiance versus distance with the conversion factor: 1 ms = 0.23 cm.

The diode array is protected with a neutral density filter to reduce background light and to prevent saturation of the detector. This filter should not be removed!

Gaussian irradiance profile:

A. The experimental arrangement is shown in Fig. 3. The variable attenuator is a circular glass plate on which an absorbing film is deposited whose thickness is a function of angle around the plate. To change the attenuation, the plate is simply rotated until the laser beam goes through a part of the film that gives the desired attenuation. There can be problems with multiple reflections within the glass plate. The attenuation should be positioned at an acute angle with respect the beam as shown in Fig. 1 and the side of the plate having the deposited film should be facing toward the laser. If the laser profile shows a series of interference fringes, the most likely cause is multiple reflections within the attenuator, which should be repositioned to eliminate them.

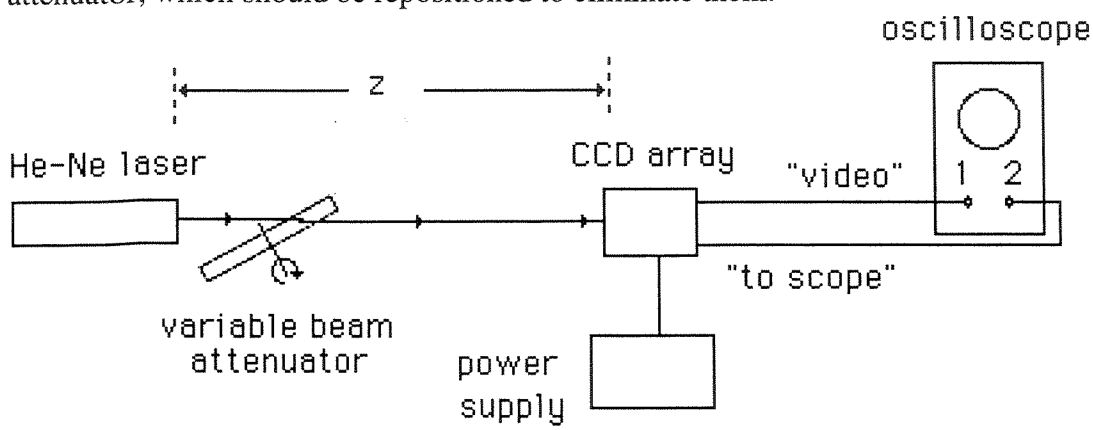

Figure 3. Experimental Setup for Measuring the Irradiance Profile

B. The detector readout is displayed on Channel 1 of the oscilloscope using the BNC output from the array labeled "video". The scope settings should be 2V/cm with a timescale of 1 ms/cm. The trigger signal from the array that stabilizes the display on the scope is labeled "to scope". It is attached to Channel 2 and the trigger source is set to Channel 2 with a trigger level of about 3.5 V. Normally Channel 2 should be switched off unless you want to observe the trigger signal. [Be sure that the switch on top of the diode array is set to "Scope" and not "Storage".]

C. Center the laser spot on the CCD array and adjust the irradiance so that the signal you see on the oscilloscope is as large as possible without saturating the CCD. (Saturation gives a flat topped signal.) The laser should be at least 2 m away from the array. With a time scale of 1 ms/cm you should see a Gaussian shaped signal on the oscilloscope. With a slower time scale of 5 ms/cm you will each multiple readouts separated by about 12 ms. With a faster timescale of 2 μs/cm you can observe the readout from the individual p-n diodes separated by about 6 μs.

D. With the time scale adjusted to give the full profile on the oscilloscope screen, press the stop button on the scope to freeze the profile. Using the measure function on the Hewlett Packard scope, measure the signal voltage V_1 and the time t_1 for a number of points along the Gaussian signal. Then block the laser beam and measure the background signal from the array due to background light. **Before changing the setup** measure the distance z from the laser to the detector.

E. Without making any changes to the laser (which might change the waist radius) repeat step D for several greatly different, larger values of z. You can change the attenuator to get a full scale signal, in which case when you fit your data to Eq. (1) you will treat I_0 as an adjustable parameter (as well as w).

F. The oscilloscope reading gives time as the horizontal axis. You need to convert this time into distance across the beam by using the conversion factor that each 6 μs of sweep reads out one 14 μm diode -- 2.33 μm/μs.

G. Set up a spreadsheet and fit your data to Eq. (1) to obtain w for each value of z for which you made measurements. Then use Eq. (4) to obtain w_0 and z_R for the laser, assuming you are in the asymptotic limit, $z >> z_R$.

Beam expander:

A. Measure the focal lengths of the two lenses provided (approximately 5 cm and 40 cm). Mount the lenses on the optical bench so that their separation is exactly equal to the sum of the focal lengths. Look through the eyepiece (the 5 cm lens) to verify that they are acting as an astronomical telescope. Adjust the separation if necessary to bring a distant object sharply in focus.

B. Use the value of w_o you measured for the laser and the focal lengths of the two lenses to calculate w_o'''' and z_R'''' for the system with the beam expander in place.

C. Measure w_o'''' (located one focal length in front of the second lens). Project the laser onto the far wall of the room and measure the beam radius for this value of z. Compare your measurement with the predicted value.

D. Adjust the location of the second lens to minimize the beam radius on the far wall. Measure the separation of the lenses and use Eq. (5) to verify your result.