Taller mecánica cuántica.

Temática: Teoría general del momento angular

Para entregar al finalizar la clase del martes 12 de noviembre.

Nombre_________________________

Código______________________

1) Demuestre que:

$$\begin{aligned} [J_z, J_+] &= \hbar J_+ \\ [J_z, J_-] &= -\hbar J_- \\ [J_+, J_-] &= 2\hbar J_z \\ [\mathbf{J}^2, J_+] &= [\mathbf{J}^2, J_-] = [\mathbf{J}^2, J_z] = 0 \end{aligned}$$

2) Demuestre que:

$$\begin{aligned} J_+ J_- &= \mathbf{J}^2 - J_z^2 + \hbar J_z \\ J_- J_+ &= \mathbf{J}^2 - J_z^2 - \hbar J_z \end{aligned}$$

La siguiente propiedad puede ser útil: $\left[\hat{A}\hat{B}, \hat{C}\right] = \hat{A}\left[\hat{B}, \hat{C}\right] + \left[\hat{A}, \hat{C}\right]\hat{B}.$

3)

1. Consider a system of angular momentum $j = 1$, whose state space is spanned by the basis $\{ |+1\rangle, |0\rangle, |-1\rangle \}$ of three eigenvectors common to $\mathbf{J}^2$ (eigenvalue $2\hbar^2$) and J_z (respective eigenvalues $+\hbar$, 0 and $-\hbar$). The state of the system is:

$$|\psi\rangle = \alpha |+1\rangle + \beta |0\rangle + \gamma |-1\rangle$$

where α, β, γ are three given complex parameters.

a. Calculate the mean value $\langle \mathbf{J} \rangle$ of the angular momentum in terms of α, β and γ.

b. Give the expression for the three mean values $\langle J_x^2 \rangle$, $\langle J_y^2 \rangle$ and $\langle J_z^2 \rangle$ in terms of the same quantities.

4)

2. Consider an arbitrary physical system whose four-dimensional state space is spanned by a basis of four eigenvectors $|j, m_z\rangle$ common to $\mathbf{J}^2$ and J_z ($j = 0$ or 1; $-j \leqslant m_z \leqslant +j$), of eigenvalues $j(j+1)\hbar^2$ and $m_z\hbar$, such that:

$$J_\pm |j, m_z\rangle = \hbar\sqrt{j(j+1) - m_z(m_z \pm 1)}\,|j, m_z \pm 1\rangle$$
$$J_+ |j, j\rangle = J_- |j, -j\rangle = 0$$

a. Express in terms of the kets $|j, m_z\rangle$, the eigenstates common to $\mathbf{J}^2$ and J_x, to be denoted by $|j, m_x\rangle$.

b. Consider a system in the normalized state:

$$|\psi\rangle = \alpha\,|j = 1, m_z = 1\rangle + \beta\,|j = 1, m_z = 0\rangle + \gamma\,|j = 1, m_z = -1\rangle + \delta\,|j = 0, m_z = 0\rangle$$

(*i*) What is the probability of finding $2\hbar^2$ and $\hbar$ if $\mathbf{J}^2$ and J_x are measured simultaneously?

(*ii*) Calculate the mean value of J_z when the system is in the state $|\psi\rangle$, and the probabilities of the various possible results of a measurement bearing only on this observable.

(*iii*) Same questions for the observable $\mathbf{J}^2$ and for J_x.

(*iv*) J_z^2 is now measured; what are the possible results, their probabilities, and their mean value?