GEOPHYSICS, VOL. 35, NO. 2 (APRIL 1970), P. 270-278, 10 FIGS., 1 TABLE

THE GRADATIONAL DENSITY CONTRAST AS A GRAVITY INTERPRETATION MODEL†

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The basic equations are presented for the gravity effect of a horizontal slab with density varying linearly between limits. A series of type curves and interpretation nomograms are presented showing the characteristics of the gradational model. A notable characteristic is the horizontal gravity gradient which is nearly constant over most of the width of the variable density zone. Maximum slope and curvature of the gravity effect vary directly with the density change, depth, and thickness of the slab and inversely with the width of the variable density zone. Practically identical gravity effects are sometimes produced by step models with different dimensions and density contrasts. However, if the variable density zone is sufficiently wide, the gravity effect cannot be approximated by a simple step model.

INTRODUCTION

This paper presents the gravity effect of a horizontal slab in which there is linear change of density between two adjacent slabs of constant density. The model is applicable where there exists a gradual horizontal change in density between two rock types, rather than a discrete density change across a single boundary. For example, it could be applied in areas where there are two rock types separated by interbedding, intrusive plutons characterized by lit-par-lit structure at their contacts, change in density through a steeply dipping sequence of beds, extensive contact metamorphism, change in regional metamorphism, or wide complicated fault zones.

The gravity effect of the gradational density contrast is similar superficially to that of the step model, but it differs in detail. Figure 1 illustrates a gradational density contrast and its gravity effect. Because the density change is gradual, the gravity effect is spread out. This results in a relatively smooth profile, lacking sharp curvature, and lacking good diagnostic features.

DERIVATION

The equation for the gravity effect of a horizontal slab with density varying in one direction is given by Novosolitskii (1965) as

$$\Delta g(x) = G\int_{-\infty}^{\infty} \rho(s) \ln \frac{H_2^2 + (x-s)^2}{H_1^2 + (x-s)^2} ds, \quad (1)$$

where

G = the universal gravity constant,
$\rho(s)$ = the density as a function of s,
s = the variable horizontal dimension,
x = a fixed point of s,
H_1 = the depth to the top of the slab,
H_2 = the depth to the bottom of the slab,
$\Delta g(x)$ = the vertical gravity effect at x,
$H_2 - H_1$ = the thickness of the slab (see Figure 1).

Equation (1) has been solved by Gendzwill (1968) who specified the following limits on the function $\rho(s)$:

$$\rho(s) = 0 \qquad -\infty < s < 0,$$
$$\rho(s) = ks \qquad 0 < s < w,$$
$$\rho(s) = kw \qquad w < s < \infty,$$

where k is a linear density gradient and w is the width of the zone of changing density. The prod-

† Manuscript received by the Editor June 5, 1969; revised manuscript received December 4, 1969.
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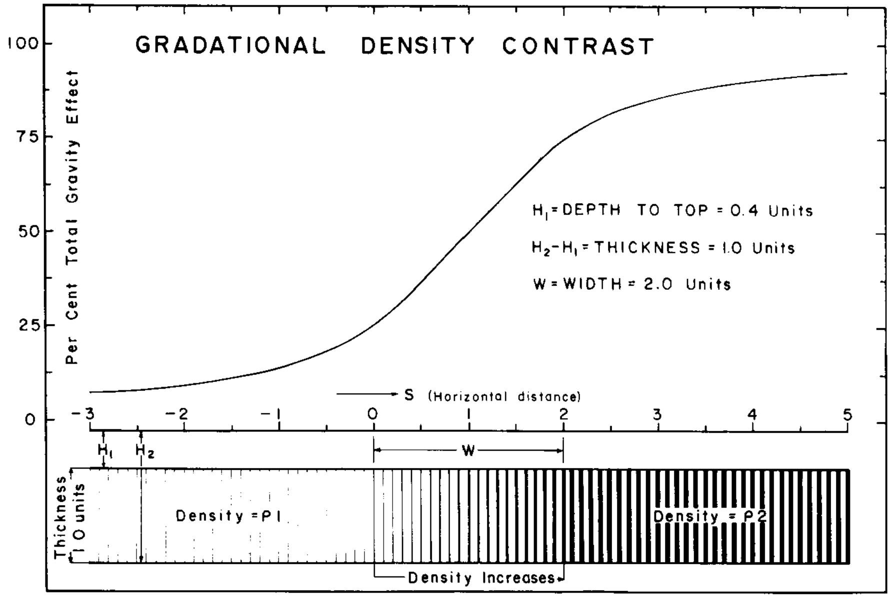

FIG. 1. Horizontal slab model with gradational density contrast, illustrating the parameters of the model.

uct kw is the total density change across the gradational zone.

With these limits the integration can be performed in a straightforward but tedious operation. The result of the integration is

$$\Delta g(x) = Gk \Bigg[\pi w(H_2 - H_1) + \frac{H_2^2 - (x-w)^2}{2} \ln (H_2^2 + (x-w)^2) - 2H_2(x-w) \tan^{-1} \frac{x-w}{H_2} - \frac{H_1^2 - (x-w)^2}{2} \ln (H_1^2 + (x-w)^2) + 2H_1(x-w) \tan^{-1} \frac{x-w}{H_1} - \frac{H_2^2 - x^2}{2} \ln (H_2^2 + x^2) + 2H_2 x \tan^{-1} \frac{x}{H_2} + \frac{H_1^2 - x^2}{2} \ln (H_1^2 + x^2) - 2H_1 x \tan^{-1} \frac{x}{H_1} \Bigg]. \tag{2}$$

The first horizontal derivative of the gravity effect is obtained by differentiating (2) with respect to x.

$$\frac{d\Delta g(x)}{dx} = Gk \Bigg[x \ln \frac{H_2^2 + x^2}{H_1^2 + x^2} - 2H_1 \tan^{-1} \frac{x}{H_1} + 2H_2 \tan^{-1} \frac{x}{H_2} - (x-w) \ln \frac{H_2^2 + (x-w)^2}{H_1^2 + (x-w)^2} + 2H_1 \tan^{-1} \frac{x-w}{H_1} - 2H_2 \tan^{-1} \frac{x-w}{H_2} \Bigg]. \tag{3}$$

Pavlovskiy and Serebryakov (1965) present equations for the horizontal gradient of gravity for several different density functions. However, their equivalent of equation (3) appears to be in error, and they do not present an equivalent of equation (2), the vertical gravity effect.

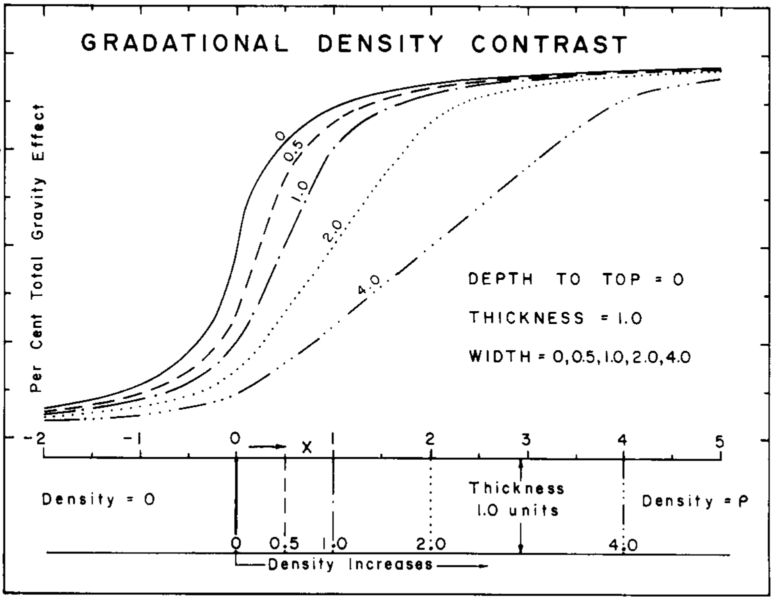

FIG. 2. Horizontal gradational density contrast slab model with top of slab at zero depth.

The second horizontal derivative of (2) is

$$\frac{d^2\Delta g(x)}{dx^2} = Gk\left[\ln\frac{H_2^2 + x^2}{H^2_1 + x^2} - \ln\frac{H_2^2 + (x-w)^2}{H_1^2 + (x-w)^2}\right]. \quad (4)$$

From this expression, it is seen that an inflection point exists at $x = w/2$. For a slab whose upper surface is at zero depth, we may determine a value of the gravity gradient at this inflection point by letting $x = w/2$, $H_1 = 0$, $H_2 = H$. Then,

$$\frac{d\Delta g(w/2)}{dx} = Gkw\left[\ln\left(\left(\frac{2H}{w}\right)^2 + 1\right) + \frac{4H}{w}\tan^{-1}\frac{w}{2H}\right]. \quad (5)$$

If $w >> H$, equation (5) may be simplified to

$$\frac{d\Delta g}{dx} = 41.9\, kH \text{ mgal/km}, \quad (6)$$

where k is in gm/cm^3/km and H is in km.

Gravity gradients computed from equation (6) will always be too large but the error will be less than 15 percent if H/w is less than 0.2.

TYPE CURVES

Equation (2) has been programmed in Fortran IV and used to calculate the collection of type curves shown in Figures 1, 2, 3, 4, 5, and 6. The thickness of the slab is taken as one unit, and the other dimensions are in proportion to the unit of thickness.

The maximum curvature and the maximum slope of the anomaly curves both decrease as the width or depth of the models increases. Maximum slopes of the curves are directly proportional to the total density contrast, kw, but this fact is not apparent because the curves are normalized. Gradational density curves resemble those of the step model, but the resemblance is close only for very narrow zones.

Grant and West (1965, p. 282–287) present characteristic curves for the interpretation of step model anomalies. They define the following parameters to be used as indices (k_1 and k_2 are not to be confused with k which is the linear density gradient):

$$k_1 = \frac{x_2 - x_{\frac{1}{2}}}{x_{\frac{1}{2}} - x_1} \quad \text{and} \quad k_2 = \frac{\Delta g_2 - \Delta g_1}{(x_2 - x_{\frac{1}{2}})s_{\max}},$$

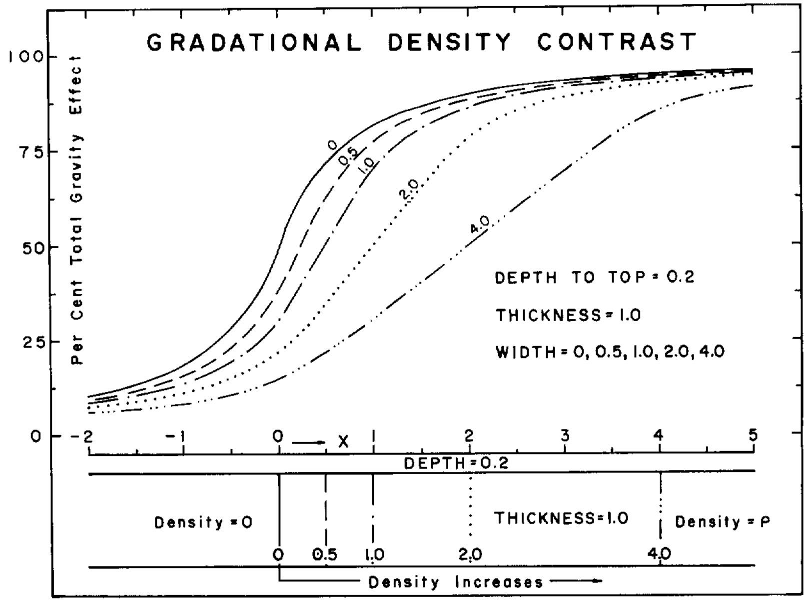

FIG. 3. Horizontal gradational density contrast slab model with top of slab at a depth of 0.2 thickness units.

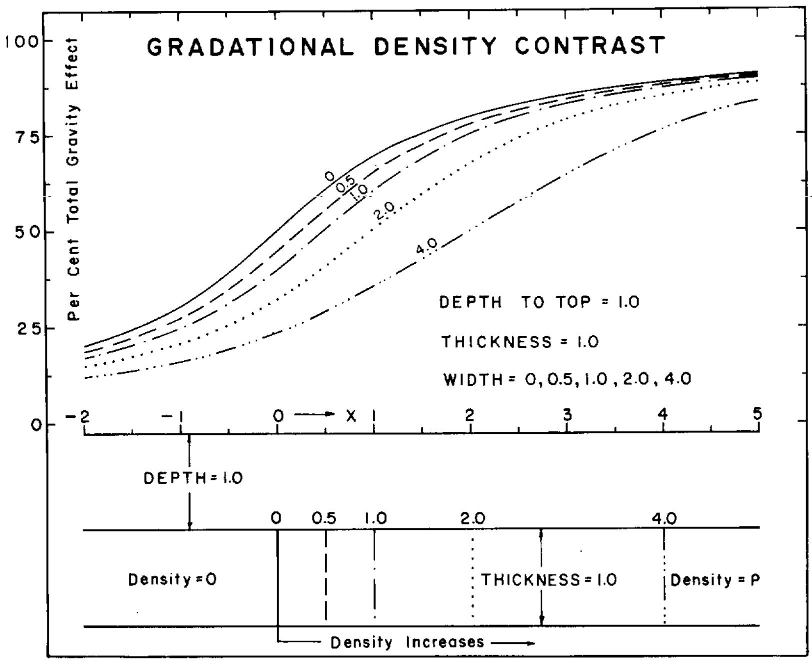

FIG. 4. Horizontal gradational density contrast slab model with top of slab at a depth of 1.0 thickness units.

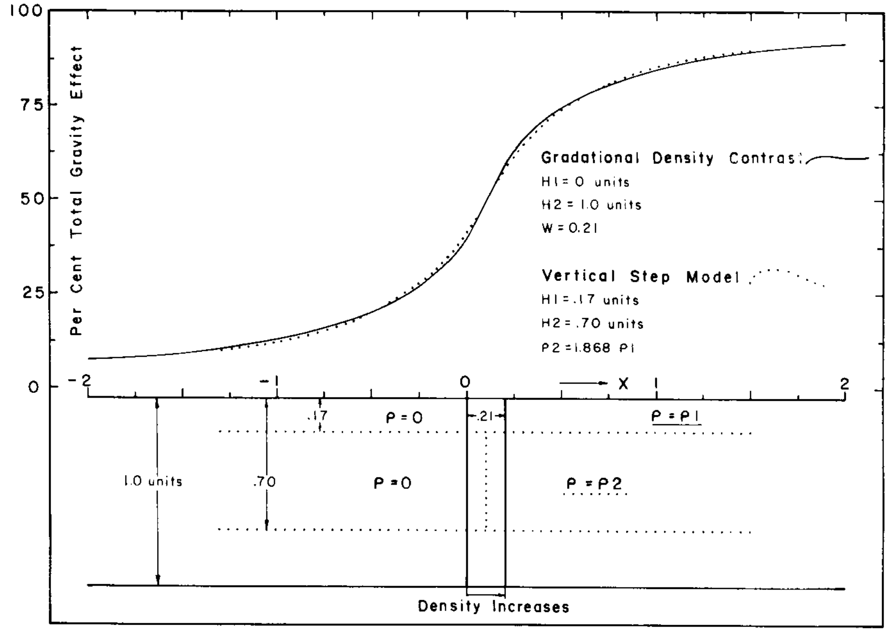

FIG. 5. Comparison between gravity effects of a gradational density contrast slab model and a vertical step model.

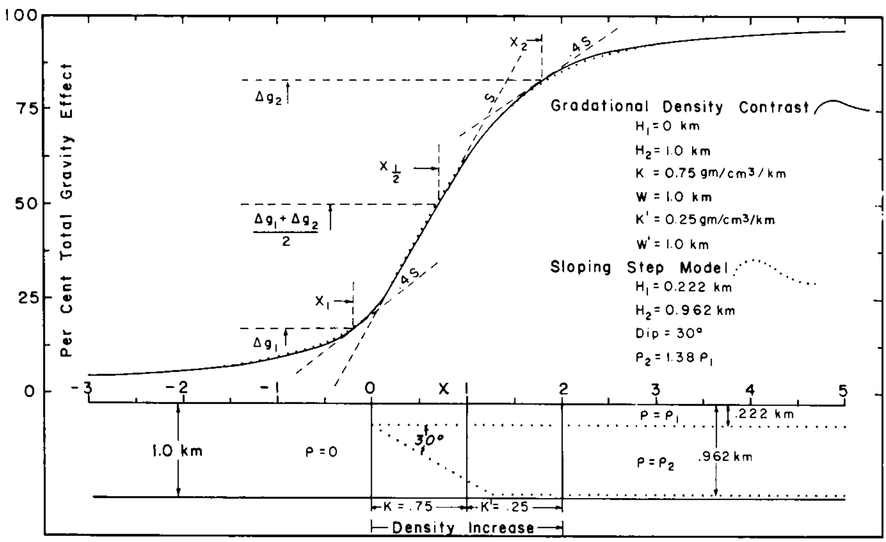

FIG. 6. Comparison between gravity effects of a gradational density contrast model with two rates of increase and a sloping step model.

where

s_{max}	is the maximum gravity slope,
Δg_1 and Δg_2	are the gravity values at the points of tangency of a line with slope equal to 0.4 s_{max},
x_1 and x_2	are the abscissae of Δg_1 and Δg_2,
$x_{1/2}$	is the abscissa of $(\Delta g_2+\Delta g_1)/2$.

Figure 6 illustrates these quantities.

The gradational density zone model always yields k_2 values greater than the vertical step model with equal depth and thickness. Figure 5 shows a gradational density model and vertical step model which both yield the same k_2 value (1.37) and nearly identical gravity effect curves. For increasing gradational width, the equivalent step model depth becomes greater, thickness decreases, and density contrast increases.

Figure 6 shows a gravity effect due to a density gradient with two rates of increase. An equivalent sloping step model and its gravity effect are superimposed on the density gradient model and its effect. The near coincidence of the curves demonstrates the similarity in the effects due to the two models. For both models $k_2=1.32$, $k_1=1.22$.

CHARACTERISTIC CURVES

For the symmetric gradational model (one rate of increase) the number of independent parameters is the same as for the sloping step model as listed in Table 1.

Table 1. Independent parameters

	Step model	Gradational model
1.	Depth	Depth
2.	Thickness	Thickness
3.	Density contrast	Density gradient
4.	Dip of face	Width of zone

The asymmetric gradational model (Figure 6) has six parameters because there are two gradients and two widths; therefore, it is too complicated for practical consideration. Unfortunately, even the symmetric model is difficult to handle because the curves lack good diagnostic features. Hence, it is impractical to use characteristic curves for a unique solution of the gradational density model. Instead, three charts are presented showing the relationship of the parameters, k_2, $(x_2-x_1)/w$, and s_{max}/kw, as they depend on H_1/w and w/H_2. The units of gravity effect, density, and length are mgal, gm/cm^3, and km. Although these are mixed systems of units, they are convenient and commonly used.

To use these charts the interpreter should make some initial assumptions, guided by geological plausibility, concerning one or more of the independent parameters. If this is not possible, several solutions based on different arbitrary assumptions should be made. The parameters k_1 and k_2 should be calculated. If k_1 is between 0.95 and 1.05, the curve may be handled as a gradational contrast with one rate of increase. If k_2 is greater than 1.44 and the curve is symmetric, it cannot be a simple step model. If H_1/w is greater than 1.5, the step model approximation is valid.

Maximum slopes are difficult to estimate, especially if w is small. For the step model at zero depth, the maximum slope is infinitely large. To estimate the maximum slope, the interpreter should therefore plot the first horizontal derivative of the gravity effect.

EXAMPLE

Figure 10 shows an actual gravity profile from northern Saskatchewan. This profile is associated with a large granodiorite body intruded into a complex of steeply dipping basic volcanic and basic intrusive rocks. The border zone is a heterogeneous mixture of basic rock and granodiorite with poorly defined contacts. The mean density of 83 samples of granodiorite is 2.67 gm/cm^3 and of 134 samples of basic rock, 2.96 gm/cm^3. The border zone can be considered as a gradational density zone between basic rock and granodiorite. The following parameters are read or calculated from the profile:

$s_{max}=5.44$ mgal/km $\quad 0.4\,s_{max}=2.18$ mgal/km

$\Delta g_2=-21.7$ mgal $\quad \Delta g_1=-38.1$ mgal

$x_2=4.83$ km $\quad x_1=8.62$ km

$x_{\frac{1}{2}}=6.76$ km

$$k_1=(8.62-6.76)/(6.76-4.83)=0.96$$

$$k_2=2(-21.7+38.1)/(8.62-4.83)/5.44 = 1.59.$$

k_1 is between 0.95 and 1.05; consequently the profile may be interpreted as a gradational density zone with a single rate of increase.

From Figures 7, 8, and 9, the following values

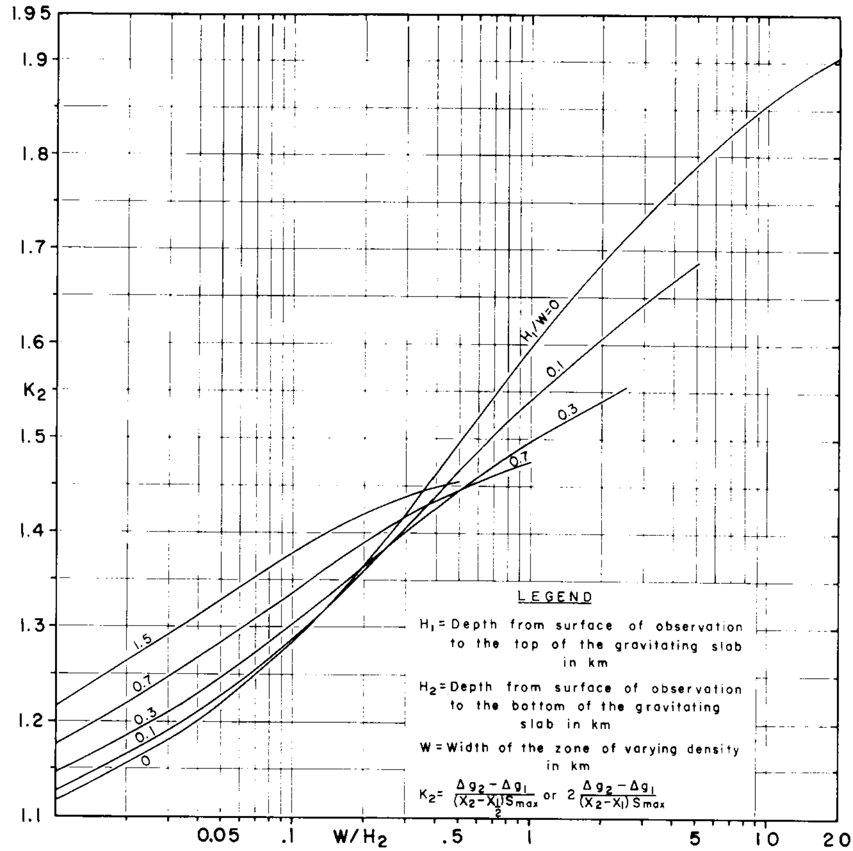

FIG. 7. Interpretation nomogram showing the relationship between K_2, H_1/w, and w/H_2 for a slab with gradational density contrast.

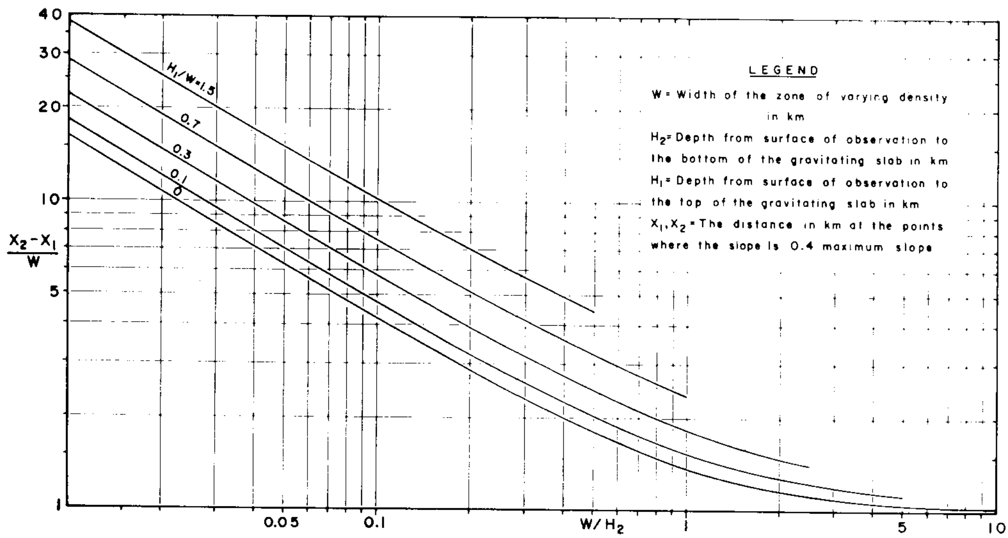

FIG. 8. Interpretation nomogram showing the relationship between $(x_2 - x_1)/w$, H_1/w, and w/H_2 for a slab with gradational density contrast.

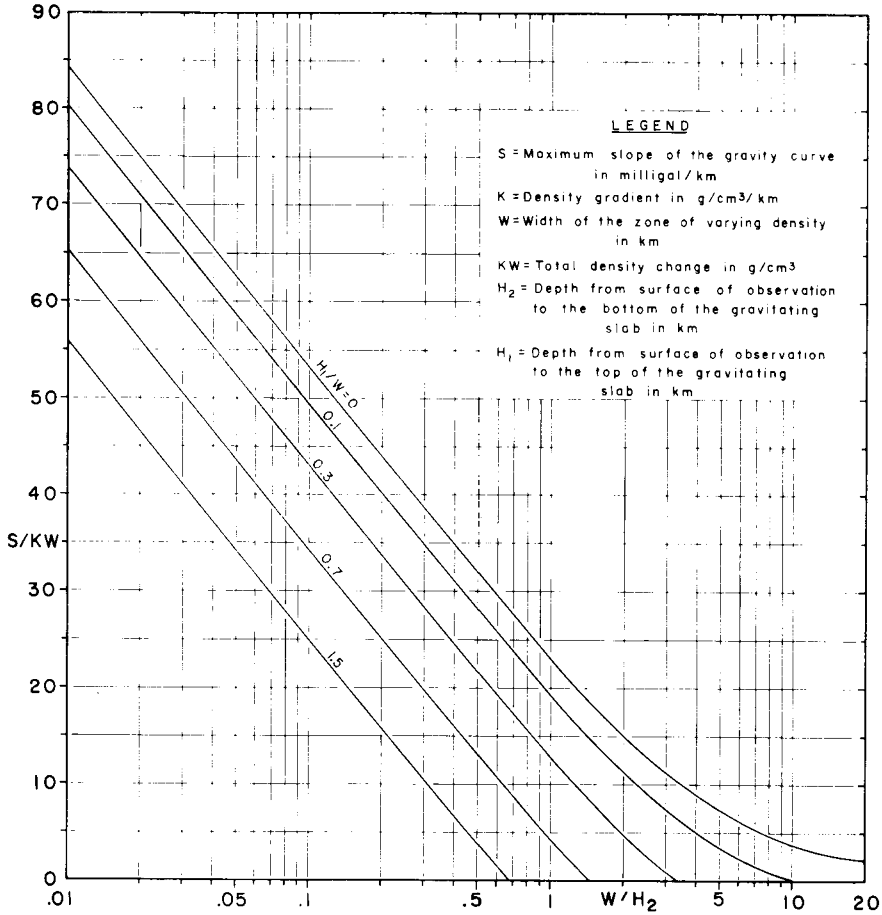

FIG. 9. Interpretation nomogram showing the relationship between s/kw, H_1/w, and w/H_2 for a slab with gradational density contrast.

may be obtained if zero depth ($H_1/w=0$) and shallow depth ($H_1/w=0.1$) are assumed in turn:

			km	$\frac{\text{mgal cm}^3}{\text{gm/km}^3}$	gm/km³	km	km
H_1/w	w/H_2	$\frac{x_2-x_1}{w}$	u	s/kw	kw	H_1	H_2
0	.956	1.38	2.75	23.5	.231	0	2.88
0.1	1.66	1.33	2.85	13.3	.409	.285	1.72

The width of the border zone on Figure 10 is 2.7 km; the density contrast estimated from samples is 0.29 g/cm³; and the rocks crop out at zero depth. Therefore, we conclude that the set of values for $H_1/w=0$ is most nearly applicable, and the density contrast between granodiorite and basic rock persists to a depth of approximately 2.88 km.

CONCLUSIONS

The gradational density contrast model is useful for interpreting the gravity effect of geological contacts where the contact is not sharp. It permits an interpretation of anomalies which resemble the step model, but whose parameters do not fit the characteristics of the step model. It is most useful in situations where the gravitating rocks are shallow or crop out. When the depth of burial is greater than 1.5 times the width of the variable density zone, the anomaly closely resembles the equivalent step model. The most indicative feature of the gradational density model is the slope of the gravity curve, since the slope is almost constant over the entire width of the gradational zone.

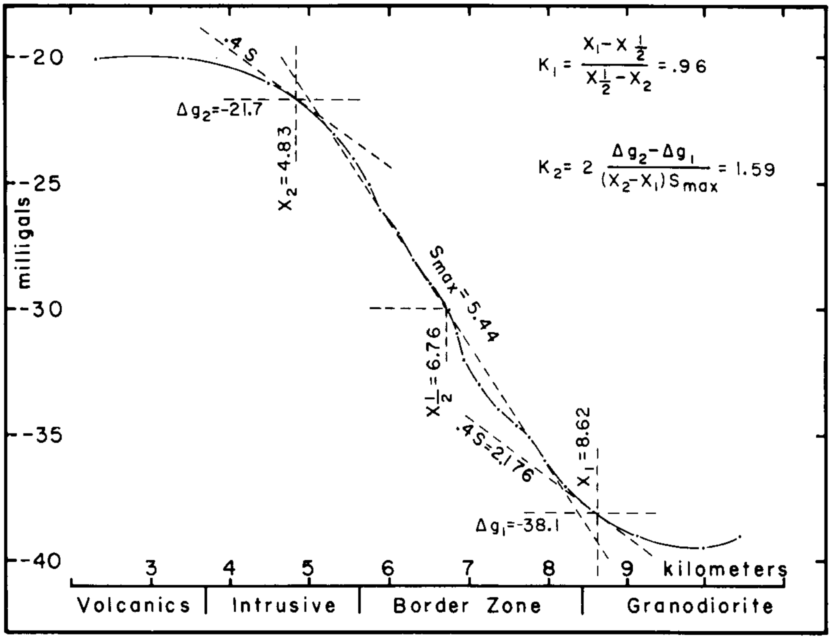

FIG. 10. Gravity anomaly over contact zone between granodiorite and basic rocks.

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