Problem 1 Valdrin Smakaj

(a)

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{9n}{5n^3} = \lim_{n\to\infty} \frac{\frac{9}{n^2}}{5} = \lim_{n\to\infty} \frac{9}{5n^2} = 0$$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{5n^3}{9n} = \lim_{n\to\infty} \frac{5}{\frac{9}{n^2}} = \lim_{n\to\infty} \frac{5n^2}{9} = \infty$$

$f(n) \in o(g)$ and $O(g)$

$g(n) \in \omega(f)$ and $\Omega(f)$

(b) $f(n) = 9n^{0.8} + 2n^{0.3} + 14\log n \qquad g(n) = \sqrt{n}$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{9n^{0.8}+2n^{0.3}+14\log n}{\sqrt{n}} = \lim_{n\to\infty} \frac{9n^{0.8}+2n^{0.3}+14\log n}{n^{0.5}} = \infty$$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{\sqrt{n}}{9n^{0.8}+2n^{0.3}+14\log n} = \lim_{n\to\infty} \frac{n^{0.5}}{9n^{0.8}+2n^{0.3}+14\log n} = 0$$

$f(n) \in \omega(g)$ and $\Omega(g)$

$g(n) \in o(f)$ and $O(f)$

(d) $f(n) = (\log(3n))^3 \qquad g(n) = 9\log n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{(\log(3n))^3}{9\log n} = \lim_{n\to\infty} \frac{\log^3(3n)}{9\log n} = \infty$$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{9\log n}{(\log(3n))^3} = \lim_{n\to\infty} \frac{9\log n}{\log^3(3n)} = 0$$

$f(n) \in \omega(g)$ and $\Omega(g)$

$g(n) \in o(f)$ and $O(f)$

(c) $f(n) = n^2/\log n \qquad g(n) = n\log n$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\frac{n^2}{\log n}}{n\log n} = \lim_{n\to\infty} \frac{n^2}{n\log^2 n} = \infty$$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{n\log n}{\frac{n^2}{\log n}} = \lim_{n\to\infty} \frac{n\log^2 n}{n^2} = 0$$

$f(n) \in \omega(g)$ and $\Omega(g)$

$g(n) \in o(f)$ and $O(f)$.