Problem 2

ADS - Homework

a) $T(n) = 36T(n/6) + 2n$

$a = 36$
$b = 6$

$n^{\log_6 36} = n^2$

$f(n) = 2n$

$f(n) = O(n^{\log_6 36 - \varepsilon}) \Rightarrow \varepsilon = \log_6 36 - 1 = 2 - 1 = 1$

$T(n) = \cancel{\ } \; \Theta(n^{\log_6 36}) = \Theta(n^2)$

b) $T(n) = 5T(n/3) + 17n^{1.2}$

$a = 5, \; b = 3$

$n^{\log_3 5} = n^{1.465}$

$f(n) = 17n^{1.2}$

$f(n) = O(n^{\log_3 5 - \varepsilon}) \Rightarrow \varepsilon = \log_3 5 - 1.2 = 0.265$

$f(n)$ is polynomially smaller than $n^{\log_3 5}$

$T(n) = \Theta(n^{\log_3 5}) \approx O(n^{1.465})$.

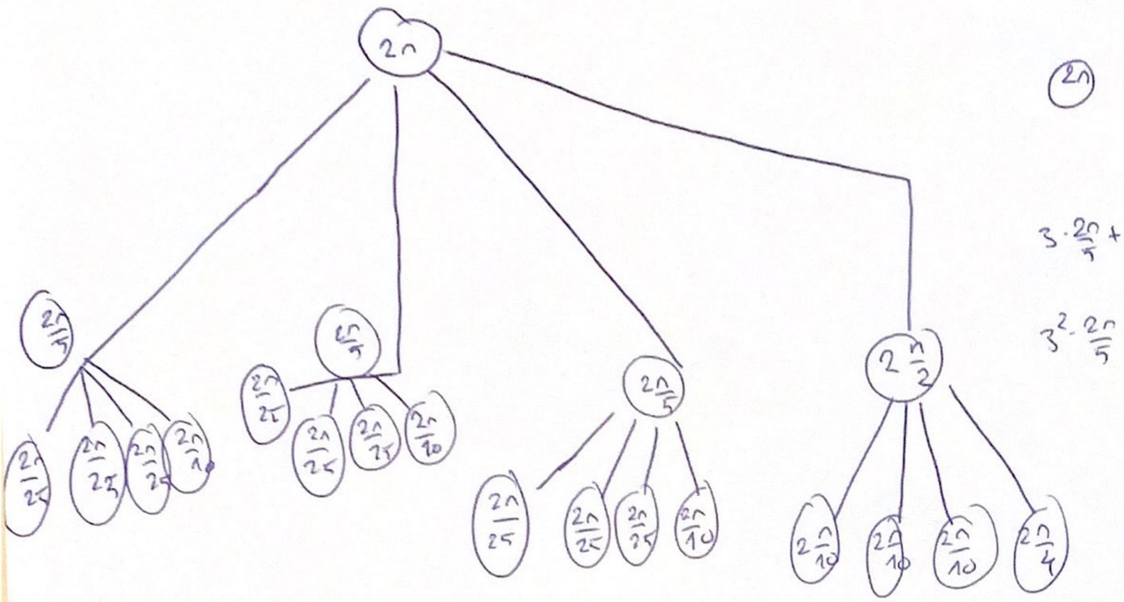
c) $T(n) = 12T(n/2) + n^2 \log n$

$a = 12, \; b = 2$

$$\left.\begin{array}{l} n^{\log_2 12} = n^{3.585} \\ f(n) = n^2 \log n \end{array}\right\} f(n) = O(n^{\log_2 12 - \varepsilon}) = O(n^{3.585 - \varepsilon})$$

$n^{3.585} > n^2 \wedge O(n) > \Theta(\log n) \Rightarrow T(n) = \Theta(n^{\log_2 5}) \approx \Theta(n^{3.8}$

d) $T(n) = 3T(n/5) + T(n/2) + 2^n$



$2n$

$3 \cdot \frac{2n}{5} + 1 \cdot \frac{2n}{2}$

$3^2 \cdot \frac{2n}{5} + 6 \cdot \frac{2n}{10} + \frac{2n}{4}$

for the tight bounds, we calculate the left and right longest path

right p $= \log_5 n = \dfrac{\log n}{\log 5} = C_1 \log n = \Omega(\log n)$

left p $= \log_2 n = \dfrac{\log n}{\log 2} = C_2 \log n = O(\log n)$

First lvl $= 2^n$

second $2^n (3 \cdot 2^{-\frac{4n}{5}} + 2^{-\frac{n}{2}})$

third $= 2^n (3^2 \cdot 2^{-\frac{24}{25} n} + 6 \cdot 2^{-\frac{9n}{10}} + 2^{-\frac{3}{4} n})$

$T(n) \leq 3c\, 2^{\frac{n}{5}} + c\, 2^{\frac{n}{2}} + 2^n$

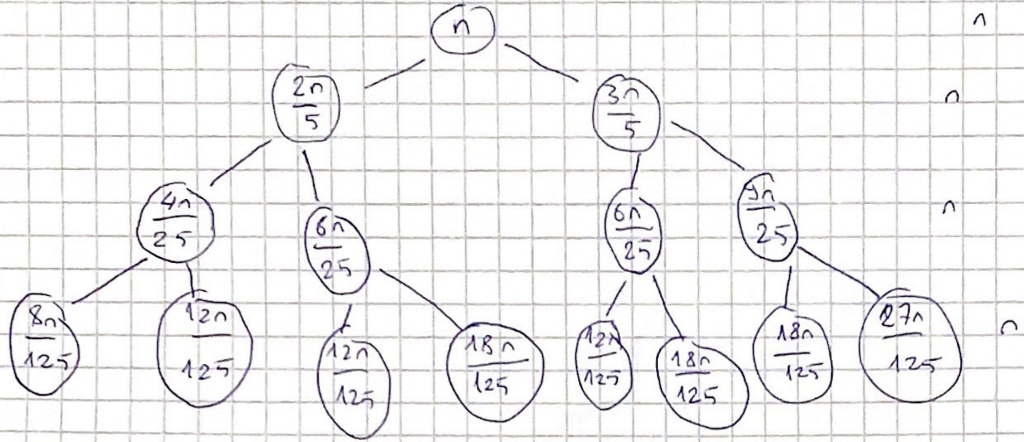
$\leq 3c^{2n} + c2^n + 2^n(1)$

$\leq c\, 2^n$

$2^n \geq 2^{n/d}$ for $d > 1$

$T_n = \Theta(2^n)$

e) $T(n) = T(2n/5) + T(3n/5) + \Theta(n)$



for the tight bound we calculate the height of tree by longest and shortest path. Longest path comes from the most right point road whereas the shortest from the longest left point.

$$h_l = \log_{\frac{5}{3}} n = \frac{\log n}{\log \frac{5}{3}} = c_1 \log n = O(\log n)$$

$$h_{sh} = \log_{\frac{5}{2}} n = \frac{\log n}{\log \frac{5}{2}} = c_2 \log n = \Omega(\log n)$$

The ~~cost~~ overall cost is calculated by the sum of all the costs of each level.

$$\sum_{i=0}^{h} n = n \cdot \sum_{i=0}^{h} 1 = n \cdot h = n \cdot \log n$$

which is proof of $T(n) = \Theta(n \log n)$