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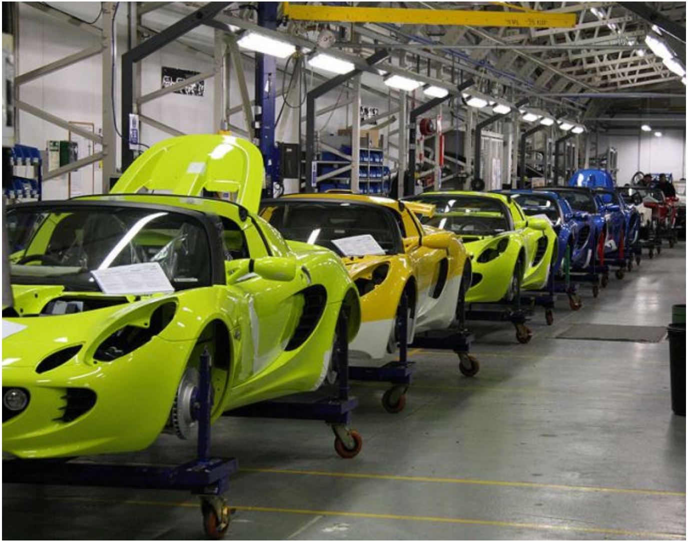
SAT Encodings for the Car Sequencing Problem

08/07/2013 at Pragmatics of SAT

Agenda

1) Car Sequencing
2) Encodings
3) Experimental Results
4) Future Work

Car Sequencing



Picture from Wikipedia

Car Sequencing from the CSPLib Benchmark

Definition

Assemble a production line of cars such that capacity constraints on the workstations are not exceeded.

Notation:

- Set of Classes C
- Demand d_i for class i
- Set of Options O
- Capacity constraint with ratio u_l/q_l for option l

Car Sequencing: Example

- $C = \{1, 2, 3\}$ with demand $3, 2, 2$
- $O = \{a, b\}$ with capacity constraints $1/2$ and $1/5$
- Class 1 no restriction
- Class 2 requires option a
- Class 3 requires option a and b

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	3	1	2	1	2	1	3
a	1	-	1	-	1	-	1
b	1	-	-	-	-	-	1

PB Model

- Boolean variable c_i^k: car $k \in C$ is at position i
- Boolean variable o_i^l: option $l \in O$ is at position i
- Demand constraints: $\forall k \in C$

$$\sum_{i=1}^{n} c_i^k = d_k$$

- Capacity constraints: $\forall l \in O$ with ratio u_l / q_l

$$\bigwedge_{i=0}^{n-q_l} \left(\sum_{j=1}^{q_l} o_{i+j}^l \leq u_l\right)$$

PB Model

And in all positions $i \in \{1 \dots n\}$ of the sequence it must hold:

- Link between classes and options: for each $k \in C$ and

$$\forall l \in O_k : \; c_i^k - o_i^l \leq 0$$
$$\forall l \in O \setminus O_k : \; c_i^k + o_i^l \leq 1$$

- Exactly one car:

$$\sum_{k \in C} c_i^k = 1$$

Modelling in CNF: the CP view

- This model with standard translation (minisat+,clasp ...) has bad performance
- More redundant constraints
- Global constraints and propagators

Sequential Counter: Auxiliary Variables

- Translation of Boolean Cardinality:

$$\sum_{i \in \{1 \ldots n\}} x_i = d$$

- x_i is true iff an object is at position i
- $s_{i,j}$ is true iff in the positions $0, 1 \ldots i$ the object exists at least j times (for technical reasons $0 \leq j \leq d + 1$).

Sequential Counter

$\forall i \in \{1 \dots n\}\ \forall j \in \{0 \dots d+1\}$:

$$\neg s_{i-1,j} \vee s_{i,j} \tag{1}$$

$$x_i \vee \neg s_{i,j} \vee s_{i-1,j} \tag{2}$$

$\forall i \in \{1 \dots n\} \forall j \in \{1 \dots d+1\}$:

$$\neg s_{i,j} \vee s_{i-1,j-1} \tag{3}$$

$$\neg x_i \vee \neg s_{i-1,j-1} \vee s_{i,j} \tag{4}$$

$$s_{0,0} \wedge \neg s_{0,1} \wedge s_{n,d} \wedge \neg s_{n,d+1} \tag{5}$$

Sequential Counter: Example

3			U	U	U	U	U	U	U	U	U
2		U	?	?	?	?	?	?	?	?	L
1	U	?	?	?	?	?	?	?	?	L	
0	L	L	L	L	L	L	L	L	L		
$s_{i,j}$	0	1	2	3	4	5	6	7	8	9	10

Setting x_2 and x_7 to 1:

3			U	U	U	U	U	U	U	U	U
2		U	0	0	0	0	0	1	1	1	L
1	U	0	1	1	1	1	1	1	1	L	
0	L	L	L	L	L	L	L	L	L		
$s_{i,j}$	0	1	2	3	4	5	6	7	8	9	10

Sequential Counter: Related Work

- Carsten Sinz Sequential Counter [4]
- Fahim Bacchus translation of AMONG by the Regular constraint [1]
- Translation through BDDs [2]

Capacity Constraints: More Global

$$(\sum_{i=1}^{n} x_i = d) \wedge \bigwedge_{i=0}^{n-q} (\sum_{l=1}^{q} x_{i+l} \leq u)$$

$\forall i \in \{q \ldots n\}, \forall j \in \{u \ldots d+1\}$:

$$\neg s_{i,j} \vee s_{i-q,j-u} \tag{6}$$

Capacity Constraints: Example

Capacity constraint $4/8$, demand $d = 12$ on a sequence of 22 variables:

13																					U	U	U
12																				U	?	?	L
11																			U	?	?	L	
10																		U	?	?	L		
9													U	U	U	U	U	?	?	L			
8												U	?	?	L	L	L	L	L				
7											U	?	?	L									
6										U	?	?	L										
5					U	U	U	U	U	?	?	L											
4				U	?	?	L	L	L	L	L												
3			U	?	?	L																	
2		U	?	?	L																		
1	U	?	?	L																			
0	L	L	L																				
$s_{i,j}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
x_i								**0**	**0**							**0**	**0**						

Capacity Constraints: Example

Partial Assignment: x_1 and x_{13} to true and x_{12}, x_{14} and x_{21} to false.

13																					U	U	U
12																				U	0	0	L
11																			U	?	1	L	
10																		U	?	1	L		
9													U	U	U	U	U	?	1	L			
8												U	0	1	L	L	L	L	L				
7											U	1	1	L									
6										U	1	1	L										
5					U	U	U	U	U	1	1	L											
4				U	?	1	L	L	L	L	L												
3			U	1	1	L																	
2		U	1	1	L																		
1	U	1	1	L																			
0	L	L	L																				
$s_{i,j}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
x_i		1	**1**	**1**			**0**	(0)	(0)	**1**	**1**	**1**	0	1	0	(0)	(0)					0	**1**

A Trick for Lower Bounds ([3])

Table: Overview of options and demands for instance 300-04

class	0	1	2	3	4	5	6	7	8	9	10	11
demand	9	4	22	2	1	62	31	4	24	4	3	36
0: 1/2	-	-	-	-	-	-	-	-	-	-	-	x
1: 2/3	-	-	-	-	-	x	x	x	x	x	x	-
2: 1/3	-	-	x	x	x	-	-	-	x	x	x	-
3: 2/5	-	x	-	-	x	-	x	x	-	-	x	-
4: 1/5	x	x	-	x	x	-	-	x	-	x	x	-

class	12	13	14	15	16	17	18	19	20	**21**	22	**23**
demand	3	25	3	8	5	2	6	21	5	**7**	11	**2**
0: 1/2	x	x	x	x	x	x	x	x	x	**x**	x	**x**
1: 2/3	-	-	-	-	-	-	x	x	x	**x**	x	**x**
2: 1/3	-	-	-	x	x	x	-	-	-	**x**	x	**x**
3: 2/5	-	x	x	-	-	x	-	x	x	-	x	**x**
4: 1/5	x	-	x	-	x	x	x	-	x	**x**	-	**x**

Results: Solved Instances

	E1	E2	E3	ASP	PB
#solved UNSAT	**17**	15	**17**	10	8
#fastest UNSAT	**5**	4	4	0	4
#solved SAT	**11**	**11**	**11**	7	2
#fastest SAT	0	4	**7**	0	0

Conclusion and Future Work

- SAT can be very competitive on certain CP benchmarks
- Spending time on the model pays off
- Learning from Constraint Programming and global constraints
- Choosing the right decomposition of cardinality constraint
- Lower bound techniques are novel

Future work:

- Exponential encoding in the number of options?
- Theoretical analysis of the decompositions and usage in other domains

End

Thank you very much

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Backup Slides

Example

Let $n = 5$, $d = 2$ with a capacity constraint of $1/2$, and let x_3 be true, then unit propagation does not force x_2 nor x_4 to false. Setting them to true will lead to a conflict through clauses (4) and (6) on positions 2, 3 and 4.

3				U	U	U
2		U	U	.	.	L
1	U	.	.	L	L	
0	L	L	L			
$s_{i,j}$	0	1	2	3	4	5
x_i		.	.	1	.	.