Yet Another SAT Encoding for Car Sequencing

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The encoding avoids a separate counter encoding for each cardinality constraint and at the same time is an alternative to [1].

The encoding is wrt. one object (car or option) for which we omit the subscript. We present an encoding for the constraint

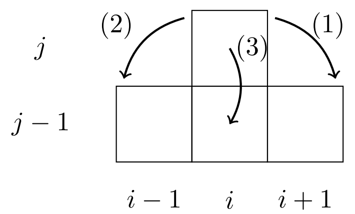
$$\bigwedge_{i=0}^{n-q} (\sum_{l=1}^{q} x_{i+l} \leq u)$$

- x_i is true (1) if the object is at position i.
- $s_{i,j}$ is true if in window $[i-q+1, \ldots i]$ there are at least j objects.

$$s_{i,j} \quad \Leftrightarrow \quad \sum_{l=i-q+1}^{i} x_l \geq j$$

The idea behind the clauses is to express the relationship between x_{i-q}, x_i and the counter $s_{i,j}$.

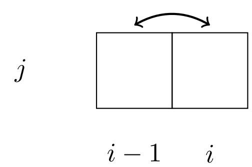
$$\neg s_{i,j} \vee s_{i+1,j-1} \tag{1}$$
$$\neg s_{i+1,j} \vee s_{i,j-1} \tag{2}$$
$$\neg s_{i,j} \vee s_{i,j-1} \tag{3}$$
$$\neg x_i \vee \neg s_{i-1,j} \vee s_{i,j} \tag{4}$$
$$x_i \vee \neg s_{i,j} \vee s_{i-1,j} \tag{5}$$
$$\neg x_{i-q} \vee \neg s_{i,j} \vee s_{i-1,j} \tag{6}$$
$$x_{i-q} \vee \neg s_{i-1,j} \vee s_{i,j} \tag{7}$$
$$\neg x_i \vee x_{i-q} \vee \neg s_{i-1,j-1} \vee s_{i,j} \tag{8}$$
$$x_i \vee \neg x_{i-q} \vee \neg s_{i,j-1} \vee s_{i-1,j} \tag{9}$$



(1) $\neg s_{i,j} \vee s_{i+1,j-1}$

(2) $\neg s_{i+1,j} \vee s_{i,j-1}$

(3) $\neg s_{i,j} \vee s_{i,j-1}$

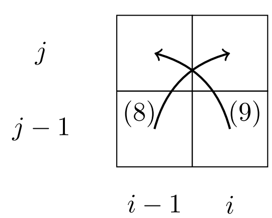


(4) $\neg x_i \vee \neg s_{i-1,j} \vee s_{i,j}$

(5) $x_i \vee \neg s_{i,j} \vee s_{i-1,j}$

(6) $\neg x_{i-q} \vee \neg s_{i,j} \vee s_{i-1,j}$

(7) $x_{i-q} \vee \neg s_{i-1,j} \vee s_{i,j}$



(8) $\neg x_i \vee x_{i-q} \vee \neg s_{i-1,j-1} \vee s_{i,j}$

(9) $x_i \vee \neg x_{i-q} \vee \neg s_{i,j-1} \vee s_{i-1,j}$

References

[1] Valentin Mayer-Eichberger and Toby Walsh. SAT Encodings for the Car Sequencing Problem. *Pragmatics of SAT*, Workshop at SAT, Helsinki, 2013.