1 Preprocessing Lower Bounds

The idea to this method goes back to the proof in [Gent, 1998] to show a lower bound of 2 for the instance 19/97. Here we will show how to generalize this technique and apply this method on all problems from the benchmark in [Gravel et al., 2005].

We start with instance 300-04 as an example. The demands and options are given in Table 1.

Table 1. Overview of options and demands for instance 300-04

class	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
demand	9	4	22	2	1	62	31	4	24	4	3	36	3	25	3	8	5	2	6	21	5	7	11	2
0: 1/2	-	-	-	-	-	-	-	-	-	-	-	x	x	x	x	x	x	x	x	x	x	x	x	x
1: 2/3	-	-	-	-	-	x	x	x	x	x	x	-	-	-	-	-	-	-	x	x	x	x	x	x
2: 1/3	-	-	x	x	x	-	-	-	x	x	x	-	-	-	-	x	x	x	-	-	-	x	x	x
3: 2/5	-	x	-	-	x	-	x	x	-	-	x	-	-	x	x	-	-	x	-	x	x	-	x	x
4: 1/5	x	x	-	x	x	-	-	x	-	x	x	-	x	-	x	-	x	x	x	-	x	x	-	x

There are two classes, 21 and 23, that require options 0, 1, 2 and 4 and sum of demands is 9. First observation is that all other classes share at least one options with these two classes. Secondly cars of class 21 and 23 have to be put at least 5 apart, so they cannot share a neighbour. Second they cannot be neighbour to any of the classes that have a $1/q$ restriction. This leaves us with the classes that only share the option 1 and for each car at most one adjacent car can have restriction 2/3. Since the first and the last car in the sequence can have any neighbour with that restriction, the number of cars that share no option is at least $9 - 2 = 7$. Since there are no such cars, the lower bound for violations (dummy cars or violated capacity constraints) is 7.

A similar argument can be made for classes 21, 22, 23 that share options 0, 1 and 2. Here the collective demand is 20 and the supply of cars that have neither of these options is $20 - 13 = 7$. This gives a lower bound of 5, which is weaker than the first case.

The general idea is to compute the demand for classes that share a subset of options such that there is a lower bound on the number of cars that do not share any options with this subset.

Proposition 1. *The following cases can be used to preprocess lower bounds.*

1. *A subset of options $B \subseteq O$ that contains only capacity constraints of the form $1/q$ and at least one with $1/2$. Let the collective demand for this set of options be k, then for a legal sequence of cars there have to be at least $k - 1$ cars that do not have any of the options in B.*
2. *A subset of options $B \subseteq O$ that contains only capacity constraints of the form $1/q$ with $q \geq 3$. Let the collective demand for this set of options be k,*

then for a legal sequence of cars there have to be at least $2 \cdot (k-1)$ cars that do not have any of the options in B.

3. *A set of options $B \subseteq O$ that contain at least one capacity constraint with $1/q$ where $q \geq 3$ and exactly one with $2/r$ where $r \geq 3$ and arbitrary many $1/s$ constraints. If the demand for this set is k, then there have to be at least $k-2$ cars that have none of these options in O.*

Proof. still working on it...

2 New Lower Bounds

The following table shows the known lower bounds (LB) and upper bounds (UB) published in the following works [Régin and Puget, 1997], [Gent, 1998]. [Gottlieb et al., 2003], [Gravel et al., 2005], [Estellon et al., 2006], as well as results from running minisat with dummy cars. The number of dummy cars ranges from 0 to the best known upper bound. Each run was limited by 1800 seconds. Take into account that the SAT approach computes bounds by adding dummy cars and such the lower bounds are also lower bounds for the other definitions of the optimization goal. Upper bounds cannot be compared, as adding dummy cars is less strict than minimizing the violated capacity constraints.

References

[Estellon et al., 2006] Estellon, B., Gardi, F., and Nouioua, K. (2006). Large neighborhood improvements for solving car sequencing problems. *RAIRO - Operations Research*, 40(4):355–379.

[Gent, 1998] Gent, I. P. (1998). Two Results on Car-sequencing Problems. In *Report APES-02-1998*.

[Gottlieb et al., 2003] Gottlieb, J., Puchta, M., and Solnon, C. (2003). A Study of Greedy, Local Search, and Ant Colony Optimization Approaches for Car Sequencing Problems. In *EvoWorkshops*, pages 246–257.

[Gravel et al., 2005] Gravel, M., Gagné, C., and Price, W. L. (2005). Review and Comparison of Three Methods for the Solution of the Car Sequencing Problem. *The Journal of the Operational Research Society*, 56(11):1287–1295.

[Régin and Puget, 1997] Régin, J.-C. and Puget, J.-F. (1997). A Filtering Algorithm for Global Sequencing Constraints. In *CP*, pages 32–46.

Table 2. Lower and upper bounds found by preprocessing (pre), by the SAT encoding and the best known.

instance	LB (pre)	LB (SAT)	sec	UB* (SAT)	sec	LB (known)	UB (known)
4/72		0		0	0.10	0	0
6/76		6	673.87	6	0.05	1	6
10/93		1	7.70	3	0.18	1	3
16/81		0		0	0.07	0	0
19/71	2		timeout	2	0.39	2	2
21/90	2	1	150.66	2	0.11	1	2
36/92		1	31.61	1	0.21	1	2
41/66		0		0	0.04	0	0
26/82		0		0	0.10	0	0
200_01		0		0	33.11		0
200_02	2	1	33.11	2	1.39		2
200_03		2	1258.10	3	28.33		3
200_04	7	1	4.36	7	3.92		7
200_05		1	1109.32	3	2.58		6
200_06	6		timeout	6	9.55		6
200_07		0		0	0.62		0
200_08	8		timeout	8	5.46		8
200_09	10	1	496.46	10	1.56		10
200_10	17	14	798.10	17	6.28		19
300_01		0		0	3.74		0
300_02				6	17.60		12
300_03	13	2	586.03	13	15.03		13
300_04	7	6	274.48	7	21.48		7
300_05	2	10	1101.27	17	899.48		27
300_06	2		timeout	2	337.12		2
300_07		0		0	1.59		0
300_08	8	1	102.26	8	30.37		8
300_09	7		timeout	7	5.81		7
300_10	3	8	1200.46	12	1426.53		21
400_01			timeout		timeout		1
400_02	15		timeout	15	62.69		15
400_03		8	1543.67		timeout		9
400_04	19	5	60.13	19	550.03		19
400_05		0		0	143.58		0
400_06		0		0	20.96		0
400_07			timeout		timeout		4
400_08	4		timeout		timeout		4
400_09		4	633.61	5	57.80		5
400_10		0		0	4.28		0