CNF Encodings for the Car Sequencing Problem

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Introduction

- Cars require different options (air-conditioning, sun-roof, etc.)
- Is there a production sequence for cars on the assembly line satisfying the sliding capacity constraints?
- CSPLib Benchmark Nr. 1

Example

- Classes $C = \{1, 2, 3\}$ with demand $d_1 = 3, d_2 = 2, d_3 = 2$
- Options $O = \{a, b\}$ with capacity constraints $1/2$ and $1/5$
- Class 1: $\emptyset$, Class 2: $\{a\}$, Class 3: $\{a, b\}$

Sequence of cars	3 1 2 1 2 1 3
Option **a**	1 - 1 - 1 - 1
Option **b**	1 - - - - - 1

PB Model

- Boolean variable c_i^k: car $k \in C$ is at position i
- Boolean variable o_i^l: option $l \in O$ is at position i
- Demand constraints: $\forall k \in C$

$$\sum_{i=1}^{n} c_i^k = d_k$$

- Capacity constraints: $\forall l \in O$ with ratio u_l/q_l

$$\bigwedge_{i=0}^{n-q_l} \left(\sum_{j=1}^{q_l} o_{i+j}^l \leq u_l\right)$$

And in all positions $i \in \{1 \dots n\}$ of the sequence it must hold:

- Link between classes and options: for each $k \in C$ and

$$\forall l \in O_k : \; c_i^k - o_i^l \leq 0$$
$$\forall l \in O \setminus O_k : \; c_i^k + o_i^l \leq 1$$

- Exactly one car:

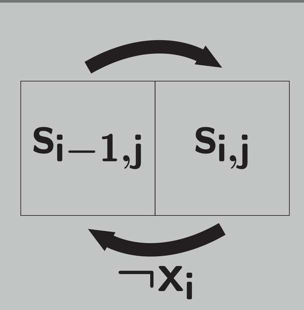
$$\sum_{k \in C} c_i^k = 1$$

SAT Approach: The Ultimate Decomposition

- ONE constraint: e.g. $a \vee b \vee \neg c$.
- ONE propagator: e.g. a and $\neg a \vee b$ then propagate b.
- Use SAT solver as a blackbox and inherit all good properties!
- Central constraint: $(\sum_{i=1}^{n} x_i = d) \wedge \bigwedge_{i=0}^{n-q} (\sum_{l=1}^{q} x_{i+l} \leq u)$
- Use cumulative sums:

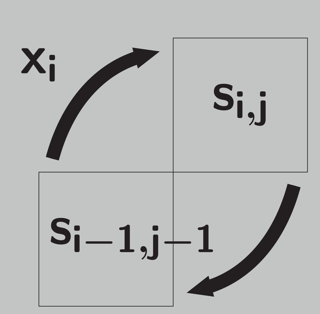
$$s_{i,j} \iff (j \leq \sum_{l=1}^{i} x_l)$$

Counter based Clause Set



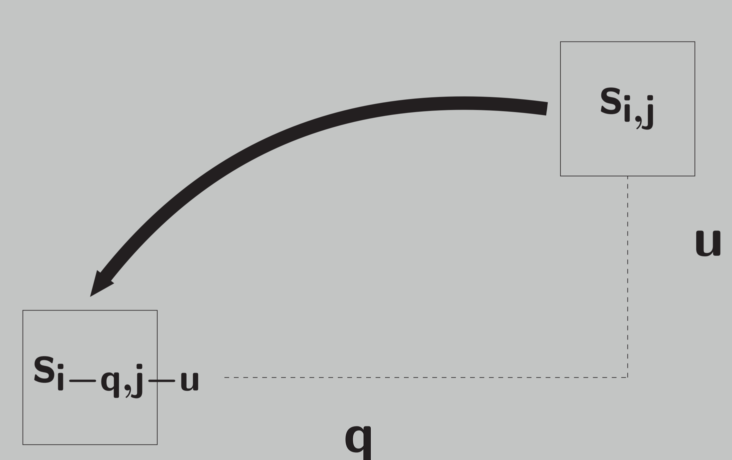
$\neg s_{i-1,j} \vee s_{i,j}$

$x_i \vee \neg s_{i,j} \vee s_{i-1,j}$



$\neg x_i \vee \neg s_{i-1,j-1} \vee s_{i,j}$

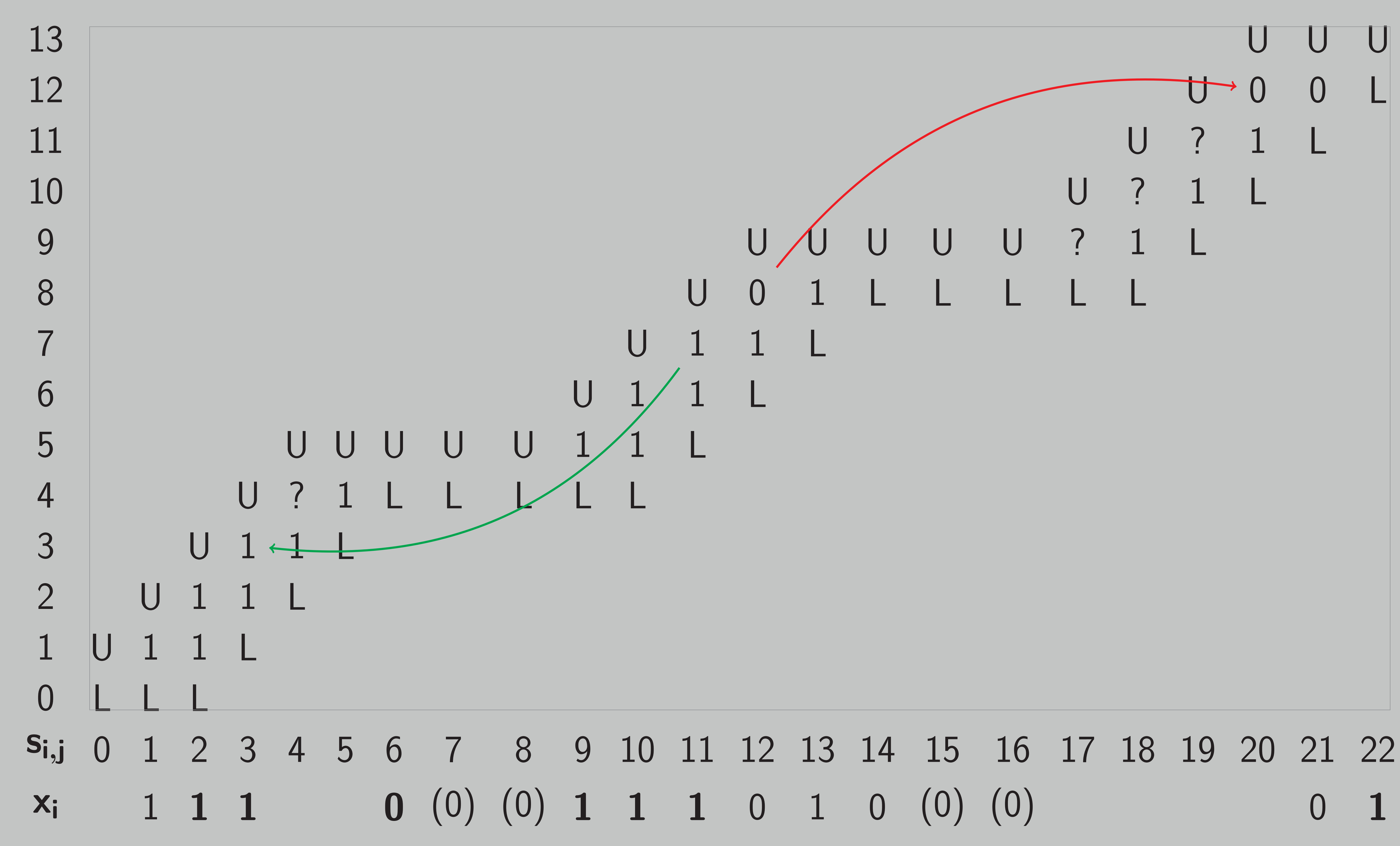
$\neg s_{i,j} \vee s_{i-1,j-1}$



$\neg s_{i,j} \vee s_{i-q,j-u}$

Example: 22 Cars, Capacity 4/8, Demand d = 12

Partial Assignment: x_1 and x_{13} to true and x_{12}, x_{14} and x_{21} to false.



Results on the harder CSPLib Instances

	E1	E2	E3	ASP	PB
#solved UNSAT	**17**	15	**17**	10	8
#fastest UNSAT	**5**	4	4	0	4
#solved SAT	**11**	**11**	**11**	7	2
#fastest SAT	0	4	**7**	0	0

E1-E3=variants of SAT, PB=minisat+, ASP=Clasp

Conclusions and Future Work

- Conclusions
  - SAT can be very competitive on CP benchmarks
  - SAT is very strong on showing unsatisfiability
  - Global Constraints motivate for encodings
  - Choosing the right encoding of cardinality constraints is crucial
- Current and Future work:
  - Fair Comparison to CP, IP, ASP, LS ...
  - Clean proof of GAC and lower bound on size
  - Idea useful in rostering, planning, scheduling?
  - Exponential encoding in the number of options?
  - New instances!

Discussion: Related Work

- Sinz: Sequential Counter CNF [5]
- Een and Soerensson: Translation through BDDs to CNF [3]
- Bacchus: Decomposition through DFAs to CNF [1]
- Brand et al: Decomposition to cumulative sums for CP [2]
- Siala et al: Linear time propagator for CP [4]

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