CNF Encodings for the Car Sequencing Problem

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Introduction

- Cars require different options (air-conditioning, sun-roof, etc.)
- Is there a production sequence for cars on the assembly line satisfying the sliding capacity constraints?
- CSPLib Benchmark Nr. 1

Example

- Classes $\mathbf{C} = \{1, 2, 3\}$ with demand $\mathbf{d_1 = 3, d_2 = 2, d_3 = 2}$
- Options $\mathbf{O} = \{\mathbf{a}, \mathbf{b}\}$ with capacity constraints $1/2$ and $1/5$
- Class 1 no restriction
- Class 2 requires option **a**
- Class 3 requires option **a** and **b**

Sequence of cars	3 1 2 1 2 1 3
Option a	1 - 1 - 1 - 1
Option b	1 - - - - - 1

PB Model

- Boolean variable $\mathbf{c_i^k}$: car $\mathbf{k} \in \mathbf{C}$ is at position $\mathbf{i}$
- Boolean variable $\mathbf{o_i^l}$: option $\mathbf{l} \in \mathbf{O}$ is at position $\mathbf{i}$
- Demand constraints: $\forall \mathbf{k} \in \mathbf{C}$

$$\sum_{i=1}^{n} c_i^k = d_k$$

- Capacity constraints: $\forall \mathbf{l} \in \mathbf{O}$ with ratio $\mathbf{u_l / q_l}$

$$\bigwedge_{i=0}^{n-q_l} \left(\sum_{j=1}^{q_l} o_{i+j}^l \leq u_l\right)$$

And in all positions $\mathbf{i} \in \{1 \dots n\}$ of the sequence it must hold:

- Link between classes and options: for each $\mathbf{k} \in \mathbf{C}$ and

$$\forall l \in O_k : \ c_i^k - o_i^l \leq 0$$
$$\forall l \in O \setminus O_k : \ c_i^k + o_i^l \leq 1$$

- Exactly one car:

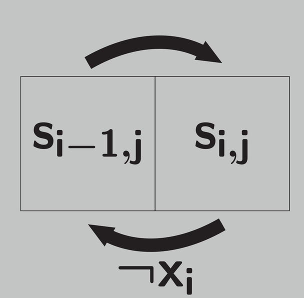
$$\sum_{k \in C} c_i^k = 1$$

The SAT Approach: The Ultimate Decomposition

- ONE constraint: e.g. $\mathbf{a} \vee \mathbf{b} \vee \neg \mathbf{c}$.
- ONE propagator: e.g. $\mathbf{a}$ and $\neg \mathbf{a} \vee \mathbf{b}$ then propagate $\mathbf{b}$.
- Using SAT solvers as blackboxes.
- Central constraint in car sequencing representing cars and options:

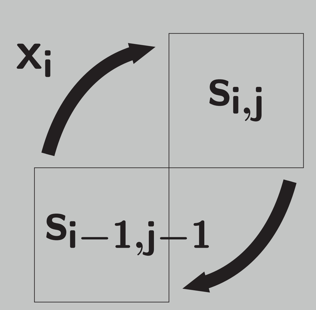
$$\left(\sum_{i=1}^{n} x_i = d\right) \wedge \bigwedge_{i=0}^{n-q} \left(\sum_{l=1}^{q} x_{i+l} \leq u\right)$$

Counter based Clause Set



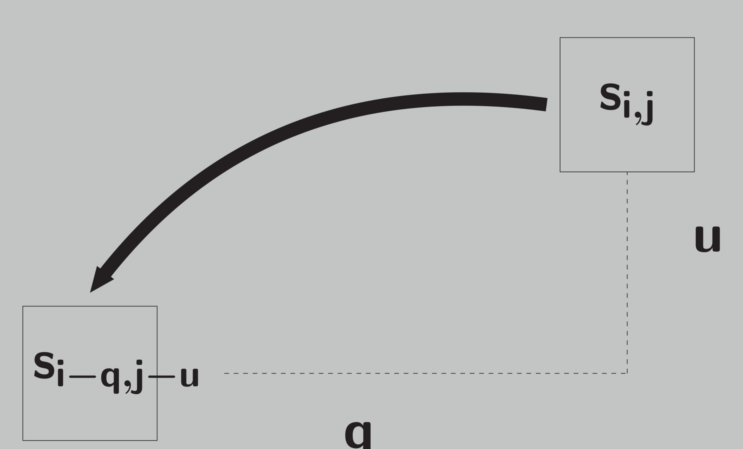
$\neg s_{i-1,j} \vee s_{i,j}$

$x_i \vee \neg s_{i,j} \vee s_{i-1,j}$



$\neg x_i \vee \neg s_{i-1,j-1} \vee s_{i,j}$

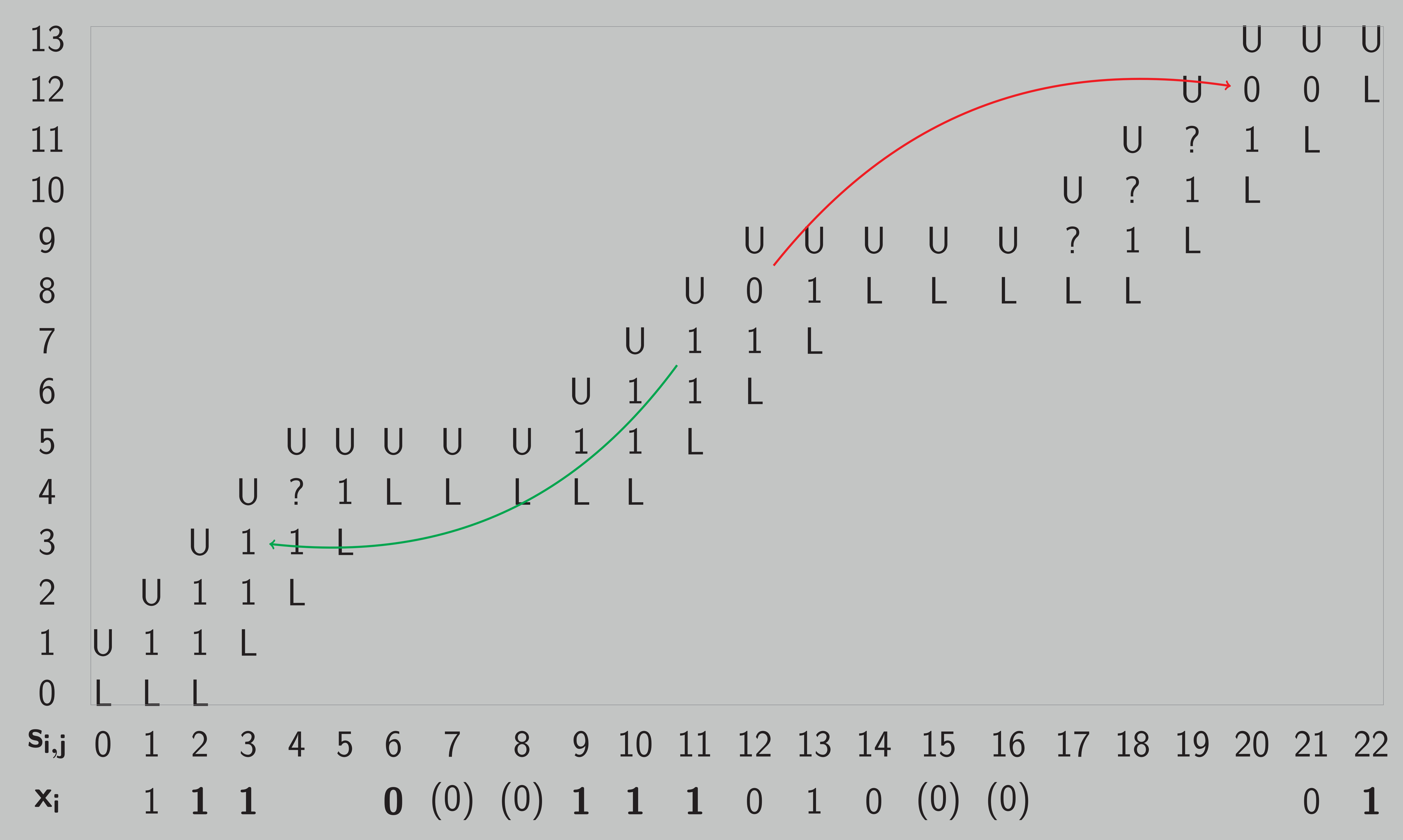
$\neg s_{i,j} \vee s_{i-1,j-1}$



$\neg s_{i,j} \vee s_{i-q,j-u}$

Example: 22 Cars, Capacity 4/8, Demand d = 12

Partial Assignment: $\mathbf{x_1}$ and $\mathbf{x_{13}}$ to true and $\mathbf{x_{12}}$, $\mathbf{x_{14}}$ and $\mathbf{x_{21}}$ to false.



Conclusions and Future Work

- Conclusion
  - SAT can be very competitive on CP benchmarks
  - SAT is very strong on showing unsatisfiability
  - Global Constraints motivate for encodings
  - Choosing the right encoding of cardinality constraints is crucial
- Future work:
  - Comparison to CP and IP
  - Theoretical analysis of the decompositions and usage in other domains
  - Exponential encoding in the number of options?

Discussion: Related Work

- Sinz: Sequential Counter CNF [?]
- Een and Soerensson: Translation through BDDs to CNF [?]
- Bacchus: Decomposition through DFAs to CNF [?]
- Brand et al: Decomposition to cumulative sums for CP [?]
- Siala et al: Linear time propagator for CP [?]

Bibliography

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