SAT Benchmark for the Car Sequencing Problem

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***Abstract*—Car sequencing occurs in the production process of the automotive industry. It addresses the problem of scheduling cars along an assembly line such that capacities of different workstations along the line are not exceeded. We provide the SAT competition with a selection of hard car sequencing problems from the CSP$_{\mathrm{LIB}}$ [1]. The encoding is based on a variant of the sequential counter encoding of cardinality constraints and the reuse of auxiliary variables.**

I. Introduction

Car sequencing deals with the problem of scheduling cars along an assembly line with capacity constraints for different stations (e.g. radio, sun roof, air-conditioning, etc). Cars are partitioned into classes according to their requirements. The stations are denoted as *options* and defined by a ratio u/q restricting the maximal number u of cars that can be scheduled on every subsequence of length q.

Example 1 *Given classes $C = \{1, 2, 3\}$ and options $O = \{a, b\}$. The demands (number of cars) for the classes are $3, 2, 2$, respectively. Capacity constraints on options are given by $a : 1/2$ and $b : 1/5$, respectively. Class 1 has no restrictions, class 2 requires option a and class 3 needs options $\{a, b\}$. The only legal sequence for this problem is $[3, 1, 2, 1, 2, 1, 3]$, since class 2 and 3 cannot be sequenced after another and class 3 need to be at least 5 positions apart.*

Car sequencing in the CSPlib contains a selection of benchmark problems of this form ranging from 100 to 400 cars. Over the years different approaches have been used to solve these instances, among them constraint programming, local search and integer programming [2][3][4][5][6].

Car sequencing has also been treated as an optimisation problem and several versions for the opimisation goal have been proposed. Most of the approaches use a variant of minimising the number of violated capacity constraints. However, for this benchmark we use the definition of [7] which transforms easily to sequence of decision problem and SAT solving can be directly applied: An unsatisfiable car sequencing problem can be made solvable by adding empty slots to the sequence. The goal is then to minimise the number of empty slots needed for a valid sequence. A lower bound lb is proven by unsatisfiability with $lb-1$ additional empty slots.

II. The Encoding

The car sequencing problem can be naturally modelled by Boolean cardinality constraints. Our approach is to translate cardinality constraints by a variant of the sequential counter encoding proposed by [8]. The key idea is then to integrate capacity constraint into the sequential counter of the demand constraints by reusing the auxiliary variables. This enforces a global view on the conjunction of these two constraints and facilitates propagation. Our own experiments show that this encodings is far better than naive approaches or an automatic translation from the pseudo Boolean model.

III. The Benchmark

A command line tool that generates CNF in DIMACS format from a problem description in the CSPlib is freely available at `github.com/vale1410/car-sequencing`. With this tool one can generate different encodings and compare the runtime of SAT solvers. For this benchmark we chose the best encoding according to our experiments and we are interested if the solvers from the competition are able to prove stronger bounds.

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