

# What is a category ?

In animated pictures ...

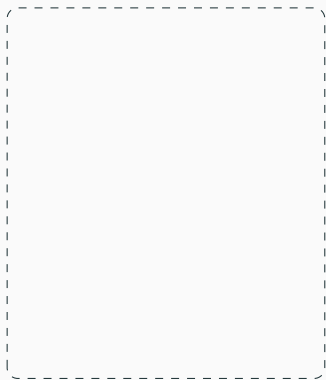
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Frédéric Peschanski

Sorbonne University – LIP6

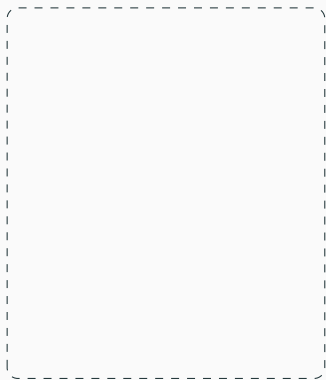
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# What is a category ?



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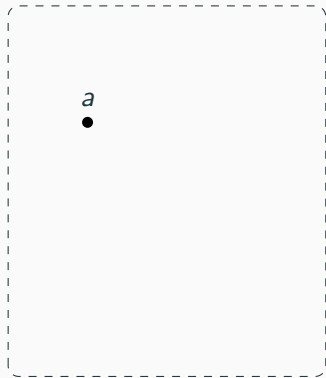
Objects



# What is a category ?

Objects

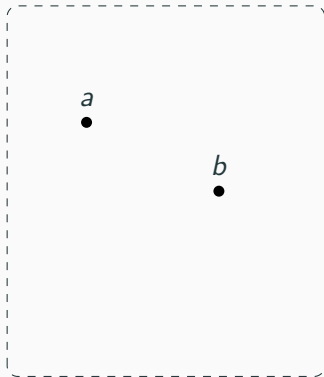
$a$



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Objects

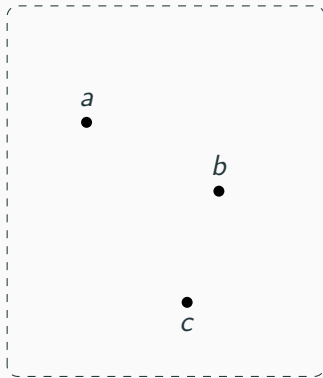
$a$     $b$   
●   ●



# What is a category ?

Objects

$a$     $b$     $c$    ...  
●   ●   ●   ...

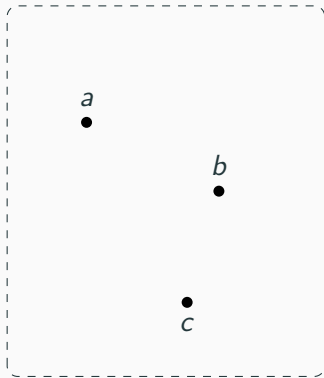


# What is a category ?

Objects

$a$     $b$     $c$    ...  
•   •   •   ...

Morphisms (arrows)



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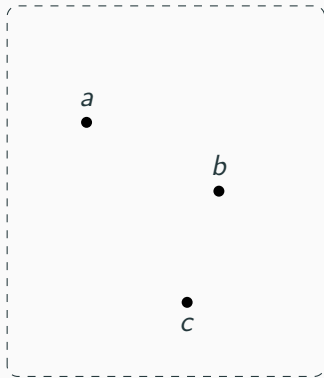
Objects

$a$     $b$     $c$    ...  
•   •   •   ...

Morphisms (arrows)

$a$     $f$     $b$   
•    $\longrightarrow$    •

$f :: a \longrightarrow b$





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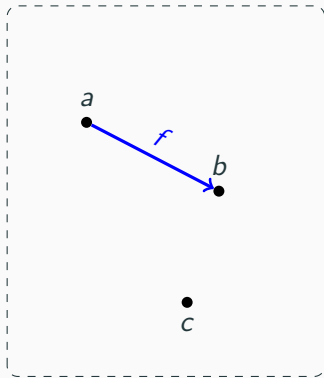
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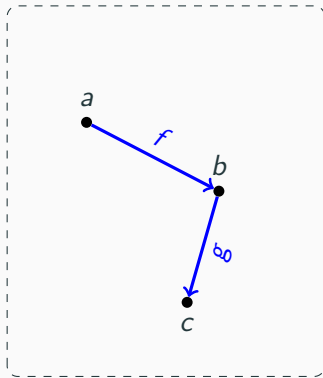
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Morphisms (arrows)



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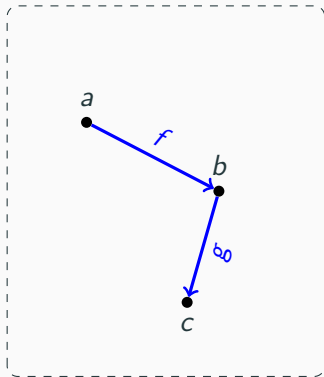
$a$     $b$     $c$    ...  
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Morphisms (arrows)

$a$     $f$     $b$   
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Identities



# What is a category ?

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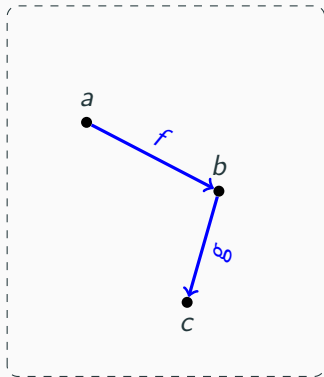
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Identities

●  
 $a$



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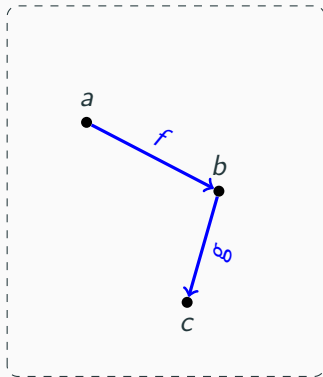
Morphisms (arrows)

$a$     $f$     $b$   
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$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} \bullet$     $id_a :: a \longrightarrow a$   
 $a$



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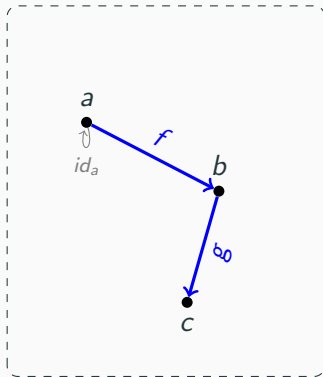
Morphisms (arrows)



$f :: a \longrightarrow b$

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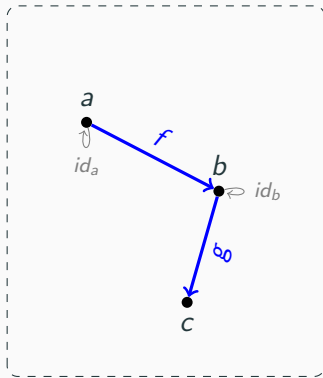
Morphisms (arrows)

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Identities

$\bullet \xrightarrow{id_a} a$     $id_a :: a \longrightarrow a$



# What is a category ?

Objects

$a \quad b \quad c \quad \dots$

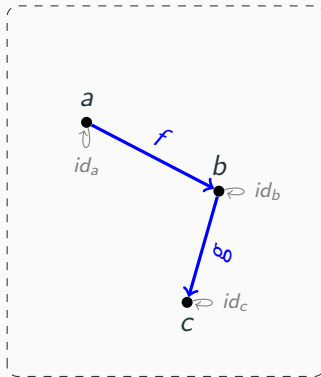
Morphisms (arrows)



$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} a \quad id_a :: a \longrightarrow a$





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$a$     $b$     $c$    ...  
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Morphisms (arrows)

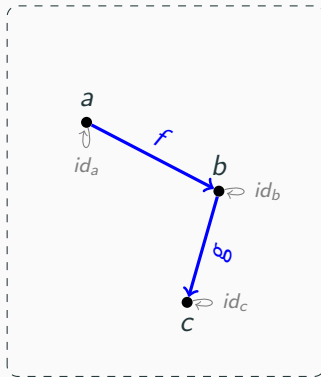
$a$     $f$     $b$   
•    $\xrightarrow{\quad f \quad}$    •

$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} a$     $id_a :: a \longrightarrow a$

Compositions



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Morphisms (arrows)

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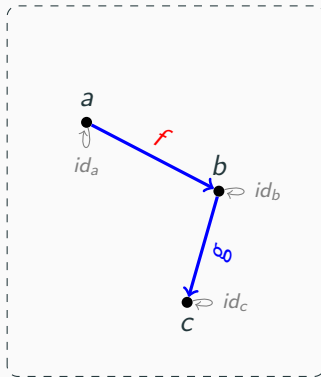
$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} a$     $id_a :: a \longrightarrow a$

Compositions

if  $f :: a \longrightarrow b$



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Morphisms (arrows)

$a$     $f$     $b$   
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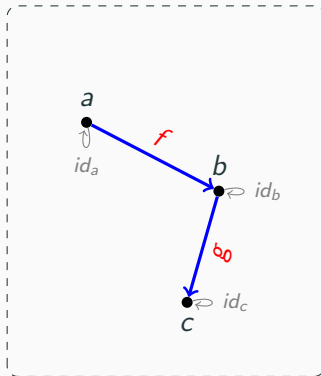
$f :: a \longrightarrow b$

Identities

$\bullet \xrightarrow{id_a} a$     $id_a :: a \longrightarrow a$

Compositions

if  $f :: a \longrightarrow b$    and  $g :: b \longrightarrow c$



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Morphisms (arrows)

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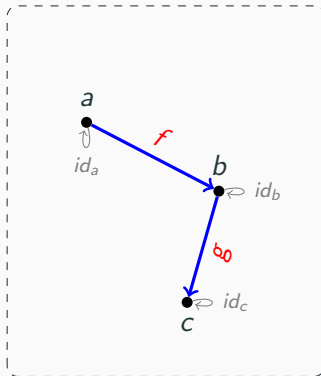
Identities

$\bullet \xrightarrow{id_a} a$     $id_a :: a \longrightarrow a$

Compositions

if  $f :: a \longrightarrow b$    and  $g :: b \longrightarrow c$

then  $g \circ f :: a \longrightarrow c$



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Morphisms (arrows)

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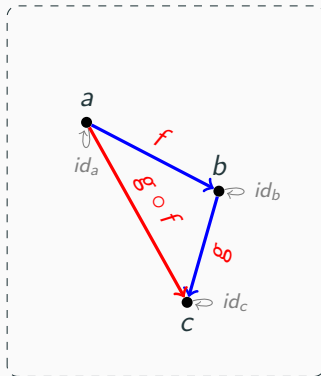
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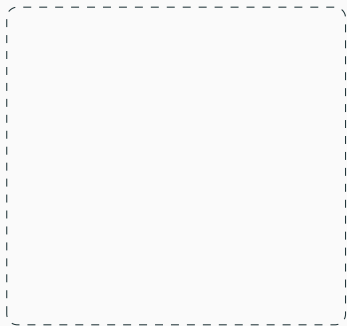
Compositions

if  $f :: a \longrightarrow b$    and  $g :: b \longrightarrow c$

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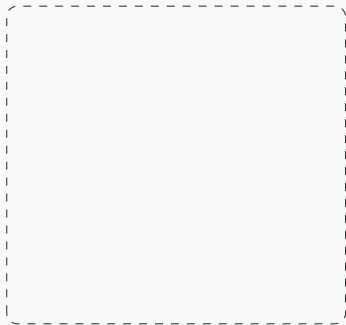


# The laws of categories



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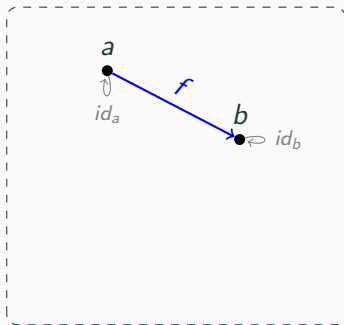
**Laws of identity :**



# The laws of categories

## Laws of identity :

For any  $f :: a \longrightarrow b$ ,



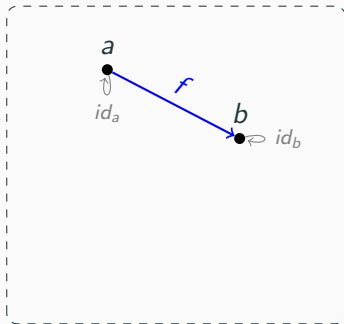


# The laws of categories

## Laws of identity :

For any  $f :: a \longrightarrow b$ ,

- $f \circ Id_a = f$

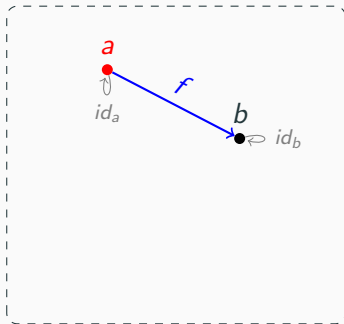


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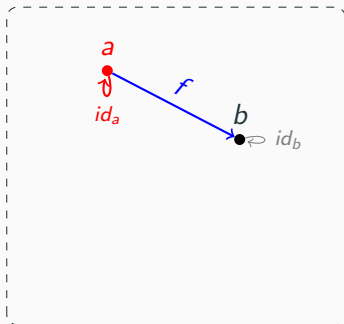


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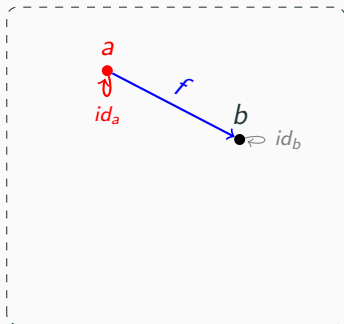


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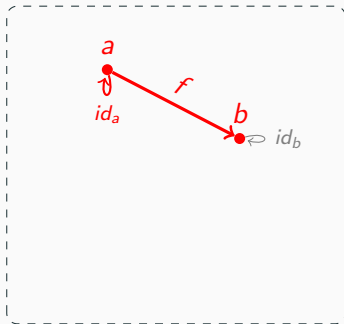


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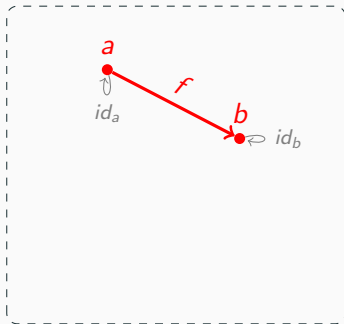


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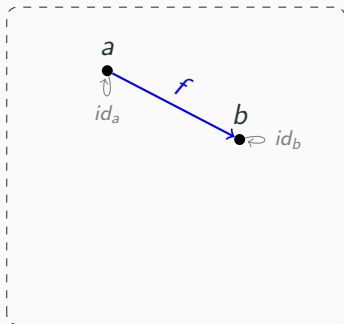


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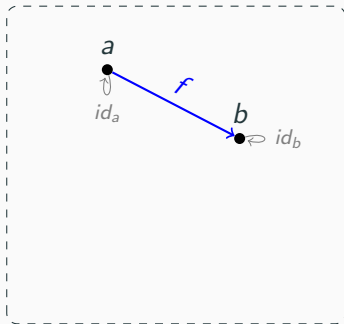


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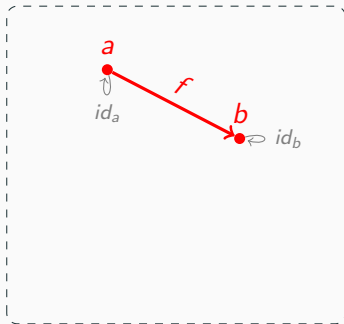


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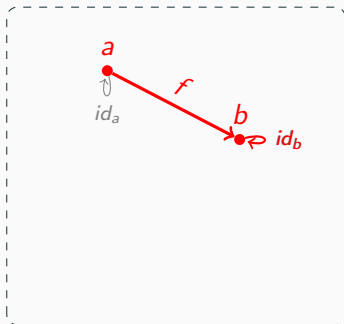


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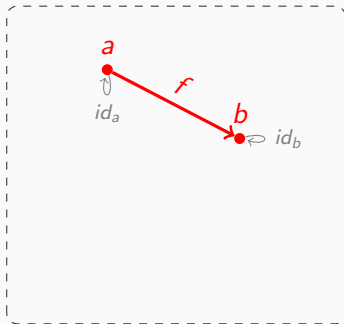


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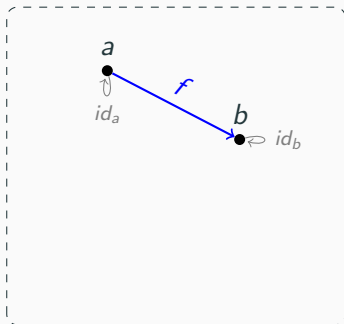
# The laws of categories

## Laws of identity :

For any  $f :: a \longrightarrow b$ ,

- $f \circ id_a = f$
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## Law of associativity :



# The laws of categories

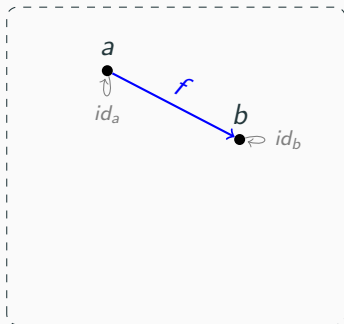
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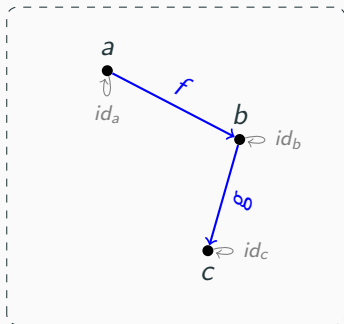
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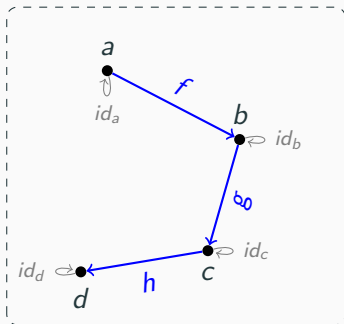
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For any  $f :: a \longrightarrow b$ ,

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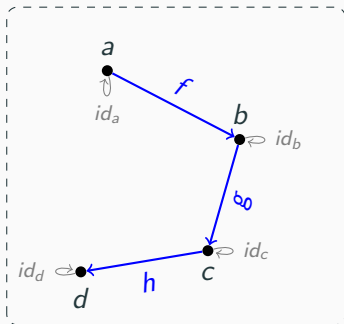
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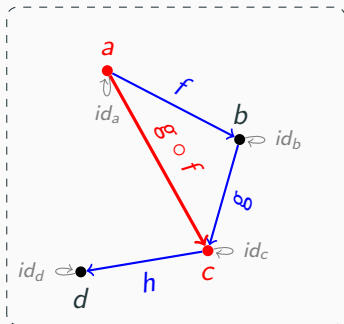
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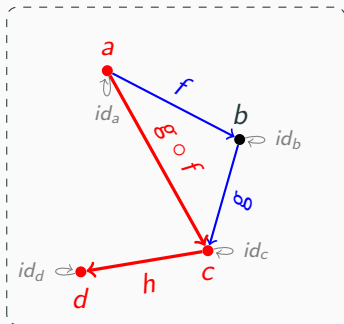
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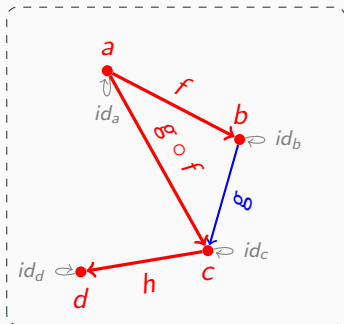
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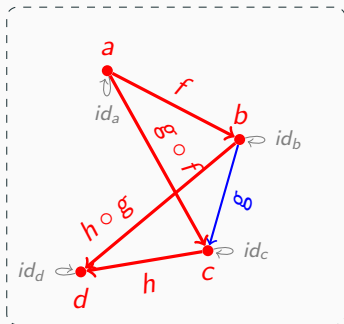
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