CS 325 – Asymptotic Analysis

Practice Problems

Big-O, Ω,Θ Examples

For each of the following pairs of functions, either f(n) is O(g(n)), f(n) is Ω(g(n)), or f(n) = Θ(g(n)). Determine which relationship is correct.

Try all the problems first before watching the solution video.

1) f(n) 0.00001n^3; g(n) = 500000n + 4000000

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$$

$$f(n) = \Omega(g(n)) \quad \text{or} \quad g(n) = O(f(n))$$

1) f(n) = logn^3; g(n) = logn + 5

$$f(n) = \log n^3 = 3\log n$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{3\log n}{\log n + 5} = 3$$

$$f(n) = \Theta(g(n)) \quad \text{or} \quad g(n) = \Theta(f(n))$$

3) f(n)=log(logn);   g(n) = logn

let $u = \log n$

compare

$\log u$ to u

$\log u = O(u)$

so $\log(\log n) = O(\log n)$

$f(n) = O(g(n))$ or

$g(n) = \Omega(f(n))$

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4) f(n) = $\log n^3$;   g(n) = $\log^3 n$

$f(n) = 3 \log n$ $g(n) = (\log n)^3$

$f(n) = O(g(n))$

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5) f(n)=nlogn;   g(n) = log(n!)

$g(n) = \log(n \cdot (n-1) \cdot (n-2) \cdots 1)$

$= \log n + \log(n-1) + \log(n-2) \cdots 1$

$\leq \log n + \log n + \log n \cdots \log n$

$g(n) \leq n\log n \Rightarrow g(n) = O(n\log n)$

By using Wolfram Alpha

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$ so $g(n) = \Theta(n\log n)$

6) f(n)=10;   g(n) = log10

Both are constants

$f(n) = \Theta(g(n)) = \Theta(1)$

7) $f(n) = 2^n$; $g(n) = 10n^2$

$f(n) = \Omega(g(n))$

o. $g(n) = O(f(n))$

8) $f(n) = 4^n$; $g(n) = 2^{2n}$; $h(n) = 2^{n+1}$

$g(n) = 2^{2n} = (2^2)^n = 4^n$

so $f(n) = \Theta(g(n))$

$$\lim_{n\to\infty} \frac{f(n)}{h(n)} = \lim_{n\to\infty} \frac{4^n}{2^{n+1}} = \lim_{n\to\infty} \frac{4^n}{2 \cdot 2^n} = \lim_{n\to\infty} \frac{1}{2}(2)^n = \infty$$

so $f(n) = \Omega(h(n))$ or

$h(n) = O(f(n))$

9) g(n) = 2^{2n}; h(n) = 2^{n^2}

$h(n) = 2^{n^2}$

$\$$ Compare exponents $2n$ to n^2

$2n = O(n^2)$

so $g(n) = O(h(n))$

or $\lim_{n\to\infty} \frac{2^{2n}}{2^{n^2}} = 0$

10. Prove or disprove (with a counterexample).

If $f_1(n) = O(g_1(n))$ and $f_2(n) = O(g_2(n))$ then $f_1(n)+f_2(n) = O(\max\{g_1(n), g_2(n)\})$.

Prove true

If $f_1(n) = O(g_1(n))$ then there exists constants c_1 & n_1 such that for all $n \geq n_1$

(I) $0 < f_1(n) \leq c_1 g_1(n)$

& since $f_2(n) = O(g_2(n))$ there exist constants c_2 and n_2 such that for all $n \geq n_2$

(II) $0 < f_2(n) \leq c_2 g_2(n)$

Let $g = \max\{g_1(n), g_2(n)\}$ for $n \geq (n_1+n_2)$

(III) $f_1(n) \leq g$

(#) $f_2(n) \leq g$

By adding III & IV

$$0 < f_1 + f_2 \leq g + g$$

$$0 < f_1 + f_2 \leq 2g \quad \text{for } n \geq (n_1 + n_2)$$

$c = 2$

Therefore

$$f_1(n) + f_2(n) \leq c\,g(n)$$

and $$f_1(n) + f_2(n) = O(g(n))$$

or $$f_1(n) + f_2(n) = O\left(\max\{g_1(n), g_2(n)\}\right)$$