# Master Method Case 3: Regularity Fails

$T(n) = aT(n/b) + f(n)$

Case 3 is root dominating $f(n) = \Omega(n^{\log_b a+\epsilon})$:

- The regularity condition $af(n/b) \le cf(n)$ for some $c < 1$, shows that total work decreases as n decreases
- Almost all functions will satisfy the regularity condition: $n^k$, $\lg^k(n)$, $2^k$, $n!$

What DOES NOT SATISFY the regularity conditions?

Idea: You want f(n) to possibly grow (or shrink by only a factor of a) when you decrease n to n/b. Functions like 1/n and $(0.5)^n$ would not be regular, but with these functions you wouldn't choose Case 3 in the first place.

It is actually a little bit tricky to find a case where we select Case 3 but then find that the regularity condition is violated. Virtually all functions you'll see in this class will satisfy the regularity condition. If the math for the following examples is to complicated you can skip it. Here is one case where the regularity condition makes a difference:

**Example 1:**

$T(n) = T(n/2) + n(2 - \cos n)$ with $a = 1$ $b = 2$ and $f(n) = n(2 - \cos n)$

$\log_2 1 = 0$ so compare f(n) to $n^0 = 1$

$n(2-\cos n) = \Omega(1)$

Case 3 : So check the regularity condition

The regularity condition is

$af(n/b) \le cf(n)$; $c<1$

$cf(n) = cn(2 - \cos n)$

$af(n/b) = n/2(2 - \cos(n/2))$

$n/2(2 - \cos(n/2)) \le cn(2 - \cos n)$ if $n = 2k\pi$ and k is odd then

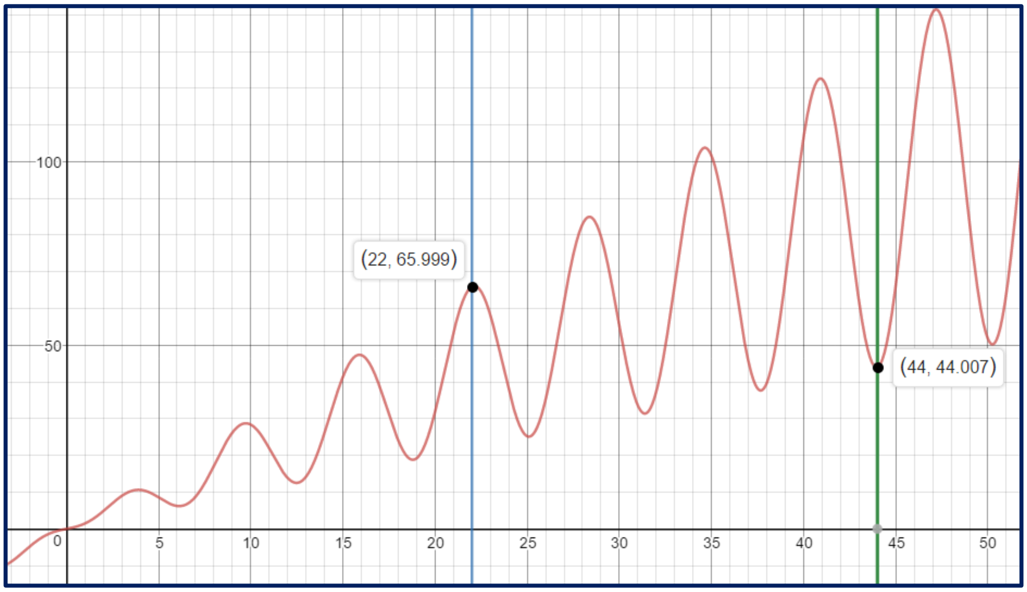
$k\pi(2 - \cos(k\pi)) \le 2ck\pi(2 - \cos(2k\pi))$

$k\pi(2 - (-1)) \le 2ck\pi(2 - 1)$

$3k\pi \le 2ck\pi$

$c \ge 3/2$

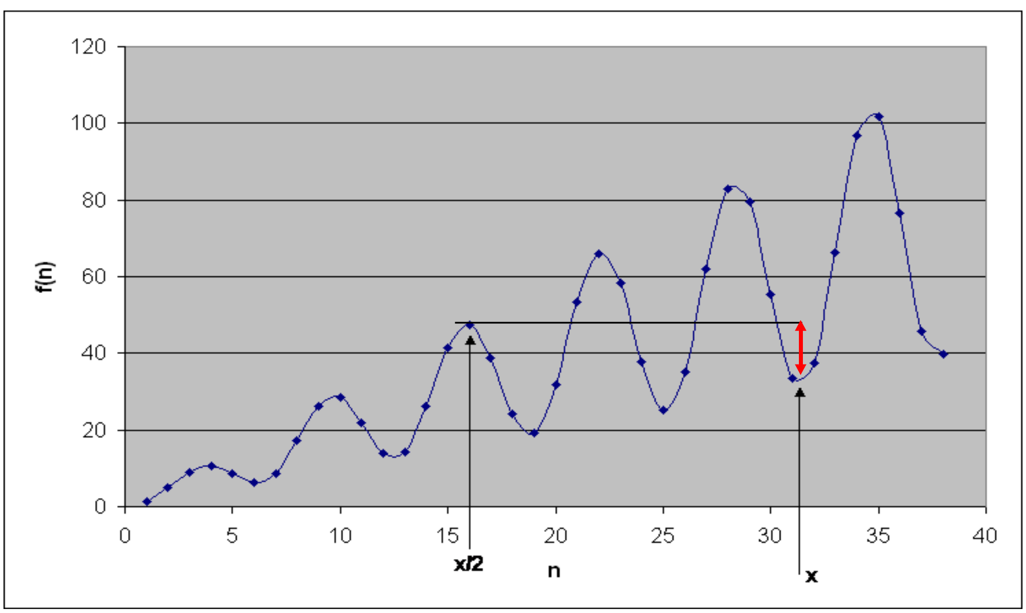
but the restriction is that $c < 1$, so regularity fails and we cannot use the master method
Below is a graph of $f(n) = n(2 - \cos n)$




**Example 2:**

$T(n) = T(n/2) + n(\sin(n - \pi/2) + 2)$

Have a look at this $f(n) = n(\sin(n-\pi/2) + 2)$. The interesting thing is that you can choose x such that $f(x/2)$ is bigger than $f(x)$, even though $f(n) \geq n$ for all n.



Here's the formal analysis:

T(n) = T(n/2) + n(sin(n - π/2) + 2)

We are in Case 3 of the Master Theorem, a=1, b=2, f(n) = n(sin(n - π/2) + 2) and the regularity condition is:

$$(n/2)\,(\sin(n/2 - \pi/2) + 2) \le c\, n(\sin(n - \pi/2) + 2)$$

$$c \ \geq (1/2)\frac{\sin\left(\frac{n}{2}-\frac{\pi}{2}\right)+2}{\sin\left(n-\frac{\pi}{2}\right)+2}$$

To see it is impossible to satisfy this for all large n, choose: n = 2 π k where k is odd.

Then sin(n/2 - π/2) = sin(π k - π/2) = sin(π/2) = 1
and sin(n - π/2) = sin(2πk - π/2) = sin(-π/2) = -1

We get

$$c \ \geq (1/2)\frac{1+2}{-1+2}$$

$$c \ \geq (1/2)\frac{3}{1}$$

$$c \ \geq 3/2$$

Thus, we can't choose c < 1 to satisfy the condition.