Visualisation and Analysis of Geographic Information

Algorithms and Data Structures

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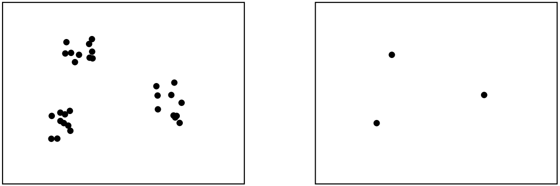
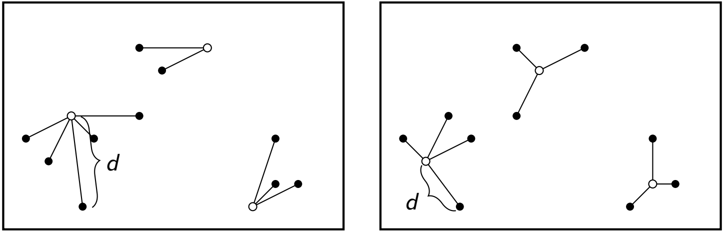
Motivation

- Reduce visual information when displaying large numbers of geographic points
- Find a representative subset of a collection of geographic points



Coverage

- Minimising Coverage



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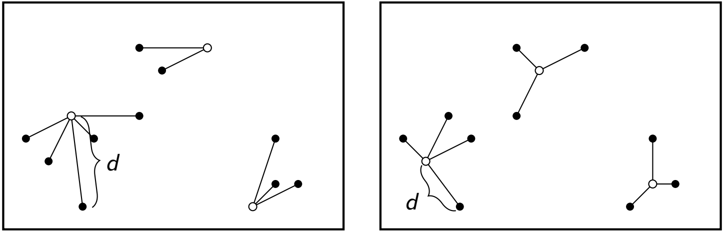

$$\min_{\substack{P \subseteq N \\ |P|=k}} \max_{n \in N} \min_{p \in P} \|p - n\|$$

Work Plan

- 1^{st} Semester
  - Literature Review: Geographic Information Systems, OGC Standards WMS, WFS, Map Projections, algorithms and heuristics for clustering and facility-location problems.
  - Development of a Branch-and-Bound approach.
- 2^{nd} Semester
  - Development of heuristic approaches.
  - Experimental analysis of the algorithms.
  - Integration of the algorithms in the visualisation framework through web-mapping standards (WMS/WFS).
  - Comparison between different approaches using Open Street Map data.

Integer Linear Programming

$$
\begin{aligned}
\text{minimise} \quad & D \\
\text{subject to} \quad & \sum_{j=1}^{N} y_j = k \\
& \sum_{j=1}^{N} x_{ij} = 1 && i = 1, \ldots, N \\
& \sum_{j=1}^{N} d_{ij} x_{ij} \le D && i = 1, \ldots, N \\
& x_{ij} \le y_j && i = 1, \ldots, N; j = 1, \ldots, N \\
& x_{ij}, y_j \in \{0, 1\} && i = 1, \ldots, N; j = 1, \ldots, N
\end{aligned}
$$

Branch-and-bound

- Branching
  - Divide search space in a binary tree
  - At each step, decide if a point is a centroid or non-centroid
  - Update objective function accordingly

Branch-and-bound

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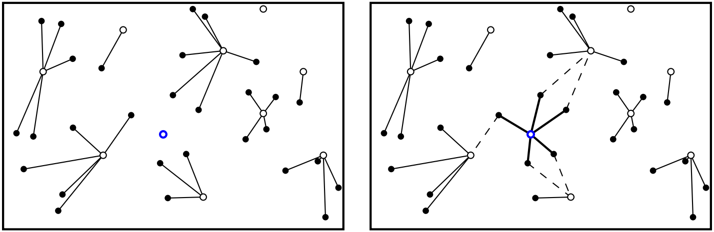
  - Divide search space in a binary tree
  - At each step, decide if a point is a centroid or non-centroid
  - Update objective function accordingly
- Bound
  - Assume best possible case
  - Prune tree

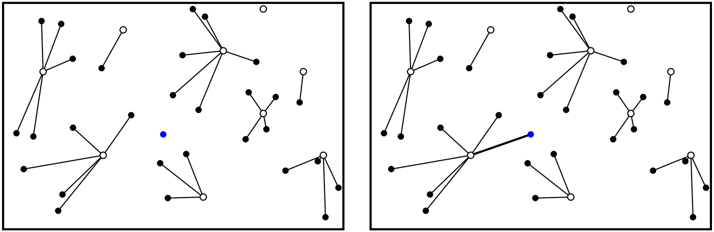
Branch-and-bound

- Inserting a Centroid
  - Search all non-centroids for assignment update
  - Smaller coverage value



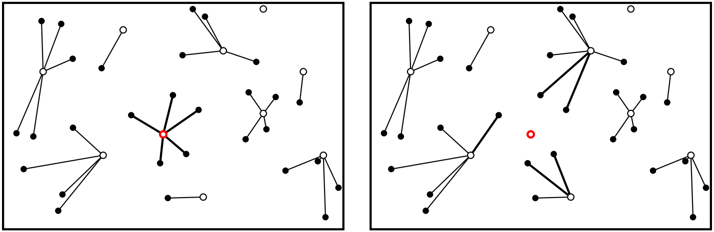
Branch-and-bound

- Inserting a Non-centroid
  - Search for closest centroid
  - Higher coverage value



Branch-and-bound

- Removing a Centroid
  - Update all non-centroids
  - Higher coverage value

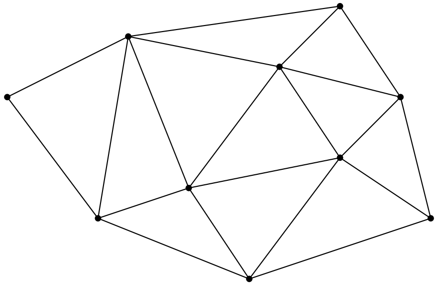


Geometric Approach

- Unnecessary number of calculations
  - Use geometric structures to speed objective function update

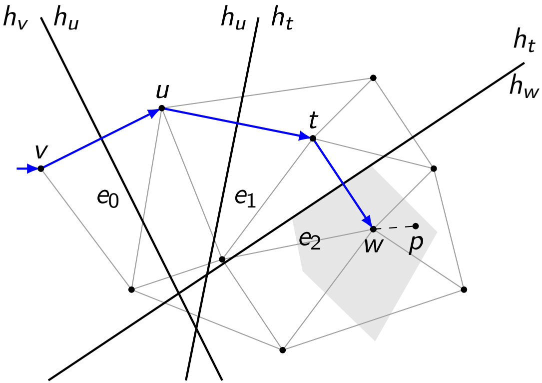
Geometric Approach

- Unnecessary number of calculations
  - Use geometric structures to speed objective function update
  - Delaunay triangulations



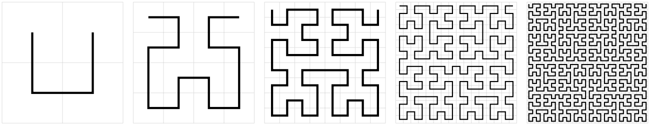
Geometric Approach

- Greedy Routing



Geometric Approach

- Greedy Routing
- Use Hilbert curves to minimise distance between consecutive routing calls



Geometric Approach

- Initialize triangulation
- Sort points using a Hilbert curve
- Inserting a Centroid
  - Insert centroid in triangulation
  - Search all neighbours for closer non-centroids for assignment update
  - Smaller coverage value
- Inserting a Non-centroid
  - Search for closest centroid using greedy routing
  - Higher coverage value

Geometric Approach

- Initialize triangulation
- Sort points using a Hilbert curve
- Inserting a Centroid
  - Insert centroid in triangulation
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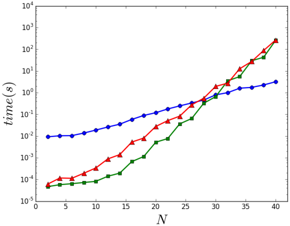
  - Smaller coverage value
- Inserting a Non-centroid
  - Search for closest centroid using greedy routing
  - Higher coverage value
- Removing a Centroid
  - Revert assignment
  - Remove centroid from triangulation
  - Higher coverage value
- Removing a Non-centroid
  - Update objective function
  - Revert assignment
  - Smaller coverage value

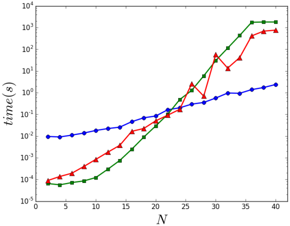
Algorithm Comparison

Algorithm	Insert		Remove	
	Centroid	Non-Centroid	Centroid	Non-Centroid
Naïve BB	$\Theta(N)$	$\Theta(K)$	$\Theta(N)$	$\mathcal{O}(1)$
Geometric BB Average Case	$\mathcal{O}(\log K + N/K)$	$\mathcal{O}(\sqrt{K})$	$\mathcal{O}(N/K)$	$\mathcal{O}(1)$
Geometric BB Worst Case	$\mathcal{O}(K + N)$	$\mathcal{O}(K)$	$\mathcal{O}(N)$	$\mathcal{O}(1)$

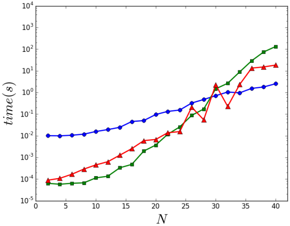
Algorithm Comparison



(a) $K = 0.25N$

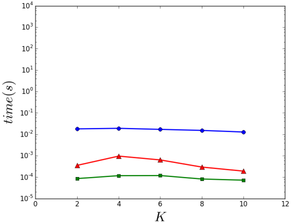


(b) $K = 0.5N$

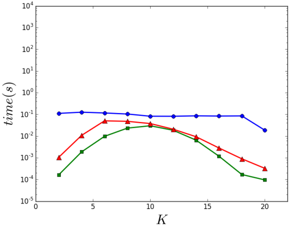


(c) $K = 0.75N$

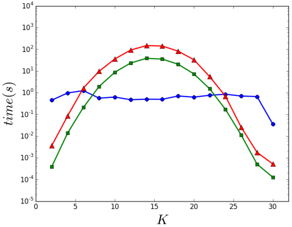
Algorithm Comparison



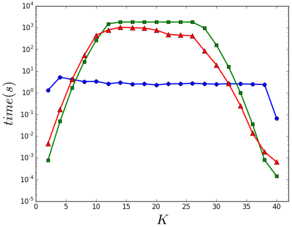
(a) $N = 10$



(b) $N = 20$



(c) $N = 30$



(d) $N = 40$

Future Work

- Integration with WFS standard
- Heuristic Approach
- Approximation Algorithms
- Benchmark with real data