Chapter 9 Application of Binary Trees

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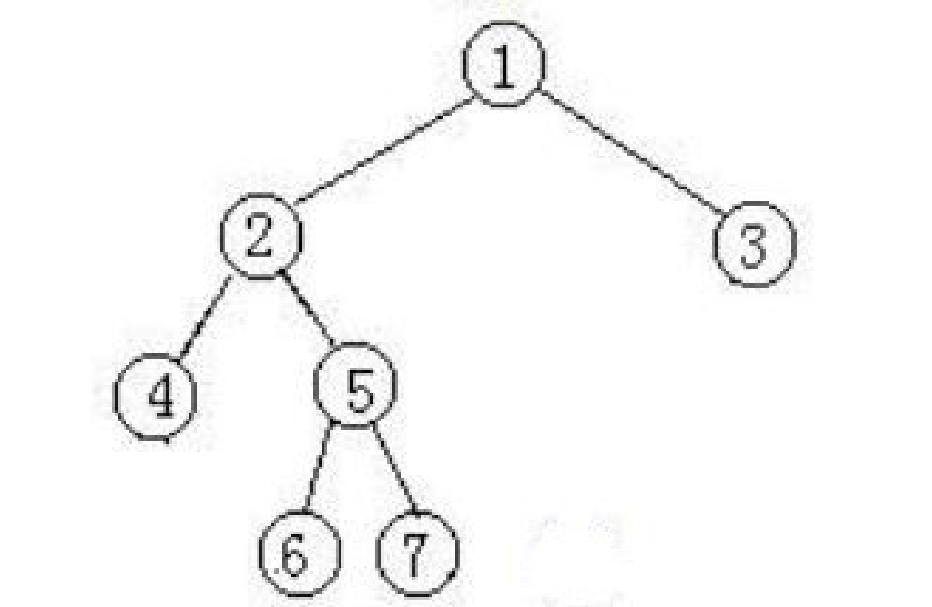
- A binary tree is a tree in which no node can have more than two children (referred to as the left child and the right child).



- 9.1 Transforming Ordered Trees into Binary Trees
- 9.2 Paths of Binary Trees
- 9.3 Traversal of Binary Trees

9.2 Paths of Binary Trees

- Paths in a tree are paths from the root to other nodes. In a tree there is no cycle, therefore there is unique path from the root to any node.

In a complete binary tree with n nodes, nodes' numbers are from 0 to n-1, and nodes are numbered top-down, and from left to right. For a node i, if it has parent, then its parent is node $\left\lfloor \frac{i-1}{2} \right\rfloor$ ($1 \le i \le n-1$); if it has left child ($2*i+1 \le n-1$), then its left child is node $2*i+1$; if it has right child ($2*i+2 \le n-1$), then its right child is node $2*i+2$. If i is even and $i \ne 0$, the left sibling for node i is node i-1; and if i is odd and $i \ne n-1$, the right sibling for node i is node $i+1$. The level where node i is at $\lfloor \log_2(i+1) \rfloor$, that is, the length of the path from node i to the root is $\lfloor \log_2(i+1) \rfloor$. Based on it, a complete binary tree can be stored in an array.

9.2.1 Binary Tree

- **Source: TUD Programming Contest 2005 (Training Session), Darmstadt, Germany**
- **IDs for Online Judge: POJ 2499**

- Binary trees are a common data structure in computer science. In this problem we will look at an infinite binary tree where the nodes contain a pair of integers. The tree is constructed like this:
  - The root contains the pair (1, 1).
  - If a node contains (a, b) then its left child contains ($a + b$, b) and its right child (a, $a + b$).
- Given the contents (a, b) of some node of the binary tree described above, suppose you are walking from the root of the tree to the given node along the shortest possible path. Can you find out how often you have to go to a left child and how often to a right child?

- **Input**
- The first line contains the number of scenarios. Every scenario consists of a single line containing two integers *i* and *j* ($1 \le i, j \le 2*10^9$) that represent a node (*i*, *j*). You can assume that this is a valid node in the binary tree described above.
- **Output**
- The output for every scenario begins with a line containing "Scenario #*i*:", where *i* is the number of the scenario starting at 1. Then print a single line containing two numbers *l* and *r* separated by a single space, where *l* is how often you have to go left and *r* is how often you have to go right when traversing the tree from the root to the node given in the input. Print an empty line after every scenario.

Analysis

- Becuase the root contains the pair (1, 1), and if a node contains (a, b) then its left child contains ($a + b$, b) and its right child (a, $a + b$); numbers in each pair are positive numbers, and for each pair, we can determine it is left child or right child by comparing the two numbers. For example, if a node contains ($a + b$, b), its parent must be (a, b) gotten by (a+b)- b. Therefore the path from the root to a node root can be gotten and how often you have to go to a left child and how often to a right child can be found out. The path is unique.

For a pair (a, b), greedy algorithm is used to calculate how often you have to go to a left child and how often to a right child.

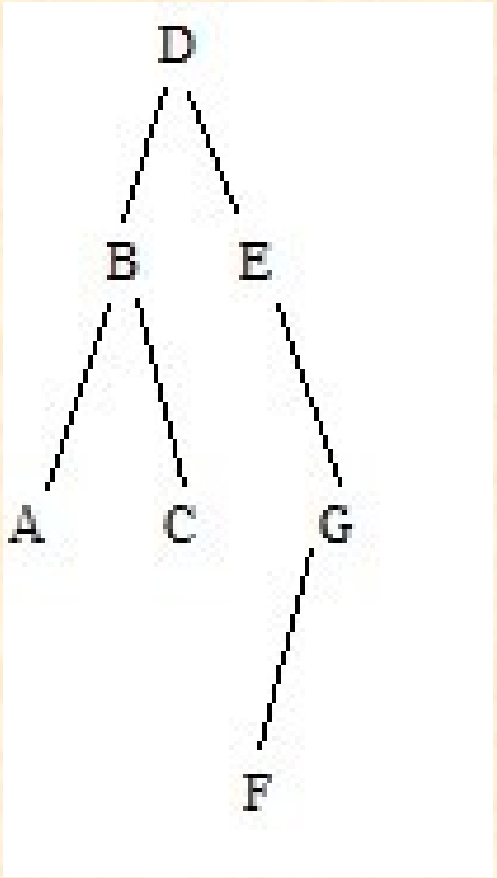
When $a>b$, then from (a, b) it takes $\left\lfloor \frac{a-1}{b} \right\rfloor$ steps to the left, in each step the left parameter – b; otherwise it takes $\left\lfloor \frac{b-1}{a} \right\rfloor$ steps to the right, and in each step the right parameter – a; finally it reaches $(1, 1)$.

9.3 Traversal of Binary Trees

- Preorder Traversal:
  - Visit the tree root;
  - Traverse the left subtree by recursively calling the preorder function;
  - Traverse the right subtree by recursively calling the preorder function;

Preorder Traversal

- DBACEGF

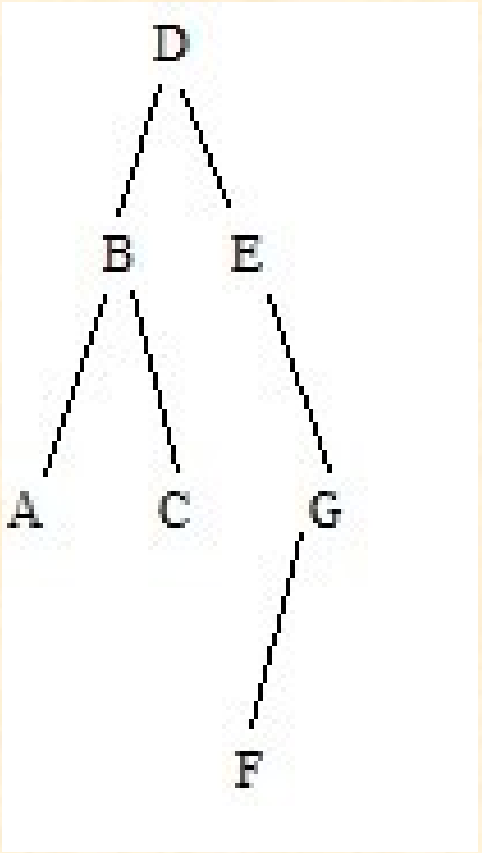


```
void PreorderTraversal(BinaryTree *pTree)
    {
        if(pTree == NULL) return;
        printf("%d ",pTree->nVaule);
        PreorderTraversal(pTree->pLeft);
        PreorderTraversal(pTree->pRight);
    }
```

- Inorder Traversal:
  - Traverse the left subtree by recursively calling the inorder function;
  - Visit the tree root;
  - Traverse the right subtree by recursively calling the inorder function;

Inorder Traversal

- ABCDEFG

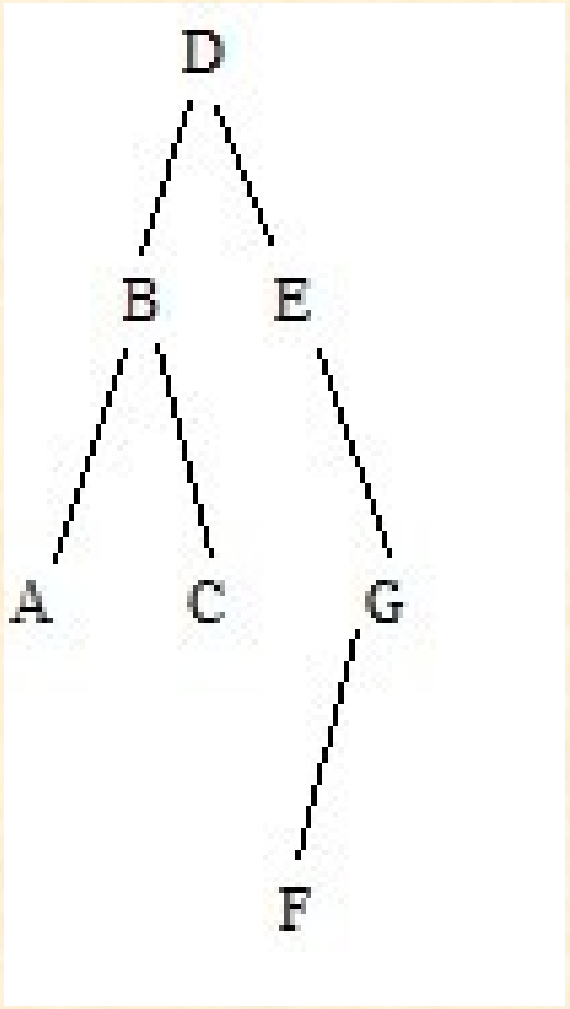


```
void InorderTraversal(BinaryTree *pTree)
    {
        if(pTree == NULL) return;
        InorderTraversal(pTree->pLeft);
        printf("%d ",pTree->nVaule);
        InorderTraversal(pTree->pRight);
    }
```

- Postorder Traversal:
  - Traverse the left subtree by recursively calling the postorder function;
  - Traverse the right subtree by recursively calling the postorder function;
  - Visit the tree root;

Postorder Traversal

- ACBFGED



```
void PostorderTraversal(BinaryTree *pTree)
 {
      if(pTree == NULL) return;
      PostorderTraversal(pTree->pLeft);
       PostorderTraversal(pTree->pRight);
       printf("%d ",pTree->nVaule);
}
```

- In preorder traversal, the tree root is visited firstly. In postorder traversal, the tree root is visited finally.
- In inorder traversal, the substring before the tree root is the result of inorder traversal for the left subtree, and the substring after the tree root is the result of inorder traversal for the right subtree.
- Therefore the results of preorder traversal and inorder traversal, and the results of postorder traversal and inorder traversal, can determine the structure of a binary tree. But the results of preorder traversal and postorder traversal can’t determine the structure of a binary tree.

From the results of postorder traversal and inorder traversal of a binary tree, the result of preorder traversal of a binary tree can be gotten.

Suppose the result of inorder traversal for a binary tree $s'=s_1' \ldots s_k' \ldots s_n'$, and the result of postorder traversal for a binary tree $s''=s_1'' \ldots\ldots s_n''$. Obviously, from $s_1'' \ldots\ldots s_n''$ produced by postorder traversal, s_n'' is the tree root. In $s_1' \ldots s_k' \ldots s_n'$ produced by inorder traversal there is a character s_k' which is equal to s_n''.

If $k>1$, then the left subtree exists and the substring $s_1' \ldots s_{k-1}'$ before s_k' is the result of inorder traversal for the left subtree, the prefix of the result of postorder traversal $s_1'' \ldots s_{k-1}''$ is result of postorder traversal for the left subtree.

If $k<n$, then the right subtree exists and the substring $s_{k+1}' \ldots s_n'$ after s_k' is the result of inorder traversal for the right subtree, and $s_k'' \ldots s_{n-1}''$ is is the result of postorder traversal for the right subtree.

If there exists the left subtree or the right subtree, the above process is called recursively.

From the results of preorder traversal and inorder traversal of a binary tree, the result of postorder traversal of a binary tree can be gotten.

Suppose $s'=s_1' \ldots s_k' \ldots s_n'$ is the result of inorder traversal of a binary tree, and $s''=s_1'' \ldots\ldots s_n''$ is the result of preorder traversal of a binary tree. Obviously s_1'', the first character of the result of preorder traversal, is the tree root of the binary tree. Suppose s_k' equals to s_1'' in $s_1' \ldots s_k' \ldots s_n'$.

If $k>1$, the left subtree exists; $s_1' \ldots s_{k-1}'$ is the result of inorder traversal of the left subtree; and $s_2'' \ldots s_k''$ is the result of preorder traversal of the left subtree.

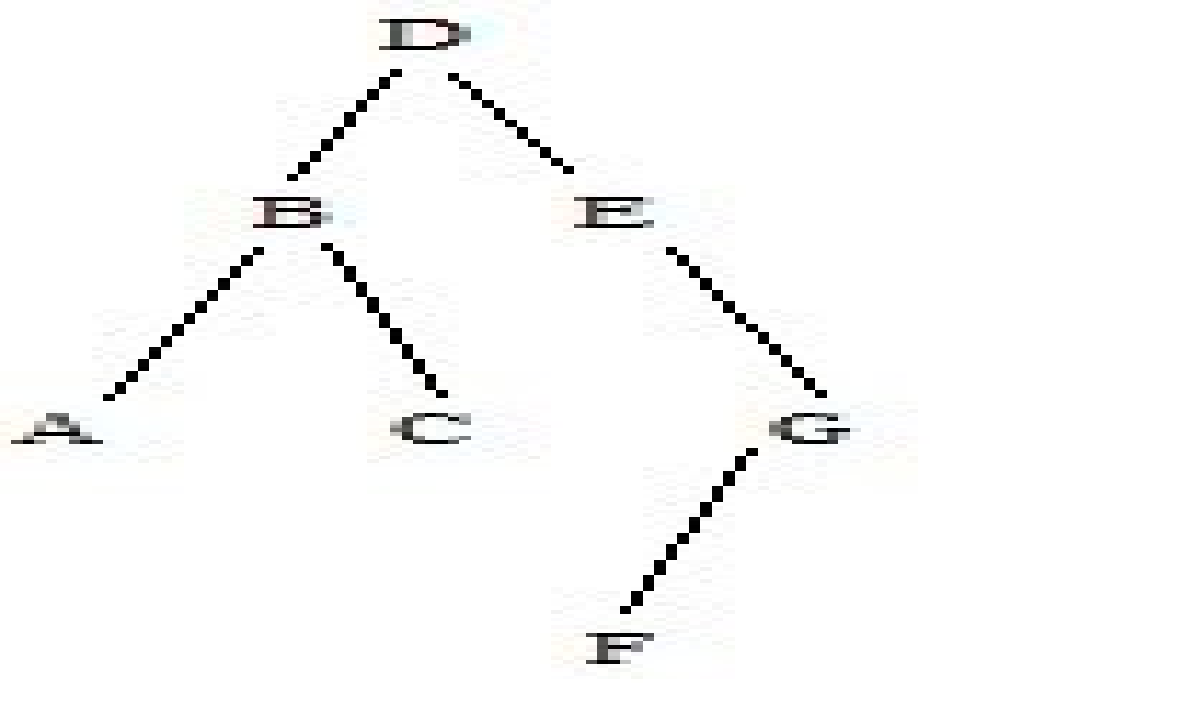
If $k<n$, the right subtree exists; $s_{k+1}' \ldots s_n'$ is the result of inorder traversal of the right subtree; and $s_{k+1}'' \ldots s_n''$ is the result of preorder traversal of the right subtree.

If there exists the left subtree or the right subtree, the above process is called recursively. Finally the root s_1'' (or s_k') is outputted.

9.3.1 Tree Recovery

- **Source: Ulm Local 1997**
- **IDs for Online Judge: POJ 2255, ZOJ 1944, UVA 536**

- Little Valentine liked playing with binary trees very much. Her favorite game was constructing randomly looking binary trees with capital letters in the nodes. This is an example of one of her creations:



- To record her trees for future generations, she wrote down two strings for each tree: a preorder traversal (root, left subtree, right subtree) and an inorder traversal (left subtree, root, right subtree). For the tree drawn above the preorder traversal is DBACEGF and the inorder traversal is ABCDEFG.
- She thought that such a pair of strings would give enough information to reconstruct the tree later (but she never tried it).
- Now, years later, looking again at the strings, she realized that reconstructing the trees was indeed possible, but only because she never had used the same letter twice in the same tree.
- However, doing the reconstruction by hand, soon turned out to be tedious. So now she asks you to write a program that does the job for her!

- **Input**
- The input will contain one or more test cases.
- Each test case consists of one line containing two strings preord and inord, representing the preorder traversal and inorder traversal of a binary tree. Both strings consist of unique capital letters. (Thus they are not longer than 26 characters.)
- Input is terminated by end of file.
- **Output**
- For each test case, recover Valentine's binary tree and print one line containing the tree's postorder traversal (left subtree, right subtree, root).

Analysis

- Based on definitions of preorder traversal and inorder traversal, for a tree, the first character in preorder traversal is the root of the tree; and in inorder traversal, the string before the character is inorder traversal of its left subtree, the string after the character is inorder traversal of its right subtree.

- A recursive function *recover*($preord_l$, $preord_r$, $inord_l$, $inord_r$) is used to produce the tree's postorder traversal based on preorder traversal and inorder traversal of the tree, where preorder traversal of the tree is *preord*, $preord_l$ and $preord_r$ are pointers for the front pointer and the rear pointer respectively; and inorder traversal of the tree is *inord*, $inord_l$ and $inord_r$ are pointers for the front pointer and the rear pointer respectively.

- Calculate the root's position in the inorder traversal of the tree (*inord*[*root*]==*preord*[$preord_l$]);
- Calculate the size of the left subtree l_l (*root*-$inord_l$) and the size of the right subtree l_r ($inord_r$- *root*);
- If the left subtree isn't empty (l_l>0), then *recover*($preord_l$, $preord_l$+l_l, $inord_l$, *root*-1), where $preord_l$ and $preord_l$+l_l are front pointer and rear pointer for preorder traversal of the left subtree; and $inord_l$ and *root*-1 are front pointer and rear pointer for inorder traversal of the left subtree;
- If the right subtree isn't empty (l_r>0), then *recover*($preord_l$+l_l+1, $preord_r$, *root*+1, $inord_r$), where $preord_l$+l_l+1 and $preord_r$ are front pointer and rear pointer for preorder traversal of the right subtree; and *root*+1 and $inord_r$ are front pointer and rear pointer for inorder traversal of the right subtree;
- Output the root *inord*[*root*]).