R_{36}

$$1\,139\,132\,346\,851\,261\,972\,844\,438\,537\,600\,339\,344\,017\,050\,909\,765\,016\,162\,665\,597\,454\,661\,514\,992\,363\,984\,953\,671\,805\,073 \div 303\,589\,256\,278\,329\,063\,734\,447\,264\,797\,494\,684\,006\,274\,034\,986\,417\,982\,381\,876\,948\,845\,277\,434\,723\,489\,378\,025\,830\,554\,019\,719\,482\,402\,795\,748\,291\,332\,712\,495\,243\,410\,005\,291\,278\,511\,103\,697\,343\,423\,866\,393\,743\,897\,177\,899\,776\,910\,159\,236\,355\,325\,952 /$$
$$382\,782\,474\,751\,194\,656\,935\,499\,150\,514\,101\,659\,066\,355\,261\,955\,881\,356\,551\,638\,008\,175\,442\,412\,332\,756\,259\,019\,106 \div 437\,335\,722\,148\,418\,426\,513\,671\,875 +$$
$$(316\,571\,246\,264\,261\,451\,513\,223\,958\,287\,182\,218\,066\,698\,186\,828\,724\,890\,033\,909\,068\,686\,721\,275\,482\,267\,295\,563\,143 \div 690\,921\,432\,494\,134\,514\,538\,725\,814\,359\,513\,133\,358\,239\,154\,093\,234\,034\,868\,353\,864\,202\,336\,721\,532\,865\,359\,711\,273 \div 364\,434\,527\,303\,029\,712\,283\,367\,265\,224\,827\,939\,621\,980\,392\,569\,302\,532\,557\,669\,597\,795\,542\,855\,514\,867\,957\,249\,581 \div 115\,963\,414\,688\,038\,912\,i) /$$
$$127\,594\,158\,250\,398\,218\,978\,499\,716\,838\,033\,886\,355\,451\,753\,985\,293\,785\,517\,212\,669\,391\,814\,137\,444\,252\,086\,339\,702 \div 145\,778\,574\,049\,472\,808\,837\,890\,625$$

This is a non-zero complex number and so R_{36} is not identically zero.

In order to prove the Elimination Lemma we still need to prove Propositin 3.3.
We have computed the polynomials F_3 and F_4 and we now declare $\beta_0 = \alpha_0$. We can now see that $F_3 = 0,\ F_4 = 0$ forms a linear inhomogeneous system on β_1 and β_2, and we can easily verify that this system has non-zero determinant.

```
Clear[λ1, λ2, α0, α1, α2]
```

```
β0 := α0
```

The following is the explicit expressions for F_3. It is clear from it that F_3 is linear on β_1, β_2.

```
Collect[F3, {β1, β2}, Simplify]
```

$$\frac{16\,(-1+\lambda_1)\,(-1+\lambda_2)\,(\alpha_1\,(-1+2\,\lambda_1+\lambda_2)\,(-1+2\,\lambda_2)+\alpha_2\,(-1+2\,\lambda_1)\,(-1+\lambda_1+2\,\lambda_2))}{(-2+\lambda_1+\lambda_2)\,(-1+\lambda_1+\lambda_2)\,(-3+2\,\lambda_1+2\,\lambda_2)\,(-1+2\,\lambda_1+2\,\lambda_2)} -$$
$$\frac{16\,(-1+\lambda_1)\,(-1+\lambda_2)\,(-1+2\,\lambda_1+\lambda_2)\,(-1+2\,\lambda_2)\,\beta_1}{(-2+\lambda_1+\lambda_2)\,(-1+\lambda_1+\lambda_2)\,(-3+2\,\lambda_1+2\,\lambda_2)\,(-1+2\,\lambda_1+2\,\lambda_2)} -$$
$$\frac{16\,(-1+\lambda_1)\,(-1+2\,\lambda_1)\,(-1+\lambda_2)\,(-1+\lambda_1+2\,\lambda_2)\,\beta_2}{(-2+\lambda_1+\lambda_2)\,(-1+\lambda_1+\lambda_2)\,(-3+2\,\lambda_1+2\,\lambda_2)\,(-1+2\,\lambda_1+2\,\lambda_2)}$$

The expression for F_4 is a bit more complicated but still linear on β_1, β_2. To see this we proceed as follows: consider the power series expansion $F_4 = c_{00} + c_{10}\,\beta_1 + c_{01}\,\beta_2 + \ldots$. We will prove that in fact F_4 coincides with its 1-jet.

```
c00 = Simplify[F4 /. {β1 → 0, β2 → 0}]
```

$$\Big(16\,(-1+\lambda_1)\,(-1+\lambda_2)\,\Big(\alpha_1\,\Big(-48+266\,\lambda_2-629\,\lambda_2^2+793\,\lambda_2^3-535\,\lambda_2^4+171\,\lambda_2^5-18\,\lambda_2^6+6\,\lambda_1^5\,\left(25-54\,\lambda_2+18\,\lambda_2^2\right)+\lambda_1^4\,\left(-715+2031\,\lambda_2-1638\,\lambda_2^2+432\,\lambda_2^3\right)+\lambda_1^3\,\left(1262-4453\,\lambda_2+5481\,\lambda_2^2-2988\,\lambda_2^3+648\,\lambda_2^4\right)+\lambda_1^2\,\left(-1021+4295\,\lambda_2-6960\,\lambda_2^2+5640\,\lambda_2^3-2376\,\lambda_2^4+432\,\lambda_2^5\right)+\lambda_1\,\left(374-1818\,\lambda_2+3598\,\lambda_2^2-3757\,\lambda_2^3+2211\,\lambda_2^4-720\,\lambda_2^5+108\,\lambda_2^6\right)+4\,\alpha_0\,(-2+3\,\lambda_1)\,\left(2-9\,\lambda_2+9\,\lambda_2^2\right)\left(8\,\lambda_1^5+4\,\lambda_1^4\,(-11+9\,\lambda_2)+2\,\lambda_1^3\,\left(45-78\,\lambda_2+32\,\lambda_2^2\right)+(-1+\lambda_2)^2\,\left(-6+19\,\lambda_2-16\,\lambda_2^2+4\,\lambda_2^3\right)+\lambda_1^2\,\left(-85+235\,\lambda_2-204\,\lambda_2^2+56\,\lambda_2^3\right)+\lambda_1\,\left(37-145\,\lambda_2+200\,\lambda_2^2-116\,\lambda_2^3+24\,\lambda_2^4\right)\right)\Big)+\alpha_2\,\Big(48-374\,\lambda_2+1021\,\lambda_2^2-1262\,\lambda_2^3+715\,\lambda_2^4-150\,\lambda_2^5-18\,\lambda_1^6\,(-1+6\,\lambda_2)-9\,\lambda_1^5\,\left(19-80\,\lambda_2+48\,\lambda_2^2\right)+\lambda_1^4\,\left(535-2211\,\lambda_2+2376\,\lambda_2^2-648\,\lambda_2^3\right)+\lambda_1^3\,\left(-793+3757\,\lambda_2-5640\,\lambda_2^2+2988\,\lambda_2^3-432\,\lambda_2^4\right)+\lambda_1^2\,\left(629-3598\,\lambda_2+6960\,\lambda_2^2-5481\,\lambda_2^3+1638\,\lambda_2^4-108\,\lambda_2^5\right)+\lambda_1\,\left(-266+1818\,\lambda_2-4295\,\lambda_2^2+4453\,\lambda_2^3-2031\,\lambda_2^4+324\,\lambda_2^5\right)+4\,\alpha_0\,\left(2-9\,\lambda_1+9\,\lambda_1^2\right)\,(-2+3\,\lambda_2)\left(4\,\lambda_1^5+24\,\lambda_1^4\,(-1+\lambda_2)+\lambda_1^3\,\left(55-116\,\lambda_2+56\,\lambda_2^2\right)+(1-2\,\lambda_2)^2\,\left(-6+13\,\lambda_2-9\,\lambda_2^2+2\,\lambda_2^3\right)+4\,\lambda_1^2\,\left(-15+50\,\lambda_2-51\,\lambda_2^2+16\,\lambda_2^3\right)+\lambda_1\,\left(31-145\,\lambda_2+235\,\lambda_2^2-156\,\lambda_2^3+36\,\lambda_2^4\right)\right)\Big)\Big) /$$
$$\Big((-2+\lambda_1+\lambda_2)^2\,(-1+\lambda_1+\lambda_2)^2\,(-3+2\,\lambda_1+2\,\lambda_2)\,(-1+2\,\lambda_1+2\,\lambda_2)\,(-5+3\,\lambda_1+3\,\lambda_2)\,(-4+3\,\lambda_1+3\,\lambda_2)\,(-2+3\,\lambda_1+3\,\lambda_2)\Big)$$