```
S5 := s*p^2*r^2 + (2*α0*σ - η)*p*r^3 + α0^2*σ*(2 - 3*σ)*s*p*r^3 + α0^2*σ^2*(η - α0*σ)*r^4
S̃5 := s*p̃^2*r^2 + (2*β0*σ - η̃)*p̃*r^3 + β0^2*σ*(2 - 3*σ)*s*p̃*r^3 + β0^2*σ^2*(η̃ - β0*σ)*r^4
c5 := -α0^4*σ^3*(1 - σ)
c̃5 := -β0^4*σ^3*(1 - σ)

S6 := p^2*r^3 + α0*(1 - 3*σ)*s*p^2*r^3 + (2*α0*σ*η - 3*α0^2*σ^2)*p*r^4 -
  α0^3*σ^2*(3 - 4*σ)*s*p*r^4 + α0^3*σ^3*(α0*σ - η)*r^5
S̃6 := p̃^2*r^3 + β0*(1 - 3*σ)*s*p̃^2*r^3 + (2*β0*σ*η̃ - 3*β0^2*σ^2)*p̃*r^4 -
  β0^3*σ^2*(3 - 4*σ)*s*p̃*r^4 + β0^3*σ^3*(β0*σ - η̃)*r^5
```

Now that above objects have been defined we proceed to making some linear computations. We first compute the polynomials $P_d(w)$ that appear in the statement of the Key lemma and the polynomials $F_d(\beta)$ that appear in the statement of the Main lemma. In order to compute the polynomials P_d(w) we use the formulas found in Propositions 6.4, 6.6, 6.8 and 6.10. In order to obtain the $F_d(\beta)$ we shall need first the polynomials R_d(w) defined in Section 3.2: "Deducing Main lemma from Key lemma". Once we have computed the R_d(w) we compute $F_d(\beta)$ using the strategy outlined in Section 7.2: "Computing the polynomials F_d ".

We proceed doing this degree by degree, from $d = 3$ to $d = 6$. We will explain in detail the process for $d = 3$. The following cases are completely analogous.

We first imput the expression for $P_3(w)$ found in Proposition 6.4

```
P3 := S̃3 - S3
```

Second, we define the linear map L_3 which appears on Section 3.2 and compute

```
L3[f_] := D[f, w]*r + 2*(s - 2*w)*f
```

The next step is to compute $R_3(w)$ by finding an inverse image of $P_3(w)$ under L_3; that is, we define $R_3(w) = L_3^{-1}(P_3(w))$.

```
A3 := Transpose[{Coefficient[L3[1], w, #] & /@ Range[0, 4], Coefficient[L3[w], w, #] & /@ Range[0, 4],
    Coefficient[L3[w^2], w, #] & /@ Range[0, 4], Coefficient[L3[w^3], w, #] & /@ Range[0, 4]}]
B3 := Drop[A3, 1]
r3 := LinearSolve[B3, Coefficient[P3, w, #] & /@ Range[1, 4]]
R3 := r3.{1, w, w^2, w^3}
```

We finally obtain the polynomial $F_3(\beta)$ by the formula $F_3 = L_3(R_3)(0) - P_3(0)$ (cf. Section 7.2).

```
v3 := Take[A3, 1]
F3 = Simplify[v3.r3 - Coefficient[P3, w, 0]][[1]];
```

We repeat the process for d=4,5,6. For these degrees we will also need the auxiliary polynomials $q_d(w)$ in order to define $P_d(w)$ (cf. Propositions 5.3, 5.4 and 5.5).

```
q4 := S4 + c2*S3*r - (c3/2)*S2*r^2
q̃4 := S̃4 + c̃2*S̃3*r - (c̃3/2)*S̃2*r^2
P4 := q̃4 - q4 - S2*R3
L4[f_] := D[f, w]*r + 3*(s - 2*w)*f

A4 := Transpose[{Coefficient[L4[1], w, #] & /@ Range[0, 6], Coefficient[L4[w], w, #] & /@ Range[0, 6],
    Coefficient[L4[w^2], w, #] & /@ Range[0, 6], Coefficient[L4[w^3], w, #] & /@ Range[0, 6],
    Coefficient[L4[w^4], w, #] & /@ Range[0, 6], Coefficient[L4[w^5], w, #] & /@ Range[0, 6]}]
B4 := Drop[A4, 1]
v4 := Take[A4, 1]
r4 := LinearSolve[B4, Coefficient[P4, w, #] & /@ Range[1, 6]]
R4 := r4.{1, w, w^2, w^3, w^4, w^5}

F4 = Simplify[v4.r4 - Coefficient[P4, w, 0]][[1]];

q5 := S5 + 2*c2*S4*r + c2^2*S3*r^2 - (2/3)*(c4 + c3*c2)*S2*r^3
q̃5 := S̃5 + 2*c̃2*S̃4*r + c̃2^2*S̃3*r^2 - (2/3)*(c̃4 + c̃3*c̃2)*S̃2*r^3
P5 := q̃5 - q5 - 2*S2*R4
L5[f_] := D[f, w]*r + 4*(s - 2*w)*f
```